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Modelling the Influence of Strain on the Yield Surface of Magnesium

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ABSTRACT: Textured magnesium alloys show strong anisotropy at room temperature. An anisotropic yield criterion was proposed by Cazacu and Barlat [Int. J. Plas. 20 (2004) 2027-45] which can reproduce individual plane stress yield surfaces of Mg. A description of the evolutionary equations for their adjustable parameters with strain is provided in this paper in the plane stress state. This involves a twinning based semi-physical analytical constitutive equation.

Key words: Magnesium; anisotropy; yield surface; twinning.

1 INTRODUCTION

Textured magnesium alloys typically display strong anisotropic plastic behaviour at room temperature. This phenomenon is usually ascribed to the low number of slip systems and the polarity of tensile twinning [1012]. Understanding the effect of twinning activity on the mechanical properties of magnesium products is crucial to the design of cold forming processes.

Recently, an anisotropic yield criterion was proposed by Cazacu and Barlat [1] which is able to accurately reproduce individual Mg plane stress yield surfaces plotted by Kelly and Hosford [2]. However, a description for the evolution of their adjustable parameters with straining is lacking in their treatment. The current paper aims to develop analytical expressions for the evolution of their adjustable parameters (single loading path). These models will be able to be incorporated into the material constitutive models in finite element (FE) packages.

2 IDENTIFICATION OF HARDENING PARAMETERS

In their original format, the Cazacu-Barlat yield criterion in the plane stress state is given as [1]:

\[ \left[ \frac{1}{6} (a_1 + a_6) \sigma_7^2 - \frac{a_3}{3} \sigma_1 \sigma_5 + \frac{1}{6} (a_1 + a_5) \sigma_6^2 \right]^\frac{3}{2} - \left[ \frac{a_1 + a_6}{27} \sigma_7^3 \right. \\
+ \left. \frac{a_1}{27} (b_1 + b_5) \sigma_1^3 - \frac{c}{9} (b_1 \sigma_1 + b_5 \sigma_5) \sigma_5 \sigma_1 \right] = \tau_0^3 \]  \tag{1}

where \( \sigma_1 \) and \( \sigma_2 \) denote the stress components along the rolling and transverse directions of a thin rolled sheet; \( c, a_1-a_3 \) and \( b_1-b_4 \) are the adjustable parameters; and \( \tau_0 \) is the yield stress in pure shear. This equation involves nine coefficients (eight adjustable parameters and one pure shear yield stress). Using the following substitutions:

\[ c_1 = a_1 / \tau_0^2; \quad c_2 = a_2 / \tau_0^2; \quad c_3 = a_3 / \tau_0^2; \]
\[ d_1 = cb_1 / \tau_0^2; \quad d_2 = cb_2 / \tau_0^2; \quad d_3 = cb_3 / \tau_0^2; \]
\[ d_4 = cb_4 / \tau_0^2. \]

Eq. (1) can be converted into the following format with only seven independent adjustable parameters:

\[ \left[ \frac{1}{6} (c_1 + c_6) \sigma_7^2 - \frac{c_3}{3} \sigma_1 \sigma_5 + \frac{1}{6} (c_1 + c_5) \sigma_6^2 \right]^\frac{3}{2} \\
- \left[ \frac{1}{27} (d_1 + d_6) \sigma_7^3 \right. \\
+ \left. \frac{1}{27} (d_1 + d_5) \sigma_1^3 - \frac{1}{9} (d_1 \sigma_1 + d_5 \sigma_5) \sigma_5 \sigma_1 \right] = 1 \]  \tag{1a}

Identification of the material hardening parameters in Eq. (1a) requires several simple tests carried out on Mg rolled sheet. It was shown that uniaxial
tension and compression tests and the channel die compression tests are useful for this purpose [2]. The following derivations for the hardening parameters can be found from these simple stress states.

Denoting the yield stresses in the uniaxial tension and compression along the rolling direction as $R_T$ and $R_C$, and stresses in the uniaxial tension and compression along the transverse direction as $T_T$ and $T_C$, the following equations can be derived from Eq. (1a) (equivalent to those in [1]):

$$R_T = \left[\left(\frac{c_1 + c_2}{6}\right)^{\frac{3}{2}} - \frac{1}{27} \left(d_1 + d_2\right)\right]^\frac{1}{3}$$  \hspace{1cm} (2)

$$R_C = \left[\left(\frac{c_1 + c_2}{6}\right)^{\frac{3}{2}} + \frac{1}{27} \left(d_1 + d_2\right)\right]^\frac{1}{3}$$  \hspace{1cm} (3)

$$T_T = \left[\left(\frac{c_1 + c_2}{6}\right)^{\frac{3}{2}} - \frac{1}{27} \left(d_3 + d_4\right)\right]^\frac{1}{3}$$  \hspace{1cm} (4)

$$T_C = \left[\left(\frac{c_1 + c_2}{6}\right)^{\frac{3}{2}} + \frac{1}{27} \left(d_3 + d_4\right)\right]^\frac{1}{3}$$  \hspace{1cm} (5)

Similarly, the yield stresses $Z_C$ and $Z_T$ in the equibiaxial tension and compression are given as:

$$Z_C = \left[\left(\frac{c_1 + c_3}{6}\right)^{\frac{3}{2}} - \frac{\left(-2d_1 + d_2 + d_3 - 2d_4\right)}{27}\right]^\frac{1}{3}$$  \hspace{1cm} (6)

$$Z_T = \left[\left(\frac{c_1 + c_3}{6}\right)^{\frac{3}{2}} + \frac{\left(-2d_1 + d_2 + d_3 - 2d_4\right)}{27}\right]^\frac{1}{3}$$  \hspace{1cm} (7)

The following expressions for the hardening parameters $c_1$, $c_2$ and $c_3$ are derived from Eqs (2)-(7):

$$c_1 = \frac{3}{2^{\frac{3}{2}}} \left[\left(\frac{1}{R_T^3} + \frac{1}{R_C^3}\right)^\frac{3}{2} + \left(\frac{1}{T_T^3} + \frac{1}{T_C^3}\right)^\frac{3}{2} - \left(\frac{1}{Z_C^3} + \frac{1}{Z_T^3}\right)^\frac{3}{2}\right]$$  \hspace{1cm} (8)

$$c_2 = \frac{3}{2^{\frac{3}{2}}} \left[\left(\frac{1}{R_T^3} + \frac{1}{R_C^3}\right)^\frac{3}{2} + \left(\frac{1}{T_T^3} + \frac{1}{T_C^3}\right)^\frac{3}{2} - \left(\frac{1}{Z_C^3} + \frac{1}{Z_T^3}\right)^\frac{3}{2}\right]$$  \hspace{1cm} (9)

$$c_3 = \frac{3}{2^{\frac{3}{2}}} \left[\left(\frac{1}{R_T^3} + \frac{1}{R_C^3}\right)^\frac{3}{2} + \left(\frac{1}{T_T^3} + \frac{1}{T_C^3}\right)^\frac{3}{2} - \left(\frac{1}{Z_C^3} + \frac{1}{Z_T^3}\right)^\frac{3}{2}\right]$$  \hspace{1cm} (10)

The following combinations of hardening parameters $d_1$-$d_4$ are also derived from Eqs (2)-(7):

$$d_1 + d_2 = \frac{27}{2} \left(\frac{1}{R_T^3} - \frac{1}{R_C^3}\right)$$  \hspace{1cm} (11)

$$d_3 + d_4 = \frac{27}{2} \left(\frac{1}{T_T^3} - \frac{1}{T_C^3}\right)$$  \hspace{1cm} (12)

$$d_1 + d_4 = \frac{9}{4} B$$  \hspace{1cm} (13)

where $B = \frac{1}{R_T^2} - \frac{1}{R_C^2} + \frac{1}{T_T^2} - \frac{1}{T_C^2} + \frac{1}{Z_C^2} - \frac{1}{Z_T^2}$.

Besides Eqs (11)-(13), an additional equation is required to find the solutions for $d_1$-$d_4$. Such an expression can be generated using the plane strain compression data. With changing the sheet orientation, the compressive loading stresses measured in this test were used to locate the stress components at which $d\sigma_3/d\sigma_1$ are 0, $\infty$, and 1 at the yield loci [2]. The following procedure shows how to find the hardening parameter expression from the channel compression test carried out on such an orientation that the compressive load is applied along the thickness direction and the sample is expanded along the transverse direction. Denoting the compressive load in this case as $ZT$, it can be shown that $ZT = \sigma_1$ [2]. Differentiating Eq. (1a) with regard to $\sigma_1$ and noting that $d\sigma_2/d\sigma_1 = 0$ leads to the following expression:

$$\frac{1}{2} \sqrt{A}\left(\sigma_1 + c_1 \sigma_1 - c_1 \cdot ZT\right) - \frac{1}{9} \left[d_1 + d_2\right] \sigma_1^3 - 2d_1 \sigma_1 \cdot ZT - d_3 ZT^3 = 0$$  \hspace{1cm} (14)

where $\sigma_1$ is unknown stress component, and

$$A = \left(\frac{c_1 + c_2}{6}\right)^3 - \frac{c_1}{3} \sigma_1 \cdot ZT + \left(\frac{c_1 + c_3}{6}\right) ZT^3$$

Eq. (13) gives

$$d_4 = \frac{9}{2} B - b_i$$  \hspace{1cm} (15)

Inserting Eq. (15) into Eq. (14) leads to:

$$d_1 = \frac{1}{2} \sqrt{A}\left(\sigma_1 + c_1 \sigma_1 - c_1 \cdot ZT\right) - \left[\left(d_1 + d_2\right) \sigma_1^3 - \frac{B}{2} ZT^3\right]/ZT^3 - 2\sigma_1 ZT$$  \hspace{1cm} (16)
Inserting Eq. (15) into Eq. (1a) gives:

\[
d_i = \left[ \frac{(d_1 + d_2)}{27} \sigma_1^4 + \frac{(d_3 + d_4)}{27} \right] \sigma_1 \cdot ZT
\]

\[
- \frac{B}{2} \sigma_1 \cdot ZT \right] + \frac{3}{A^3 - 1} \left\{ (\sigma_1 - ZT) \sigma_1 \cdot ZT \right\}
\]

Equation (17)

Matching Eq. (16) with (17) yields an implicit expression for \(\sigma_1\) and it can be solved by numerical methods. Finally, \(d_i\) can be found from Eq. (16) or (17). In the present work, we leave \(d_i\) undetermined and set out instead to obtain a description of the evolution of key parameters with strain.

3 ANALYTICAL FLOW STRESSES

Twinning is a major deformation mode in magnesium and its effect on the flow stress \(\sigma\) can be described by a law of mixtures [3]:

\[
\sigma = (1 - X_T) \sigma_{x} + X_T \sigma_T
\]

(18)

where \(X_T\) is the volume fraction of twins, \((10\overline{1}2)\) twinning is expected, and \(\sigma_o\) is the stress in the twins in which twinning does not occur:

\[
\sigma_x = \beta E^* \sigma_{x}^{1-n} \varepsilon^n
\]

(19)

where \(\beta\) is a texture based constant; \(E\) is the Young's modulus; \(\sigma_{x}^{1-n}\) is a reference stress; \(\varepsilon\) is the hardening index; \(n\) is the imposed total macroscopic strain. The stress in the twins in which twinning does occur is given by:

\[
\sigma_T = \chi \sigma_{x}^{1-n} + (1 - \chi) \xi E^* \sigma_{x}^{1-n} (\varepsilon - \chi \varepsilon)\]

(20)

which is a law of mixtures combination of the stress in the as yet un twisted material, \(\sigma_{x}\) (a parameter that is considered to be insensitive to strain) and the stress due to slip in the twinned volume. The constant \(\xi\) reflects the effect twinning has on the stress required for glide, \(\varepsilon\) is the strain directly attributable to twinning and \(\chi\) is the twinning fraction (over the grains expected to twin) whose evolution with strain can be expressed as:

\[
\chi = 1 - \exp \left[ - \frac{4}{\varepsilon} \left( \varepsilon / \varepsilon_1 \right) \right]
\]

(21)

where \(a\) is a rate exponent and \(\varepsilon_1\) is the strain at which the twinning activity is 98% complete.

Figs 1-3 show how well Eq. (18) fits the experimental data obtained for pure Mg [2]. The values of the parameters used are listed in Table 1.

<table>
<thead>
<tr>
<th>(ZT)</th>
<th>(TZ)</th>
<th>(R_T)</th>
<th>(R_C)</th>
<th>(T_T)</th>
<th>(T_C)</th>
<th>(Z_C)</th>
<th>(Z_T)</th>
</tr>
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<tr>
<td>0</td>
<td>0.9</td>
<td>0.1</td>
<td>0.8</td>
<td>0</td>
<td>0.8</td>
<td>0</td>
<td>1.0</td>
</tr>
<tr>
<td>(\sigma)</td>
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<td>0.15</td>
<td>0.18</td>
<td>4.5</td>
<td>4.5</td>
<td>1.25</td>
<td>1.0</td>
</tr>
<tr>
<td>(\sigma_{x}^{1-n})</td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>(\sigma_{x}^{1-n})</td>
<td>4.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\xi)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\chi)</td>
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<td>1.05</td>
<td>0.75</td>
<td>1.0</td>
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<td></td>
</tr>
</tbody>
</table>

*: units in ksi.

![Fig. 1 Compressive loading stresses in the plane strain states](image1)

![Fig. 2 Yield stresses in the uniaxial tension and compression](image2)
7Z in Fig. 1 denotes the compressive load stress in the channel die compression in which the compressive load is applied along the transverse direction and the sheet is expanded along the thickness direction. The data points for Z in Fig. 3 are found from the yield surface [2] as no test data are available.

Higher values of can be seen in the case in which a tensile load is applied along the transverse directions. This is caused by the greater spread of the texture in the transverse direction such that more "easy" basal slip operates [2]. Higher values of X are obtained for the compressive loadings in the plane of the sheet. This reflects the enhanced activation of twinning in the samples oriented for extension along the crystallographic c-axis.

4 DISCUSSIONS AND CONCLUSIONS

Using the yield stresses predicted by Eq. (18) under different stress states, the evolution of the adjustable parameters in Eqs (8-13) with strain are shown in Figs 4-5. Also shown are points taken from [1]. The agreement between two is reasonable for the values of the "c" parameters. The "d" parameter combinations are not so well described. This will be explored in the future work.

It can nevertheless be concluded that the proposed evolution equations for the yield surface capture the main physical activities in Mg deformation and provide an analytical basis for the material model which can be implemented in the FE simulation using Cazacu and Barlat yield criterion.

REFERENCES

