EFFECTIVE MATHEMATICS LEARNING ENVIRONMENTS: PRIMARY TEACHERS' BELIEFS AND PRACTICES

Susie Groves and Brian Doig

Abstract
A first priority for changing teaching practice is to make problematic for teachers aspects of their current practice. As part of a project on improving mathematics and science teaching in the middle years of schooling, teachers were asked to rate their practice against components of effective teaching and learning and to rate each of these according to their perceived importance. Findings suggest that primary teachers endorsed the components as representing effective practice, scoring most components higher than their actual practice. Gaps were particularly evident for items relating to challenging students conceptually and higher-order thinking, with these becoming the basis for some of the action planning for change.

Introduction
There needs to be ... a much stronger emphasis on ... approaches to teaching and learning that stress higher order thinking and critical literacy, greater depth of knowledge and understanding and increases in overall intellectual demand and expectations of middle years students. (Luke et al., 2003, p. 8)

Improving teaching and learning in schools is a focus in all Australian education systems, with the middle years of schooling (Years 5 to 9) receiving particular attention because of the coincidence of the disengagement of students with the significance of these years for the preparation of students for their future role in society.

A prior condition for improvement in any subject is agreement on the nature of effective teaching and learning. The Improving Middle Years Mathematics and Science: The role of subject cultures in school and teacher change project is investigating the role of mathematics and science knowledge and subject cultures in mediating change processes in the middle years of schooling. The project has its roots in the Science in Schools research project (SiS), which developed a successful strategy for improving teaching and learning science based on two major aspects: the SiS Components—a framework for describing effective teaching and learning in science—and the SiS Strategy—a strategic process for planning and implementing change (see, for example, Gough & Tytler, 2001). The IMYMS project is exploring the extent to which the SiS Components and Strategy can transcend subject boundaries by working with four clusters of schools from urban and rural regions of Victoria, that are funded through the Victorian state government’s Schools for Innovation and Excellence initiative.

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Improving Middle Years Mathematics and Science: The role of subject cultures in school and teacher change (IMYMS) is funded by an Australian Research Council Linkage Grant, with Industry Partner the Victorian Department of Education and Training. The Chief Investigators are Russell Tytler, Susie Groves and Annette Gough.
Based on reviews of the literature on effective teaching (Doig, 2001; 2003) and a series of interviews with fifteen teachers nominated as being effective teachers of middle years mathematics (Tytler, 2004), the IMYMS project team redeveloped the SiS Components to produce the IMYMS Components of Effective Teaching and Learning (see Figure 1).

A major factor that informs most contemporary change strategies in science and mathematics is an emphasis on the role and responsibility of learners in constructing meaning and regulating their learning. This implies a need for teachers to re-examine their assumptions about the nature of learning and teaching (Borko & Putman, 1995; Goldsmith & Schifter, 1997). However, a first priority for changing teaching practice is to make problematic for teachers aspects of their current practice (Yackel, 1994).

This paper explores the gaps between 46 Years 5 and 6 teachers’ beliefs regarding the importance for mathematics of a range of classroom practices incorporated in the IMYMS Components and their perceptions of their current practices in mathematics. In the broader context of the project, such gaps are the driving force for the action planning process for implementing change.

Methodology

The IMYMS Component Mapping process is based on an interview between individual teachers and their school IMYMS coordinator. Each sub-component is discussed, with the teacher and interviewer agreeing on a score representing the degree of exemplification of that sub-component in the teacher’s practice. Scores out of 5 are given separately for practice in science and mathematics, and for the importance of each sub-component in each subject (Tytler, 2004). For example, a score of 5 on sub-component 5.2 corresponds to a teacher agreeing with the statement “Students regularly engage in developing explanations and argument based on evidence. I encourage and support students to express their ideas and opinions, to question evidence, to raise issues and to speculate”.

The mapping process is personal, and non-judgmental, with acknowledgement that a score of 5 is not necessarily possible or even appropriate. As well as providing data for the project, it serves as a major focus for a productive conversation about practice. All teachers in the project take part in the mapping process at the beginning and end of their two-year involvement in the project.

During 2004, the component mapping process was carried out with 44 secondary and 59 primary teachers of mathematics, and 31 secondary and 52 primary teachers of science. The data discussed below comes from the 46 primary teachers of mathematics who completed the scoring for mathematics for both their practice and their beliefs regarding importance on the five-point scale.
1. **The learning environment promotes a culture of value and respect**
   1.1 The teacher builds positive relationships through knowing and valuing each student.
   1.2 The learning environment is characterised by a sense of common purpose and collaborative inquiry.
   1.3 The learning environment provides a safe place for students to take risks with their learning.
   1.4 Persistence and effort are valued and lead to a sense of accomplishment.

2. **Students are encouraged to be independent and self motivated learners**
   2.1 Students are encouraged and supported to take responsibility for their learning.
   2.2 Students are encouraged to reflect on their learning.

3. **Students are challenged to extend their understandings**
   3.1 Subject matter is conceptually complex and intriguing, but accessible.
   3.2 Tasks challenge students to explore, question and reflect on key ideas.
   3.3 The teacher clearly signals high expectations for each student.

4. **Students are supported to develop meaningful understandings**
   4.1 Teaching strategies explore and build on students’ current understandings.
   4.2 Individual students’ learning needs are monitored and addressed.
   4.3 Students are supported to make connections between key ideas.
   4.4 Teaching sequences promote sustained learning that builds over time.
   4.5 Learning sequences involve an interweaving of the concrete and the abstract/conceptual.

5. **Students are encouraged to see themselves as mathematical and scientific thinkers**
   5.1 Students are explicitly supported to engage with the processes of investigation and problem solving.
   5.2 Students engage in mathematical/scientific reasoning and argumentation.

6. **Mathematics and science content is linked with students’ lives and interests**

7. **Assessment is an integral part of teaching and learning**
   7.1 Learners receive feedback to support further learning.
   7.2 Assessment practices reflect all aspects of the learning program.
   7.3 Assessment criteria are made explicit.

8. **Learning connects strongly with communities and practice beyond the classroom**
   8.1 The learning program provides opportunities to connect with local and broader communities.
   8.2 Learners engage with a rich, contemporary view of mathematics and science knowledge and practice.

9. **Learning technologies are used to enhance student learning**

*Figure 1: The IMYMS Components of Effective Teaching and Learning*
Results and discussion

An initial analysis of teachers’ responses by sub-component shows that teachers scored the importance of each sub-component (SC) of the IMYMS Components more highly than their practice. For example, Figure 2 shows that a much greater proportion of teachers rated the importance of both SC5.1 and SC5.2 as “highly important” (a score of 5) than those who rated their own practice as being at the highest level (again a score of 5). Similarly, while no one rated either sub-component as “not important” or “slightly important” (a score of 1 or 2), a small proportion of teachers rated their own practice as 2 or 1 (for SC5.2 only).

![Comparison of teachers' practice (P) and beliefs about importance (I) for the sub-components of IMYMS Component 5](image)

Results for all sub-components followed a similar pattern, with only five of the twenty-three sub-components having any teachers scoring their practice as 1 (sub-components 1.2, 3.1, 5.2, 7.3 and 9), and only a total of six of the twenty-three sub-components having any teachers scoring importance as 1 (sub-components 3.2, 7.3, 8.1 and 8.2) or 2 (sub-components 1.2, 2.2, 3.2 and 7.3). Space prevents all of the comparisons from being included here in the format shown in Figure 2.

In order to provide a better analysis and reporting of the results, teachers’ responses to both their view of their practice and the importance of each of the twenty-three sub-components were analysed using the Master’s Partial Credit Model with the Item Response Theory (IRT) software Quest (Adams & Khoo, 1993). The item response approach (see, for example, Bond & Fox, 2001) does not assume, like traditional analyses, that it is equally easy, or hard, to endorse every stimulus statement. For example, it may be much easier to regard SC1.4 “Persistence and effort are valued ...” as “highly important” than to regard SC8.2 “Learners engage with a rich contemporary view of mathematics ...” as “important”. The result of this form of analysis is an interval scale that provides
indications of the ease, or difficulty, that respondents have in agreeing with the stimulus statements of a rating scale, with both the difficulty of selecting a particular category of response and the “attitude” of each respondent measured on the same scale. This enables us to estimate the likelihood of the category a respondent would select for any item, if we know their overall score. More importantly, it also allows a more rigorous analysis of the responses than that provided by the sole use of bar graphs such as those given in Figure 2.

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Figure 3: Item map of primary teachers’ practice and beliefs (importance) for the IMYMS sub-components

Figure 3 shows an item map of the 46 primary teachers' practice and beliefs for the IMYMS sub-components, based on an analysis using Quest. Each X on the (vertical) "logit" scale represents one teacher. For each of the 46 items (the 23 sub-components for practice and importance) each "threshold" is also mapped against the same logit scale. So, for example, P4.2.5 indicates the threshold at which a responder is more likely than not to score themselves as 5 for practice on SC4.2. It also shows that only the teacher who was most positive about the mapping overall (mark X just above 3.0) has a probability of 0.5 of rating their practice for SC4.2 as 5. This teacher would have a probability less than 0.5 of rating their practice on SC1.2 as 5, and a probability greater than 0.5 of rating all other sub-components as 5. The fact that the lowest threshold shown is P4.1.3 indicates that all teachers scored their practice at least at 2 on SC4.1.

<table>
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<th>Threshold</th>
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<tr>
<td>-2.0</td>
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</tbody>
</table>

Each X represents 1 teacher.
While space does not allow a complete analysis of the data shown in Figure 3, a number of observations can be made. Perhaps most importantly, as commented earlier, Figure 3 shows that for almost every sub-component and every level of response, teachers’ views of the importance exceeded their scores for their own practice. An extreme example of this occurs for SC4.2 “Individual students’ learning needs are monitored and addressed”, where P4.2.5 is located above 3.0 on the logit scale indicating that very few teachers – in fact one teacher only – had a probability of at least 0.5 for scoring their practice at level 5, while the corresponding logit value for I4.2.5 is below zero and the scale shows that every teacher’s probability for scoring importance at level 5 was greater than 0.5. This is an interesting result in itself, showing that teachers attach a high importance to this aspect of their teaching but are not able to rate their practice equally highly (although a score of 4 for practice was relatively frequent as shown by the position of P4.2.4 just above zero).

Moreover, teachers’ views can be seen as validating the importance of the sub-components for effective teaching and learning, with views of importance being negative only extremely rarely. This can be seen by the fact that for importance a total of only six sub-components had 2 or 3 as the lowest threshold, with these also occurring low on the logit scale showing that very few teachers held these views.

Among the sub-components themselves, apart from SC4.2, the sub-components placed highest on the logit scale for scoring 5 on teacher practice were 1.2, 5.2, 3.2, 3.1 and 4.5. This is of some concern as these sub-components deal with challenging students’ conceptual understandings and expecting them to engage in higher-order mathematical thinking. These sub-components also had moderately high thresholds for scores of 5 for importance on the logit scale (between about 0.25 and 0.7).

Of particular interest also are SC8.1 “The learning program provides opportunities to connect with local and broader communities” and SC8.2 “Learners engage with a rich, contemporary view of mathematics ... knowledge and practice”. These not only have a high threshold for a score of 5 for practice, but are also the highest placed for scores of 4 for practice, indicating that relatively few teachers gave themselves a “positive score” of 4 or 5. These two sub-components are also highest on the logit scale for a score of 5 for importance, indicating that relatively few teachers believed these to be highly important aspects of their teaching. This suggests that these aspects are more closely linked to teachers’ beliefs and practice in science than mathematics.

Conclusion

As reported above, the results of the data analysis suggest that teachers endorse the IMYMS Components as representing aspects of effective mathematics teaching, at least at the primary school level. Furthermore, while
results were reported in a different format to teachers, schools and clusters, the
gaps between teachers’ views of effective teaching and learning and their actual
practice have resulted in schools and clusters developing action plans for change
focusing on many of the aspects discussed above. In particular, there has been a
focus on higher-order thinking, promoting student reflection, assessment
(particularly in relation to assessment criteria and rubrics, which can also be
seen from Figure 3 to be an area where there is a gap), and developing
community links (particularly in science, but also in some instances in
mathematics at the secondary level).

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