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Mechanical Behavior of Irregular Fibers

Part 3: The Flexural Buckling Behavior

Weiyu He and Xungai Wang

School of Engineering & Technology
Deakin University, Geelong VIC. 3217 Australia

ABSTRACT
Fiber buckling behavior is associated with fabric-evoked prickle, which affects the clothing comfort and aesthetics. In this paper, the flexural buckling behavior of irregular or non-uniform fibers is studied, using the finite element method (FEM). Fiber dimensional irregularities are simulated with sine waves of different magnitude, frequency and initial phase. The critical buckling loads of the simulated fibers are then calculated from the FE model. The results indicate the increasing the level of irregularity will decrease the critical buckling load of fibers, but the effect of frequency and initial phase of irregularity on fiber buckling behavior is complicated and is affected by the fiber diameter and effective length.

1. INTRODUCTION
The buckling behavior of fibers is related to the fabric-evoked prickle. Early studies [4, 7] have revealed that reducing the buckling load of protruding fiber ends would contribute to a reduction in the fabric prickle and that the fiber diameter and the protruding length of fiber ends are the major factors affecting the buckling load. Naylor [8] and Veitch and Naylor [12] applied the buckling theory of thin rods to uniform fibers and proposed that the critical buckling load is proportional to $d^4/l^2$ ($d$ is fiber diameter and $l$ is fiber length). The calculations also show that a fiber diameter of 30 μm and a protruding length of 2 mm will give a critical buckling load of 0.75 mN. Naylor [8] used a subjective test method to indicate that coarse fiber edge is an important variable. For example, he revealed
that the prickliness is markedly different when the percentage of fibers with a diameter
greater than 30 µm is 1% and 2.4%, respectively. Naylor et al. [11] also used this method
to prove that the fine (19 µm) wool fabrics are non-prickly and the prickle increases
significantly as the mean fiber diameter increases. Dolling et al. [3] carried out further
work in this area. They concluded that the fiber diameter distribution (coefficient of
variation of diameter $C_{V_d}$) is a more important factor than the mean fiber diameter,
because they found that a knitted fabric with a mean diameter of 23.2 µm and $C_{V_d}$ of
16.4% was less prickly than a similar fabric with a mean fiber diameter of 21.5 µm and
$C_{V_d}$ of 21.7%. Meanwhile Naylor and Stanton [10] found that the time of shearing the
sheep also affected the fiber diameter characteristics of the fiber ends for Western
Australian (WA) wools. They found that fibers with narrow fiber ends from autumn
shearing would lead to better skin comfort in the garment than fibers with wide fiber ends
from spring shearing. Recently, Naylor et al. [9] examined fabric-evoked prickle using a
range of worsted single jersey knitted wool fabrics with different fiber diameter
characteristics. The previous studies have applied the buckling theory for uniform thin
rods to fibers, without considering the effect of single fiber diameter irregularities on the
fiber buckling behavior.

In parts 1 and 2 of this series [5, 6], we used Finite Element model (FEM) to investigate
both linear and non-linear tensile behavior of fibers with dimensional irregularities. In this
part, we use this technique with ABAQUS software package to simulate dimensional
irregularities of fibers and analyze their buckling behavior.

2. FINITE-ELEMENT MODEL
2.1 Assumptions of fiber specimen
In this paper, we only consider fiber specimens with dimensional irregularities. We also
assume that the fiber cross-section is circular along fiber length and the fiber is
axisymmetric, so that fiber diameter variation can represent its dimensional irregularity.
The different fiber diameter variations follow the sine wave pattern, which is expressed as:

$$Y = 0.015 \cdot [1 - a \cdot SIN(f \cdot X - \theta)] \ (mm)$$  (1)
Where: \( a \) is the level or magnitude of fiber diameter variation

\( f \) is the frequency of fiber diameter variation, which represents the number of sine wave in a simulated fiber length.

\( \theta \) is the initial phase, which represents the position of the thin segment of fiber.

We can change these parameters and obtain different simulation cases discussed below. Figure 1 (c) gives one case for 50% diameter variation (\( a = 50\%, \ f = 1, \ \theta = 0 \)). Table 1 lists the relevant parameters concerning the fiber specimens we used for the simulations.

Table 1. The parameters for FE model in buckling analysis

<table>
<thead>
<tr>
<th>Properties</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young's modulus ( E ) (MPa)</td>
<td>3000</td>
</tr>
<tr>
<td>Poisson's ratio ( \gamma )</td>
<td>0.35</td>
</tr>
<tr>
<td>Specimen average diameter ( d ) (( \mu )m)</td>
<td>30</td>
</tr>
<tr>
<td>Specimen length ( L ) (mm)</td>
<td>1.0</td>
</tr>
</tbody>
</table>

2.2 Description of Finite Element Model (FEM)

Although the fiber is axisymmetric, its deformation is not asymmetric in the buckling analysis. So we used three-dimensional (3-D) elements for this simulation. We divide the fiber specimen models into 84 elements in cross-section area [(see Figure 1 (a)) and a total of 120 layers along the fiber specimen length [(see Figure 1 (b), only three layers are shown]. We used a mesh density of 10080 elements per mm of fiber in simulation model. The shape of elements is cuboid. There are eight nodes on each element, one on each corner of the cube, and each node has six degrees of freedom (three translations and three rotations). In the buckling analysis, one end (bottom) of the fiber is fixed and the other end (top) is subject to an axial compressed force and can be free to rotate but no sideways movement (pinned-end). Figure 1 (d) shows the original and displaced 3-D fiber specimen.

3. RESULTS AND DISCUSSION

3.1 Comparing the classical theory and FE model of buckling

For a uniform fiber specimen with the load and end conditions as indicated in section 2.2, we can use the equation below to calculate its critical buckling load for mode \( n \) :
\[ P_{cr} = \frac{n^2 \pi^2 EI}{L_e^2} \]

Where \( E \) is Young’s modulus, \( I \) is the moment of inertia \((I = \frac{\pi d^4}{64}, \ d \) is fiber diameter\) and \( L_e \) is the effective length of the fiber specimen. For this study, the lowest mode is of significance, namely, \( n = 1 \), while \( L_e = 0.699L \) [1] for the fiber specimen with fixed-end and pinned-end. Using the parameters given in Table 1, the calculated theoretical buckling load is 245 mg, and the estimated buckling load from the FE model is 240 mg. The agreement between these two methods is quite good, with an approximate error of 2% only.

### 3.2 Determination of the effective length of non-uniform fiber.

The effective length of a column, physically, is the distance between points of zero moment when the column is deflected in its fundamental elastic buckling mode \((n = 1)\) [2]. According to this definition of the effective length, we calculate the effective length of a non-uniform fiber specimen. The effect of axial loading on an initially straight slender specimen may deform it into a curved shape. The deformation can be expressed in terms of the deflection curve of the specimen. At any point \((x, y)\) of the curve, as in Figure 2 (a), the equilibrium of the specimen is:

\[
\frac{d^2 y(x)}{dx^2} = -\frac{M(x)}{E \cdot I(x)}
\]

Where: \( E \) is the Young’s modulus, which is a constant.

\( I(x) \) is the moment of inertia, \( I(x) = \frac{\pi d^4(x)}{64} \), where \( d \) is fiber diameter, which is a variable with \( x \) for non-uniform fiber specimen.

\( M(x) \) is the bending moment with respect to \( x \).

Rearranging Equation (1):

\[
M(x) = -E \cdot \frac{\pi d^4(x)}{64}.
\]

When \( M(x) = 0 \),
\[
\left( \frac{\pi E}{64} \right) \cdot d^4(x) \cdot \frac{d^2 y(x)}{dx^2} = 0
\]

Then we must have \( \frac{d^2 y(x)}{dx^2} = 0 \). So we can calculate the inflective point from the equation \( y(x) \) of the buckled curve, and the distance from the inflective point to the top of the fiber specimen \( M(x) = 0 \) is the effective length of the specimen. In many cases of buckling, especially for non-uniform specimen, deflections are difficult to express in equations directly. However using the FE model, we can obtain the deflected shape of a specimen under buckling easily. For example, Figure 2 (b) represents the central deflection of an irregular fiber with 30% diameter variation. We use a polynomial to fit the shape of the buckled fiber and obtain the equation of the deflection curve, and we then can calculate the effective length of the fiber.

### 3.3 Effect of level of variation on fiber buckling behavior

We simulate two different cases here. Case 1 represents a uniform fiber specimen. Case 2 simulates three irregular fibers of the same average diameter but with three different levels of diameter variation, namely, \( a \) is 10%, 30% and 50%, respectively, and \( f \) is 1, \( \theta \) is zero [see Equation (1)]. Table 2 (I) lists the critical buckling loads for different cases. The results indicate that the critical buckling load of fibers decreases significantly with increased level of diameter variation. For different levels of diameter variation, the minimum diameter of the fiber is an important factor, which will influence the critical buckling load of the fiber specimen. As the level of irregularity increases, the minimum fiber diameter decreases markedly, and the calculated effective length of the fiber increases (0.738L, 0.761L and 0.782L at 10%, 30% and 50% variation respectively). The critical buckling load decreases as a consequence.

### 3.4 Effect of frequency of irregularity on fiber buckling behavior

As indicated in Table 2 (II), we simulated three different cases, \( a \) is 10%, 30% and 50% respectively. To examine the effect of the frequency of irregularity, we choose 1, 5, 10 as the values of \( f \) and zero for \( \theta \) [see Equation (1)]. Figure 3 gives a graphical representation of the fiber specimens for Case 4. In these three cases, when \( f \) increases
from 1 to 5, the critical buckling load of the fibers decreases. But increasing the $f$ to 10 sine waves will increase the critical buckling load slightly [see Table 2 (II)]. For the different frequency of diameter variation, we know that their minimum diameters of fiber are the same, so we use the average diameter within the effective length to evaluate the results. As an example in Case 4, when the $f$ increases from 1, 5 to 10, the effective length decreases from 0.76L, 0.73L to 0.70L, and the average diameter within the effective length also decreases from 31.8, 30.6 to 30.0µm, respectively. As the critical buckling load relates to $d^4/L_e^2$, the combined effects of the average diameter and the effective length will lead to a reduction of the critical buckling load, and then a slight increase again. This result is also consistent with the buckling theory.

3.5 Effect of position of the thin segment on fiber buckling behavior
We used different initial phases ($\theta$) of the sine wave to depict the different positions of the thin segment along fiber. Table 2 (III) summarizes the simulation condition and results. In cases 6, 7 and 8, $a$ is still 10%, 30% and 50%. In each level, $f$ equals 1 but $\theta$ is 0, $\pi/2$, $\pi$ and $3\pi/2$, respectively. Figure 4 gives a graphical depiction of fiber specimens with different initial phases for Case 7. We note that, for these three cases, increasing $\theta$ from 0, $\pi/2$ to $\pi$ reduces the critical buckling load. But when $\theta$ increases further to $3\pi/2$, the critical buckling load increases significantly [see Table 2 (III)]. Figure 4 also gives the effective length of the fibers. The average diameters within the effective lengths are 31.8, 28.4, 24.6 and 31.7µm, respectively, when $\theta$ increases. Since the critical buckling load is proportional to $d^4/L_e^2$, the combined effect of diameter and effective length explains the results here also. Figure 5 shows the critical buckling load versus the initial phase for the simulated irregular fiber specimens. We also note that the inflective point of buckled curve does not occur at the thinnest segment of irregular fiber.

3.6 Effect of diameter variation related to shearing time on fiber buckling behaviour
As mentioned in the introduction, Western Australian (WA) wools have narrow fibre ends from autumn shearing and have coarse fibre ends from spring shearing. We simulated these two cases here. In case A, $f$ is $1/2$ and $\theta$ is $\pi/2$. In case B, $f$ is $1/2$ but $\theta$ is $3\pi/2$. 
We also use three different levels of diameter variation. Table 3 gives the simulation results. For different levels of variation, the critical buckling load for case A is lower than for case B. These results indicate that fibers with a finer tip and a coarser base buckle more easily than fibers with a coarser tip and a finer base. This is consistent with the experimental results obtained by Naylor and Stanton [10].

Table 3: Data for additional cases

<table>
<thead>
<tr>
<th>Case</th>
<th>Level of variation (%)</th>
<th>Frequency of variation</th>
<th>Critical buckling load (mg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10</td>
<td>1/2</td>
<td>223.3</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td>233.6</td>
</tr>
<tr>
<td>A</td>
<td>30</td>
<td></td>
<td>167.0</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td>184.3</td>
</tr>
<tr>
<td>A</td>
<td>50</td>
<td></td>
<td>94.4</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td>100.1</td>
</tr>
</tbody>
</table>

4. CONCLUSIONS
A three dimensional finite-element model has been utilized to investigate the buckling behavior of fiber specimens with simulated dimensional irregularities. The following conclusions can be drawn from this study:

- The level of diameter variations significantly influences the buckling behavior of non-uniform fibers. As the level of variation increases, the critical buckling load decreases.
- The critical buckling load is related to the shape of the fiber specimen and fibers with a thinner tip buckle more easily than fibers with a coarser tip under the conditions examined.
- The effective length and the average diameter within the effective length of a non-uniform fiber are the key factors to influence the critical buckling load of the fiber. These two parameters can explain the effect of frequency and initial phase on the critical buckling load of fibers with the same mean diameter and coefficient of variation of diameter.
Acknowledgment

We wish to acknowledge the support of an Australian Postgraduate Award for the first author. We also wish to thank Dr Shaorui Zhang for his valuable assistance and constructive discussion during the course of this study.

REFERENCES


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Figure 1: 3-D finite element mesh used for analyzing fiber buckling behavior

Figure 2: The deflection curve of the central axis of a buckled specimen

Figure 3: Graphical depiction of the un-buckled and buckled specimens at different frequency of irregularity (30% variation is used)

Figure 4: Buckled specimens with different initial phases (at 30% variation)

Figure 5: Critical buckling load versus initial phase for the simulated irregular specimen
Table 2: Data for fiber specimens with different diameter variations

<table>
<thead>
<tr>
<th>Different variation in the simulation</th>
<th>Simulation conditions</th>
<th>Results</th>
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<tr>
<td></td>
<td>Level of variation $a$ (%)</td>
<td>Frequency of variation $f$</td>
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<td>(I) Level</td>
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</tr>
<tr>
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<td>Case 2</td>
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<tr>
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<td>1</td>
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<tr>
<td>(II) Frequency</td>
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<td>Case 8</td>
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<td>1</td>
</tr>
<tr>
<td></td>
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<td>3</td>
</tr>
</tbody>
</table>
Figure 1

50% variation used 
\( a = 50\%, f = 1, \theta = 0 \)
(a). The specimen with fixed-pinned ends

(b). Polynomial fit the shape of the buckled specimen

Figure 2
(X:Y:Z=1:1.5)

Figure 3
Figure 4
Figure 5