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# **Distribution of Power Requirements during Yarn Winding in Ring Spinning**

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## **ABSTRACT**

A model of a yarn package is established for a ring spinning system. The yarn layer, surface area and mass of the yarn package are formulated with respect to the diameters of empty bobbin and full yarn package, yarn count and yarn winding-on time. Based on the principles of dynamics and aerodynamics, the models of the power requirements for overcoming the skin friction drag, increasing the kinetic energy of the yarn package (bobbin and wound yarn) and overcoming the yarn wind-on tension are developed. The skin friction coefficient on the surface of a rotating yarn package is obtained from experiment. The power distribution during yarn packaging is discussed based on a case study. The results indicate that overcoming the skin friction drag during yarn winding consumes the largest amount of energy. The energy required to overcome the yarn wind-on tension is also significant.

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## Introduction

Ring spinning has been the dominant system of manufacturing high quality yarns for apparel applications. In recent years, leading European textile machinery manufacturers, such as Rieter (Switzerland) and Cognetex (Italy), have invested heavily in further developing the ring spinning system to improve the quality of yarns. But two outstanding problems of ring spinning – high power consumption and low productivity – remain a challenge to engineers and research scientists as they have been for over 170 years.

The process of ring spinning drafts an input strand of fibre and then simultaneously inserts the required amount of twist and winds the spun yarn onto a bobbin (yarn package). Figure 1 [4] is a simplified diagram of the system. There is a drafting zone at **A**, and the drawn and twisted fibres pass through lappet **L**, through a traveller **T** on the ring **R**, and onto the yarn package **P**. The spindle is rotated by a tape drive and the package built by a controlled rise and fall of the ring-rail **Q**.

In the only reported trial we can find from literature [9], the power to rotate one frame package of continuous-filament yarn was recorded; the package was then wrapped manually with one layer of heavy-count woollen yarn, and the power required was five times the initial result. However, almost no work on power distribution analysis of a yarn packaging has been reported in the literature.

In this paper, we are mainly interested in the energy required to build up a yarn package in ring spinning, because about two thirds of power supplied to the ring spinning machine is consumed by the spindles [7]. In particular, we will evaluate the respective contribution to

power requirement made by skin friction drag, yarn wind-on tension, package rotation and build-up.

## A Geometrical Model of a Bobbin with Full Yarn Package

### PROFILE OF A YARN PACKAGE

Based on the winding method of the cop build in a ring spinning system, we use a model of a bobbin with full yarn package as shown in Figure 2 where, for simplicity in this study, we consider the profiles of both the top and bottom conical ends to be straight/linear, and the upper and lower chases have the same angle of  $\beta$  ( $0^\circ < \beta < 90^\circ$ ). Furthermore, we ignore the effects of yarn hairiness and the skin friction drag on the exposed surface of the bobbin because the surface of the empty bobbin is quite smooth comparing to the surface of a yarn package. Let  $d_0$  [m] be the diameter of the empty bobbin,  $d_p$  [m] the maximum diameter of the full yarn package,  $h_0$  [m] the total height of the full yarn package, and  $h_m$  [m] the height of main part of the full yarn package, from Figure 2(a),

$$h_m = h_0 - (d_p - d_0)\cot\beta \quad (1)$$

In the cop build, a complete yarn layer comprises a main (up) winding and a cross (down) winding. The main winding, which fulfils the primary yarn take-up function, is formed during the slow rise of the ring rail, whereas the more open cross-winding forms during the rapid descend. The ratio between the length of yarn in the main (up) and cross (down) windings is of about 2:1 [7].

Grosberg (1971) proposed a relationship between yarn diameter and count [4]:

$$d_y = 4.44 \times 10^{-5} \sqrt{\frac{N}{D}} \quad (2)$$

where  $d_y$  [m] is yarn diameter,  $N$  [tex] is yarn count and  $D$  [g/cm<sup>3</sup>] is fiber density.

In order to analyze the process of winding yarn onto a bobbin, the yarn region of a full yarn package wound with cop build, as shown in Figure 2(b), can be divided into a main part and a

base part. In the main part, the number of complete layers,  $L_y$ , is equal to  $\left[ \frac{h_m \sin \beta}{2d_y} \right]$  and the

number of yarn coils in each main (up) winding layer,  $C_m$ , is equal to  $\left[ \frac{d_p - d_0}{2d_y \sin \beta} \right]$ . The first

coil has a yarn length of  $(d_p - d_y)\pi$  (approximately) and the last coil has a yarn length of  $(d_0 + d_y)\pi$  (from lower to upper). The yarn length of each complete layer in the main part will be

$$L_m = 1.5(d_p - d_y)\pi C_m - \frac{1.5(d_p - d_0 - 2d_y)\pi}{C_m - 1} \sum_{i=1}^{C_m} (i - 1) \quad (3)$$

In the base part, the number of complete layers,  $L_b$ , is equal to  $\left[ \frac{(d_p - d_0)}{4d_y} \right]$ . For the main

(up) winding of the first complete layer, the number of yarn coils,  $C_1$ , is equal to

$\left[ \frac{(d_p - d_0)\cotan \beta}{2d_y} \right]$  and each coil has a yarn length of  $(d_0 + d_y)\pi$ . The yarn length of the  $i$ th

complete layer ( $i = 1 \rightarrow L_b$ ) will be:

$$L_i = 1.5 (d_0 + d_y)\pi C_1 + (i - 1) \times \frac{L_m - 1.5(d_0 + d_y)\pi C_1}{L_b - 1} \quad (4)$$

We have provided the detailed derivations for  $L_y$ ,  $C_m$ ,  $L_m$  and  $L_i$  in the Appendix.

#### RELATIONSHIPS RELATED TO WINDING-ON TIME

Let  $v_w$  [m/s] be linear winding-on speed (i.e., front roller delivery speed), then

$$T_p = [(\sum_{i=1}^{L_b} L_i + L_m \times L_y)/v_w] \quad (5)$$

where  $T_p$  [s] is the total time taken to wind a full yarn package,  $L_b$  is the number of complete layers in the base part and  $L_y$  is the number of complete layers in the main part of a full yarn package, and  $L_m$  and  $L_i$  are given by Equations 3 and 4.

We divide the total time taken to wind a full yarn package into three parts and then calculate the area of bobbin surface having yarn for each of the parts. The surface area of a yarn package in ring spinning can be formulated as

$$S_p(t) = \begin{cases} \frac{(d_0 + 4d_y)\pi d_y C_1}{T_1} t & (0 \leq t \leq T_1) \\ (d_0 + 4d_y)\pi d_y C_1 + \left( \frac{d_p - d_0}{2 \sin \beta} (d_0 + d_p + 2d_y)\pi - (d_0 + 4d_y)\pi d_y C_1 \right) \frac{t - T_1}{T_b - T_1} & (T_1 < t \leq T_b) \\ \frac{d_p - d_0}{2 \sin \beta} (d_0 + d_p + 2d_y)\pi + \frac{d_p \pi h_m}{T_p - T_b} (t - T_b) & (T_b < t \leq T_p) \end{cases} \quad (6)$$

where  $S_p(t)$  [m<sup>2</sup>] is the surface area of yarn package at time  $t$  [s],  $T_1$  [s] and  $T_b$  [s] are times used to wind the first complete layer and the base part, respectively ( $T_1 = [1.5(d_0 + d_y)\pi C_1/\nu_w]$  and  $T_b = [(\sum_{i=1}^{L_p} L_i)/\nu_w]$ ).

The mass of a yarn package in ring spinning can be expressed as:

$$m_p(t) = N\nu_w t \times 10^{-6} + m_0 \quad (0 \leq t \leq T_p) \quad (7)$$

where  $m_p(t)$  [kg] is the mass of yarn package at time  $t$  [s], and  $N$  [tex] is yarn count,  $\nu_w$  [m/s] is linear winding-on speed and  $m_0$  [kg] is the mass of an empty bobbin.

### Computation of Power Requirements

During a typical ring spinning process, the speed of spindle rotation varies quite simply – from zero to full speed  $V_0$  [rps] (beginning of yarn packaging), full speed and from full speed to zero (end of yarn packaging when a full yarn package is completed). Assuming that  $t_0$  [s] is the initial time taken by the spindle to accelerate from zero to full speed  $V_0$ , then  $t_0$  is usually very small (i.e. less than 10 seconds) in normal ring spinning, compared to the total time (over 1.5 hours, see Table 1) taken to wind a full yarn package. For ease of the calculation, we assume that the spindle speed  $V(t)$  increases a constant acceleration and have

$$V(t) = \begin{cases} \frac{V_0}{t_0} t & (0 \leq t \leq t_0) \\ V_0 & (t_0 < t \leq T_p) \end{cases} \quad (8)$$

## POWER REQUIREMENT FOR SKIN FRICTION DRAG OF A YARN PACKAGE

In order to further simplify the calculation, the skin friction coefficient on the surface of a yarn package can be considered to be the skin friction coefficient on the surface of the main part of this yarn package, i.e., the skin friction coefficient  $C_{fp}$  on the surface of a circular-cylinder having diameter  $d_p$ .

While winding a yarn package, we can imagine that the bobbin with the wound yarn is fixed and the air around the bobbin moves around the surface of the bobbin. Then the air forms airflow around the surface during winding the yarn package. For a bobbin rotating at the given spindle speed  $V(t)$ , the skin friction drag on the package surface can be obtained [1] by

$$D_f(t) = \frac{1}{2} \rho (\pi d_p V(t))^2 S_p(t) C_{fp} \quad (0 \leq t \leq T_p) \quad (9)$$

So the power requirement for overcoming the skin friction drag on the surface of a rotating yarn package at an given spindle speed  $V(t)$  is

$$P_f(t) = \frac{1}{2} \rho (\pi d_p V(t))^3 S_p(t) C_{fp} \quad (0 \leq t \leq T_p) \quad (10)$$

where  $P_f(t)$  [W] is the power requirement,  $\rho$  [kg/m<sup>3</sup>] is air density and equals 1.197,  $C_{fp}$  [dimensionless] is the skin friction coefficient,  $V(t)$  [rps] is spindle speed and  $S_p(t)$  [m<sup>2</sup>] is the surface-area of a yarn package at time  $t$  [s],  $d_p$  [m] is the maximum diameter of a full yarn package.

## KINETIC ENERGY OF A YARN PACKAGE

Considering the dynamic system of ring spinning, the kinetic energy stored in a rotating yarn package (including a bobbin and wound yarn) is

$$E_k(t) = \frac{1}{2} \mathbf{I}(t) \omega^2(t) \quad (11)$$

where  $E_k(t)$  [J] is kinetic energy of the rotating package,  $\omega(t)$  [rad/s] is the angular velocity of the package [6] and  $\mathbf{I}(t)$  [ $\text{kg} \cdot \text{m}^2$ ] is the moment of inertia which can be expressed as [3]:

$$\mathbf{I}(t) = \frac{1}{2} m_p(t) r_p^2(t) \quad (12)$$

in which  $m_p(t)$  [kg] is the mass of a yarn package,  $r_p(t)$  [m] is the distance from centre line of the rotating package to winding-on yarn.

Substituting Equation 12 into Equation 11,

$$E_k(t) = \frac{1}{4} m_p(t) r_p^2(t) \omega^2(t) \quad (13)$$

Equation 13 shows that the yarn package will absorb energy when the speed and/or mass of the package increase(s). The yarn package will release energy when the speed decreases.

### *Generating Acceleration of a Rotating Bobbin (in Start-up Stage)*

Since there is rapid acceleration in every early stage of a bobbin (yarn package) rotation (if  $t_0 = 5$  seconds, then  $t_0/T_p < 0.1\%$ ) when a yarn is wound onto a bobbin for the first layer of yarn package,  $r_p(t) = \frac{1}{2}d_0$  and  $m_p(t)$  can be considered to be  $m_0$  (the mass of an empty bobbin).

Therefore Equation 13 can be written as

$$E_{ka}(t) = \frac{1}{4} m_0 d_0^2 \pi^2 V^2(t) \quad (14)$$

Substituting Equation 8 into Equation 14 and performing derivation for the two sides,

$$P_{ka}(t) = \begin{cases} \frac{1}{2t_0^2} m_0 d_0^2 \pi^2 V_0^2 t & (0 \leq t \leq t_0) \\ 0 & (t_0 < t \leq T_p) \end{cases} \quad (15)$$

where  $P_{ka}(t)$  [W] is the power required to accelerate the bobbin (package) at time  $t$  [s],  $t_0$  [s] is time spent increasing spindle speed from zero to full speed  $V_0$  [rps] and  $m_0$  [kg] is the mass of an empty bobbin.

### *Increasing Mass of a Yarn Package (in Full-speed Stage)*

The spindle rotation is on full speed over 99% of yarn packaging time, the spindle speed  $V(t)$  can be considered to be full speed  $V_0$  when a yarn is winding onto a bobbin, hence  $\omega(t)$  can

be considered to be a constant (i.e.,  $\omega(t) = 2\pi V_0$ ). The variation of package mass (including bobbin and wound yarn) relates to the winding-on time  $t$ , so Equation 13 can be written as,

$$E_{ks}(t) = \pi^2 V_0^2 m_p(t) r_p^2(t) \quad (0 \leq t \leq T_p) \quad (16)$$

While winding yarn with cop build,  $r_p(t)$  varies between  $\frac{1}{2}d_0$  and  $\frac{1}{2}d_p$  within every complete layer in the main part and time spent on winding the main part is over 93%. Therefore,  $r_p(t)$  can be considered to be a function with respect to winding-on time  $t$  with period of  $T_m$  throughout the yarn packaging process:

$$r_p(t) = \begin{cases} \frac{d_p}{2} - \frac{3}{4T_m}(d_p - d_0)(t - kT_m) & \left( kT_m < t \leq kT_m + \frac{2}{3}T_m \right) \\ \frac{d_p}{2} - \frac{3}{2T_m}(d_p - d_0)((k+1)T_m - t) & \left( kT_m + \frac{2}{3}T_m < t \leq (k+1)T_m \right) \end{cases} \quad (17)$$

where  $k = 0, 1, 2, \dots, \left\lceil \frac{T_p}{T_m} \right\rceil - 1$ ,  $d_0$  [m] is the diameter of an empty bobbin and  $d_p$  [m] is the maximum diameter of a yarn package,  $T_p$  [s] is the total time taken to wind a full yarn package and  $T_m$  [s] is time used to wind a complete layer in the main part of the yarn package.

To simplify the calculation, we use the maximum value of  $r_p(t)$ ,  $d_p/2$ , to substitute  $r_p(t)$  in Equation 16 and using Equation 7,

$$E_{ks}(t) = \frac{1}{4} (N\omega_w t \times 10^{-6} + m_0) \pi^2 V_0^2 d_p^2 \quad (0 \leq t \leq T_p) \quad (18)$$

Performing derivation for the two sides of Equation 18, the power requirement for increasing mass of a yarn package (including bobbin and wound yarn):

$$P_{ks}(t) = \frac{1}{4} N v_w \pi^2 V_0^2 d_p^2 \times 10^{-6} \quad (0 \leq t \leq T_p) \quad (19)$$

Combining Equations 15 and 19

$$P_k(t) = \begin{cases} \frac{1}{2t_0^2} m_0 d_0^2 \pi^2 V_0^2 t + \frac{1}{4} N v_w \pi^2 V_0^2 d_p^2 \times 10^{-6} & (0 \leq t \leq t_0) \\ \frac{1}{4} N v_w \pi^2 V_0^2 d_p^2 \times 10^{-6} & (t_0 < t \leq T_p) \end{cases} \quad (20)$$

where  $P_k(t)$  [W] is the power to be absorbed by a rotating yarn package (including bobbin and wound yarn) at time  $t$  [s],  $t_0$  [s] is time spent increasing spindle speed from zero to full speed  $V_0$  [rps],  $T_p$  [s] is time taken to wind a full yarn package,  $d_0$  [m] is the diameter of an empty bobbin and  $d_p$  [m] is the maximum diameter of a yarn package,  $m_0$  [kg] is the mass of an empty bobbin and  $N$  [tex] is yarn count.

#### YARN WIND-ON TENSION

The power required to overcome the yarn wind-on tension will be

$$P_F(t) = F(t) \omega_1(t) r_p(t) \quad (0 \leq t \leq T_p) \quad (21)$$

where  $P_F(t)$  [W] is power requirement,  $F(t)$  [N] is yarn wind-on tension,  $\omega_1(t)$  [rad/s] is angular velocity of traveller,  $r_p(t)$  [m] is the distance from the centre line of the rotating bobbin to the yarn winding-on yarn at time  $t$  [s].

The power calculated here includes the power required to overcome air drag on the balloon as well as yarn/traveller and traveller/ring friction as all of these effects are included in calculating the yarn wind-on tension.

Since the value of  $\omega_1(t)/\omega(t)$  is greater than 0.96, we can use  $\omega(t)$  (spindle angular velocity) instead of  $\omega_1(t)$ :

$$P_F(t) = 2\pi F(t)V(t)r_p(t) \quad (0 \leq t \leq T_p) \quad (22)$$

where  $P_F(t)$  [W] is the power required to overcome the yarn wind-on tension  $F(t)$  [N],  $V(t)$  [rps] is spindle speed,  $T_p$  [s] is time taken to wind a full yarn package,  $r_p(t)$  [m] is given by Equation 17.

## Case Study

### BASIC DATA

Supposing the diameter of an empty bobbin  $d_0$  is 0.025 m, the mass of empty bobbin  $m_0 = 0.08362$  kg, the maximum diameter of a full package (including bobbin and wound yarn)  $d_p$  is 0.045 m, the total height of a full package  $h_0$  is 0.245 m and the upper/lower chase angle in the cop of a full yarn package  $\beta$  is  $32^\circ$ . If the yarn to be wound is made of cotton fibre which

density  $D = 1.52 \text{ g/cm}^3$  [8] and the linear-winding on speed (i.e., front roller delivery speed)  $v_w$  is  $0.3 \text{ m/s}$ , then the data in Table 1 can be obtained from Equations 2 – 5. Next, we will analyze power requirements for yarn packaging in ring spinning a cotton yarn of 38 tex at full spindle speed of 167 rps.

**Table 1 Yarn diameter, layer number, yarn length and time spent winding a package**

Yarn count ( $N$ ) [tex]	10	25	38	45	60	70	80
Diameter ( $d_y$ ) [ $\times 10^{-4}$ m]	1.14	1.80	2.22	2.42	2.79	3.01	3.22
Layer number of a yarn package	538	340	276	253	219	203	190
Total length of yarn [m]	14412	5742	3809	3205	2384	2044	1789
Time taken to wind a full yarn package ( $T_p$ ) [second]	43236	17224	11427	9613	7151	6133	5369

#### POWER REQUIRED TO OVERCOME SKIN FRICTION DRAG

We measured the power level during the rotation of a single yarn package using an experimental set-up [5]. In order to remove the yarn hairiness effect on energy consumption, we singed the yarn package to eliminate the hair fibers on the package surface hairiness before the experiment. We calculated the skin friction coefficient on the surface of a rotating yarn package based on experimental data and the result depends on the spindle speed and the diameter of yarn package. For this case study, the skin friction coefficient of the yarn package surface is 0.1554 [10].

Substituting the relevant data into Equation 6,

$$S_p(t) = \begin{cases} 5.20 \times 10^{-5} t & (0 \leq t \leq 25) \\ 1.3 \times 10^{-3} + 3.77 \times 10^{-6} (t - 25) & (25 < t \leq 745) \\ 4.01 \times 10^{-3} + 2.82 \times 10^{-6} (t - 745) & (745 < t \leq 11427) \end{cases} \quad (6a)$$

Furthermore, from Equation 10, the power requirement to overcome skin friction drag on the yarn package surface can be expressed as

$$P_f(t) = \begin{cases} 5.10 \times 10^{-4} t^4 & (0 \leq t \leq 5) \\ 6.38 \times 10^{-2} t & (5 < t \leq 25) \\ 1.59 + 4.61 \times 10^{-3} (t - 25) & (25 < t \leq 745) \\ 4.91 + 3.45 \times 10^{-3} (t - 745) & (745 < t \leq 11427) \end{cases} \quad (24)$$

#### POWER REQUIRED TO STORE KINETIC ENERGY ONTO BOBBIN

Substituting relevant data into Equation 20, the power requirement to store in rotating yarn package can be expressed as

$$P_k(t) = \begin{cases} 0.29t + 1.77 \times 10^{-3} & (0 \leq t \leq 5) \\ 1.77 \times 10^{-3} & (5 < t \leq 11427) \end{cases} \quad (25)$$

#### POWER REQUIRED TO OVERCOME YARN WIND-ON TENSION

The yarn wind-on tension during winding of a cop layer is variable. For the sake of simplicity in this study,  $F(t)$  can be considered to be the maximum yarn wind-on tension and can be

taken as approximately 0.4 N [2, 7]. Substituting  $F(t)$  and related data into Equation 22, the power requirement to overcome yarn wind-on tension can be expressed as

$$P_F(t) = \begin{cases} 26.72\pi r_p(t)t & (0 \leq t \leq 5) \\ 133.61\pi r_p(t) & (5 < t \leq 11427) \end{cases} \quad (26)$$

where  $r_p(t)$  is given by

$$r_p(t) = \begin{cases} 2.25 \times 10^{-2} - 3.57 \times 10^{-4}(t - 42k) & (42k < t \leq 42k + 28) \\ 2.25 \times 10^{-2} - 7.14 \times 10^{-4}(42(k + 1) - t) & (42k + 28 < t \leq 42(k + 1)) \end{cases} \quad (17a)$$

where  $k = 0, 1, 2, \dots, 271$ .

#### A COMPARISON OF POWER DISTRIBUTION WHILE WINDING A YARN PACKAGE

Equations 24 - 26 give power requirements for a yarn package during the ring spinning process. A comparison of power requirements for skin friction drag, kinetic energy and yarn wind-on tension is shown in Figure 3. While winding a yarn package in the initial stage, from Figure 3a, the most power will be consumed by the yarn wind-on tension, followed by the skin friction drag, then increasing package mass. However, after the initial winding stage, the power required for overcoming the skin friction drag becomes dominant, as shown in Figure 3b. The power requirement for increasing yarn package mass is “very small”, the reason may

be that the variation of package mass is quite slow. In contrast, the power requirement for the initial acceleration is “very large” because the acceleration of bobbin rotation is very fast.

## Conclusion

A geometrical model of a bobbin with full yarn package is developed. The yarn layer, surface area and mass of a yarn winding package are formulated with respect to the diameters of empty bobbin and full yarn package, yarn count and winding-on time. The skin friction coefficient on the surface of a rotating yarn package was obtained from experiment.

The power requirements for a yarn package process are addressed. It is found that power required to overcome skin friction drag is over 74% (on average) and that required to overcome yarn wind-on tension is about 25% (on average). The power consumed by the stored kinetic energy into a rotating yarn package can be neglected because it is less than 1% (on average). However the power required at the spinning start-up is also quite high (about 14%), due to the rapid package acceleration over a short period of time.

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## Appendix: Derivations for $L_y$ , $C_m$ , $L_m$ and $L_i$

### DERIVATION FOR $L_y$

In the main part, from Figure 2, each of the layers occupies a space of  $\frac{d_y}{\sin \beta}$  in an upright direction. Since one complete layer consists of two layers (one up and one down windings), the number of complete layers in the main part of a full yarn package will be:

$$L_y = \left[ \frac{h_m}{2d_y / \sin \beta} \right] = \left[ \frac{h_m \sin \beta}{2d_y} \right].$$

### DERIVATION FOR $C_m$

From Figure 2, the number of yarn coils for each of the complete layers in main part is:

$$C_m = \left[ \sqrt{\left( \frac{d_y}{2} - \frac{d_0}{2} \right)^2 + \left( \left( \frac{d_y}{2} - \frac{d_0}{2} \right) \cot \beta \right)^2} / d_y \right] = \left[ \frac{d_p - d_0}{2d_y \sin \beta} \right].$$

### DERIVATION FOR $L_m$

The first coil has a yarn length of  $(d_p - d_y)\pi$  (approximately) and the last coil has a yarn length of  $(d_0 + d_y)\pi$  (from lower to upper). The yarn length of the  $i$ th coil ( $i = 1 \rightarrow C_m$ , from lower to upper) will be

$$\begin{aligned}
& (d_p - d_y)\pi - (i-1) \times \frac{(d_p - d_y)\pi - (d_0 + d_y)\pi}{C_m - 1} \\
& = (d_p - d_y)\pi - \frac{(i-1)(d_p - d_0 - 2d_y)\pi}{C_m - 1}.
\end{aligned}$$

So the yarn length of each complete layer in the main part is

$$\begin{aligned}
L_m & = 1.5 \sum_{i=1}^{C_m} \left\{ (d_p - d_y)\pi - \frac{(i-1)(d_p - d_0 - 2d_y)\pi}{C_m - 1} \right\} \\
& = 1.5(d_p - d_y)\pi C_m - \frac{1.5(d_p - d_0 - 2d_y)\pi}{C_m - 1} \sum_{i=1}^{C_m} (i-1).
\end{aligned}$$

DERIVATION FOR  $L_i$

Since the ratio between the length of yarn in the main (up) and cross (down) windings is of about 2:1 [7], the first complete layer has a yarn length of  $1.5(d_0 + d_y)\pi C_1$ . The coil number and yarn length of the last complete layer (from lower to upper) in the base part are the same as that of one complete layer in the main part. Therefore the yarn length of the  $i$ th complete layer ( $i \rightarrow L_b$ ) in the base part is

$$L_i = 1.5 (d_0 + d_y)\pi C_1 + (i-1) \times \frac{L_m - 1.5(d_0 + d_y)\pi C_1}{L_b - 1}.$$

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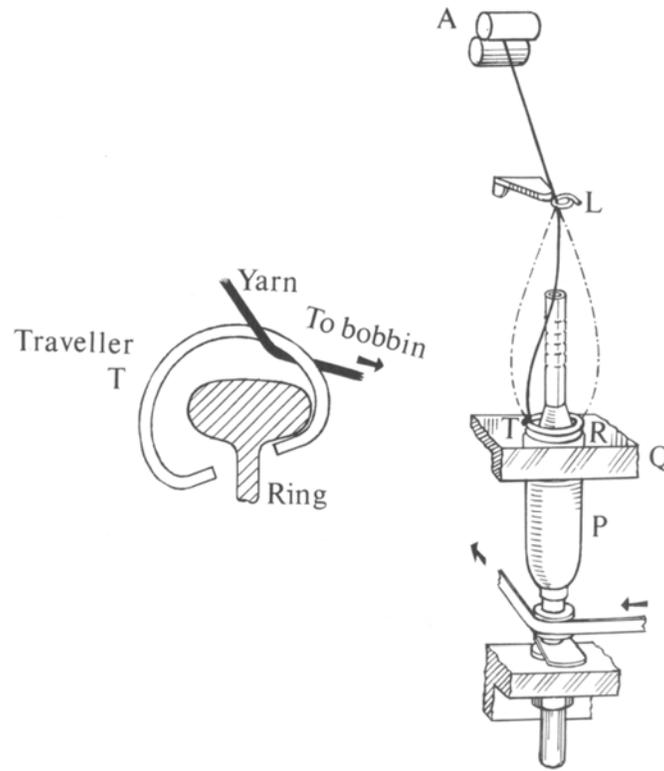


Figure 1. The ring spinning frame [4].

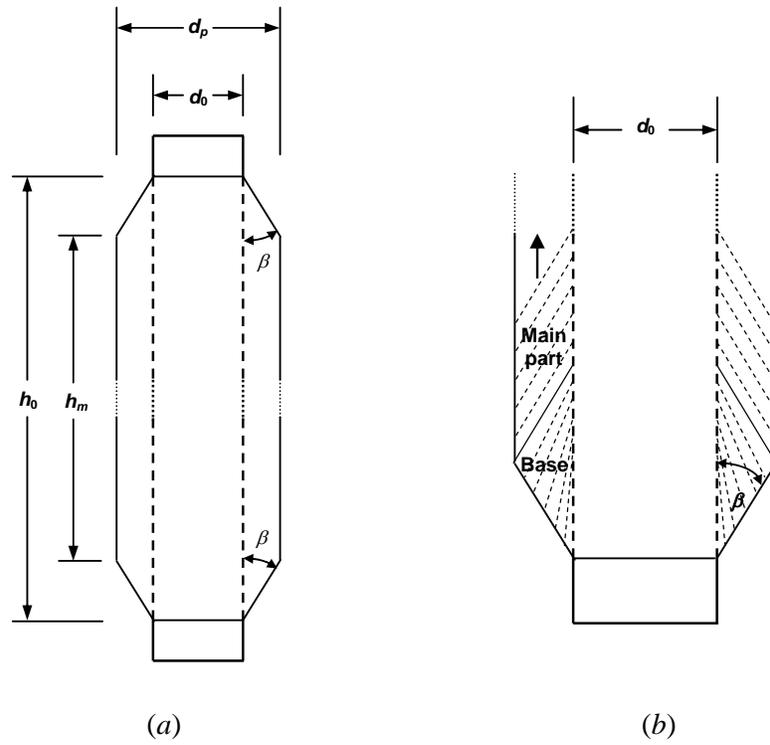


Figure 2. (a) A model of bobbin with full yarn package and (b) a part of the lateral section of a winding on yarn package with cop build method.

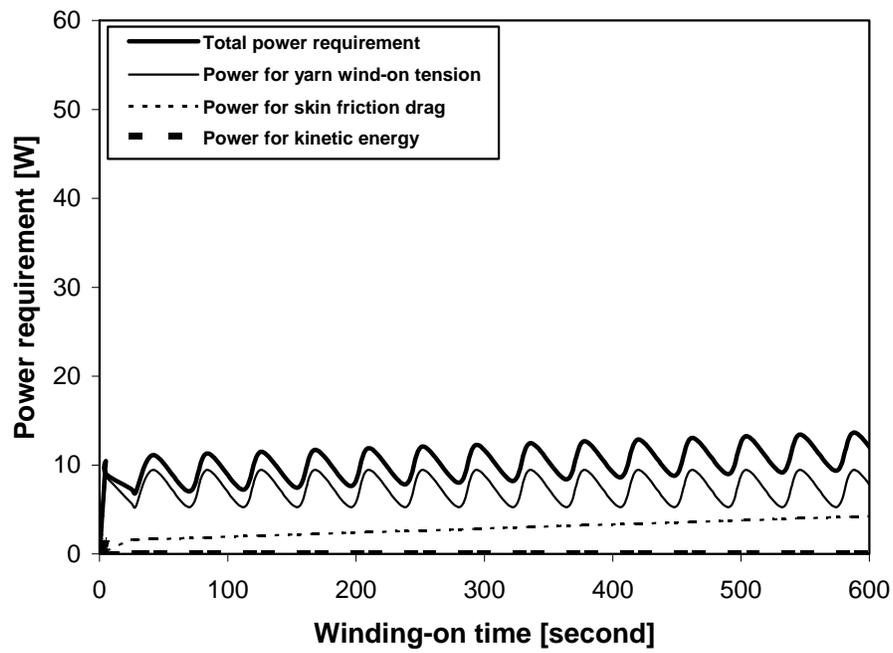


Figure 3a . Comparison of power requirements for skin friction drag, kinetic energy and yarn wind-on tension during yarn packaging (initial range).

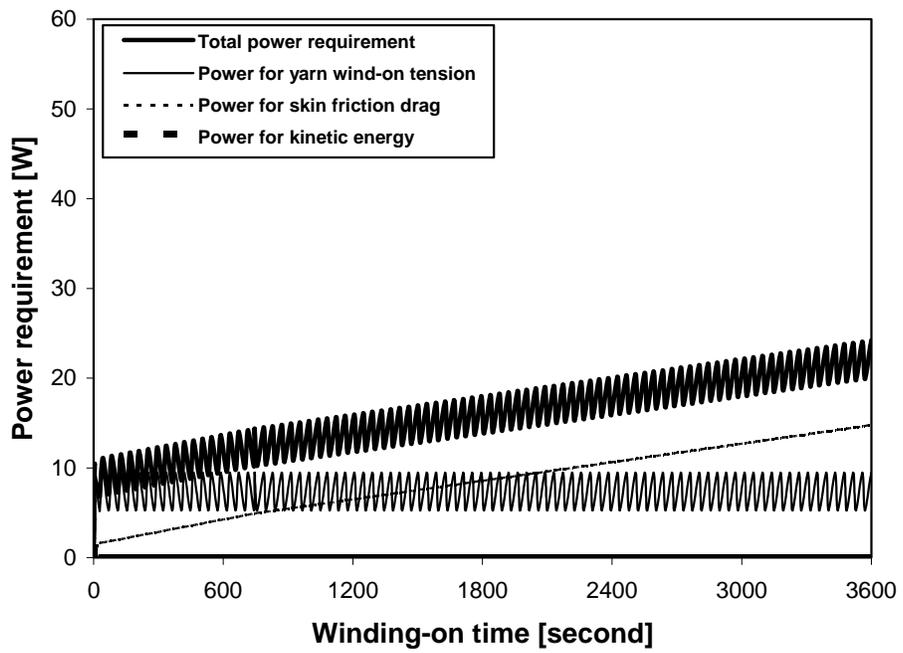


Figure 3b . Comparison of power requirements for skin friction drag, kinetic energy and yarn wind-on tension during yarn packaging (first 3600s).