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Energy Consumption Per Unit Yarn Production in Ring Spinning

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1 Introduction

Ring spinning is the key system of yarn manufacturing in the textile industry. The principle of ring spinning is depicted in Figure 1. A bundle of parallel fibres (the roving) is fed to the attenuating or drafting zone. The difference in surface velocity of the front (faster) and back (slower) drafting rollers will attenuate the roving to a thinner strand of parallel fibres, under the control of the double aprons. The thin strand of parallel fibres emerging from the front rollers is then simultaneously twisted and wound onto a yarn package (i.e., cop) mounted on a driven spindle. The twisted thin strand of fibres, now called a yarn, is threaded through a traveller and a yarn guide and balloons out between these two elements during normal spinning.

Because of the careful fibre control during the spinning process, ring spun yarns have a very high quality. The quality of ring spun yarns has been used as a benchmark against which the quality of yarns produced on other spinning systems is judged [7]. However, because twist is inserted into the tiny strand of fibres by the rotation of the relatively massive yarn package, the power consumption in ring spinning is much higher than that in other spinning systems for per unit yarn production.

2 Yarn Production and Power Distribution

On the assumptions that (a) the yarn package is wound with the cop-build method in ring spinning; (b) the profiles of both the top and bottom conical ends are straight/linear; (c) the upper and lower chases have the same angle of $\beta$ ($0^\circ < \beta < 90^\circ$); and (d) the slight tapering at both ends of the bobbin and the effects of yarn hairiness are ignored, the yarn production and power requirements for overcoming...
skin friction drag and yarn wind-on tension can be formulated as follows [8].

2.1 Yarn Production. Yarn production is related to yarn linear density or count, which is usually expressed in tex (N) – mass in grams per kilometre of the yarn. During ring spinning, the yarn count is pre-determined. So the amount of yarn produced within a given time is:

\[ m_p(t) = N\nu_o \cdot t \quad (0 \leq t \leq T_p) \]  

where \( m_p(t) \) [mg] is the mass of the yarn wound onto a bobbin at time \( t \) [s], and \( N \) [tex] is yarn count, \( \nu_o \) [m/s] is linear winding-on velocity (front delivery velocity) and \( T_p \) [s] is the total time taken to wind a full yarn package.

2.2 Power Requirement for Overcoming Skin Friction Drag. The power requirement for overcoming the skin friction drag on the surface of a rotating yarn package, \( P_s(t) \), can be expressed as:

\[ P_s(t) = \frac{1}{2} \rho p (\pi d_p V(t))^2 S_A(t) C_f(t) \quad (0 \leq t \leq T_p) \]  

where \( \rho \) [kg/m³] is a constant and is equal to 1.197, \( C_f(t) \) [dimensionless] is the skin friction coefficient on the surface of the yarn package, \( d_p \) [m] is the maximum diameter of the yarn package, \( V(t) \) [rps (revolutions per second)] is spindle speed and \( S_A(t) \) [m²] is the surface-area of the yarn package at time \( t \) [s].

Since the surface of a tube (empty bobbin) is quite smooth compared to the surface of a yarn package, the skin friction drag caused by the exposed surface of the bobbin will be ignored here.

2.3 Power Requirement for Overcoming Yarn Wind-on Tension. The power requirement for overcoming the yarn wind-on tension in ring spinning, \( P_c(t) \), can be expressed as:

\[ P_c(t) = 2\pi F(t) V(t) r_p(t) \quad (0 \leq t \leq T_p) \]  

where \( F(t) \) [N] is the yarn wind-on tension at time \( t \), \( V(t) \) [rps] is spindle speed, \( r_p(t) \) [m] is the distance from the centre line of the rotating bobbin to the yarn wind-on point.

3 Energy-consumption Per Unit Yarn production

In order to establish a model which can be conveniently applied in practice, we will analyze and simplify the variables in Equations (2)-(4) as below.

3.1 Spindle Speed and Winding-on Velocity. The spindle rotation is on full speed over 99% of yarn packaging time. The spindle speed \( V(t) \) can be considered to be at full speed \( V_o \) when yarn is wound onto a bobbin. Thus

\[ V(t) \approx V_o \quad (0 \leq t \leq T_p) \]  

The linear winding-on velocity \( \nu_w \) [m/s] is proportional to the full spindle speed \( V_o \) [rps], since the twist level in the yarn is pre-determined before spinning. Let \( \nu_w = k_v V_o \), from Equation (2)

\[ m_p(t) = k_v V_o N t \quad (0 \leq t \leq T_p) \]  

3.2 Skin Friction Coefficient on the Surface of a Yarn Package. Let \( d_0 \) [m] be the maximum diameter of a yarn package, \( d_b \) [m] the minimum diameter of the yarn package. For a full yarn package, the surface area of the main part (having diameter of \( d_b \)) is about 90% of the yarn package. In order to simplify the calculation, the skin friction coefficient on the surface of the yarn package can be considered to be the skin friction coefficient on the surface of the main part of this yarn package.

The skin friction coefficient of package surface depends on spindle speed and package diameter. Let \( C_{fp} \) be the skin friction coefficient on the surface of the package having diameter \( d_0 \), \( C_f \) the skin friction coefficient on the surface of the package having diameter \( d_p \) and \( V_o \) a given full spindle speed, then \( C_{fp} \) is equal to \( \frac{d_b}{d_p} C_f \) and \( C_f \) approximates to \( a V_o^b \), where \( a \) and \( b \) are constants which can be determined from experiments (the details will be published in our further work), thus

\[ C_f(t) = \frac{d_b}{d_p} a V_o^b \quad (0 \leq t \leq T_p) \]  

3.3 Surface Area of a Yarn Package. After winding the first complete layer or yarn, the surface area of a yarn package is

\[ S_b = (d_0 + 4d_b)\pi \frac{d_p - d_b}{2} \cot \theta \]  

The surface area of the full yarn package will be

\[ S = \left[ h_b - (d_p - d_b)\cot \theta \right] \pi d_p \]  

\[ + \frac{d_p - d_b}{2\sin \beta} (d_b + d_p + 2d_b) \pi \]  

Yarn diameter \( d_b \) in the Equations (8) and (9) can be estimated [5] by

\[ d_b = k_d \sqrt{N} \]  

where \( d_b \) [m] is yarn diameter, \( N \) [tex] is yarn count, \( k_d \) [g/cm³] is a constant [5].

For ease of statement, it is assumed that the surface area of a yarn package increases evenly from \( S_b \) to \( S \) throughout the yarn packaging process. Furthermore, since the time taken to wind the first complete layer is less than 0.66% of \( T_p \) (the time used to wind a full yarn package), then

\[ S_p(t) \approx \frac{S_b - S}{T_p} t \quad (0 \leq t \leq T_p) \]  

3.4 Yarn Wind-on Tension and Distance from Centre Line to Yarn Wind-on Point. For the sake of simplicity in this study, the average yarn wind-on
tension can be taken as approximately 0.72 g-wt per tex [1], so 
\[ F(t) = 7.06N \times 10^{-3} \quad (0 \leq t \leq T_p) \quad (12) \]

While winding yarn with the cop-build [8], \( r_p(t) \) varies between \( d_i/2 \) and \( d_j/2 \) within every complete layer in the main part. The time used to wind the main part is over 93% of the total time used to wind the full yarn package. Therefore, \( r_p(t) \) can be considered to vary between \( d_i/2 \) and \( d_j/2 \) throughout the yarn packaging process. Furthermore, the \( r_p(t) \) evenly varies from \( d_i/2 \) to \( d_j/2 \) during main (up) winding and from \( d_j/2 \) to \( d_i/2 \) during cross (down) winding, respectively. So, when calculating the energy consumed on yarn wind-on tension (i.e., integrating over the interval \( 0 \leq t \leq T_p \)) using Equation (4), we can use that
\[ r_p(t) = \frac{d_j - d_i}{4} \quad (13) \]

3.5 Mathematical Formulation. From Equation (6), the yarn production on a full yarn package will become
\[ m_s = k_1 V_o NT_p \quad (14) \]

Substituting Equations (5), (7) and (11) into Equation (3), and then integrating over the interval \( 0 \leq t \leq T_p \), the energy consumed by overcoming skin friction drag on the package surface for winding a full yarn package in ring spinning can be written as
\[ E_f = \frac{d_j - d_i}{4} \rho x \dot{V}^2 (S_y - S_x) \eta V_0^2 T_p \quad (15) \]

where \( \eta \) and \( \delta \) are constants which can be determined by experiments, \( S_y \) and \( S_x \) can be obtained from Equations (8)–(10):
\[ S_y = (d_j - 4k_2 \sqrt{N}) \pi \frac{d_j - d_i}{2} \cot \beta \]
\[ S_x = (b_0 - (d_j - d_i) \cot \beta \pi) \sum_{p=1}^{m} \pi d_p \rightarrow \frac{d_j - d_i}{2} \sin \beta \]

Substituting Equations (5) and (12) into Equation (4), and using Equation (13), the energy consumed in overcoming yarn wind-on tension for winding a full yarn package in ring spinning can be written as
\[ E_f = 3.53 \pi (d_j - d_i) \rho V_0^2 T_p \times 10^{-3} \quad (16) \]

Substituting Equations (14)–(16) into Equation (1), we obtain a model for the ratio of energy-consumption to yarn-production:
\[ R = \frac{3.53 \pi (d_j + x_1) \times 10^{-3}}{k_1} \]
\[ + \frac{a d_j \rho \pi x_1^{2+3} x_1^2 (d_j + 4k_2 \sqrt{x_1}) (x_1 - d_i) \cos \beta}{8k_4 x_4 \sin \beta} \]
\[ + \frac{a d_j \rho \pi x_1^{2+3} x_1^2 (d_j + 2k_2 \sqrt{x_1} + x_1) (x_1 - d_i)}{8k_4 x_4 \sin \beta} \]

over the set
\[ S = \{(x_1, x_2, x_3) : 350 > x_1 > 50, 225 > x_2 > 5, 0.065 > x_3 > d_0 \} \]

where \( R [\text{J/mg}] \) is the ratio of energy-consumption to yarn-production, \( x_1 [\text{rps}] \) is full spindle speed (\( V_0 \), \( x_2 [\text{tex}] \) is yarn count (\( N \), \( x_3 [\text{m}] \) is the maximum diameter of a yarn package (\( d_j \), \( d_i \) [m] is the minimum diameter of the yarn package, \( h_o \) [m] is the total height of the full yarn package, \( \beta [^\circ] \) is chase angle of the full package, \( \rho [\text{kg/m}^3] \) is air density and is equal to 1.197, \( k_1 [\text{m/rev}] \) is constant which relates to winding-on velocity and full spindle speed, \( k_2 \) \([\text{g/cm}^3]^{-0.5} \) is a constant which relates to yarn diameter and yarn count, \( a \) and \( b \) are constants which can be determined from experiments.

Now, the minimizing energy consumption on yarn package in ring spinning is the question of seeking the minimum value from Equation (17).

4 Case Study
When \( d_j = 0.025 \text{ m} \), \( h_o = 0.245 \text{ m} \), \( \beta = 32^\circ \), \( \rho = 1.197 \text{ kg/m}^3 \), \( k_1 = 0.002 \text{ m/rev} \) (i.e., winding-on velocity is equal to 20 m/min when full spindle speed \( V_0 \) is equal to 167 rps), and for the yarn spun from cotton fibres, \( k_2 = 3.60 \times 10^{-5} \text{ (g/cm)}^{-0.5} \), \( a = 1660000 \) and \( b = -3.0267 \) (obtained from experiments), then Equation (17) becomes
\[ R = 0.1386 + 5.5449x_3 + 
\[-397.2756 x_1^{-1.0267} x_2^{-1.4} x_3^4 \]
\[-0.028x_1^{-1.0257} x_2^{-0.5} x_3^4 \]
\[+ 160.2699 x_1^{-1.0267} x_2^{-1.4} x_3^4 \]
\[-0.00072 x_1^{-0.5} x_2^{0.5} x_3^4 \]
\[-0.0540 x_1^{-0.5} x_2^{0.5} x_3^4 \] \times 10^6 \quad (18) \]

over the set
\[ S = \{(x_1, x_2, x_3) : 350 > x_1 > 50, 225 > x_2 > 5, 0.065 > x_3 > 0.025 \} \]

4.1 Constant Spindle Speed. If the spindle speed in ring spinning is given as 167 rps (i.e., \( x_1 = 167 \)), then Equation (18) becomes
\[ R_1 = 0.1386 + 5.5449x_3 - 282.0530 x_1^{-1.4} x_3^4 \]
\[-3.7606 x_2^{-0.5} x_3^4 + 837120 x_1^{-1.4} x_3^4 \]
\[-149.3836 x_2^{-0.5} x_3^4 - 2075100 x_1^{-1.4} x_3^4 \]

over the set
\[ S_1 = \{(x_2, x_3) : 225 > x_2 > 5, 0.065 > x_3 > 0.025 \} \]

Figure 2 shows that the ratio of energy-consumption to yarn-production for a full package is about 1.5 J/mg for most of the area in \( S_1 = \{(x_2, x_3) : \}

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225 > x_2 > 5, 0.065 > x_1 > 0.025) when a full spindle speed is given as 167 rps during yarn winding in ring spinning. In particular, when yarn count is 80 tex and package diameter is 0.035 m, the ratio is 0.7568 J/mg.

However, the ratio is very high if yarn count is less than 20 tex and package diameter is greater than 0.05 m. For example, the ratio will be 15.7477 J/mg when yarn count is 10 tex and package diameter is 0.06 m.

4.2 Constant Yarn Count. If the yarn count in ring spinning is given as 38 tex (i.e., x_2 = 38), then Equation (18) becomes

\[ R_2 = 0.1386 + 5.5449 x_3 - 1537.8994 x_1^{0.0267} x_3^2 + 4212960.5 x_1^{-0.0267} x_3^3 - 10455000 x_1^{-1.0267} x_3^4 \]

over the set

\[ S_2 = \{ (x_1, x_3): 350 > x_1 > 50, 0.065 > x_3 > 0.025 \}. \]

Figure 3 shows that the ratio of energy-consumption to yarn-production for a full package is about 2.5 J/mg for most of the area in \( S_2 \) = \{ (x_1, x_3): 350 > x_1 > 50, 0.065 > x_3 > 0.025 \} when yarn count is selected as 38 tex during yarn winding in ring spinning. In particular, when spindle speed is 200 rps and yarn count is 80 tex, the ratio is 1.0404 J/mg.

However, the ratio is very high if spindle speed is less 100 rps and yarn count is less than 20 tex. For example, the ratio will be 14.3253 J/mg when spindle speed is 50 rps and yarn count is 15 tex.

4.3 Constant Package Diameter. If the yarn package diameter in ring spinning is given as 0.045 m (i.e., x_3 = 0.045), then Equation (18) becomes

\[ R_3 = 0.3881 - 4.0642 x_1^{0.0267} x_3^{2.5} + 12866.55 x_1^{-0.0267} x_3^{-1} \]

over the set

\[ S_3 = \{ (x_1, x_2): 225 > x_1 > 50, 225 > x_2 > 5 \}. \]

Figure 4 shows that the ratio of energy-consumption to yarn-production for a full package is about 2 J/mg for most of the area in \( S_3 \) = \{ (x_1, x_2): 350 > x_1 > 50, 225 > x_2 > 5 \} when yarn package diameter is selected as 0.045 m during yarn winding in ring spinning. In particular, when spindle speed is 300 rps and yarn count is 80 tex, the ratio is 0.8472 J/mg.

However, the ratio is very high if spindle speed is less 100 rps and yarn count is less than 20 tex. For example, the ratio will be 15.8231 J/mg when spindle speed is 50 rps and yarn count is 15 tex.

5 Conclusion

A model for predicting the ratio of energy-consumption to yarn-production for a full package during yarn winding in ring spinning is established. Three cases are discussed when spindle speed, yarn count and package diameter are kept constant, respectively. It is found that

1) For a given full spindle speed, increasing yarn count and/or decreasing yarn package diameter will reduce energy consumption because it will shorten the time used to wind a full package of a given mass.

2) When yarn production and yarn count are given, decreasing spindle speed and/or increasing yarn package diameter will increase energy consumption, because it will take longer to produce a given amount yarn.

3) For a chosen yarn package diameter, selecting a higher spindle speed and/or larger yarn count will reduce the ratio of energy-consumption to yarn-production; on the contrary, selecting lower spindle speed and/or smaller yarn count will increase the ratio of energy-consumption to yarn-production.

It should be noted that in this study, the time used to replace the yarn bobbin and the energy consumed on replacing the bobbin (re-start up, etc.) are not considered.
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FIGURE 3. Ratio of energy-consumption to yarn-production for a full package during yarn winding in ring spinning when yarn count is constant (38 tex).

FIGURE 4. Ratio of energy-consumption to yarn-production for a full package during yarn winding in ring spinning when package diameter is constant (0.045 m).

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References