Ma, Xiang, Barnett, Matthew and Davies, C.H.J. 2005, Modelling the influence of strain on the yield surface of magnesium, *in Proceedings of the 8th ESAFORM Conference on Material Forming*, Publishing House of the Romanian Academy, Bucharest, Romania, pp. 395-398.

Modelling the Influence of Strain on the Yield Surface of Magnesium

X. Ma[1], M.R. Barnett[1], C.H.J. Davies[2]

[1]*School of Engineering and Technology, Deakin University, Geelong, VIC 3217, Australia*
URL: http://www.deakin.edu.au/engineering/research/ammp/?188
e-mail: xma@deakin.edu.au; barnettm@deakin.edu.au

[2]*School of Physics and Materials Engineering, Monash University, Clayton, VIC 3800, Australia*
URL: http://www.spme.monash.edu.au/staff/davies.html *e-mail: chris.davies@spme.monash.edu.au*

ABSTRACT: Textured magnesium alloys show strong anisotropy at room temperature. An anisotropic yield criterion was proposed by Cazacu and Barlat [Int. J. Plas. 20 (2004) 2027-45] which can reproduce individual plane stress yield surfaces of Mg. A description of the evolutional equations for their adjustable parameters with strain is provided in this paper in the plane stress state. This involves a twinning based semi-physical analytical constitutive equation.

Key words: Magnesium; anisotropy; yield surface; twining.

1 INTRODUCTION

Textured magnesium alloys typically display strong anisotropic plastic behaviour at room temperature. This phenomenon is usually ascribed to the low number of slip systems and the polarity of tensile twinning $\{10\bar{1}2\}$. Understanding of the effect of twinning activity on the mechanical properties of magnesium products is crucial to the design of cold forming processes.

Recently, an anisotropic yield criterion was proposed by Cazacu and Barlat [1] which is able to accurately reproduce individual Mg plane stress yield surfaces plotted by Kelly and Hosford [2]. However, a description for the evolution of their adjustable parameters with straining is lacking in their treatment. The current paper aims to develop analytical expressions for the evolution of their adjustable parameters (single loading path). These models will be able to be incorporated into the material constitutive models in finite element (FE) packages.

2 IDENTIFICATION OF HARDENING PARAMETERS

In their original format, the Cazacu-Barlat yield criterion in the plane stress state is given as [1]:

$$\left[\frac{1}{6}(a_1+a_3)\sigma_1^2-\frac{a_1}{3}\sigma_1\sigma_2+\frac{1}{6}(a_1+a_2)\sigma_2^2\right]^{\frac{3}{2}}-\left[\frac{c}{27}(b_1+b_2)\sigma_1^3+\frac{c}{27}(b_3+b_4)\sigma_2^3-\frac{c}{9}(b_1\sigma_1+b_4\sigma_2)\sigma_1\sigma_2\right]=\tau_Y^3 \quad (1)$$

where σ_1 and σ_2 denote the stress components along the rolling and transverse directions of a thin rolled sheet; c, a_1-a_3 and b_1-b_4 are the adjustable parameters; and τ_Y is the yield stress in pure shear. This equation involves nine coefficients (eight adjustable parameters and one pure shear yield stress). Using the following substitutions:

$c_1=a_1/\tau_Y^2$; $c_2=a_2/\tau_Y^2$; $c_3=a_3/\tau_Y^2$;
$d_1=cb_1/\tau_Y^3$; $d_2=cb_2/\tau_Y^3$; $d_3=cb_3/\tau_Y^3$;
$d_4=cb_4/\tau_Y^3$.

Eq. (1) can be converted into the following format with only seven independent adjustable parameters:

$$\left[\frac{1}{6}(c_1+c_3)\sigma_1^2-\frac{c_1}{3}\sigma_1\sigma_2+\frac{1}{6}(c_1+c_2)\sigma_2^2\right]^{\frac{3}{2}}-\left[\frac{1}{27}(d_1+d_2)\sigma_1^3+\frac{1}{27}(d_3+d_4)\sigma_2^3-\frac{1}{9}(d_1\sigma_1+d_4\sigma_2)\sigma_1\sigma_2\right]=1 \quad (1a)$$

Identification of the material hardening parameters in Eq. (1a) requires several simple tests carried out on Mg rolled sheet. It was shown that uniaxial tension and compression tests and the channel die compression tests are useful for this purpose [2]. The following derivations for the hardening parameters can be found from these simple stress states.

Denoting the yield stresses in the uniaxial tension and compression along the rolling direction as R_T and R_C, and stresses in the uniaxial tension and compression along the transverse direction as T_T and T_C, the following equations can be derived from Eq. (1a) (equivalent to those in [1]):

$$R_T=\left[\left(\frac{c_1+c_3}{6}\right)^{3/2}-\frac{1}{27}(d_1+d_2)\right]^{-1/3} \quad (2)$$

$$R_C=\left[\left(\frac{c_1+c_3}{6}\right)^{3/2}+\frac{1}{27}(d_1+d_2)\right]^{-1/3} \quad (3)$$

$$T_T=\left[\left(\frac{c_1+c_2}{6}\right)^{3/2}-\frac{1}{27}(d_3+d_4)\right]^{-1/3} \quad (4)$$

$$T_C=\left[\left(\frac{c_1+c_2}{6}\right)^{3/2}+\frac{1}{27}(d_3+d_4)\right]^{-1/3} \quad (5)$$

Similarly, the yield stresses Z_C and Z_T in the equibiaxial tension and compression are given as:

$$Z_C=\left[\left(\frac{c_2+c_3}{6}\right)^{\frac{3}{2}}-\frac{(-2d_1+d_2+d_3-2d_4)}{27}\right]^{-\frac{1}{3}} \quad (6)$$

$$Z_T=\left[\left(\frac{c_2+c_3}{6}\right)^{\frac{3}{2}}+\frac{(-2d_1+d_2+d_3-2d_4)}{27}\right]^{-\frac{1}{3}} \quad (7)$$

The following expressions for the hardening parameters c_1, c_2 and c_3 are derived from Eqs (2)-(7):

$$c_1=\frac{3}{2^{2/3}}\left\{\left(\frac{1}{R_T^3}+\frac{1}{R_C^3}\right)^{\frac{2}{3}}+\left(\frac{1}{T_T^3}+\frac{1}{T_C^3}\right)^{\frac{2}{3}}-\left(\frac{1}{Z_C^3}+\frac{1}{Z_T^3}\right)^{\frac{2}{3}}\right\} \quad (8)$$

$$c_2=\frac{3}{2^{2/3}}\left\{\left(\frac{1}{T_T^3}+\frac{1}{T_C^3}\right)^{\frac{2}{3}}+\left(\frac{1}{Z_C^3}+\frac{1}{Z_T^3}\right)^{\frac{2}{3}}-\left(\frac{1}{R_T^3}+\frac{1}{R_C^3}\right)^{\frac{2}{3}}\right\} \quad (9)$$

$$c_3=\frac{3}{2^{2/3}}\left\{\left(\frac{1}{R_T^3}+\frac{1}{R_C^3}\right)^{\frac{2}{3}}+\left(\frac{1}{Z_C^3}+\frac{1}{Z_T^3}\right)^{\frac{2}{3}}-\left(\frac{1}{T_T^3}+\frac{1}{T_C^3}\right)^{\frac{2}{3}}\right\} \quad (10)$$

The following combinations of hardening parameters d_1-d_4 are also derived from Eqs (2)-(7):

$$d_1+d_2=\frac{27}{2}\left(\frac{1}{R_C^3}-\frac{1}{R_T^3}\right) \quad (11)$$

$$d_3+d_4=\frac{27}{2}\left(\frac{1}{T_C^3}-\frac{1}{T_T^3}\right) \quad (12)$$

$$d_1+d_4=\frac{9}{2}B \quad (13)$$

where $B=\frac{1}{R_C^3}-\frac{1}{R_T^3}+\frac{1}{T_C^3}-\frac{1}{T_T^3}+\frac{1}{Z_C^3}-\frac{1}{Z_T^3}$

Besides Eqs (11)-(13), an additional equation is required to find the solutions for d_1-d_4. Such an expression can be generated using the plane strain compression data. With changing the sheet orientation, the compressive loading stresses measured in this test were used to locate the stress components at which $d\sigma_2/d\sigma_1$ are 0, ∞, and 1 at the yield loci [2]. The following procedure shows how to find the hardening parameter expression from the channel compression test carried out on such an orientation that the compressive load is applied along the thickness direction and the sample is expanded along the transverse direction.

Denoting the compressive load in this case as ZT, it can be shown that $ZT=\sigma_2$ [2]. Differentiating Eq. (1a) with regard to σ_1 and noting that $d\sigma_2/d\sigma_1=0$ leads to the following expression:

$$\frac{1}{2}\sqrt{A}\left[(c_1+c_3)\sigma_1-c_1\cdot ZT\right] -\frac{1}{9}\left[(d_1+d_2)\sigma_1^2-2d_1\sigma_1\cdot ZT-d_4 ZT^2\right]=0 \quad (14)$$

where σ_1 is unknown stress component, and

$$A=\frac{(c_1+c_3)}{6}\sigma_1^2-\frac{c_1}{3}\sigma_1\cdot ZT+\frac{(c_1+c_2)}{6}ZT^2$$

Eq. (13) gives

$$d_4=\frac{9}{2}B-b_1 \quad (15)$$

Inserting Eq. (15) into Eq. (14) leads to:

$$d_1=\left\{\frac{1}{2}\sqrt{A}\left[(c_1+c_3)\sigma_1-c_1\cdot ZT\right] -\left[\frac{(d_1+d_2)}{9}\sigma_1^2-\frac{B}{2}ZT^2\right]\right\}\Big/\left(ZT^2-2\sigma_1 ZT\right) \quad (16)$$

Inserting Eq. (15) into Eq. (1a) gives:

$$d_1 = \left\{\left[-\frac{(d_1+d_2)}{27}\sigma_1^3 + \frac{(d_3+d_4)}{27}ZT^3 - \frac{B}{2}\sigma_1 \cdot ZT\right] + A^{\frac{3}{2}} - 1\right\} \Big/ (\sigma_1 - ZT)\sigma_1 \cdot ZT \quad (17)$$

Matching Eq. (16) with (17) yields an implicit expression for σ_1 and it can be solved by numerical methods. Finally d_1 can be found from Eq. (16) or (17). In the present work, we leave d_1 undetermined and set out instead to obtain a description of the evolution of key parameters with strain.

3 ANALYTICAL FLOW STRESSES

Twinning is a major deformation mode in magnesium and its effect on the flow stress σ can be described by a law of mixtures [3]:

$$\sigma = (1 - X_T)\sigma_S + X_T\sigma_T \quad (18)$$

where X_T is the volume fraction of grains in which $\{10\bar{1}2\}$ twinning is expected, and σ_S is the stress in the grains in which twinning does not occur:

$$\sigma_S = \beta E^n \sigma_{0r}^{1-n} \varepsilon^n \quad (19)$$

where β is a texture based constant; E is the Youngs modulus; σ_{0r} is a reference stress; n is the hardening index; ε is the imposed total macroscopic strain. The stress in the grains in which twinning does occur is given by:

$$\sigma_T = \chi\sigma_{0t} + (1-\chi)\xi E^n \sigma_{0r}^{1-n}(\varepsilon - \chi\varepsilon_t)^n \quad (20)$$

which is a law of mixtures combination of the stress in the as yet untwinned material, σ_{0t} (a parameter that is considered to be insensitive to strain) and the stress due to slip in the twinned volume. The constant ξ reflects the effect twinning has on the stress required for glide, ε_t is the strain directly attributable to twinning and χ is the twinning fraction (over the grains expected to twin) whose evolution with strain can be expressed as:

$$\chi = 1 - \exp\left[-4\left(\varepsilon / \varepsilon_1\right)^a\right] \quad (21)$$

where a is a rate exponent and ε_1 is the strain at which the twinning activity is 98% complete.

Figs 1-3 show how well Eq. (18) fits the experimental data obtained for pure Mg [2]. The values of the parameters used are listed in Table 1.

Table 1. Values of parameters used for Eq. 18 in Figs 1-3

	ZT	TZ	R_T	R_C	T_T	T_C	Z_C	Z_T
a	3.0							
ε_1	0.15							
n	0.18							
σ_{0t}*	13.5							
σ_{0r}*	4.5							
ξ	1.25							
X_T	0	0.9	0.1	0.8	0	0.8	0	1.0
β	1.0		0.64		1.05	0.7	0.75	1.0

*: units in ksi.

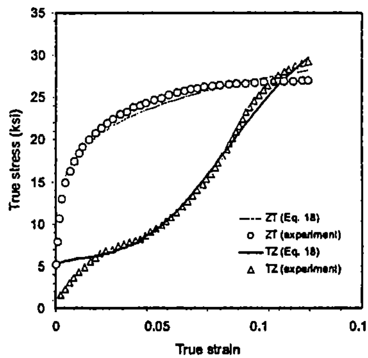

Fig. 1 Compressive loading stresses in the plane strain states

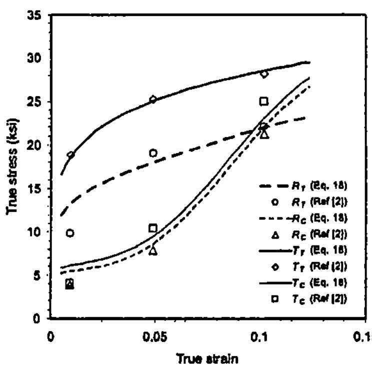

Fig. 2 Yield stresses in the uniaxial tension and compression

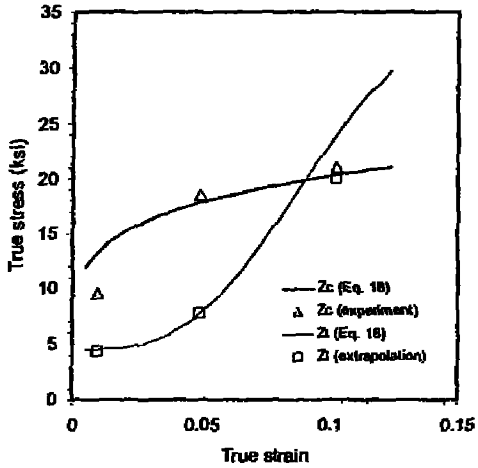

Fig. 3 Yield stresses in the equibiaxial tension and compression

TZ in Fig. 1 denotes the compressive load stress in the channel die compression in which the compressive load is applied along the transverse direction and the sheet is expanded along the thickness direction. The data points for Z_T in Fig. 3 are found from the yield surface [2] as no test data are available.

Higher values of can be seen in the case in which a tensile load is applied along the transverse directions. This is caused by the greater spread of the texture in the transverse direction such that more "easy" basal slip operates [2]. Higher values of X_T are obtained for the compressive loadings in the plane of the sheet. This reflects the enhanced activation of twinning in the samples oriented for extension along the crystallographic c-axis.

4 DISCUSSIONS AND CONCLUSIONS

Using the yield stresses predicted by Eq. (18) under different stress states, the evolution of the adjustable parameters in Eqs (8-13) with strain are shown in Figs 4-5. Also shown are points taken from [1]. The agreement between two is reasonable for the values of the "*c*" parameters. The "*d*" parameter combinations are not so well described. This will be explored in the future work.

It can nevertheless be concluded that the proposed evolution equations for the yield surface capture the main physical activities in Mg deformation and provide an analytical basis for the material model which can be implemented in the FE simulation using Cazacu and Barlat yield criterion.

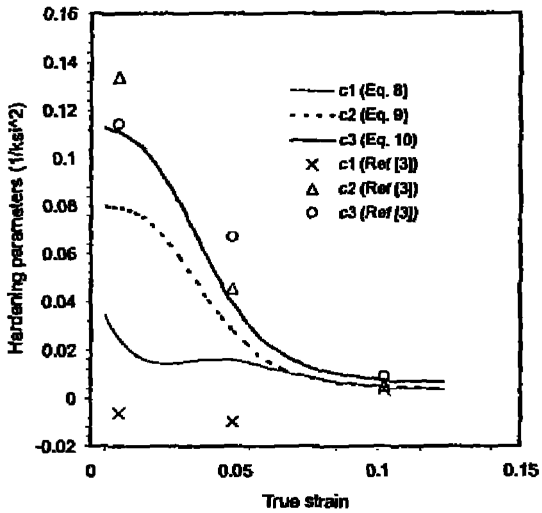

Fig. 4 Evolution curves for c_1-c_3

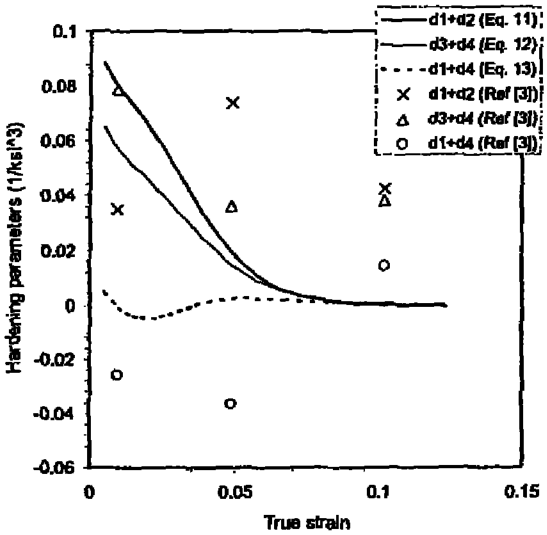

Fig. 5 Evolution curves for the combinations of d_1-d_4

REFERENCES

1. O. Cazacu, and F. Barlat, A criterion for description of anisotropy and yield differential effects in pressure-insensitive metals. *International Journal of Plasticity* 20 (2004) 2027-45.
2. E.W. Kelley, and J. Hosford, W. F., The Deformation characteristic of Textured Magnesium. *Transactions of the Metallurgical Society of AIME* 242 (1968) 654-60.
3. M.R. Barnett, C.H.J. Davies, and X. Ma, An analytical constitutive law for twinning dominated flow in magnesium. *Scripta Materialia* 52 (2005) 627-32.