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TEACHERS’ PRE-CONCEPTIONS
OF YOUNG CHILDREN’S CONCEPTIONS OF NUMBER

Brian Doig and Susie Groves

Abstract
This paper uses data from several sources to argue that while teachers of young children are reasonably accurate in their predictions of when the majority of children will achieve mastery of specific objectives in mathematics, they are much less likely to be aware of the conceptions of high achieving children, and that, as a result, their classroom activities constrain these students’ learning.

The positive effect on student learning outcomes of high teacher expectations is well established, with teachers being encouraged to have high expectations — see, for example National Council of Teachers of Mathematics (1991) and Doig (2002). Further, recent research into educationally productive classroom environments suggests that setting high expectations of students is a key characteristic of such classrooms (Gore, 2000).

Many studies, however, have found that young children’s mathematical skills are consistently underestimated by teachers, with the consequence that a high percentage of tasks set by teachers fail to challenge many children (see, for example, Young-Loveridge, 1989). Moreover, while effective teaching depends on teachers’ knowledge of children’s mathematical understandings and the way this is acquired by young learners, teachers of children in their first year of schooling have been found to take little account of children’s mathematical knowledge gained in “out-of-school, problem-solving situations” (Aubrey, 1995, p. 5).

This paper examines data from several sources in order to further illustrate the extent of agreement (and disagreement) between teachers’ expectations of young children’s learning of number and their actual performance, and discusses research findings which suggest that teachers need opportunities and strategies for assessing children’s mathematical knowledge first hand.

* Deakin University, Burwood, Victoria, 3125, Australia; e-mail: badoig@deakin.edu.au, grovesac@deakin.edu.au
Two comparisons of teachers' expectations and children's performance

As part of the nationwide *Curriculum and Organisation in the Early Years of School* project, teachers assessed young children's performance from pre-school to Year 2 of school. Data were collected from 131 teachers on their expectations of children's mathematical development at the end of pre-school and the first three years of formal schooling. The left hand portion of Table 1 shows the percentage of teachers who indicated each of the year levels as being the earliest at which a majority of children would be expected to achieve mastery for each of the five items related to counting — see de Lemos (1999a) for further details of the project and for results from the pre-school teachers only. Pre-school children in the middle and at the end of the year were also assessed in this project using the *Who Am I?* instrument (de Lemos & Doig, 1999). The end of year assessment provided the data on pre-school children's ability to recognise and write the numerals from 1 to 10 (Item 2, right hand portion of Table 1).

The evaluation of the *Victorian First Steps Pilot Project for the First Three Years of Schooling* (de Lemos, 1999b) included teachers using an extensive checklist with over 3000 Victorian children aged from about 4 to 7 years to assess whether or not the children had achieved or partially achieved specific objectives in mathematics. The right hand portion of Table 1 for Prep to Year 2 shows the percentage of children reported as achieving mastery of each of these objectives. Only children assessed as having achieved full mastery are included.

Table 1  Year levels at which teachers expect majority of children to first achieve mastery of number concepts and skills and actual performance

<table>
<thead>
<tr>
<th>No</th>
<th>Number concepts and skills</th>
<th>Earliest level majority of children expected to achieve mastery</th>
<th>Percentage of children reported as achieving mastery at each year level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>PS</td>
<td>Prep</td>
</tr>
<tr>
<td>1</td>
<td>Ability to say number words in sequence from 1 to 10.</td>
<td>48</td>
<td>48</td>
</tr>
<tr>
<td>2</td>
<td>Ability to recognise &amp; write the numerals from 1 to 10.</td>
<td>8</td>
<td>86</td>
</tr>
<tr>
<td>3</td>
<td>Ability to count up to 10 objects.</td>
<td>18</td>
<td>79</td>
</tr>
<tr>
<td>4</td>
<td>Correct use of terms first, second, third etc. up to ten.</td>
<td>0</td>
<td>66</td>
</tr>
<tr>
<td>5</td>
<td>Ability to order whole numbers up to 999.</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

1 Percentage of teachers. Rows do not total 100%, as some teachers did not respond to all items.

Source: Unpublished data from the *Curriculum and Organisation in the Early Years of School* project.

2 Source: *Who Am I?* assessment of pre-school children (de Lemos, 1999a)

3 Source: *Victorian First Steps Pilot Project for the First Three Years of Schooling* (de Lemos, 1999b)
As Table I shows, by and large teachers' expectations of young children's ability to achieve the selected number objectives corresponded well to children's ability to do so. An example of a task in which teachers under-estimated children's abilities was Item 2, where only 8% of teachers expected the majority of pre-school children to be able to recognise and write the numerals from 1 to 10, while 60% of pre-school students in the same study achieved this by the end of their pre-school year. Unfortunately similar performance data at the pre-school level is not available for the other four items. However, for Item 4, almost a third of the teachers expected Year 1 to be the first year in which the majority of children would be able to correctly use the terms first, second, third, etc up to ten, while the majority of children were able to achieve this in Year 1.

Apart from Item 5, where a small proportion of teachers (11%) appeared to overestimate the ability of Year 1 students by predicting that the majority — rather than almost a third — would be able to order whole numbers up to 999, the data from the studies examined above suggests that for items well within the "normal" curriculum range, children achieved at least as well as, if not better than, predicted by their teachers.

The study reported below looked at student achievement in what, by and large, may have been considered to be performance "outside" the normal curriculum, in order to look at the impact of calculator use in young children's learning in the area of number.

The Calculators in Primary Mathematics project was a long-term investigation into the effects of the introduction of calculators from the first year of school on the learning and teaching of primary mathematics. The project, which worked with 1000 children and 80 teachers in six schools over a four year period was based on the premise that calculators have the potential to significantly change mathematics curriculum and teaching by providing children with a rich mathematical environment to explore. Changes in teachers' expectations of children's mathematical performance and consequent changes in the mathematics curriculum was one of the major foci for the project (see, for example, Groves & Cheeseman, 1993, 1995).

All teachers completed a written questionnaire consisting of 121 randomly ordered items relating to number, at levels of difficulty ranging from what would be usually accepted as relevant to children in their first year of school to ones which would normally not be seen as part of the primary mathematics curriculum. At the beginning of each year of their involvement in the project, teachers were asked to indicate, for each of the items, their expectations of children at the beginning and end of that year, by indicating whether they expected all, most, some or none of the children to be successful on the item. Their own completed questionnaires were returned to them at the end of the year to indicate, in the same manner, children's actual performance by the end of the year.

For the purpose of reporting the data, responses relating to the number of children expected or actually able to perform a task were mapped onto a linear numerical scale as follows: None = 0, Some = 1, Most = 2, All =3. The data reported in Table 2 comes from the seven teachers who taught children in their first year of school (Prep) for two or more years of their participation in the project. Years 1 and 2 indicate teachers' first and second years in the project, respectively. Only data on items related to counting are reported here — for further details see Groves and Cheeseman (1995).

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22 The Calculators in Primary Mathematics was funded by the Australian Research Council, Deakin University and the University of Melbourne.
Table 2  Teachers’ expectations of children’s performance* and their actual performance on items related to counting

<table>
<thead>
<tr>
<th>No</th>
<th>Task description</th>
<th>Yr 1 Beg</th>
<th>Yr 2 Beg</th>
<th>Yr 1 End</th>
<th>Yr 2 End</th>
<th>Yr 1 Act</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>Count aloud to 120</td>
<td>0.43</td>
<td>0.57</td>
<td>1.14</td>
<td>1.43</td>
<td>1.57</td>
</tr>
<tr>
<td>16</td>
<td>Count from 389 to 407</td>
<td>0.00</td>
<td>0.14</td>
<td>0.43</td>
<td>0.71</td>
<td>0.86</td>
</tr>
<tr>
<td>22</td>
<td>Know the number 1 more than 17</td>
<td>0.57</td>
<td>1.00</td>
<td>2.0</td>
<td>1.75</td>
<td>2.28</td>
</tr>
<tr>
<td>23</td>
<td>Count backwards from 10 to 1</td>
<td>1.00</td>
<td>1.43</td>
<td>2.57</td>
<td>2.85</td>
<td>2.85</td>
</tr>
<tr>
<td>40</td>
<td>Count by 2s from 1 to 17</td>
<td>0.14</td>
<td>0.43</td>
<td>1.00</td>
<td>1.28</td>
<td>1.00</td>
</tr>
<tr>
<td>42</td>
<td>Count by 10s from 7 to 77</td>
<td>0.14</td>
<td>0.14</td>
<td>0.85</td>
<td>0.57</td>
<td>1.28</td>
</tr>
<tr>
<td>47</td>
<td>Count by 10s from 10 to 100</td>
<td>0.57</td>
<td>0.57</td>
<td>2.00</td>
<td>1.71</td>
<td>2.14</td>
</tr>
<tr>
<td>83</td>
<td>Count by 10s from 960 to 1050</td>
<td>0.00</td>
<td>0.00</td>
<td>0.14</td>
<td>0.43</td>
<td>0.57</td>
</tr>
<tr>
<td>88</td>
<td>Count by 2s from 2 to 28</td>
<td>0.14</td>
<td>0.43</td>
<td>1.00</td>
<td>1.14</td>
<td>1.43</td>
</tr>
<tr>
<td>92</td>
<td>Continue: 5, 10, 15, 20, ...</td>
<td>0.28</td>
<td>0.71</td>
<td>1.57</td>
<td>1.57</td>
<td>2.43</td>
</tr>
<tr>
<td>100</td>
<td>Count backwards in 10s from 100 to 0</td>
<td>0.14</td>
<td>0.28</td>
<td>0.71</td>
<td>1.14</td>
<td>1.14</td>
</tr>
<tr>
<td>107</td>
<td>Count by 5s from 5 to 50</td>
<td>0.00</td>
<td>0.28</td>
<td>1.57</td>
<td>1.43</td>
<td>2.28</td>
</tr>
<tr>
<td>111</td>
<td>Know that 10 more than 42 is 52</td>
<td>0.00</td>
<td>0.14</td>
<td>0.57</td>
<td>0.85</td>
<td>1.00</td>
</tr>
</tbody>
</table>

* Mean of responses of the 7 teachers’ expectations of first year of school children at the beginning and end of year and actual performance using the scale of None = 0, Some = 1, Most = 2, All = 3

The questionnaire results showed an increase in expectations for most of the 13 items dealing with counting for both the beginning and end of the year between the first and second years of teachers’ involvement in the project. In general, teachers’ increased expectations reflected their observations of children’s performance during the previous year. Nevertheless, teachers’ predictions remained conservative compared to actual levels of performance during the first year.

The presence of calculators and an awareness of the ways in which children used them in their classrooms did however stimulate teachers to re-assess their expectations of children. One teacher commented:

My expectations have changed enormously. It has changed and challenged both the children and myself. There now seems to be no limits to what we can do in our theme work. I am surprised by the children's understanding of concepts.

Another teacher commented:

Many more children were using larger numbers than I expected. ... This calculator project has enabled the brighter children to extend their thoughts on number work through open-ended tasks and problem solving activities.

Conclusions

Hartnett and Gelman (1998) argue that we underestimate the ability of young children to learn about the abstract, stating that “abstractness in and of itself need not be a problem for young learners. The barriers to learning are better understood as mismatches between the structure of what is known and what is to be learned” (p. 372).
Selter (1998, p. 1) comments that while mathematics educators are aware that children do not come to school with "empty heads", there is evidence that children often have considerable knowledge of topics before they are taught in schools. Selter proposes "progressive mathematisation" as a means of connecting children's informal knowledge with formal mathematics, stating that "all learning strands should begin with the informal, context-bound methods of children" (Selter, 1998, p. 2).

The data reported above suggest that teachers of young children are reasonably accurate in their predictions of when the majority of children will achieve mastery of specific objectives, but are much less likely to be aware of the conceptions of higher achieving children. One of the outcomes of the Calculators in Primary Mathematics project was that teachers were confronted with a much wider range of understandings of number in their classes than their pre-conceptions suggested, encouraging them to adopt more challenging and child-centred classrooms. These data underscore the need for teachers to explore the extent of children's understandings and raise their expectations accordingly.

The construction of developmental frameworks, based on extensive research, particularly in the early years, has been a major feature of recent Australian research. For example, Clarke, Sullivan, Cheeseman and Clarke (2000) report that, as part of the Early Numeracy Research Project, an Emergent Numeracy Profile was constructed (see also Clarke, 2001). This was used as a basis for the design of structured, numeracy-specific teaching and learning materials to scaffold a hierarchy of skills, strategies and dispositions concerned with mathematical thinking and problem solving. This was further developed into a comprehensive research-based learning and assessment framework — a framework of growth points of early number learning. Clarke and his colleagues report that this framework took into account major research in the learning of number as well as previous attempts to develop such frameworks. Data collected from individual interviews with over 5000 children was used to refine the framework. By using task-based interviews, based on developmental frameworks, teachers can gain first hand knowledge of children's mathematical understanding on which to base their expectations.

Research, such as that reported here, underscores the need for not only mapping students' progress, but also for developing appropriate teacher professional development, which examines key mathematical concepts and their development.

References


DOES A TEACHER UNDERSTAND STUDENT’S PRE-CONCEPT?

Stanislaw Domoradzki •

Abstract

That paper shows examples of understanding by students’ teachers pre- onceptions in the field of shapes methods in calculus memory among 11 years’ old pupils. This article came into the being as the result of collaboration of the author and the Department of Mathematics and Didactics of Mathematics of Pedagogical Faculty of Charles University in Prague. Some methods of analysis, which are developed in Prague, were used in this work particularly thanks to Prof. M. Hejñ. Keywords: pre-concept, interaction teacher – pupil, instructional teaching, the reasons of mistakes, empathy, and mental computation.

Zoltan Dines in 1960 introduced the idea that many mathematical concepts arose from mathematical processes. Many researchers pay attention to dependence: process → concept. This connection characterises the dual way of being of mathematical concepts, relation, and situation according to child’s everyday imagination. To process suits: changes, motion, construction, appearance and disappearance, division and addition, joining etc. To concept suits: state, being, existence, invariability, permanence. In D.Tall’s and E.Grey’s publications about moulding of mathematical concepts this connection is on the central place. It’s not the

• University of Rzeszów, Faculty of Mathematical and Natural Sciences, Poland; e-mail: domoradz@atena.univ.rzeszow.pl