THE LOGICAL STATUS OF LACAN'S
"FORMULAE OF SEXUATION"

Geoff Boucher

In this article, I propose to investigate the logical status of Lacan’s formulae of sexuation, as expounded in Seminar XX.\(^1\) According to Lacan, ‘the sexed being ... results from a logical exigency in speech’.\(^2\) But the question of the relation between “the logical demands of language” and human sexuality is the locus for a series of objections to the psychoanalytic project. According to the contemporary doxa, the individual finds himself located within the multiple language games that constitute social reality. This involves the radically contingent construction of gender identity through dramatic performances of social roles.\(^3\) Such critics allege that there are as many sexualities as there are language games. They oppose the “radical translation” between incompatible social worlds to sexual difference as a transhistorical reality.\(^4\) To insist, as Lacan does, that the relation of the subject to language necessarily includes an unconscious, masculine or feminine stance, seems, to such critics to be a naturalisation of culturally constructed gender characteristics.\(^5\) So do Lacan’s formulae of sexuation represent a return to biological essentialism? What are the epistemological claims raised by Lacan’s metapsychological formalisations? I depart from the assumption that a relevant indicator of

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2 op. cit. p. 10.
the scientific status of a theory is its crossing of certain "thresholds of formalisation". I briefly explain the relevant aspects of Lacan's two distinct unconscious "grammars". Then, I consider these proposals in the light of the formalization of "transconsistent logic" in the last twenty years. Recent work in logical analysis reveals that it is possible to render so-called transconsistent systems of propositions, ones that include contradictions or undecidability, as formal logical orders. My paper will test the hypothesis that Lacan's formulae have a valid - albeit highly general - logical status and can be regarded as valid theoretical formalisations. A complete formal reconstruction in logical notation will not be essayed. Instead, the paper will turn to the discursive implications of the logical status of sexuation. Once we have described the formulae logically, we can consider their relevance to human sexuality, and therefore the appropriateness of Lacan's decision to designate these orders "masculine" and "feminine". In what sense can "masculine" or "feminine" be ascribed to logical systems, or, rather, why would masculine and feminine unconscious processes exhibit logical differences?

In Seminar XI, Lacan claims that the epistemological ambition of his research is to constitute psychoanalysis as a science, based on the fundamental postulate that 'the unconscious is structured like a language'. This 'conjectural science of the subject', however, has a distinctive object, which is identified with a break in regularity. Furthermore, a scientific object is abstracted from the Real of nature and symbolised mathematically, whereas the object of psychoanalysis, the cause of the subject, is the remainder of the operation of symbolisation/mathematisation. Within a mathematically formalised discourse, the determinations of the object are systematic - that is, the object only exists

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7 op. cit. p. 43.
as a mathematical abstraction with a logical discourse. By these standards, the objects of psychoanalysis are non-scientific and the project of psychoanalysis is merely discursive.

Instead of regarding the impossibility of mathematical abstraction as the abandonment of the project of a formalised psychoanalysis grounded in the postulate that the unconscious is structured like a language, however, I suggest we should consider it the paradoxical realization of that project. It should be recognised that formalisation, as quantification on the model of "mathematical physics," is only one possibility. Another possibility is that Lacan's mathemes generate a formal qualification of the psychoanalytic experience. The "matheme" condenses conceptualisation into a qualitative relation that can be transmitted (that is, the letter is transmitted, but the signifier remains polyvalent). It is a symbolisation of the Real - a nonquantitative formalisation - that might be thought of as a "literalisation," as opposed to a mathematisation.9

It is because Lacanian psychoanalysis describes the unconscious as a sequence of material letters, obedient to a combinatorial, that it can claim to have aspects of a formal theory.10 As Bruce Fink comments, "Lacan offers a definition of science in terms of the kinds of signifiers, of formal vocabularies, that it adds to the objects it studies and that, in turn, transform those objects into the objects of science."11 Thus, through the formalisation of the mathemes, psychoanalysis includes logical elements and intersects with scientific research, without being reducible to them. As Fink states: "psychoanalysis is not a science, but a discourse that allows us to understand the structure and operation of scientific discourse at a certain fundamental level."12 In Lacan's formulae of sexuation, by contrast, the formal properties of the logical object are preserved. This makes them something other than either an

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12 op. cit. p. 145.
imagination of properties or a symbolisation of relation - it makes them 
structures, existing in the Real. My suggestion is that the unconscious, as 
an automatic system,\textsuperscript{13} displays systematic properties of incompleteness or 
inconsistency, and that these fundamental properties determine the 
"sexuation" of the unconscious.

**How Lacan's formulae work**

I want to demonstrate this hypothesis by investigating the logical 
properties of Lacan's formulae of sexuation. Before beginning, we need to 
dispense with the common misconception that the formulae prescribe a 
compulsory heterosexual norm built in to the structure of human 
sexuality. In Lacanian terms, this means sexuality is defined as an 
imaginary complementarity between man and woman, whereas Lacan 
expressly states that "there is no sexual relation". Likewise, we have to 
abandon the notion that the formulae deal with a symbolic difference, 
where the presence of one sex is the absence of the other. Instead of 
symbolic difference, the formulae of sexuation explicitly reject any 
mutually constitutive opposition between woman and man, proposing, on 
the contrary, that sexuation results in two independent realities, defined 
not in relation to one another (as in symbolic difference), but by means of 
a third function. Masculine and feminine define separate discursive 
universes, or incommensurable paradigms, which only relate via a third 
term, the phallic function. The formulae of sexuation, then, designate a 
real division that, while it cannot be symbolised, never ceases to inscribe 
itself - that is, it is as ineluctable as it is impossible.

At first glance, the formulae have nothing to do with sexuation, for 
they present the logical paradoxes of totality\textsuperscript{14}. The key to the "formulae of

\textsuperscript{13} ibid. pp. 153-172.

sexuation” lies in how two asymmetrical forms of totalisation generate the (non-)existence of the universal. Where the universal exists (logically, not ontically), implying the closure of a field, it does so only through a constitutive exception, that is, at the cost of a contradiction. The exclusion of contradiction, by contrast, results in the non-existence of the universal, meaning the lack of an exception and the non-closure of the field. The formulae of sexuation presented in Seminar XX\(^{15}\) can be glossed, based on Joan Copjec’s reading\(^{16}\), as follows:

<table>
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<tr>
<th>Masculine Structure</th>
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<td>$\exists x . \Phi x$: There exists an $x$ that is not submitted to the phallic function</td>
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<tr>
<td>$\forall x . \Phi x$: Every $x$ is submitted to the phallic function</td>
<td>$\forall x . \Phi x$: Not every $x$ is submitted to the phallic function</td>
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The symbol phi is the function of castration internal to language, while x is the speaking being, so that $\Phi x$ means: ‘the submission of the speaking being to castration in the field of speech’. Each of the four formulae presents a logical proposition concerning the qualitative and the quantitative dimensions of the phallic function. The subject of the proposition is the quantification term and the predicate is the phallic function. The symbols $\exists$ and $\forall$ are quantifiers. $\forall$ is a universal quantifier, meaning ‘any/every’. $\exists$ is an existential quantifier, denoting ‘there exists an/at least one …’. The operator ‘not’ determines the quality of the predicate, whether affirmative (unmarked) or negative (marked). Note that the “masculine” and “feminine” sides of the formulae represent

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different distributions of the phallic function and not two alternative functions of masculinity and femininity.

At first glance, masculine and feminine are simple negations of one another, and each side is internally composed of a contradiction. Likewise, a superficial reading might suppose that the diagonally opposed terms are equivalent, that is:

$$\exists x . \Phi x = \forall x . \Phi x$$

and:

$$\forall x . \Phi x = \exists x . \Phi x$$

That is, is not 'there ex-sists a constitutive exception to symbolic castration upon which masculine structure is based' the same as 'not all of a woman comes under the law of the signifier - some part of woman escapes symbolic castration'? And is not 'the whole of a man falls under the phallic function – man is altogether determined by symbolic castration' the same as 'every woman is partially determined by symbolic castration – there ex-sists no exception'? The answer is no. There is an asymmetry between universal quantification of the phallic function and existential quantification of its negation. Lacan's discussion of the feminine "not-all"\(^{18}\), together with the wealth of Lacanian commentary on feminine sexuation\(^ {19}\), make it clear that the logic of not-all implies a non-closed field, rather than a contradictory one. Discursively, the result of the formulae can be described as a position of enunciation in relation to the phallic function. According to the formulae, the inclusion of all men under the phallic

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\(^{17}\) S. Žižek. op. cit. p. 36.


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function depends upon the contradictory inclusion of at least one man who escapes it. Likewise, not-all women are subjected to the phallic function thanks to a fundamental undecidability wherein every woman’s subjection to the phallic function consists.

The relation of the sexes to the phallic function determines their antagonistic relation to one another: ‘the universal function implies a constitutive exception; the lack of exception to the function Φ prevents its universal span’\textsuperscript{20}. The female side relates castration to enjoyment, suggesting that a supplementary enjoyment emerges beyond the phallic function. The male side relates the signifier to a constitutive exception where all the enjoyment is located. \textit{The key is the inclusion/exclusion of an exceptional-universal term}\textsuperscript{21}. This is the key to the notational distinction introduced by Lacan. Accordingly, the propositions mean:

\hspace{1cm} \exists x \cdot \Phi x: \text{there exists a constitutive exception to symbolic castration upon which masculine structure is based;}
\hspace{1cm} \forall x \cdot \Phi x: \text{the whole of a man falls under the phallic function - man is altogether determined by symbolic castration;}
\hspace{1cm} \exists x \cdot \neg \Phi x: \text{there exists no Universal Woman, no exception to symbolic castration;}
\hspace{1cm} \forall x \cdot \neg \Phi x: \text{every woman is at least in part defined by symbolic castration.}

Everything depends upon the interpretation of this last term. Does it mean that some part of woman escapes the signifier - that is, that there is a mysterious part of woman external to the symbolic order? Or does it mean, more radically, that (as Lacan asserts), the feminine is entirely submitted to the signifier - but that there are things said about the feminine that are true but unprovable, that is, undecidable? As Suzanne Barnard says, ‘she is in the phallic function \textit{altogether}', but this function

\textsuperscript{20} S. Žižek. op. cit. p. 36.
totalises openly, without generating a closed totality - the lack of an exception is equivalent to the absence of a limit\textsuperscript{22}. Working with this latter interpretation yields:

(1) The masculine formulae work according to the logic of an exception that is a contradiction. \( \forall x . \Phi x \): Every \( x \) is submitted to the phallic function implies a field that is completely described by the phallic function. \( \exists x . (\Phi x) \): The condition of this closure is that an element be elevated to the status of a constitutive exception, generating a contradiction (an inconsistency) within the system defined by the phallic function.

(2) The feminine formulae work according to the logic of a system without closure. \( \exists x . \overline{\Phi x} \): Every element of the system is submitted to the phallic function, implying that the field is consistent. \( \forall x . \Phi x \): Nonetheless, the field is incomplete, because the phallic function is incapable of entirely determining the speaking being without the universal quantifier.

At least in this context, "masculine" and "feminine" structure have nothing to do with sexuality. They represent the logical possibilities once we step beyond classical two-value logic. The concept that woman is "not-all" means only that this discursive structure conforms to the logical requirements of a consistent, incomplete field. The constitutive exception informing masculine universality conforms to the protocols of an inconsistent, complete field.

\textsuperscript{22} S. Barnard. op. cit. p. 178.
Inconsistency and incompleteness, masculine and feminine

In his discussion of the formulae, Lacan is not developing a general theory of totalisation. Instead, he claims that the formulae result from a specific problem: they spring ‘from a logical exigency in speech’. The emphasis here, however, should be on the word “logical”. I suggest that Lacan is not speaking descriptively but formally of a logical problem concerning language. More precisely, it is a logical problem to do with metalinguistically determining whether language is “all,” whether language can “say it all” and whether “all of language” holds together without exception. Now we - we readers of Lacan - know in advance that his answer to this question is negative, for that is part of the point of the formulae of sexuation and of Lacan’s “linguistrickery”. But what exactly is this logical problem, as opposed to the descriptive observation that language fails and that a signifier is lacking?

Before examining the logical demand inherent in language of which Lacan speaks, however, we have to understand that he is operating beyond traditional two-value logic. According to a tradition that dates from Aristotle, a logically well-defined field (say, for instance, arithmetic operations with the natural numbers) is both “complete” and “consistent”. By “complete” is meant that the field contains no undecidable propositions, or, what amounts to the same thing, no true theorems that lack a proof. Every “statement” in arithmetic (every equation, loosely speaking, but we mean here also the propositions of number theory) should be able to be decided, proven, by correctly using the axioms, operations and elements (numbers) of the arithmetical field. By “consistent” is meant that propositions in the field are either true or false. It is called “two-value logic” because every proposition must have one of the two values, true or false - but not both (contradiction, that is, inconsistency), nor neither (undecidability, that is, incompleteness).

Now, Aristotle thought that the business of philosophy was to systematise and rationalise popular opinion, and the two-value logic turns out to be nothing more than an imaginary ratification of common sense.
The imaginary appeal of the two-value logic helps explain the prejudice that admitting contradiction into a system means that you can prove everything from anything (because you are allowed to contradict yourself), and that accepting undecidability means renouncing determinacy (because uncertainty spreads throughout the system). Of course, in some circles, undecidability has gained acceptance as an exotic procedure by virtue of deconstruction, but the logic of dialectics, which rests upon contradiction, is still frequently dismissed as meaningless. Recent developments in formal logic, however, have challenged this ancient model by demonstrating that it is possible to have logics which are inconsistent or incomplete. Formal logics of contradiction and undecidability are both "transconsistent" - meaning that they break with the traditional two-value logic, in which every proposition can only be true or false, but not "both" (contradiction), nor "neither" (undecidable). Indeed, it transpires that it is the assumption of the two-value logic, that a well-defined field is both complete and consistent, which is impossible - not logics of inconsistency and incompleteness.

According to the celebrated incompleteness theorem of the mathematician Kurt Gödel, for formal systems of sufficient complexity, a system is either complete or consistent\(^\text{23}\). Gödel demonstrated that in any consistent (non-contradictory) formal field of sufficient complexity, there exists an element (a proposition) that is undecidable on the terms of this field. By complexity, Gödel meant, for instance, arithmetic, and he demonstrated that within the field of arithmetic there exists at least one true proposition that does not have a proof within the field. In other words, Gödel showed that the perfectly well-defined field of arithmetic was absolutely consistent (no proof was both true and false) - but at the cost of the field including at least one proposition whose proof was undecidable. Consistency led to incompleteness! From this, the

impossibility of a sufficiently complex formal system that is both complete and consistent follows immediately. Worse yet, it is not as if this maverick proposition could be pinned down once and for all, localised and kept in a theoretical quarantine, for the problem has to do with recursive operations performed on themselves. Incompleteness manifests itself the moment a field tries to "grasp itself," to "prove itself" in a self-reflexive, totalising operation in which it applies its own operations to itself. The details of this operation (Gödel numbering and so forth) are not relevant here: what matters is the outcome (which we can accept as a demonstrated result), namely, the search for the problematic proposition cannot be finished. One way of understanding the result of Gödel's theorem is to say that it means that (for such a system), there are things said about the system that are true but cannot be proven, and things proposed that cannot be proven true or false. Even more frighteningly, as the logician Graham Priest points out, the field could be made complete, by starting at the other end and assuming that proofs existed for every proposition. The "cost" would be that at least one proposition would turn out to be both true and false - in other words, the price of completeness would be inconsistency. Following Gödel's incompleteness theorem, therefore, systems can be either inconsistent-and-complete, or incomplete-and-consistent. Consistency and completeness are involved in a trade-off: a system can be incomplete and consistent, or complete and inconsistent. In either case, the traditional two-value logic is finished.

Notions of incompleteness and inconsistency are sometimes employed promiscuously in the humanities, as if they were synonyms. Far from being equivalent notations, they designate incommensurable paradigms: consistency implies incompleteness, and completeness implies inconsistency. The distinction between incompleteness and inconsistency can be tabulated:

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<td><strong>Inconsistency</strong> - the field includes a logical contradiction</td>
<td><strong>Consistency</strong> – the field does not include a contradictory exception</td>
</tr>
<tr>
<td><strong>Completeness</strong> – the field is closed: every proposition has a proof; all operations can be decided</td>
<td><strong>Incompleteness</strong> – the field is open: not every proposition has a proof; not all of the operations can be decided</td>
</tr>
<tr>
<td>(Logic of contradiction e.g. Dialectics)</td>
<td>(Logic of undecidability e.g. Deconstruction)</td>
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In the 1930s, Gödel had already demonstrated to the satisfaction of the philosophical and scientific community that regarding consistent fields as incomplete was not a logical defect, but a strict logical requirement. In a way, this did not involve a "paradigm shift": fields that had been thought of as complete turned out to be incomplete, and the implications of this had to be thought out – but they remained consistent, they continued to operate as before. The dream of a final theory of mathematics, based on a demonstration of the completeness of the field, was over. But incompleteness did not threaten to destroy logic as we know it in the way that inconsistency did. It is only recently that Graham Priest has logically demonstrated that the existence of a contradiction within a formal field does not entail the extension of contradictions throughout the entire field.\(^{25}\) With certain restrictions operative upon the scope of a formalized

inconsistent field, the contradictory element remains a *singularity* - that is, a unique point of contradiction within an otherwise consistent field.

The distinction between the two sides concerns the status of an unusual operation - let us call this operation "Inclosure" (for reasons to be discussed in a moment) and describe it in terms of reflexive totalisation. On the side of inconsistency, everything (every proposition) is submitted to this operation and decided accordingly. But this is only on condition that one proposition has the status of a singularity: this proposition is exceptional in that it is both true and false, and therefore excluded from the set of results of the normal decision procedure that generates a true/false outcome. On the side of incompleteness, although there is not any proposition that cannot be submitted to the operation (because this field is consistent, and therefore generates no contradictory proposition), not every proposition submitted to the operation yields a true/false decision.

I propose to identify the logic of incompleteness with the "feminine" aspect of sexuation and to identify the logic of inconsistency with the "masculine" aspect of sexuation. Accordingly, this interpretation can be filled in on the table above:

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</tr>
<tr>
<td>$\forall x . \Phi x$: Every $x$ is submitted to the phallic function <em>Completeness</em> – the field is closed</td>
<td>$\forall x . \Phi x$: Not every $x$ is submitted to the phallic function <em>Incompleteness</em> – the field is open</td>
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*Inconsistent-and-complete* | *Incomplete-and-consistent* |
To carry the day with this interpretation, I have to demonstrate that Lacan's "phallic function" is logically the same as the operation, based on Gödel's theorem, that I called "Inclosure".

I now want to show that the operation of totalisation necessarily leads to the alternatives of consistency-with-incompleteness and inconsistency-with-completeness. Then I will be able to explain how Lacan's phallic function and the logical theorem of Inclosure are identical operations (on different objects). And I further want to propose that the relevance of transconsistent logic to psychoanalytic theory can be demonstrated by considering the "logical demands of language" in the case of linguistics.

I shall do this by following Priest's development of what he calls the "Inclosure Theorem," a theorem relating to totalisation, explained in a moment²⁶. Priest's basic logical strategy is to demonstrate that totalisation generates a contradiction at the limit of thought, where one is forced to claim that a recursive decision procedure, reflexively applied, leads to a result that is both inside the field (Closure) and beyond it (Transcendence)²⁷.

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²⁷ For those who worry about the charge of "intellectual imposture" in relation to this use of Gödel's theorem, some more discussion may be needed regarding whether the incompleteness theorem applies to natural languages. I rely here on Priest's demonstration that it does. See G. Priest. op. cit. 1987. pp. 15-20, 49-64. Briefly, the demonstration depends upon Tarski's formalisation of natural language semantics, so what is being formalised is not words, but phrases which have truth-values. See A. Tarski. Logic, Semantics, Metamathematics: Papers From 1923 to 1938. Oxford, Clarendon Press, 1956. pp. 152-278. The formalisation must meet certain conditions, easily satisfied by a natural language. Once these are satisfied, the set of phrases, with their truth-values, can be given a Gödel numbering and the incompleteness theorem applies. See G. Priest. op. cit. 1987.
The inclosure theorem and the phallic function

Let us suppose that the fundamental theorem of structural linguistics is the hypothesis of value, according to which the value of any sign is differentially determined from its context. This means that signs do not have permanently fixed values: any signifier (word) is assigned a signified (meaning) by its context, because in that context this signifier functions as the binary opposite of some other signifier, whose absence its presence implies. According to Saussure, therefore, the meaning of $S_2$ is "not-$S_2$," for instance, in one context the meaning of /black/ might be "not white". But in another context it might be "not good," and in another "not happy". We can then say, more generally, that every utterance is differentially determined through its context.

As Lacan recognised, this implies a certain "floating" of the signifier and "flowing" of the signified, that is, that the signification of an utterance is not only provisional and revisable, but it must be assigned retrospectively in relation to the temporality of the utterance. I take this to be the meaning of Lacan's claim that 'it is in the chain of the signifier that the meaning 'insists' but that none of its elements 'consists' in the signification of which it is at the moment capable," and that "we are forced, then, to accept the notion of an incessant sliding of the signified under the signifier'. This is because 'the sentence completes its signification only with its last term, each term being anticipated in the construction of the others, and, inversely, scaling their meaning by its retroactive effect'. But as Lacan saw, this leads to a paradox.

Consider a chain of signification, which we can represent in a shorthand way as $S_1$. By inquiring into the context, we can provisionally "fix" the meaning of the signifiers. For instance, I read the semantically ambiguous sentence, "the shooting of the hunters was terrible," but only discover the sense of the genitive when I read that the article is about

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hunting accidents, not hunters' inaccuracy. But that is equally to say that
the determining context has to be linguistically formulated as another
differential string of signifiers, $S_2$.

The determination of the meaning of $S_1$ therefore takes the form of a
statement of $S_1$ plus a supplementary utterance, $S_1 - S_2$. Clearly, this
procedure invokes a regress whose general form is $S_1 - S_2 - \ldots - S_n$, where
$S_n$ is the result of the $(n-1)^{th}$ iteration. At every step of the process, the
totality of the signifiers in the string of utterances, $S_1 - S_2 - \ldots - S_n$, acts as a
provisional synchronous order, assigning a value to $S_1$ until the next step
in the process.

The problem that Lacan identified is that this procedure is open-
ended. How then does any meaning emerge from this flux of
indeterminacy, if signification is provisional and retroactive? How do we
know that just this pause for breath or full stop is the one that ends the
supplementary utterance for step n-1, and not just an internal division
within the supplementary utterance? In other words, how do we know in
advance what is significant, when the whole purpose of the exercise is to
determine signification retroactively? As is well known, Lacan’s answer is
the “quilting point,” or phallic signifier. This signifier “sutures” the
diachrony of successive utterances to the synchrony of a totality of
contexts, enabling an “impossible” operation, namely, the coming to a
provisional stop of an endless process.

What permits this operation - why is the “phallic signifier” a
legitimate supposition and not a wild fancy? Lacan’s most important
insight into this problem is metalinguistic.

Consider the totality of all possible contexts - Saussure’s
“Language” (Langue). We obtain this at the limit of the string of utterances,
$S_1 - S_2 - \ldots - S_n$.

Now, the totality of contexts is not another chain of signification,
but the synchronic order of Language. In Priest’s terms, this means,
Transcendence.

But its determination is the result of a regular, diachronic
procedure. In Priest’s terms, this means, Closure.
The totality of contexts both is and is not accessible through the diachronic succession of the utterance of all possible contexts. In other words, the operation of totalisation that produces Language is at once necessary and impossible: the character of the signifier presupposes it and therefore it is implied as the horizon of expectation of every utterance, but at the same time, it “never arrives,” it “does not exist”. Hence the profound implications of Lacan’s claim that the phallic signifier ‘is the signifier intended to designate as a whole the effects of the signified, in that the signifier conditions them by its presence as a signifier”\textsuperscript{30}\textsuperscript{30}. The phallic signifier represents the possibility of the emergence of meaning at the limit of signification because it indicates a necessary yet impossible operation of totalisation. This operation is one meaning of Lacan’s “phallic function”.

We can state this more formally, with reference to Russell’s paradox.

_Given a property \( \alpha \) and a function \( \Phi \), such that, if \( v \) belongs to all members of \( u \), always exists, has the property \( \alpha \), and is not a member of \( u \); then the supposition is that there is a class \( \Omega \) of all terms having the property \( a \) and that \( \Phi(\Omega) \) exists leads to the conclusion that \( \Phi(\Omega) \) both has and has not the property \( a \).\textsuperscript{31}\textsuperscript{31}_

Russell’s paradox is a paradox of reflexive totalisation. It “works” (i.e., generates a paradoxical result) when we perform an operation of totalisation upon the relevant set of everything.

Consider the following application of Russell’s paradox. (The symbols \( \in \) and \( \not\in \) denote “is an element of” and “is not an element of,” respectively.)

Let \( a \) be the property “having a signified”.
Let \( y \) be any signifier.

\textsuperscript{30} ibid. p. 285.

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Let $\Phi$ be the function "differential determination of the value of the signifier".

Let $\Omega$ be the "treasury of the signifier," the set of all possible significations – Saussure’s Language.

Now we can write $x = \{y; \sigma(y)\}$ to mean the set of all of the signifiers and their signifieds in any determinate context, $x$ – for instance, the romance *The Death of Arthur* = \{white; “not black”, knight: “not Lady”, ...\}.

This set, $x$, is susceptible to rearticulation: by simply applying $\Phi$ to $x$ we generate a supplementary “utterance,” say, $z$, that is not a part of $x$. For instance, I perform a Marxist interpretation of Mallory’s *The Death of Arthur* by adding a new context, the feudal class situation relevant to the poem’s production. Then I generate the following: *The Death of Arthur: A Marxist Interpretation* = \{white; “not pagan”, knight; “not peasant”, ...\}.

With the help of this example, it is pretty easy to see that some formal conditions hold for the relations between $x$ - a subset of all possible contexts - and $\Omega$ - the set of all possible contexts.

1) Existence. $\Omega = \{y; \sigma(y)\}$ exists - that is, $\Omega$ is the set of all significations, $y$, for which a meaning exists.
2) Transcendence. If $x$ is a subset of $\Omega$: (a) $\Phi(x) \in x$
3) Closure. (b) $\Phi(x) \in \Omega$

Excuse me? Well, (1) is *ex hypothesi*: the supposition of Language is that there is such a set of all possible significations. So supposing the existence of such a set, and given the operation that generates it, $\Phi$, what can be said of it? (2) Consider a subset of $\Omega$, called $x$: whenever we want to differentially determine the meaning of $x$, we just apply the operation $\Phi$ to generate a larger subset, $z$. (a) To write $\Phi(x) \in x$ is to say that the set of significations, $z$, resulting from doing $\Phi$ to $x$, is not just a part of $x$ itself, but something beyond $x$, transcending $x$. (b) And insofar as this new set, $z$, which results from the operation $\Phi(x)$, is a set of significations, it too is a part of the ultimate set - the Language - that we have denoted $\Omega$. So the
operation $\Phi(x)$, instead of resulting in going beyond signification into nonsense, stays within the set of significations: Closure.

Now consider not $\Phi(x)$, but a more dangerous operation, $\Phi(\Omega)$. To find what happens, all we have to do to obtain Russell’s paradox in the language that Priest uses is to “plug the numbers into the formula”.

2) Transcendence. (Noting that any set is its own subset.) (a) $\Phi(\Omega) \sim \varepsilon \Omega$
3) Closure. (b) $\Phi(\Omega) \varepsilon \Omega$

These conditions generate a contradiction. The corollaries (a) and (b), applied to $\Omega$, entail that $\Phi(\Omega) \sim \varepsilon \Omega$ and $\Phi(\Omega) \varepsilon \Omega$. This is the Inclosure Theorem, which Priest develops out of Russell’s paradox. $^{32}$

In the terms of our example that means: the application of the fundamental law of language to Language both works and does not work. Reflexive totalisation of Language (i.e., metalanguage) results in sense and nonsense. But this is all a consequence of the operation of totalisation applied to a field that is assumed to be both complete and consistent. Take Language, for instance. Every sign in Language can be determined by the differential procedure outlined in the hypothesis of value - completeness. No sign both has and does not have a value - consistency.

There are then two, quite different, ways out of this paradox. Incompleteness: the first is to deny the possibility of a limit. The price to be paid for this solution is that there will be some element, $a$, whose meaning remains indeterminate within the field of the signifier, $\Omega$. To be exact, the field $\Omega$ does not have the power to justify the interpretation of $a$. This element will therefore appear as an object that resists signification. So the first alternative involves choosing consistency at the price of incompleteness. Inconsistency: alternatively, the field can be closed, but only if the contradiction is affirmed. The difficulty with this solution is that it generates an element, $\varphi$, that is contradictory, that is, that appears

as a tautological signifier lacking a signified. Such an alternative amounts
to choosing completeness as the cost of inconsistency.

To conclude, the Inclosure Theorem is the logical function of
totalisation. In relation to language, assuming the hypothesis of value, this
designates the operation of the determination of meaning. But this is
exactly one role of what Lacan refers to as the phallic function. (For Lacan,
the phallic signifier is there to indicate the existence of a signified33). This
does not exhaust the phallic function, but it does confirm that the
alignment of the Inclosure Theorem with the phallic function in relation to
language is justified.

Meaning and reference, masculine and feminine

I now want to investigate the two logical alternatives that result
from the application of the phallic function to the speaking being. To do
this, I will turn to a logician’s treatment of the problems of meaning and
reference, in the form of Willard Quine’s celebrated essay, Ontological
Relativity34. I do this both because of the exemplary clarity of Quine’s
reflections on the logical demands of language and because this essay has
been invoked as support for the rejection of the Lacanian Real. In this
investigation, I follow Priest’s analysis of Quine in terms of the Inclosure
Theorem35. Then I proceed further, demonstrating that, unbeknownst to
Quine, two asymmetrical solutions to the paradoxes of meaning and
reference are outlined in his essay.

The kernel of Quine’s demonstration of “ontological relativity” is
the hypothesis of the “indeterminacy of translation,” also known as
Radical Meaning Variance. Like Saussure, Quine subscribes to a radical
meaning holism, according to which expressions acquire a meaning only
contextually, by means of a detour through the entire field of signification.

34 W. van Quine. Ontological Relativity and Other Essays. New York, Columbia University
The foundation for this position is a rejection of "semantic correlates," or the correspondence theory, according to which every statement has a determinate semantic correlate that defines its meaning and reference. For Quine, "uncritical semantics," is the 'myth of a museum in which the exhibits are the meanings and the words are the labels"\(^{36}\). In other words, this pre-Critical semantics maintains the illusion of a natural bond between sign and referent, and accordingly believes that 'the words and sentences of a language have their determinate meanings"\(^{37}\). Quine therefore shares essentially the same assumptions as Saussure regarding language.

For uncritical semantics, translation between languages is merely a question of swapping the labels. In Lacanian terms, this relation involves the imaginary complementarity of meaning and reference. But Quine proposes that this theory cannot be sustained. His thought experiment of "radical translation" involves an effort to guess the meaning of the utterances of a radically unknown language by means of an elementary pragmatic analysis. The idea is that by forming a core of analytic hypotheses we can attain a basic grasp of the language and then begin to decode the foreign speech \textit{in extenso}. The problem is that we cannot be certain that by holding up things and asking "what do you call this?" that we have really understood the response. For instance, the term "gavagai" might mean rabbit, rabbit-part, stage-of-rabbit-development, and so forth. The "apparatus of individuation" of our language - its grammatical constructions and linguistic particles - impose a categorial order on the world that entails an ontology. Because we cannot step outside this frame, it is impossible to know with certainty that the translation manual we have devised has not systematically misrecognised the language in question. Indeed, the existence of alternative, empirically equivalent translation manuals seems probable. It follows that, when we have a stable background ontology and inquire into an unfamiliar ontology, several empirically equivalent translation manuals can be generated. Equally, when we have a fixed translation manual (for instance, homophonic

\(^{36}\) W. van Quine. op. cit. pp. 26-68.

\(^{37}\) ibid. pp. 26-68.
translation in the native language), a number of constructions of the background ontology become possible.

The consequence of Quine’s position is that all meaning is indeterminate. This has a specific definition: there will be different sets of analytic hypotheses for the whole language that are consistent with the empirical evidence and yet translate the utterances of the language quite differently. There are ‘rival systems of analytic hypotheses [that] can conform to all speech dispositions … and yet dictate … utterly different translations’\textsuperscript{38}. This is Quine’s restatement of the notion of the theory-ladenness of observation statements, which implies that every thesis is underdetermined even by all possible evidence, even when this includes contradictory accounts that are, because of the empirical underdetermination of theory, impossible to discriminate between. There will exist several “empirically equivalent systems of the world”: reality is always multiple. Thus, because there is no means of finally fixing meaning, there can be no guarantee of the uniqueness of a translation. The situation where multiple translations exist is known as Radical Meaning Variance, and it can be stated in a universal form: every utterance is radically indeterminate; there is no fact of the matter about what statements mean. The encounter with a radically unknown language is therefore merely an explanatory device, because in principle this situation obtains in every language community and radical translation, therefore, ‘begins at home’\textsuperscript{39}. Because of the indeterminacy of meaning, it becomes impossible to determinately refer to entities. The Thesis of the Inscrutability of Reference suggests that one cannot refer determinately (uniquely) to objects, with the entailment that ‘reference would seem to become nonsense, not just in radical translation, but at home’\textsuperscript{40}.

Just at this point, Quine tries to anchor meaning to reference by means of the background, or natural language.

\textsuperscript{39} W. van Quine. op. cit. 1969. pp. 26-68.
\textsuperscript{40} ibid.
It is meaningless to ask whether, in general, our terms ‘rabbit,’ ‘rabbit-part,’ etc. really refer to rabbits, rabbit-parts, etc. rather than something else ... It is meaningless to ask this absolutely; we can ask meaningfully only relative to some background language. ... The background language gives the query sense, if only a relative sense; sense relative, in turn, to it, the background language41.

As Priest notes, this is a revealing slip of the pen: Quine is discussing reference by means of a claim regarding the determinacy of the background meaning. What this indicates is Quine’s assumption that the statements of the background language do, despite radical meaning variance, have a determinate sense42. The tautological self-reference of the background language indicates that a (relatively) determinate meaning does emerge - through a sort of trick. By holding meaning fixed, Quine can examine various theoretical ontologies and develop the Indeterminacy of Reference Thesis, and vice versa: Radical Meaning Variance makes intuitive sense only relative to a stable background ontology, whereby diverse pragmatic communities refer to the same objects in different ways. This problem seems to me relatively straightforward: if there are several distinct theoretical systems that construct the world as consisting of different entities, then this is not a situation of relativity, but of a multiplicity of separate worlds coexisting side-by-side. We only have relativism of meaning if we can say that different theoretical systems are actually construing one set of worldly entities in radically different ways; we only have indeterminacy of reference if we can say that a single theoretical system is capable of constructing the entities of the world in several different ways.

Quine proposes that the threatening abyss of nonsense is “resolved” by the relativity of frameworks: by holding fixed a theoretical frame, a relation emerges between meaning and reference. ‘The relativistic

41 ibid.
42 G. Priest. op. cit. 1995, p. 221.
thesis is ... it makes no sense to say what the objects of a theory are, beyond saying how to interpret or reinterpret that theory in another\footnote{W. van Quine. op. cit. 1969. pp. 26-68.}

Before examining Quine's arguments for relativity (as opposed to multiplicity), let us briefly note that his general hypothesis entails a contradiction that can be highlighted by means of the Inclosure Theorem. Quine's master statement, that "all meaning is indeterminate," involves a logical contradiction similar in form to the familiar paradoxes of statements such as "I am lying," that is, they involve a contradiction between the statement and the enunciation. Quine claims that meaning is radically variant, yet this statement is supposed to be wholly determinate. The theoretical statement of the universal proposition, in other words, immediately snarls itself in a metalinguistic regress.

We can once again apply the logic of the Inclosure Theorem that was developed using Saussure's Language as its example. Consider the following, based on Priest's discussion of Quine\footnote{G. Priest. op. cit. 1995. p. 221.}.

Let $\alpha$ be the property "having a meaning relative to some background language".

Let $\Phi$ be the function of radical translation, "indetermination of the statement".

(Note that radical translation results not in nonsense, but in mutually exclusive meanings, that is, it is a polyvalence operator, not a meaning annihilator.)

Let $\Omega$ be the set of all possible meaningful statements.

1) Existence. $\Omega = \{y; \alpha(y)\}$ exists - that is, $\Omega$ is the set of all statements, $y$, for which a meaning exists.
2) Transcendence. If $x$ is a subset of $\Omega$: (a) $\Phi(x) \neg \varepsilon x$
3) Closure. (b) $\Phi(x) \varepsilon \Omega$
From these assumptions, it follows that the radical translation operation, applied to $\Omega$, entails that $\Phi(\Omega) \not\in \Omega$ and $\Phi(\Omega) \in \Omega$ - contradiction (in line with the *Inclusion Theorem*).

"There is no sexual relation"

Rather than seeing such a paradoxical result as a conceptual fault, we should rather regard this contradiction as an index of theoretical truth. If metalanguage is impossible, it is also ineluctable. It is crucial to note that this metalanguage is founded on a paradox, which emerges when Quine comes to discuss the relation between meaning and reference. What Quine is unaware of is that he proposes two, distinct and incompatible solutions for this problem - one for meaning and the other for reference.

On the side of meaning, the completeness of language is salvaged by means of an inconsistency. Quine is fully aware that the problem concerns the existence of the universal once we consider the totality. 'One is tempted to say,' he explains, 'that meaninglessness sets in when we try to pronounce on everything in our universe'\textsuperscript{45}. But this is premature, because the frame of reference provided by a background language (the "apparatus of individuation") can 'accommodate all existing speech dispositions'\textsuperscript{46}. In other words, everything can be meaningfully expressed in language - the universal exists. Indeed, 'what makes ontological questions meaningless when taken absolutely is not universality but circularity'\textsuperscript{47}. That is, the universal exists only because there is a "circular" exception to the universal rule. 'In practice,' Quine notes of the paradox of radical translation, 'we end the regress of background languages ... by acquiescing in our mother tongue and taking its words at face value'\textsuperscript{48}. This is done very specifically, by means of the problem of identity. The danger posed by radical translation is that we will end up by rejecting self-identity, which 'surely,' he says, 'is not to be

\textsuperscript{45} W. van Quine, op. cit. 1969. p. 51.
\textsuperscript{46} Ibid. pp. 26-68.
\textsuperscript{47} Ibid. p. 53.
\textsuperscript{48} Ibid. p. 49.
rejected as meaningless". Quine's modal particle, "surely," must be understood psychoanalytically as the marker for a radical uncertainty, for, according to his own argument, self-identity must be both meaningful and meaningless. In other words, the ability of the natural language to pronounce on everything in the universe is secured by means of a tautological element that pronounces on nothing at all (self-identity), because it is inherently contradictory. The field of meaning exists and is both inconsistent and complete.

On the side of reference, the solution to the Inscrutability of Reference happens by means of a field that is incomplete, but consistent. The elementary matrix of the problem has already been given in the example of the invented word "gavagai" - is this the abstract singular (rabbit individual) or the concrete general (some rabbit)? According to the thesis of the Inscrutability of Reference, 'the ostensive indistinguishability of the abstract singular from the concrete general turns upon what might be called 'deferred ostension,' as opposed to direct ostension". It is deferred, because every ostension must "detour" through its contextual determination by the entire theoretical system in which it takes place. In principle, then, all ostension is deferred. Because of Radical Meaning Variance, the result of this detour is equivocal. Nonetheless, Quine affirms that ultimately, 'we end the regress of coordinate systems by something like pointing". This sounds like a contradiction, until he explains that the discussion turns on how the background language, with its 'primitively adopted and ultimately inscrutable ontology', is in the final analysis 'uncritically accepted". We can end the regress of coordinate systems by pointing because we "make believe" that the ontology of the background language outlines a stable and univocal set of entities that directly correspond to the words of that language. In

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9 ibid. p. 52.
50 ibid. p. 39.
51 ibid. pp. 41-44.
52 ibid. p. 49.
53 ibid. p. 51.
54 ibid. p. 53.
other words, we need the illusion of an “uncritical semantics” in order to
end relativistic dilemmas and get on with pointing at things as if this
solved the problems of reference. This means that every entity can be
pointed at using a word in the background language - unlike the field of
meaning (where self-identity is threateningly inconsistent), in the field of
reference, it is possible to consistently refer to things (through uncritical
ostension). But, Quine claims, that only implies that some of the things we
confidently point to do not actually have names. In other words, the field
of reference is consistent, but incomplete.

Quine places this claim on a firm logical footing in the second half
of the essay, discussing the problems of shifting from one theoretical
universe, \textit{U}, to another theoretical universe, \textit{V}, by means of a ‘proxy
function,’ that is, ‘a function mapping one universe into part or all of another’\textsuperscript{55}. He is trying to determine what happens in ostension - are there nameless
objects that we nonetheless point to (on the face of it, an astonishing
claim)? Obviously, based on the \textit{Inscrutability of Reference Thesis}, there is no
point in wandering around like Adam and seeing whether words fail
when trying to name things. What we must do instead is to point to
everything using one system of reference (say, an object theory such as
Newtonian physics) and then count all of these “pointings to” relative to
another system of reference (say, the background ontology of common-
sense experience implied by the natural language) and see whether there
are the same number of objects. Clearly, this is a mapping between two
sets of objects that must happen numerically, not discursively. We do it by
correlating expressions from physics, for instance, that describe various
theoretical entities, with objects in the natural world (i.e., with the
background ontology), just by naming them. The necessary logical
background to this discussion is Quine’s earlier demonstration that any
“proto-syntax” of a natural language, consisting of expressions
categorised according to formal symmetry\textsuperscript{56}, can be mapped onto natural
numbers by numbering the syntactical expression categories. So it is

\textsuperscript{55} ibid. p. 55.
\textsuperscript{56} ibid. pp. 41-42.
possible to perform this mapping (in theory, at least). Of course, Quine is
interested in numbering the basic existential propositions of a language by
ordering them into syntactical classes - e.g., "it is raining," "there is a bird"
- since he is not concerned with the empirical question of how many birds
there are at present, but with the counting of natural kinds.

Although Quine's discussion rests upon the Löwenstein-Skolem
theorem, not Gödel, his discussion of reference nonetheless aims towards
its formalisation57. The result of this operation would be a series of
numbered syntactical expression categories corresponding to a series of
numbered theoretical propositions, that could be fitted into an open
sentence that says that such-and-such type of name corresponds to such-
and-such theoretical description. ("We describe rainbows and other
phenomena of the refraction of light using the laws of optics," for
instance.) But it follows, as soon as this formalisation is performed, that
the results of Gödel's incompleteness theorem apply to this mapping
operation using proto-syntax between theory and entities. The world of
the theory will be a consistent field, and Gödel's Theorem states that a
consistent field includes at least one proof for which no demonstration
exists. That means that there is a "law of physics" lacking a
demonstration and consequently a syntactical expression category without
an object. Even in the simplest case, where all we are doing is mapping an
object theory onto a background ontology by denotation, "if we succeed in
showing that every result of substituting a name for the variable in a certain open
sentence is true of the theory, but at the same time we disprove the universal
quantification of the sentence, then certainly we have shown that the universe of
the theory contained some nameless objects"58.

Quine proposes - on the basis of logical arguments that do not
concern us here, and whose validity we shall just assume - that 'a theory
might attest to its own nameless objects ... by showing how some open sentence
became true under all constant substitutions but false under universal

57 ibid. pp. 57-60.
58 ibid. p. 64.
quantification"\(^{59}\). That is to say, Quine concludes that the universe of referential proto-syntax is, in Lacanian terms, not-all: the universal quantifier does not exist, but the theory can embrace nameless objects because there is not one referential sentence in the theory that does not submit to the operation of substitutional quantification.

Quine's therefore proposes that 'ontology is ... doubly relative'\(^{60}\). 'Specifying the universe of a theory only makes sense relative to some background theory, and only relative to some choice of a manual of translation'\(^{61}\). What Quine shows (but does not make explicit) is that the background ontology and the manual of translation operate in logically incommensurable modalities: on the one hand, everything can be meaningfully stated, and so the universal quantification holds; but on the other hand, referential employment of language to specify an ontology invokes a situation where universal predication does not hold. It follows that the operation of transcoding meaningful statements into referential quantification is impossible. At the same time, it is necessary, for as Quine acknowledges, without this operation, language itself becomes nonsense. Quine's solution is an imaginary operation, whereby first one, and then the other, field is fixed by means of an "uncritical acceptance" of meaning and reference. In Lacanian terms, there is no sexual relation, except in the imaginary relation of complementarity.

Conclusion

Part of the reply to the postmodern critic of psychoanalysis who objects that "radical translation" between language games precludes any "transcendental" sexual difference is: it is precisely the logical requirement of radical translation that generates sexual difference. The other part of the reply is to demonstrate that it is legitimate to align a logical asymmetry with the distribution of sexes. But this lies beyond the scope of my paper. I

\(^{59}\) ibid. p. 66.
\(^{60}\) ibid. p. 54.
\(^{61}\) ibid. p. 55.
will therefore only briefly outline how I think that Ellie Ragland’s work on the logic of sexuation supplies this part of the argument. Ragland points out that ‘the difference between the sexes is situated by language at a signifying level where oedipal identifications create sexed positions in terms of unconscious desire’\(^{62}\). These Oedipal identifications relate four logically possible interpretations of the phallus and castration\(^{63}\), where the phallus is interpreted in terms of the movement of difference rather than “phallic” power\(^{64}\), to the imaginary schema of the body and the real of the drives. ‘Human sexuality is perforce constituted on the bias of language, which remains, de facto, an imaginary “body” which is “beyond” or Other to the jouissance of the body qua physical organism’\(^{65}\). It follows that the bond between the logics of inconsistency and incompleteness and human sexuality is motivated - that is, it is neither arbitrary, nor natural, but the result of a culturally specific signification of the phallus.

**Address for Correspondence:**

Room Ad 26  
Arts Faculty  
Deakin University  
Waurn Ponds 3127  
Victoria  
Australia

email: boucher@deakin.edu.au

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\(^{63}\) ibid, p. 38.  
\(^{64}\) ibid, p. 27.  
\(^{65}\) ibid, p. 71.