KINEMATIC AND DYNAMIC MODELING OF A ROBOTIC HEAD WITH LINEAR MOTORS

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ABSTRACT

Cutting heads have a great impact on cutting strategy. Currently, most cutting heads have 2 degrees-of-freedom (DOF). For this reason, these cutting heads are not suitable to solve three-dimensional cutting problems. To meet the present demands of industrial products it is necessary to develop a 3-DOF cutting head. Our present research focuses on kinematic and dynamic modeling of a 3-DOF robotic cutting head for the next generation of CNC machines. The robotic cutting head is one kind of parallel manipulator of 3-PUU type, which has a high flexibility of motion in three-dimensional space. The parallel manipulator consists of three linear servomotors, which drive three connecting rods independently according to the cutting strategy. Being a parallel manipulator, the robotic cutting head has higher stiffness and position accuracy; consequently, higher velocities and accelerations can be achieved. A very suitable application of this mechanism is as a cutting head of a precision machine tool for three-dimensional cutting problems.

Key words: Cutting Head, Parallel Manipulators, Singularity, 3D Machining, Linear Servo Motors.

1. INTRODUCTION

Parallel manipulators offer significant advantages over current serial manipulators; better stiffness and accuracy, lighter weight, greater load-bearing, higher velocities and accelerations, and less powerful actuators. The disadvantages are; smaller workspace and more complex kinematics and dynamics. Parallel manipulators were proposed by Hunt (1978). Cox and Tesar (1981) compared the relative merits of serial and parallel robots. Hunt (1983) conducted preliminary studies of various parallel robot configurations. Some configurations have been built and also a numerous works analyze kinematics, dynamics, workspace, and control of parallel manipulators by many researchers (Daniali et al., 1995; Gosselin, 1996). Clavel (1988) proposed a high-speed parallel DELTA robot equipped with spatial 3-DOF. It is used in industry for high speed handling applications but its application is limited because its degrees of freedom are too small to perform a complicated task. Tsai (1996) proposed the kinematics of a 3-DOF platform. Hervé (1992) proposed the Star structure, and the Prism structure (1995). Pierrot (1990) proposed a HEXA parallel robot with the expansion of DELTA mechanism. However, HEXA robot suffers from its high price and complexity. Mobility analysis of the 3-UPU parallel mechanism was done by Gregorio and Castelli (1999). Giberti (2001) has carried out some significant work on 3-PUU parallel manipulator with pneumatic drives. A linear DELTA mechanism with linear drives has been carried out by Company et. al., (2000). Kim and Tsai (2003) have developed a 3-RPS parallel manipulator.

A new and simple type parallel manipulator has been proposed to use widely in industries, especially for the precision machine tool for three-dimensional cutting environment, which has simple but stable kinematic and dynamic behaviour. This paper deals with the development of a Three Slider Manipulator (TSM) of 3-PUU type. This type of cutting head consists of three linear motors, where the moving platform is connected with the actuators by three links. The link rods are connecting with the actuators and base platform with universal joints that allow higher repeatability. Being a parallel mechanism with linear motors, the main advantage of the proposed system is robustness against external forces. Therefore, such kind of robotic cutting head will be able for three-dimensional machining tasks peculiar to industrial automation sectors for quality and mass production in all aspects. The kinematic and dynamic models of the new system have been developed. Simulations have been carried out to show the effective of the proposed mechanism.

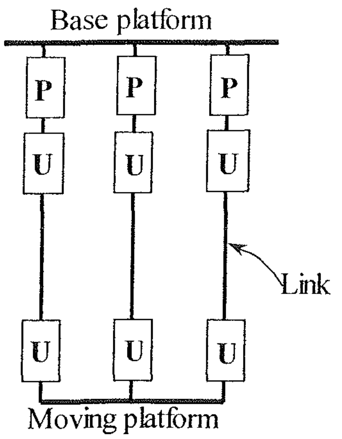

Figure 1. Architectural scheme of 3-PUU.

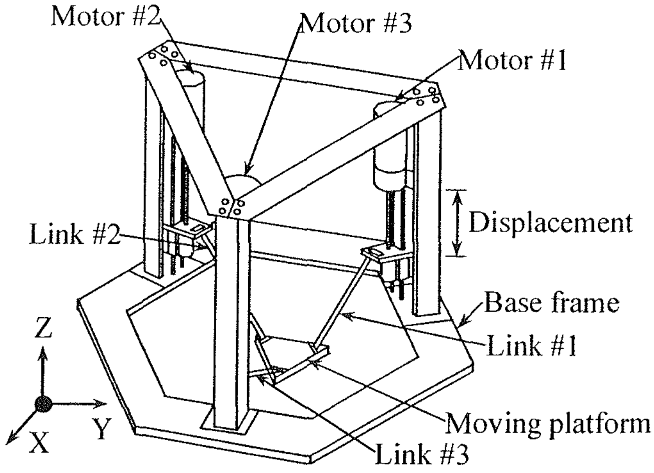

Figure 2. Robotic cutting head.

The organization of this paper is as follows: Section 2 briefly describes the robotic cutting head, which is a Three Slider Manipulator (TSM) with 3-PUU configuration. Section 3 discusses the kinematics. Dynamic and simulation results are shown in section 4. Finally, conclusions are given in section 5.

2. THREE SLIDER MANIPULATOR

Most of the parallel systems are very complex in structure. For this reason, it is very difficult solve its inverse kinematics. In this research a new and simple parallel manipulator, Three Slider Manipulator (TSM), has been proposed for robotic cutting head for industrial automation to solve the three-dimensional machining problems. In general, the sliders move by the forces of the prismatic actuated joints (P). The mechanical links are of constant lengths and are connected to the sliders through universal joints (U). Finally, mechanical links are connected to the mobile platform through universal joints (U). The basic concept of TSM is described by a simple architectural scheme illustrated in Figure 1, where joints are represented by rectangles and links between those joints are represented by lines. Here, P and U represent prismatic and universal joint, respectively.

The parallel manipulator proposed in this research for cutting head is symmetric and composed of three identical parts. Each part consists of one linear motor and a link. One end of the link is connected to the linear motor with universal joint. The other ends of the links are connected to the triangular moving platform with universal joints. The actuators are the heavy parts of a manipulator, which are fixed at the base. Consequently, higher velocities and accelerations of the mobile platform can be achieved. Selecting thin rods for links reduces the risk of mechanical link interference problem. The main advantage of such kind of robotic cutting head is the flexibility of motion in three-dimension space. Figure 2 shows a schematic diagram of robotic cutting head.

3. KINEMATICS OF MANIPULATOR

In this section, the inverse and forward kinematics of 3-PUU type parallel manipulator are described. The Jacobian matrix is used to analyze the singularity problem. Figure 3 shows the geometric structure of robotic cutting head.

3.1 Geometric Description

The parameters d_i, l_i, a_i, b_i, α_i, β_i, and p_z, are the linear displacement of the $i-$th linear motor in the vertical direction, the length of the $i-$th link, the distance between the center of base frame and the location of $i-$th linear motor, the distance between the center of moving platform and the $i-$th mechanical link, rotating angle of the $i-$th link, orientation angle of the moving platform with the $i-$th mechanical link, and the movable vertical distance of moving platform, respectively. Figure 4 shows the parameters of a 3-PUU robotic cutting head.

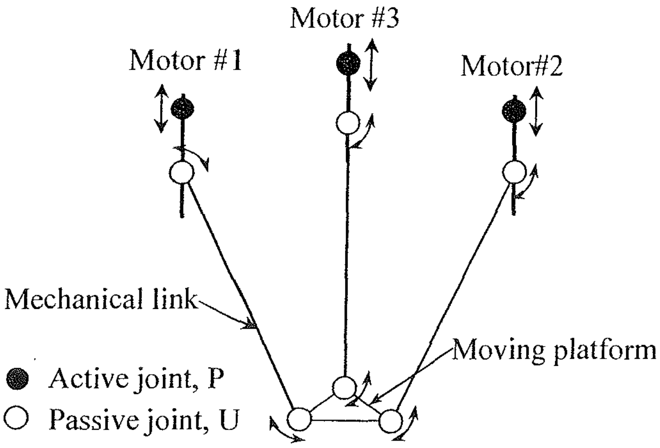

Figure 3. Geometric structure of robotic cutting head.

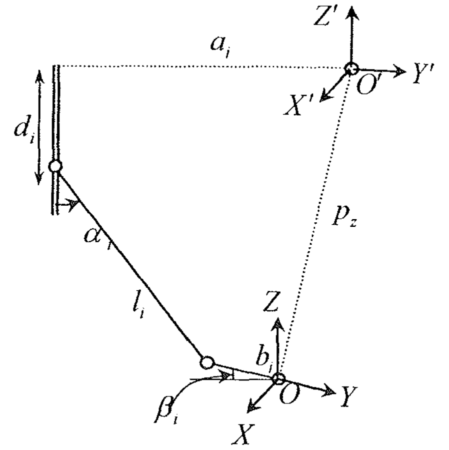

Figure 4. Parameters of robotic head.

3.2 Inverse Kinematics

The inverse kinematics involves solving the inverse transformation equations to find the relationships between the links of the manipulator from the location of the end-effector in space.

Here, the constrained equation of the system is as follows:

$$e_i + d_i - p = l_i \tag{1}$$

where $e_i = a_i - b_i$ and $i = 1,2,\text{and } 3$.

After some mathematical manipulation, the inverse kinematics equation is derived as follows:

$$d_i = p_z \pm \sqrt{-e_i^2 - p_x^2 - p_y^2 + 2e_{ix}p_x + 2e_{iy}p_y + l_i^2} \tag{2}$$

3.3 Forward Kinematics

The forward kinematics involves solving the forward transformation equations to find the location of the end-effector in space in terms of the displacements between the links.

The forward kinematics equations are derived as follows:

$$p_x = \frac{\Delta e_{2y}(k_3 - p_z(\Delta e_{3z} + \Delta d_3)) - \Delta e_{3y}(k_2 - p_z(\Delta e_{2z} + \Delta d_2))}{\Delta e_{3y}\Delta e_{2x} - \Delta e_{3x}\Delta e_{2y}} \tag{3}$$

$$p_y = \frac{\Delta e_{2x}(k_3 - p_z(\Delta e_{3z} + \Delta d_3)) - \Delta e_{3x}(k_2 - p_z(\Delta e_{2z} + \Delta d_2))}{\Delta e_{3y}\Delta e_{2x} - \Delta e_{3x}\Delta e_{2y}} \tag{4}$$

$$p_z = d_1 - \sqrt{-e_1^2 - p_x^2 - p_y^2 + l_1^2 + 2e_{1x}p_x + 2e_{1y}p_y} \tag{5}$$

where $\Delta e_2 = (e_2 - e_1)$, $\Delta e_3 = (e_3 - e_1)$, $\Delta d_2 = (d_2 - d_1)$, $\Delta d_3 = (d_3 - d_1)$,

$k_2 = p_x\Delta e_{2x} + p_y\Delta e_{2y} + p_z\Delta e_{2z} + p_z\Delta d_2$, and $k_3 = p_x\Delta e_{3x} + p_y\Delta e_{3y} + p_z\Delta e_{3z} + p_z\Delta d_3$

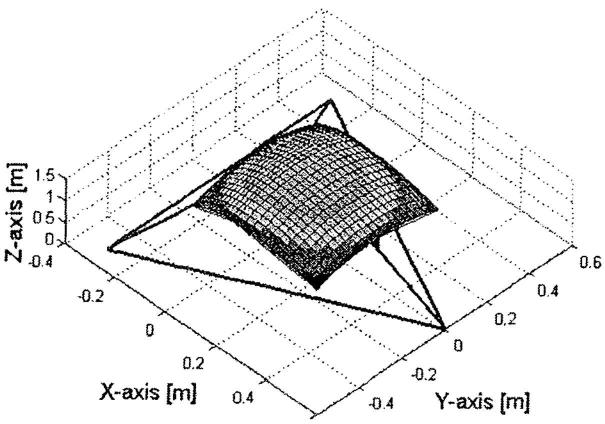

Figure 5. Total workspace of TSM.

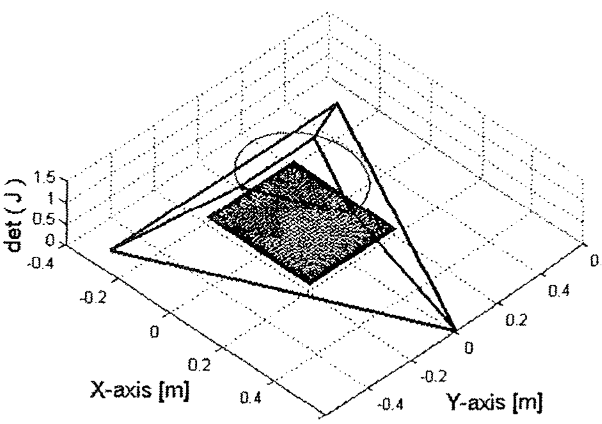

Figure 6. Determinant of Jacobian in XY plane.

3.4 Workspace and Singularity Problems

The geometric workspace is an important characteristic of a manipulator since a small workspace can limit the possible applications of a given manipulator architecture. The workspace of a manipulator is the reachable area of the end-point of the manipulator. At singular configuration, the manipulator cannot move in certain direction. The TSM has two types of singularity, under mobility and over mobility. At under mobility, some links are in the same plane. On the other hand, at over mobility, some links are in parallel. To avoid both types of singularity problems it is essential to optimized the over all parameters of parallel manipulator. The Jacobian matrix of the TSM is derived as follows:

$$J = \begin{bmatrix} \dfrac{(e_{1x}-p_x)}{\sqrt{-e_1^2-p_x^2-p_y^2+2e_{1x}p_x+2e_{1y}p_y+l_1^2}} & \dfrac{(e_{1y}-p_y)}{\sqrt{-e_1^2-p_x^2-p_y^2+2e_{1x}p_x+2e_{1y}p_y+l_1^2}} & 1 \\ \dfrac{(e_{2x}-p_x)}{\sqrt{-e_2^2-p_x^2-p_y^2+2e_{2x}p_x+2e_2p_y+l_2^2}} & \dfrac{(e_{2y}-p_y)}{\sqrt{-e_2^2-p_x^2-p_y^2+2e_{2x}p_x+2e_{2y}p_y+l_2^2}} & 1 \\ \dfrac{(e_{3x}-p_x)}{\sqrt{-e_3^2-p_x^2-p_y^2+2e_{3x}p_x+2e_{3y}p_y+l_3^2}} & \dfrac{(e_{3y}-p_y)}{\sqrt{-e_3^2-p_x^2-p_y^2+2e_{3x}p_x+2e_{3y}p_y+l_3^2}} & 1 \end{bmatrix} \quad (6)$$

Figure 5 shows the workspace of TSM when the following parameters were considered: link lengths, $l_i = 1.0$[m], the distance between the center of base frame and the location of $i-$th linear motor, $a_i = 0.6[m]$, the distance between the center of moving platform and the $i-$th mechanical link, $b_i = 0.1[m]$. The rotating angle, $0 < \alpha_i < 45°$, orientation angle, $0 < \beta_i < 45°$. The simulation result shows a good working volume. Figure 6 shows that the determinant of Jacobian matrix when the manipulator moves on a circular trajectory with the conditions: center $(x = 0, y = 0)$, radius $r = 0.1[m]$, at a vertical height, $p_z = 1.2[m]$. This simulation result shows that the manipulator can move freely on the commanded circular trajectory.

4. DYNAMICS OF MANIPULATOR

The dynamics of the parallel manipulator describes how the manipulator moves in response to the linear displacement of the linear motors. The inverse dynamics problem finds out the required vector of linear displacement of linear motors from a given vector of positions, velocities, and accelerations, which is important to move the manipulator on a commanded trajectory. On the other hand, the forward dynamics problem finds out the resulting manipulator motions from a given vector of linear displacement of linear motor, which is important for simulation studies. Using the Eular-Lagrange equation of motion the dynamic equation of the manipulator is as follows:

$$F = M(\ddot{d}+g) - 2\lambda(d-p_z) \quad (7)$$

where F is the force matrix of the linear motors, M is the mass matrix of the parallel system, $\ddot{d}$ is the acceleration matrix of the liner motors, g is the gravity force matrix, λ is the Lagrange multiplier, d is the liner displacement matrix of the liner motor, and p_z is the displacement of the end-effector in the vertical direction.

4.1 Control Method of Parallel Manipulator

The control method is shown in Figure 7. In the simulation process, the implemented control scheme is based on the PID theory to reduce the errors of the displacements of the linear motors, so that manipulator can move on a desire trajectory with a very small error. The feed forward control scheme based on PID control theory is as follows:

$$F = M[\ddot{d}^d - kp(d-d^d) - kv(\dot{d}-\dot{d}^d) - ki\int_0^t (d-d^d)dt] + Mg - 2\lambda(d-p_z) \quad (8)$$

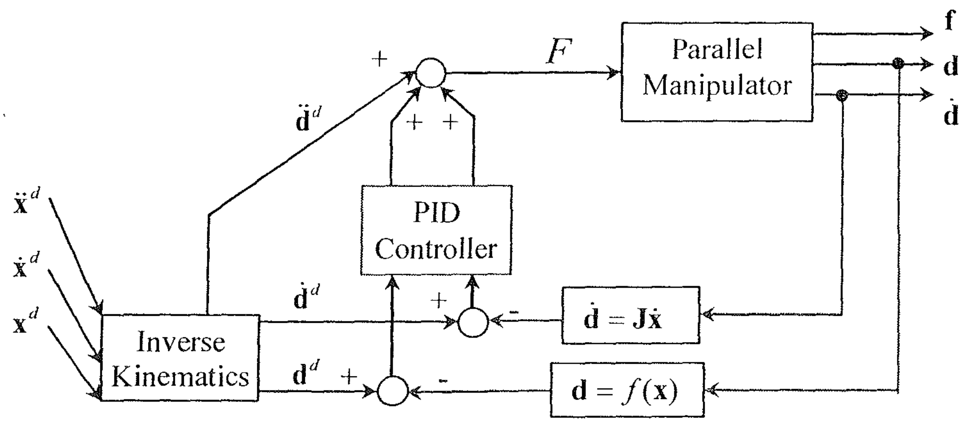

Figure 7. Control scheme of parallel manipulator.

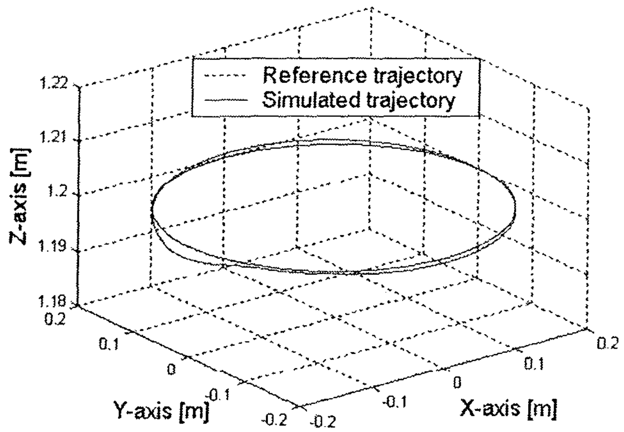

Figure 8. Position trajectory.

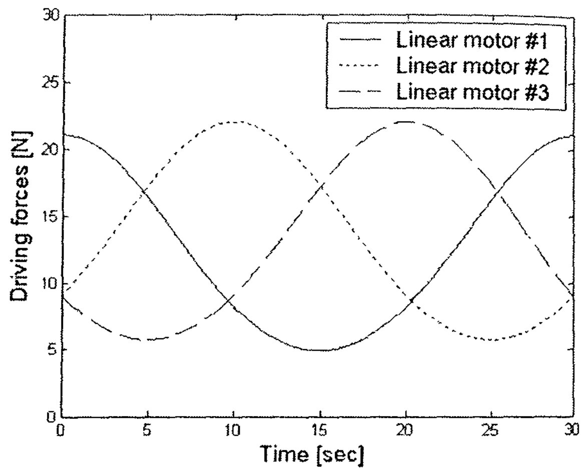

Figure 9. Driving forces of linear motors.

4.2 Simulation Results

In the simulation process, the following properties of the manipulator were considered: mass of the links, $m_l = 0.4[kg]$, mass of the moving platform, $m_p = 0.8[kg]$, gravity force, $g = 9.81[m/\sec^2]$. In the simulation process the manipulator tracks on a circular trajectory with the following conditions: center $(x = 0, y = 0)$, radius $r = 0.1[m]$, at a vertical height, $p_z = 1.2[m]$. The manipulator moves on the circular trajectory with a tool of mass, $m_t = 2.0[kg]$. The total processing time is considered as 30.0[*sec*], and the tracking velocity is 20.94[*mm/sec*]. The controller gain coefficient is selected as $kp = 0.98; kv = 0.705; ki = 0.001$. Figure 8 shows that the manipulator moves on the commanded reference trajectory with a maximum position error of $\pm 0.5[mm]$. Figure 9 shows the driving forces of linear motors, which shows that the system required three linear motors of minimum 25.0[N] force capacity. These simulation results show the stable dynamic behaviour of the system and also the satisfactory control method of the parallel manipulator with our proposed controller.

5. CONCLUSION

A new model of TSM has been proposed which is a 3-PUU type parallel manipulator with linear motors. It has simple forward and inverse kinematics. The kinematic and dynamic models for the new type of manipulator have been developed. The Jacobian matrix is used to analyze the workspace and singularity problem. The parameters of parallel manipulators have been optimized from the kinematic simulation. The dynamic behaviour of the manipulator is investigated from the simulation results. The simulation results show that such kind of robotic cutting head will be able for three-dimensional machining tasks peculiar to industrial automation sectors for quality and mass production in all aspects due to its stable kinematics and dynamic behaviour. A very suitable application of such kind of mechanism is as the cutting head of a precision machine tool to solve the three-dimensional machining problems.

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