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Mechanical Behavior of Irregular Fibers

Part 1: Modeling the Tensile Behavior of Linear Elastic Fibers

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ABSTRACT

Fiber irregularities are inherent to textile fibers, natural fibers in particular. This series of papers examines the impact of fiber irregularity on the mechanical behavior of textile fibers. In the first part, the effect of fiber dimensional irregularities on the tensile behavior of linear elastic fibers is examined, using the finite element method (FEM). Fiber dimensional irregularities are simulated with sine waves of different magnitude and frequency. The results indicate that increasing the level or magnitude of irregularity will decrease the breaking load, breaking elongation and method Young’s modulus of the fiber, while increasing the frequency of irregularity will decrease the breaking load and method Young’s modulus, but the breaking elongation will increase. Fiber dimensional irregularity and the gauge length effect are also simulated in this study.
1. INTRODUCTION

Fiber irregularities fall into two main categories: dimensional irregularity (external) and structural irregularity (internal). Diameter variation along fiber length is a typical example of fiber dimensional irregularity, while the presence of flaws inside a fiber signifies structural irregularity. The effect on fiber tensile properties of fiber irregularities has long been recognized, as entailed in the truism that the strength of a chain is that of its weakest link [11]. Banky and Slen [2] conducted experiments on irregular wool fibers, and reported large difference in the amount of extension experienced by segments of the same fiber, with the thinner segments extending more than the thicker segments. Kenny and Chaikin [6] examined the stress-strain-time relationships of non-uniform textile materials analytically, and demonstrated the profound effects of fiber dimensional non-uniformity. For example, they revealed that a typical keratin fiber with a 16% coefficient of variation in cross section will extend 60% more than a uniform fiber having the same average cross-sectional area, subject to a particular load. Further work in this area was carried out by Collins and Chaikin [3, 4]. Since then, very little has been published on fiber irregularity or non-uniformity. This may be due to the tedious work involved in collecting data on fiber irregularity and a general lack of appreciation of the significance of fiber irregularity. In recent years, there has been a renewed interest in fiber irregularity, particularly the diameter variation of wool. Instruments have been developed to measure diameter variation along fiber length [8, 12]. The direct relationship between the diameter variation of wool and the variation of its breaking load is also established [13, 14], suggesting that the weakest link for wool is usually the
section of the fiber with the minimum diameter. However, many questions remain unanswered. For instance, how does the level and frequency of dimensional irregularity affect fiber strength? What is the effect of fiber irregularity on fiber tensile modulus? How does fiber irregularity contribute to the gauge length or size effect in tensile testing? What difference does fiber irregularity make to the static and impact test results of fibers? What is the flexural and torsional behavior of irregular fibers? How can the fiber irregularities be incorporated in the analysis of fibrous assemblies such as fiber reinforced composites and twisted yarns?

Numerical modeling, using the finite element method, has been successfully applied to many fiber and textile related problems [5, 7, 9, 10]. This series of paper attempts to use this technique to address some of the questions related to fiber irregularities. We use the ABAQUS software package for the numerical modeling, and consider the simple case of fibers obeying a linear stress-strain relationship in this first part of the series.

2. FINITE-ELEMENT MODEL

2.1 Assumptions and model representation

Only dimensional irregularity of fibers with linear elastic behavior is considered in this paper. We assume the internal structure of fiber is flawless and the fiber is axisymmetric. We also assume that fiber cross-section is circular along fiber length, so that fiber diameter variation can represent its dimensional irregularity. Based on these assumptions, the three-dimensional configuration of a fiber can be transformed into a two-dimensional
problem in the model. We only need to model a plane (A plane) through the fiber, as shown in Figure 1. This plane can represent a complete three-dimensional fiber specimen.

To examine the effect on fiber tensile behavior of the level and frequency of dimensional irregularity, sine waves of different magnitude and frequency have been used to simulate the different fiber irregularities. The gauge length effect is also investigated using different levels of variation of sine waves. Relevant details concerning the fiber specimen are listed in Table 1. The values used are arbitrary values only.

<table>
<thead>
<tr>
<th>Properties</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus (MPa)</td>
<td>3000</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.35</td>
</tr>
<tr>
<td>Maximum equivalent stress at break (MPa)</td>
<td>300</td>
</tr>
<tr>
<td>Specimen diameter (μm)</td>
<td>20</td>
</tr>
<tr>
<td>Specimen length (mm)</td>
<td>2, 4*</td>
</tr>
</tbody>
</table>

*The length is used in testing gauge length effect

2.2 Definition of equivalent stress and method Young’s Modulus

When a fiber specimen is extended, using the FE model, the stress of every element in the specimen can be calculated. Since axisymmetric elements provide for the modeling, there are four non-zero components of stress in each element. The equivalent stress is calculated to consider these components using equation (1) [1]:

\[
\sigma_{eq} = \sqrt{\sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2 + 2\tau_{xy}^2 + 2\tau_{yz}^2 + 2\tau_{zx}^2}
\]
\[
\overline{\sigma} = \frac{3}{\sqrt{2}} s_{ij} s_{ij}
\]  
(1)

Where \( s_{ij} = \sigma_{ij} - p \delta_{ij} \) and \( p = \frac{1}{3} \sigma_{ij} \)

- \( s_{ij} \): Deviatoric stress
- \( \sigma_{ij} \): Stress tensor
- \( p \): Spherical stress tensor
- \( \delta_{ij} \): Kronecker delta
- \( i, j \): 1, 2, 3

Since the fiber specimen is not uniform in cross-section, the equivalent stress along the length of the fiber will vary from element to element. The maximum equivalent stress is most important in determining the tensile failure of irregular fiber. When the equivalent stress of a point in the fiber reaches the maximum equivalent value, the fiber will break. For this work, the maximum equivalent stress is assumed to be 300 MPa, as indicated in Table 1.

The Young’s modulus of 3000 MPa is used in the simulation (see Table 1). It is the true modulus inherent to the fiber simulated in this study. Because of fiber dimensional irregularity along fiber length, the Young’s modulus derived from empirical tensile stress-strain curve will be different than its theoretical value. We denote this modulus as the method Young’s modulus. We will examine how the fiber irregularity affects the method Young’s modulus.
2.3 Determination of the number of elements or mesh density

The finite element process is a method of approximation to a continuum problem. The continuum (or fiber in this case) can be divided into a finite number of parts or elements, the behavior of which is specified by a finite number of parameters. The number of elements will influence the results obtained from a FE model. It is therefore necessary to determine the appropriate number of elements needed for a particular problem.

Using information given in the previous section, we plot the maximum equivalent stress of the irregular fiber at 2% extension against the number of elements in Figure 2. Figure 2 shows that the tensile stress begins to approach a stable value when the number of elements is greater than approximately 3000 per mm. The shape of elements in the model is quadrilateral. Based on these results, we have chosen a mesh density of 3000 elements per mm of fiber specimen for the following analyses.

3. RESULTS AND DISCUSSION

3.1 Effect of level of variation on fiber tensile properties

As indicated in Table 2(I), we simulated two different cases here. Case 1 represents a uniform fiber specimen (2 mm) with a diameter of 20 μm (see Table 1). Case 2 simulates three irregular fibers of the same average diameter but with three different levels of diameter variation, which is represented by a complete sine wave in each fiber (Figure 3).
The results for case 1 and case 2 suggest that for fibers of the same average diameter, the breaking load, breaking extension and method Young’s modulus all decrease with the increase in level of diameter variation (irregularity). As the level of irregularity increases, the minimum fiber diameter decreases, the fiber weakens and its breaking load decreases as a consequence. The results are therefore consistent with the weakest link theory. In addition, Table 2 (I) also shows that increasing the irregularity level will decrease the method Young’s modulus of the fiber, with the thinnest segment of fiber specimen experiencing the highest breaking extension. This result supports previous experimental results obtained by Banky and Slen [2].

3.2 Effect of frequency of variation on fiber tensile properties
Table 2(II) summarizes the simulation conditions and results. The simulated fiber specimens both have the same average diameter (20 μm) and length (2 mm). The diameter variation also follows the sine wave pattern. To examine the effect of the frequency of irregularity or diameter variation, different numbers of sine wave variations have been added into the fiber specimen. The results are listed in Table 2(II). For both levels of variation, increasing the frequency of diameter variation (irregularity) will decrease the breaking load and method Young’s modulus, but the breaking extension will increase. Figure 4 shows finite element meshes for a 0.4 mm specimen length for case 3, with the arrows indicating the location of minimum fiber diameter. As the frequency of variation increases, an increased number of thin fiber segments contributes to the slightly decreased breaking load and increased fiber extension. Consequently, the method Young’s modulus also decreases slightly. The results in Table 2 (I) and (II) also show
that the level of diameter variation affects the fiber tensile properties more than the frequency of diameter variation.

### 3.3 Fiber irregularity and the gauge length effect

To examine the gauge length effect, we simulated four cases. They all have the same average diameter (20 \( \mu \text{m} \)) and the diameter variation also follows a sine wave. Case 5 simulates an irregular fiber with 2 mm length and 30% variation. Case 6 to case 8 represent three irregular fibers with the 4 mm length. The first half of each of these fibers is identical to the fiber in case 5, but the second half has a level of variation of 10%, 30% and 50% respectively. Figure 5 gives a graphical depiction of these cases, and their tensile properties are given in Table 2 (III).

When the gauge length is doubled, in case 6 and case 7, the breaking load is the same as in case 5 because the thinnest segments in all three cases are the same. The minimum fiber diameters are not changed, but the breaking extension in case 7 is higher than in case 6 because of its higher variation in the second half and the fiber becomes easier to stretch. The method Young’s modulus is lower than in case 6 as a result. In case 8, the breaking load, breaking extension and method Young’s modulus significantly decrease with the increased gauge length. This is because a thinner segment appears in the second half of the fiber. Figure 6 shows the load-extension curves of these cases. The results indicate that gauge length effect is a direct result of the presence of weaker spot in the increased specimen length, which concurs with the weakest-link theory postulated by Peirce [11]. It is interesting to note that in case 5 and case 7, the values of breaking load,
breaking extension and method Young’s modulus are all the same despite of the difference in gauge length. Their load-extension curves also overlap as indicated in Figure 6. We do need to reiterate that the fibers simulated in this study are free from structural defects, and we only consider the fiber dimensional irregularities.

4. CONCLUSIONS

A linear-elastic finite-element model has been utilized to investigate the tensile behavior of fiber specimens with simulated dimensional irregularities. The following conclusions can be drawn from this preliminary study:

- The level of diameter variations significantly influences the tensile behavior of the fiber. As the level of variation increases, the breaking load, breaking extension and method Young’s modulus all decrease.
- The higher the frequency of diameter variation, the lower the breaking load and method Young’s modulus, but the higher the breaking extension.
- Increasing the fiber gauge length does not necessarily lead to deterioration in fiber tensile properties. Deterioration occurs when a weaker (thinner) spot appears in the increased length, as suggested by the weakest-link effect.

We stress that fibers with linear elastic tensile behavior are considered in this paper. In part 2, we will investigate the non-linear tensile behavior of irregularity fibers.
Acknowledgment

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REFERENCES


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