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Blind Separation of Cyclostationary Signals from Instantaneous Mixtures

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***Abstract*—This paper presents a new approach for blind separation of unknown cyclostationary signals from instantaneous mixtures. The proposed method can perfectly separate the mixed source signals so long as they have either different cyclic frequencies or clock phases. This is a weaker condition than those required by the algorithms presented in [1] and [11]. The separation criterion is to diagonalize a polynomial matrix whose coefficient matrices consist of the correlation and cyclic correlation matrices, at time delay $\tau = 0$, of multiple measurements.**

***Key words*: Blind source separation; Second order statistics; Cyclostationary signals; Instantaneous mixture.**

I. INTRODUCTION

Blind source separation (BSS) from instantaneous mixtures is a fundamental problem encountered in many applications such as wireless communication, remote sensing, speech enhancement and medical data analysis. It is motivated by practical multiple-input multiple-output scenarios where *m* source signals impinge on an array of *n* sensors. Given the measurements from the sensors, BSS aims to estimate both the structure of the linear combinations and the source signals. For BSS to be possible, something extra must be known about the source signals. Under the assumption that the source signals are stationary and temporally coherent, some efficient second-order statistics (SOS) based algorithms [2], [12] have been proposed. In this paper, the extra assumption is that the source signals are cyclostationary [8].

Cyclostationarity is shared by most communication signals as a result of periodic switching, gating, or mixing operations at the transmitter [8]. For example, cyclostationarity is induced at multiples of the baud rate in BPSK, QPSK and QAM signals, and at twice the carrier frequency in DSB-AM signals. The behavior of current SOS based methods for cyclostationary signals is investigated in [4], [7]. However, by exploiting the cyclostationarity of the source signals, it is not necessary for the source signals to be temporally coherent. While the higher-order cyclostationarity of signals can be utilized to accomplish source separation [3], this paper restricts its attention to methods based on second-order cyclostationary statistics.

In some practical situations where the source signals overlap both in time and frequency, their spectral redundancy functions are nonoverlaping because their cyclic frequencies are distinct. This is a result of signals having distinct carrier frequencies and/or pulse rates or keying rates, even when occupying the same spectral band. The cyclic frequencies are usually known by the receivers since they are related to the frequencies of periodic phenomena involved in the construction of the signals. The frequency diversity of source signals is assumed in [1] with the derivation of algorithms TH1 and TH3. Furthermore, in many multiuser systems such as the US Digital Cellular standard IS-54, the user signals have different clock phases on which the phase self-coherence restoral (P-SCORE) algorithm is built [11]. This can be caused by either randomly generated local clock signals or the result of cooperative timing control among users.

This paper presents a method which jointly utilizes the phase and frequency redundancy of source signals. A second-order separation criterion is proposed to achieve the BSS, which is implemented through diagonalizing a set of zero-delay correlation and cyclic correlation matrices of the measurements. Unlike most existing SOS based techniques, our method does not require the source signals to be temporally coherent.

II. PROBLEM FORMULATION

Consider the following blind separation problem:

$$\mathbf{x}(t) \quad \mathbf{A}\mathbf{s}(t) \quad \mathbf{w}(t)$$

with

$$\begin{aligned} \mathbf{s}(t) &\quad s_1(t),\ s_2(t), \cdots,\ s_m(t)\ ^T \\ \mathbf{x}(t) &\quad x_1(t),\ x_2(t), \cdots,\ x_n(t)\ ^T \\ \mathbf{w}(t) &\quad w_1(t),\ w_2(t), \cdots,\ w_n(t)\ ^T \end{aligned}$$

where $\mathbf{s}(t)$ is the $m \times 1$ complex *source vector*, $\mathbf{x}(t)$ is the $n \times 1$ (n) complex *output vector*, $\mathbf{w}(t)$ is the $n \times 1$ complex *noise vector*, $\mathbf{A}$ is the $n \times m$ full

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column rank *mixing matrix*, and the superscript ^{T} denotes the transpose operator. The source signal vector $\mathbf{s}(t)$ is modeled as a cyclostationary complex stochastic process whose components $s_i(t)$, $1 \leq i \leq m$ are assumed to be mutually uncorrelated with zero mean. The additive noise $\mathbf{w}(t)$ is modeled as a stationary, zero-mean complex random process independent of the source signals and satisfying $E[\mathbf{w}(t)\mathbf{w}(t)^H] = \sigma^2\mathbf{I}$, where ^{H} is the conjugate transpose operator, E the expectation operator and $\mathbf{I}$ the identity matrix.

Assume that there are d distinct groups of sources each of which contains d_i source signals with same nonzero cyclic frequency β_i, $i = 1, 2, \cdots, d$ Write

$$\mathbf{s}(t) \quad \mathbf{s}_1(t)^T, \mathbf{s}_2(t)^T, \cdots, \mathbf{s}_d(t)^T]^T$$

where each column vector $\mathbf{s}_i(t)$ contains the d_i source signals with cyclic frequency β_i. Clearly, $m = d_1 + d_2 + \cdots + d_d$. Without loss of generality, we use s_i to represent the kth signal of group i, $i = 1, 2, \cdots, d$ and $k = 1, 2, \cdots, d_i$. Using these notations and the definition of second-order cyclostationarity given in [8], one has

$$\langle \mathbf{s}_i(t)\mathbf{s}_j(t)^H e^{j \quad _i t} \rangle = \mathbf{0}, \qquad \text{if } i \neq j \tag{1}$$
$$\langle \mathbf{s}_i(t)\mathbf{s}_i(t)^H e^{j \quad _j t} \rangle = \mathbf{0}, \qquad \text{if } \beta_j \neq \beta_i \tag{2}$$
$$\langle \mathbf{s}_i(t)\mathbf{s}_i(t)^H e^{j \quad _i t} \rangle = \boldsymbol{\varrho}_i, \qquad \forall\, i \tag{3}$$

with

$$\boldsymbol{\varrho}_i = \mathrm{d} \quad \rho_{i\,1}, \rho_{i\,2}, \cdots, \rho_{i \quad _i}) \tag{4}$$
$$\rho_i = \langle s_i \quad s_i^* \quad e^{j \quad _i t} \rangle \tag{5}$$

for $i = 1, 2, \cdots, d$ and $k = 1, 2, \cdots, d_i$. Here $j = \sqrt{-1}$, * is the conjugate operator, and $\langle \quad$ denotes the time averaging operator defined as

$$\langle f(t) \rangle \triangleq \lim_{N \to \infty} \frac{1}{N} \sum_{t=0}^{N-1} f(t).$$

Obviously, all diagonal elements of $\boldsymbol{\varrho}_i$ are nonzero as the source signals in group i have cyclic frequency β_i. Furthermore, if these source signals have distinct clock phases, $\boldsymbol{\varrho}_i$ will be diagonal and have distinct nonzero diagonal elements.

The aim of blind source separation is to find an $m \times n$ separating matrix $\mathbf{B}$ such that $\mathbf{Bx}(t)$ is an estimate of the source signals. However, complete identification of separating matrix $\mathbf{B}$ is impossible in the blind context [1] because the exchange of a fixed scalar between the source signal and the corresponding column of $\mathbf{A}$ does not affect the measurements. We can take advantage of this indeterminacy of BSS to assume that the source signals have unit variance, i.e.,

$$\mathbf{R}_s \triangleq E[\mathbf{s}(t)\mathbf{s}(t)^H] = \mathbf{I}. \tag{6}$$

III. Separation Criterion

We start with the singular value decomposition of the mixing matrix $\mathbf{A}$:

$$\begin{aligned} \mathbf{A} &= [\mathbf{U}_r \ \mathbf{U}_0] \begin{bmatrix} \boldsymbol{\Lambda}_r & \\ & \mathbf{0} \end{bmatrix} \mathbf{V}^H \\ &= \mathbf{U}_r \boldsymbol{\Lambda}_r \mathbf{V}^H \end{aligned}$$

where $\boldsymbol{\Lambda}_r$ is a $m \times m$ diagonal matrix with nonnegative diagonal elements in decreasing order, and $\mathbf{U}_r$, $\mathbf{U}_0$ and $\mathbf{V}$ are unitary matrices of dimensions $n \times m$, $n \times (n - m)$ and $m \times m$ respectively. Then the correlation matrix of the array output $\mathbf{x}(t)$ takes the following structure:

$$\begin{aligned} \mathbf{R}_x &\triangleq E[\mathbf{x}(t)\mathbf{x}(t)^H] \\ &= \mathbf{U}_r \boldsymbol{\Lambda}_r^2 \mathbf{U}_r^H + \sigma^2 \mathbf{I}. \end{aligned} \tag{7}$$

It is well known that an estimate of the noise variance is the average of the $n - m$ smallest eigenvalues of $\mathbf{R}_x$. In practice, $\mathbf{R}_x$ will be replaced by its estimate $\hat{\mathbf{R}}_x$ with finite sample size N, which is defined as

$$\hat{\mathbf{R}}_x = \frac{1}{N} \sum_{t=0}^{N-1} \mathbf{x}(t)\mathbf{x}(t)^H. \tag{8}$$

By removing the noise contribution from (7), we obtain the noiseless correlation matrix of $\mathbf{x}(t)$:

$$\mathbf{R}'_x \approx \mathbf{U}_r \boldsymbol{\Lambda}_r^2 \mathbf{U}_r^H. \tag{9}$$

Thus, the estimates of $\mathbf{U}_r$ and $\boldsymbol{\Lambda}_r$ can be obtained from the eigenvalue decomposition of the noiseless correlation matrix $\mathbf{R}'_x$. The relationship between the original and estimated ones is as follows:

$$\hat{\boldsymbol{\Lambda}}_r = \mathbf{P} \boldsymbol{\Lambda}_r \mathbf{P} \tag{10}$$
$$\hat{\mathbf{U}}_r = \mathbf{U}_r \mathbf{Q} \mathbf{P} \tag{11}$$

where $\mathbf{P}$ is an $m \times m$ permutation matrix with $\mathbf{P} = \mathbf{P}^H$ and $\mathbf{P}^2 = \mathbf{I}$, and $\mathbf{Q}$ is an $m \times m$ diagonal matrix with elements of the form e^{j} . It is easy to check that $\hat{\mathbf{U}}_r \hat{\boldsymbol{\Lambda}}_r^2 \hat{\mathbf{U}}_r^H = \mathbf{U}_r \boldsymbol{\Lambda}_r^2 \mathbf{U}_r^H$.

Denote

$$\boldsymbol{\Phi} \triangleq \hat{\mathbf{U}}_r \hat{\boldsymbol{\Lambda}}_r^{-1} \tag{12}$$
$$\mathbf{y}(t) \triangleq \boldsymbol{\Phi}^H \mathbf{x}(t). \tag{13}$$

Substituting equations (10)-(12) into (13) results in

$$\mathbf{y}(t) \quad \mathbf{Hs}(t) \quad \boldsymbol{\Phi}^H \mathbf{w}(t) \tag{14}$$

where

$$\mathbf{H} = \mathbf{P} \mathbf{Q}^H \mathbf{V}^H$$

is an $m \times m$ unitary matrix, which reduces the n-dimensional array output vector $\mathbf{x}(t)$ to an m-dimensional vector $\mathbf{y}(t)$. It is easy to show that the multiplication of any two unitary matrices is always nonsingular.

Lemma 1: Given two $m \times m$ unitary matrices $\mathbf{U}_1$ and $\mathbf{U}_2$, then matrix $\mathbf{U}_1\mathbf{U}_2$ is nonsingular.

Proof: The proof is straightforward from the definition of unitary matrix. ∎

The remaining task is to find an $m \times m$ unitary matrix $\mathbf{G}$ such that $\mathbf{G}\mathbf{y}(t)$ is an estimate of the original source signal vector $\mathbf{s}(t)$, or

$$\mathbf{C} \triangleq \mathbf{G}\mathbf{H} \tag{15}$$

is diagonal up to a row permutation. This problem can not be solved by utilizing only the second-order statistics of $\mathbf{y}(t)$ because changing $\mathbf{H}$ in (14) to any other unitary matrix leaves the correlation matrix of $\mathbf{y}(t)$ unchanged. We will show next that the second-order cyclostationary information of the source signals can be utilized to solve the BSS problem.

Define the cyclic correlation matrices of source vector $\mathbf{s}(t)$ by

$$\mathbf{R}_s^{\beta_i} \triangleq \langle \mathbf{s}(t)\mathbf{s}(t)^H e^{j \ \ _i t} \rangle \tag{16}$$

where β_i, $i = 1, 2, \cdots, d$ represent the cyclic frequencies. From equations (1)-(3), we have

$$\mathbf{R}_s^{\beta_i} = \begin{bmatrix} \mathbf{0} & \cdots & & \mathbf{0} \\ & \ddots & & \\ \vdots & & \boldsymbol{\varrho}_i & \vdots \\ & & \ddots & \\ \mathbf{0} & \cdots & & \mathbf{0} \end{bmatrix}. \tag{17}$$

Clearly, if $\beta_i \neq \beta_j$, the diagonal entries in $\boldsymbol{\varrho}_i$ and $\boldsymbol{\varrho}_j$ will not overlap. Furthermore, we denote by $\mathbf{R}_s(z)$ the following polynomial matrix:

$$\mathbf{R}_s(z) \triangleq \mathbf{R}_s + \sum_{i=1}^{d} \mathbf{R}_s^{\beta_i} z^i. \tag{18}$$

It can be seen from (6) and (17) that $\mathbf{R}_s(z)$ is diagonal. Similarly, define the polynomial matrix corresponding to $\mathbf{y}(t)$ by

$$\mathbf{R}_y(z) \triangleq \mathbf{R}_y + \sum_{i=1}^{d} \mathbf{R}_y^{\beta_i} z^i \tag{19}$$

where

$$\mathbf{R}_y = E(\mathbf{y}(t)\mathbf{y}(t)^H) - \mathbf{\Phi}^H \mathbf{\Phi} \tag{20}$$

$$\mathbf{R}_y^{\beta_i} \triangleq \langle \mathbf{y}(t)\mathbf{y}(t)^H e^{j \ \ _i t} \rangle. \tag{21}$$

Substituting (14) into (19) leads to

$$\mathbf{R}_y(z) \quad \mathbf{H}\mathbf{R}_s(z)\mathbf{H}^H. \tag{22}$$

Assume $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \cdots, \alpha_m]$ is a $1 \times m$ constant row vector. Denote the diagonal elements of $\mathbf{R}_s(z)$ by $r_1(z), r_2(z), \cdots, r_m(z)$. We have the following lemma:

Lemma 2: Provided that the source signals with same cyclic frequency have distinct clock phases, then

$$\alpha_1 r_1(z) \quad \alpha_2 r_2(z) \quad \cdots + \alpha_m r_m(z) \not\equiv 0 \tag{23}$$

for any nonzero vector $\boldsymbol{\alpha} \neq 0$.

Proof: It is clear from (18) that if two source signals $s_k(t)$ and $s_l(t)$, $1 \leq k \neq l \leq m$ either have different cyclic frequencies or distinct clock phases, then the coefficient vector of $r_k(z)$ is linearly independent of that of $r_l(z)$. This implies that (23) always holds if $\boldsymbol{\alpha} \neq 0$. ∎

As we mentioned in Section 1, the phase clock diversity of source signals exists in many multiuser systems, for example the US Digital Cellular standard IS-54, because of either the random generation of local clock signals or the cooperative timing control among users.

Recall that

$$\mathbf{R}_s(z) \quad d \quad (r_1(z),\ r_2(z), \cdots,\ r_m(z)$$

and $\mathbf{C} = \mathbf{G}\mathbf{H}$. Since $\mathbf{G}$ and $\mathbf{H}$ are $m \times m$ unitary matrices, it is known from Lemma 1 that $\mathbf{C}$ is a nonsingular matrix. Then we present another lemma as follows.

Lemma 3: Provided that c_i is the *(i,j)*th element of matrix $\mathbf{C}$ with $1 \leq i \quad \leq m$. The diagonalization of $\mathbf{C}\mathbf{R}_s(z)\mathbf{C}^H$ results in $\mathbf{C}_i\mathbf{D}_i = \mathbf{0}$, $i = 1, 2, \cdots, m$ where

$$\mathbf{C}_i = \mathbf{C} \text{ w} \qquad it$$

$$\mathbf{D}_i = d \quad (c_{i\,1},\ c_{i\,2}, \cdots,\ c_i \quad)$$

Proof: Let us consider the ith column of the diagonal matrix $\mathbf{C}\mathbf{R}_s(z)\mathbf{C}^H$. The $m-1$ zero elements of the ith column imply

$$\mathbf{C}_i\mathbf{R}_s(z) \begin{bmatrix} c_{i\,1} \\ c_{i\,2} \\ \vdots \\ c_i \end{bmatrix} \equiv \mathbf{0}. \tag{24}$$

Denote

$$\mathbf{r}(z) \triangleq [r_1(z),\ r_2(z), \cdots,\ r_m(z)\ ^T.$$

Then (24) can be written as

$$\mathbf{C}_i\mathbf{D}_i\mathbf{r}(z) \equiv \mathbf{0}. \tag{25}$$

We know from Lemma 2 that $\boldsymbol{\alpha}\mathbf{r}(z) \equiv 0$ holds only if $\boldsymbol{\alpha} = \mathbf{0}$. Applying this result to (25) implies $\mathbf{C}_i\mathbf{D}_i = \mathbf{0}$ for $i = 1, 2, \cdots, m$ ∎

Based on these lemmas, a second-order blind source separation criterion is presented here.

Theorem: Let $\mathbf{G}$ be an $m \times m$ unitary matrix. The diagonalization of $\mathbf{G}\mathbf{R}_y(z)\mathbf{G}^H$ implies that of $\mathbf{C}$ up to a row permutation if the source signals with same cyclic frequency have different clock phases.

Proof: From (22), we have

$$\mathbf{G}\mathbf{R}_y(z)\mathbf{G}^H = \mathbf{C}\mathbf{R}_s(z)\mathbf{C}^H.$$

It is shown by Lemma 3 that the diagonalization of $\mathbf{G}\mathbf{R}_y(z)\mathbf{G}^H$ (or equivalently the diagonalization of $\mathbf{C}\mathbf{R}_s(z)\mathbf{C}^H$) gives $\mathbf{C}_i\mathbf{D}_i = \mathbf{0}$, $i = 1, 2, \cdots, m$ if the source signals with same cyclic frequency have distinct clock phases.

For $i = 1$, $\mathbf{C}_1\mathbf{D}_1 = \mathbf{0}$ is equivalent to

$$\begin{bmatrix} c_{1,1}c_{2,1} & c_{1,2}c_{2,2} & \cdots & c_{1,} \;\; c_{2,} \\ c_{1,1}c_{3,1} & c_{1,2}c_{3,2} & \cdots & c_{1,} \;\; c_{3,} \\ \vdots & \vdots & \ddots & \vdots \\ c_{1,1}c_{m,1} & c_{1,2}c_{m,2} & \cdots & c_{1,} \;\; c_{m,} \end{bmatrix} = \mathbf{0}. \qquad (26)$$

Equation (26) implies that for each column of $\mathbf{C}$, either the first element of that column is zero or all the elements except for the first one of that column are zero. Furthermore, one of the elements in the first row of $\mathbf{C}$ must be nonzero because of the nonsingularity of $\mathbf{C}$. In other words, there is a column of $\mathbf{C}$ whose first element is nonzero but the rest elements of that column are zero.

For $i = 2$, we can similarly see that another column of $\mathbf{C}$ must be of all zero elements except for one. This nonzero element must be in a different row from the first row as $\mathbf{C}$ is nonsingular. By considering $\mathbf{C}_i\mathbf{D}_i = \mathbf{0}$ for all i, we conclude that matrix $\mathbf{C}$ must be diagonal up to a row permutation. ■

The diagonalization of $\mathbf{G}\mathbf{R}_y(z)\mathbf{G}^H$ can be implemented by minimizing the following cost function

$$\mathbf{J}(\mathbf{G}) \qquad \mathbf{G}\mathbf{R}_y\mathbf{G}^H) \quad \sum_{i=1}^{d} \text{off } \mathbf{G}\mathbf{R}_y^{\beta_i}\mathbf{G}^H) \qquad (27)$$

with $\mathbf{G} \in C^{m \times m}$ being unitary and

$$\text{off } X) \quad \sum_{i \neq j} |x_i \;\; |^2$$

where x_i is the $(i \;\;)$th element of matrix X. It is known that a square matrix $\mathbf{M}$ is said to be normal if $\mathbf{M}\mathbf{M}^H = \mathbf{M}^H\mathbf{M}$. Obviously, all coefficient matrices of $\mathbf{R}_y(z)$ are normal matrices. Thus, the minimization of $\mathbf{J}(\mathbf{G})$ can be achieved using the JADE (Joint Approximate Diagonalization of Eigen-matrices) method [5] which is optimal in the least squares sense [13]. The JADE method minimizes (27) by successive Givens rotations on $m(m-1)/2$ sets of 2×2 matrices. Each set consists of $d + 1 \;\; 2 \times 2$ matrices. Other methods which can deal with this optimization problem with a unitary constraint include those in [6], [9], [10].

REFERENCES

[1] K. Abed-Meraim, Y. Xiang, J. H. Manton, and Y. Hua, "Blind source separation using second-order cyclostationary statistics", *IEEE Trans. on Signal Processing*, Vol.49, No. 4, pp. 694–701, April 2001.

[2] A. Belouchrani, K. Abed-Meraim, J. F. Cardoso, and E. Moulines, "Blind source separation using second order statistics," *IEEE Trans. on Signal Processing*, Vol. 45, No. 2, pp. 434–444, Feb. 1997.

[3] L. Castedo and A. R. Figueiras-Vidal, "An adaptive beamforming technique based on cyclostationary signal properties", *IEEE Trans. on Signal Processing*, vol. 43, no. 7, pp. 1637–1650, July 1995.

[4] P. Chevalier, A. Ferréol, and L. Albera, "On the behavior of current second order blind source separation methods for first and second order cyclostationary sources-application to CPFSK sources", *Proc. of 2002 IEEE International Conference on Acoustics, Speech and Signal Processing*, pp. 3081–3084, Orlando, USA, May 2002.

[5] J.-F. Cardoso and A. Souloumiac, "Blind beamforming for nonGaussian signals," *IEEE Proceedings-F*, Vol. 140, No. 6, pp. 362–370, Dec. 1993.

[6] A. Edelman, T. Arias, and S. T. Smith, "The geometry of algorithms with orthogonality constraints," *SIAM J. Matrix Analysis Applics*, Vol. 20, No. 2, pp. 303–353, 1999.

[7] A. Ferréol and P. Chevalier, "On the behavior of current second and higher order blind source separation methods for cyclostationary sources", *IEEE Trans. on Signal Processing*, Vol.48, No. 6, pp. 1712–1725, June 2000.

[8] W. Gardner, "Exploitation of the spectral redundancy in cyclostationary signals," *Signal Processing Mag.*, Vol. 8, No. 2, pp. 14–37, 1991.

[9] J. H. Manton, "Optimization algorithms exploiting unitary constraints," *IEEE Trans. on Signal Processing*, Vol. 50, No. 3, pp. 635–650, Mar. 2002.

[10] M. Nikpour, J. H. Manton, and G. Hori, "Algorithms on the Stiefel manifold for joint diagonalization", *Proc. of 2002 IEEE International Conference on Acoustics, Speech and Signal Processing*, pp. 1481–1484, Orlando, USA, May 2002.

[11] S. V. Schell. "An overview of sensor array processing for cyclostationary signals," *Cyclostationarity in Communications and Signal Processing* (W. A. Gardner, ed.), ch.3, IEEE Press, pp. 168–239, 1994.

[12] L. Tong, R. W. Liu, V. C. Soon, and Y. F. Huang, "Indeterminacy and identifiability of blind identification," *IEEE Trans. on Circuits and Systems*, Vol. 38, No. 5, pp. 499–509, May 1991.

[13] M. Wax and J. Sheinvald, "A least-squares approach to joint diagonalization", *IEEE Signal Processing Letters*, Vol.4, No. 2, pp. 52–53, Feb. 1997.