The Minimum Diameter Distribution and Strength Variation of Top Dyed Wool

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Summary

Wool fibres are well known for their diameter variations, and the minimum diameter usually determines the strength of the fibre. This paper first examines the distributions of the minimum fibre diameters and the breaking force of wool from a dyed wool top. The quantitative relations between the coefficient of variation (CV) of breaking force and that of minimum diameter at different gauge lengths are then corroborated with experimental data. The results suggest that after top making and dyeing of the wool, its breaking force variation can still be predicted from the variation in minimum fibre diameters, even though the top making and dyeing processes would have caused some damage to the fibres. The results also imply that most processed fibres break at their thinnest position, particularly when the gauge length is long. This is similar to early findings concerning un-processed wool. The effect of strain rates on the results is also briefly discussed in this paper.

Keywords: Minimum diameter variation, variation in breaking force, gauge length

Introduction

Top dyeing is still a common practice in many worsted mills. While much research has been carried out on the strength and variation of un-processed wool, the behaviour of top-dyed wool has received little attention. This information is important in the understanding of fibre/yarn relationship where yarns are produced from top-dyed wool. It is also important in understanding the impact of top dyeing on the changes in fibre tensile properties.

Wool fibres are different from synthetic fibres in that wool fibres exhibit considerable variations in fibre diameter. To obtain the information of the average fibre strength, fibre bundle test is normally employed commercially since single fibre tensile test is tedious and time-consuming. However, because of the different strength of every single fibre in fibre bundle, the fibre bundle strength is always lower than the average fibre strength (Coleman 1958). The discrepancy is mainly determined by the variation of single fibre strength. Fibre breaking force is particularly sensitive to the changes in the minimum diameter. For dyed wool top, the influence of the variation of fibre diameter on the variation of fibre strength is still unknown. Because of the additional processing conditions of dyed wool top, the relation is worth to be further explored.

The diameter distributions of raw wool and wool from the top were deemed to follow lognormal distribution (Henon 1952, Monfort 1960a and b, Linhart and van Der Westhuyzen 1963, Lunney and Brown 1985, Wang and Wang 1998). According to the linear relationship between fibre breaking force and the square of fibre diameter (Smuts *et al* 1981 a and b), the distribution of breaking force should obey lognormal distribution as well (Wang and Wang 1998). In addition, the relationship between the coefficient of variation of fibre breaking force (CV_{BF}) and that of fibre diameter (CV_{FD}) has been established statistically as follows (Wang and Wang 1998).

$$CV_{BF}=\sqrt{[1+(CV_{FD})^2]^4-1} \quad (1)$$

Because most fibres break at the position of minimum diameter, it is proposed further that for a small sample size, the CV of minimum fibre diameters (CV_{mFD}) should be employed instead of CV of diameter (as measured on OFDA for a large sample) (Wang *et al* 1998).

$$CV_{BF}=\sqrt{[1+(CV_{mFD})^2]^4-1} \quad (2)$$

Equation 2 was verified to fit observed data better than equation 1 especially for small samples of scoured wool (Wang *et al* 1998). However, just like equation 1, two assumptions are made in establishing this relationship. First, there is a linear relationship between fibre breaking force and the square of **minimum fibre diameter**. Second**,** there should be a lognormal distribution of the **minimum diameters** of the fibres.

Using a different approach and the first assumption only, a simpler relationship (equation 3) between CV of breaking force and CV of minimum diameter has been be derived and verified empirically with scoured wool (Wang 2000).

$$CV_{BF} \approx 2\ CV_{mFD} \qquad (3)$$

Processed wool is different from raw wool in that many fibres would have been damaged during processing. In addition, shorter and weaker fibres may be removed during processes such as carding and combing. After the early stage of processing (scouring, carding, gilling and combing), it has been found that the diameter distribution still follows a lognormal distribution. However, the distribution of breaking force might be altered after carding or gilling. Interestingly, the breaking force exhibits a lognormal distribution again after combing due to the removal of some short and weak fibres (Wang and Wang 1998). During dyeing, wool fibres may experience additional damage, physically or geometrically.

This paper examines the relationship between variations in fibre minimum diameter and breaking strength of wool fibres sampled from a dyed top under different gauge lengths. It aims to clarify if the previously established relationships between the fibre diameter and strength variations can also be applied to processed and dyed wool fibres at different gauge lengths.

Materials and methods

A dyed (black) merino wool top was used for this work. After the tops were conditioned for more than 24 hours at 20 ±2°C and 65%±2% RH, individual fibres were withdrawn randomly and gently from the top for testing. Then the diameters of each single fibre were first measured with 1cN pretension at 40 μm intervals along its length on the Single Fibre Analyzer (SIFAN).

When measuring fibre diameters on the SIFAN, the fibre ends near two jaws are not accessible to the CCD camera on the instrument. This may lead to inaccurate results for along-length diameter variations, especially for fibres with a short measuring length. To eliminate this problem, fibres to be tested at a short length (i.e.10 mm and 20 mm) were prepared for fibre diameter measurement at a gauge length of more than 20 mm. The real measuring section was marked on the fibre, and only the marked part in the middle of the fibre, which was fully scanned for fibre diameter, was used for the tensile test.

After the diameter measurements, the single fibres were then tested for tensile properties on an INSTRON with a crosshead speed of 20 mm/min, at a gauge length of 10mm, 20mm, 50mm and 100mm respectively. All tests were conducted under standard conditions (20 $\pm$ 2 ^{O}C and 65 $\pm$ 2% RH).

Results and discussion

Summary of results

Results for the fibre diameter and tensile strength as well as their CV values are summarised in Table 1 below.

Table 1 Summary of diameter and tensile test results

G.L.	**N**		**BF**	**IFS**	**BS**	**D_{min}**	**D_{aver}**	**CV_D along**
10mm	126	Mean	66.85	213.79	50.81	19.71	24.89	9.12
		CV	41.17	17.61	28.35	17.66	17.05	19.52
20mm	153	Mean	62.49	213.44	37.10	19.02	25.01	10.20
		CV	43.61	17.84	37.36	19.97	18.41	18.86
50mm	37	Mean	50.49	205.38	25.57	17.50	24.45	11.91
		CV	32.60	14.02	44.00	15.86	13.07	19.34
100mm	51	Mean	49.00	201.36	19.06	17.49	26.01	13.90
		CV	36.42	18.28	49.37	18.07	15.11	17.97

$D_{min:}$ Minimum fibre diameter (μm)

D_{aver}: Average fibre diameter (μm)

CV_D along: CV of fibre diameter along fibres (%)

BF: Breaking force (mN)

BS: Breaking strain (%)

IFS: Intrinsic fibre strength ($4BF/\pi D^2_{min}$) (MPa)

CV: Coefficient of variation (%)

The results in Table 1 indicate that as the gauge length increases, fibre breaking force (BF), breaking strain (BS) and the intrinsic fibre strength (IFS) all decrease. This is most likely caused by the presence of a much thinner spot along the fibre at the longer gauge length, as implied from the increased fibre diameter CV along the fibre length (CV_D along). In other words, the minimum diameter of a fibre along its length is very important in governing the fibre tensile behaviour. The following sections discuss the minimum diameter and its effect on fibre tensile properties in more detail.

The distribution of minimum diameter

As mentioned in the introduction, the relationship between CV of minimum diameter and CV of break force has been derived and verified for unprocessed scoured wool. It is also suggested that for a small fibre sample size, the minimum fibre diameters should be employed in predicting variations in fibre breaking force (Wang 2000). However, it is worth noting that in previous studies, fibre diameter was measured on either OFDA or a projection microscope. The resultant diameter distribution may not reflect the distribution of **minimum** fibre diameters. Since one of the assumptions mentioned earlier is that the fibre minimum diameter should follow lognormal distribution, the distribution of minimum diameter warrants further examination. The SIFAN instrument can measure fibre diameter along a single fibre at every 40 μm and the minimum diameter of each fibre can thus be obtained. The distributions of minimum fibre diameters at four different gauge lengths and their fits to lognormal distributions are illustrated in Fig. 1.

Fig. 1: Fits to the lognormal distribution of the minimum diameters at 10mm, 20mm, 50mm and 100mm gauge lengths

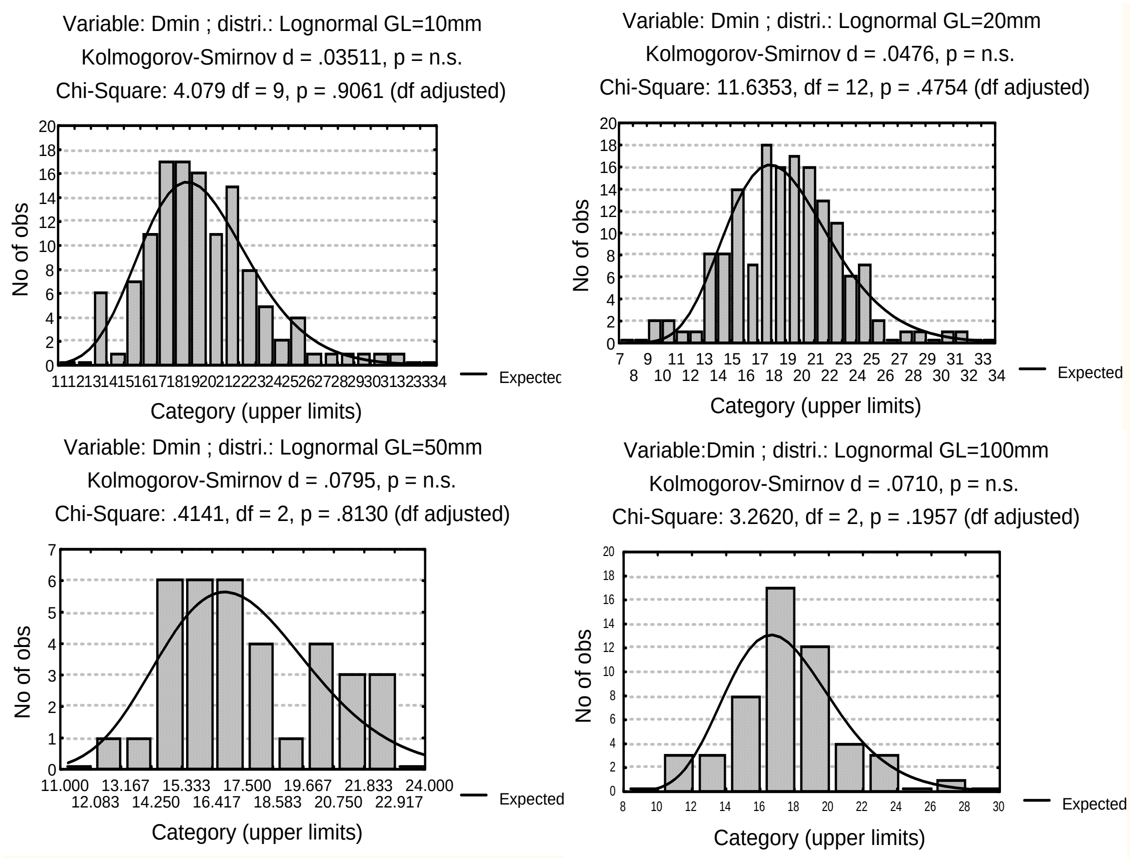
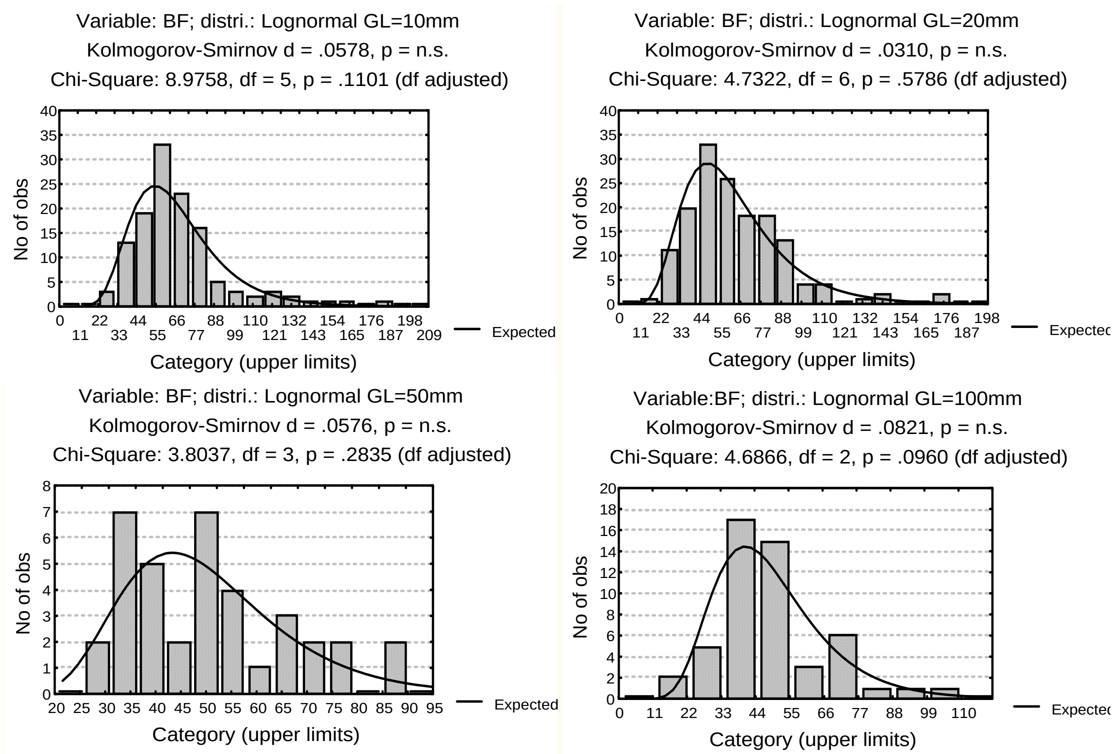


Fig. 1 shows all the p values of Kolmogorov-Smirnov and Chi-Square goodness of fit are not significant (P>0.05), thus rejecting the hypothesis that each distribution is not lognormal distribution. In other words, the experimental data are consistent with the minimum fibre diameter obeying a lognormal distribution.

The distribution of fibre breaking force

According to the linear relationship between break force and the square of minimum diameter, the same distributions should apply to both of them. That is: fibre break force should also obey lognormal distribution. Fig. 2 gives the distributions of break force and their fits to lognormal distribution.

Fig. 2: Fits to lognormal distributions of fibre break force at different gauge lengths



The p values given in Fig. 2 indicate that the distributions of break force at different gauge lengths follow the lognormal distribution.

The linear dependency of breaking force on the square of minimum diameter

Using different measurement methods, many researchers have found a strong linear relationship between breaking force and the square of mean fibre diameter (Hunter *et al* 1983, Meybeck and Gianola 1955, Smuts *et al* 1981 (a) and (b)), the square of diameter at position of break (Dollin *et al* 1995, Thompson *et al* 1995), and the square of minimum diameter (Orwin *et al* 1980). It has been found also that about 85% of tender wools and 70% of sound wools break at the position of minimum diameter (Orwin *et al* 1980). The relationships of breaking force and the square of minimum diameter at different test lengths for wool from the dyed top are examined as shown in Fig. 3.

Fig. 3: The linear dependency of breaking force on the square of minimum diameter at different gauge lengths

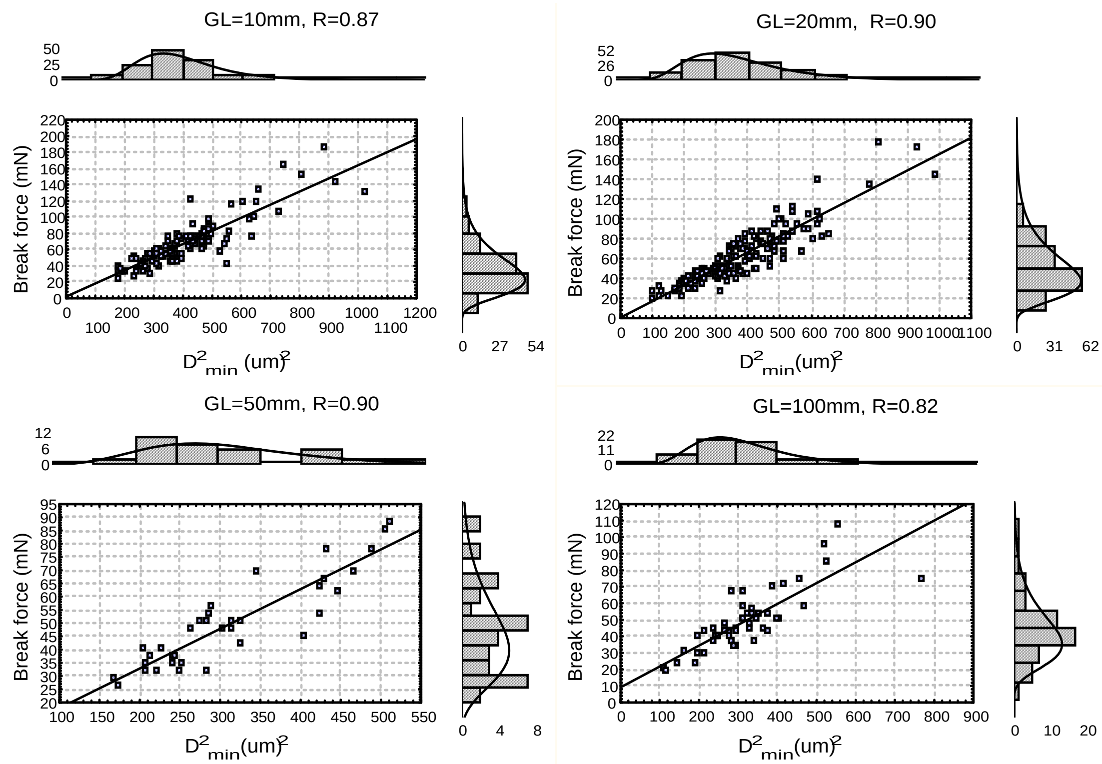


Fig. 3 indicates that like unprocessed wool, the breaking force of this top-dyed merino wool exhibits a very strong linear correlation with the square of minimum fibre diameter (correlation coefficients range from 0.82 to 0.90) regardless of the fibre length.

The correlation between the variations in minimum fibre diameters and in single fibre breaking force

Because the minimum diameters of the top dyed wool fit the lognormal distribution and a linear relationship exists between the breaking force and the square of minimum fibre diameters, the CV of break force of this wool may be predicted from the CV of minimum fibre diameters, similar to the prediction for unprocessed wool. In other words, equations 2 and 3 can be used for both processed and unprocessed wool fibres, even though they are derived initially for the unprocessed wool. The CVs of measured and predicted single fibre breaking force as well as the relative errors (δ=[predicted value-measured value]/ measured value) for the top dyed wool are listed in Table 2.

Table 2: The measured and predicted coefficients of variation of breaking force as well as their relative errors

Gauge length	CV of break force (%)		
	Measured	Predicted (2) $CV_{BF}=\sqrt{[1+(CV_{mFD})^2]^4-1}$	Predicted (3) $CV_{BF}\approx 2\ CV_{mFD}$
10mm	41.17	36.15 (δ= -12.19%)	35.32 (δ= -14.21%)
20mm	43.61	41.15 (δ= -5.64%)	39.94 (δ= -8.42%)
50mm	32.60	32.32 (δ= -0.86%)	31.72 (δ= -2.70%)
100mm	36.42	37.03 (δ= 1.67%)	36.14 (δ= -0.77%)

* CV_{BF} : CV of break force CV_{mFD} : CV of Minimum fibre diameter

Table 2 shows that most of the predicted values are slightly smaller than the measured values, which is not always true for scoured wool as reported in the previous work (Wang 2000).

It is evident that some errors exist between predicted and measured CV values in Table 2. The CV values predicted by equation 2 are generally closer to experimental results than equation 3, which is reasonable because equation 3 used approximation to simplify the relationship (Wang 2000).

In addition, it appears that the difference between predicted and measured values increases as the gauge length decreases even though the measured value at 50mm and 100mm gauge lengths is much less than that at shorter gauge lengths (10mm and 20mm). In other words, as the gauge length gets shorter, factors other than fibre diameter variations may impact on the variation in fibre breaking force. This may also imply that fibres usually break in their thinnest place (position of minimum diameter) at longer gauge length. In contrast, at shorter gauge lengths, fibres are generally more uniform and some fibres may break in other positions along the fibre where structural defects or flaws might exist. This and previous results (Wang and Wang 1998, Wang 2000) suggest that variations in break force of scoured and processed wool are mostly determined by the variation of fibre diameters. The effect of fibre internal structural defects on the variation of break force is very small, particularly at long fibre lengths.

It should be mentioned that other factors might have some bearing on the discrepancies between predicted and measured values. First, wool fibres are not strictly circular in cross section. Fibre crimps and the associated development of torsional stress in the fibre during stretching can also affect the tensile strength of the fibre. Nevertheless, the results reported in this paper suggest that the combined impact of these factors is very small.

The effects of strain rate

Wool fibres are visco-elastic. Therefore, unlike brittle fibres, time effects should be considered when studying their mechanical properties. The time needed to break fibres is different when they are stretched at different strain rates. The tensile strength of fibres increases as the strain rate increases because of the decreasing time interval allowed for the fibres to break. Others argue that time sustained by fibres during stretching has little effect on the break force and time effect may be ignored (Meybeck and Gianola 1955).

Different strain rates apply in this work, because of the constant extension speed (20 mm/min) but different gauge lengths used for the tensile tests. From the ASTM standard (ASTM D3822-95a and 96), 60%/min extension rate should be chosen for fibres with break strain from 8% to 100%. More brittle fibres should be measured under a lower extension rate. The strain rates used in this work are listed in Table 4.

Table 4:The strain rates at different gauge lengths with 20mm/min crosshead speed

Gauge Length	10mm	20mm	50mm	100mm
Strain Rate (%/min)	200	100	40	20

At the same crosshead speed, longer gauge length gives a smaller strain rate. That means it will take more time to break the fibre at longer gauge length when testing fibres with constant rate of elongation on the INSTRON. Nevertheless, there is a strong linear dependency between breaking force and the square of minimum diameter for fibres measured at the different gauge lengths and strain rates, as shown in Figure 3. The strain rates used have little impact on the prediction of CV of break force as shown in Table 2.

Conclusion

This work first examines the distribution of the minimum fibre diameter and the break force for processed wool from a dyed top. The Kolmogorov-Smirnov and Chi-Square goodness of fit demonstrates that both the minimum fibre diameter and the break force obey lognormal distribution and that there exists a strong linear relationship between them.

For processed and dyed wool, its CV of break force can also be predicted from the CV of minimum diameters as for unprocessed wool. In other words, most fibres break at the position of minimum diameter, especially when the gauge length is long. Other factors, such as structural defects, may have some influence on fibre break force at short gauge lengths.

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