IMPETUS TO EXPLORE: APPROACHING OPERATIONAL DEFICIENCY OPTIMISTICALLY

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‘Impetus to explore’ is studied through post-lesson video-stimulated student interviews. This impetus focuses ‘spontaneous’ decisions to explore unfamiliar mathematics. Uncertainty, quests for elegance and curiosity have been found to contribute to this impetus. This study identified other contributing factors: inability to undertake the teacher’s task, ‘optimistic explanatory style’ (Seligman, 1995), and identifying relevant complexities (Williams, 2002). Explanatory style affects how a person perceives successes and failures. Enactment of optimism in this study illuminated its role in creating an impetus to explore. Re-conceptualising ‘operational deficiency’ (Chen & Siegler, 2000) to include absence of resources to undertake a task illuminated a task design feature that can increase exploration.

INTRODUCTION

This is a study of how an inability to proceed with the teacher’s task can sometimes lead to ‘spontaneous’ exploratory activity. It focuses upon the factors that contribute to a spontaneous decision to explore in such cases. The term ‘spontaneous’ refers to student-directed activity over a time interval when there is no mathematical input from external sources (see Williams, 2004). This study is part of a broader study of creative mathematical thinking and the social and personal influences upon this thinking (Williams, 2005b). One case is used to illustrate the findings from this study. Some other cases have previously been reported (e.g., Williams, 2003b, 2005a).

THEORETICAL FRAMEWORK

Uncertainty, quests for elegance, and curiosity can lead to an ‘incentive to enquire’ or ‘impetus’ to move to a higher level of thinking (see, Goodson-Espy, 1998). This impetus to explore has been identified as an area requiring further study (Goodson-Espy, 1998). Chen and Siegler (2000) found that exploratory activity sometimes occurred when toddlers experienced difficulty manipulating a tool to drag a toy (operational deficiency) towards them on across a table (e.g., they tried another tool, or leaned over to try to reach the toy). This study examines how exploratory activity can result when students are unable to carry out a mathematical task to meet a teacher-set goal. To examine why some students ‘spontaneously’ explored, Seligman’s (1995) dimensions of optimism/resilience (permanent-temporary, pervasive-specific, personal-external) were employed to find how students perceived the failures they encountered. It was hoped that evidence of students enacting ‘optimism’ during lessons would be found. A child with an optimistic explanatory style (Seligman, 1995) perceives successes as ‘permanent’ (“I
succeeded, I can do this”), ‘pervasive’ (“I succeeded, I am good at this”), and ‘personal’ (“I achieved this”), and failures as ‘temporary’, ‘specific’, and ‘external’ (“I failed this time, I will examine the situation to see what I can change to increase my chances of succeeding next time”). Seligman linked the building of optimism with success in flow situations. Flow (Csikszentmihalyi, 1992 in Williams, 2002) is a state of high positive affect during creative activity. This state occurs during mathematical exploration when a student identifies a mathematical complexity, spontaneously decides to explore it, and subsequently develops new conceptual knowledge (Williams, 2002). This success in creating new knowledge is seen as optimism building. Thus, impetus to explore is linked to the creation of flow situations.

This study examines the nature of the ‘failures’ students encountered when they tried to undertake the mathematical task set by the teacher. It also examines the personal characteristics of the students and the ways they found to circumvent the failures they encountered. The study is focused by the research question: “What factors contribute to an impetus to explore when a student is unable to undertake the mathematical task as set by the teacher?”

**RESEARCH DESIGN**

Data was generated as part of the international Learners’ Perspective Study. Six Year 8 classes (from Australia and the USA) were studied to find evidence of creative student thinking. Data was collected from each classroom over at least ten consecutive lessons. Three cameras simultaneously captured the activity of the teacher, a different pair of focus students each lesson, and the whole class. A mixed video image was produced during the lesson (focus students at centre screen and teacher as an insert in the corner). This mixed video image was used to stimulate student reconstruction of their thinking in post-lesson interviews and the video image from the teacher tape stimulated teacher discussion. Students (and the teacher) were asked to identify parts of the lesson that were important to them, and discuss what was happening, and what they were thinking and what they were feeling. Through this process, students who explored mathematical complexities to generate novel mathematical ideas and concepts were identified and social and personal influences upon their thinking were made explicit through their discussion of the lesson video.

Ericsson and Simons (1980) have shown that verbal reports can provide valid data when attention is given to research design. The interview probes fitted with Ericsson and Simon’s (1980) findings about how to generate high quality verbal data associated with cognitive activity. Salient stimuli (mixed image lesson videos) were used to stimulate student reconstruction, probes focused on lesson activity and what students were thinking (rather than the interviewer asking general questions), and students focused the content of the interview through what they attended to in the lesson video. Ericsson and Simons have shown that where the researcher asks specific questions that include constructs the subject has not previously reported,
the subject is more likely to “generate answers without consulting memory traces” (Ericsson & Simons, 1980, p. 217). On the other hand, if a subject spontaneously “described one or more specific sub-goals, and these were both relevant to the problem and consistent with other evidence of the solution process” (Ericsson & Simons, 1980, p. 217) there was stronger evidence that the reported activity occurred.

Interviews, in conjunction with the lesson video, were used to identify intervals of time from when students first encountered difficulties with the teacher’s task to when they spontaneously explored an identified complexity. To identify social and personal influences contributing to the impetus to explore in such situations, simultaneous analysis of student enacted optimism (Seligman, 1995) and the social and cognitive elements of the process of abstracting (Dreyfus, Hershkowitz, & Schwarz, 2001) was undertaken. These are elaborated through the illustrative case.

SITES, SUBJECTS AND CONTEXT

Only 5 of 86 students were identified creatively developing novel mathematical ideas and concepts. These five students varied in their mathematical performances, cultural backgrounds, socio-economic status, gender and the classes they attended. Other students in these classes may have engaged in spontaneous exploratory activity during the research period and not been identified because two students were the primary focus of study each lesson. There were occasions (including this illustrative case) where the student identified spontaneously exploring was not the focus student that lesson. In such cases, the student was selected as a focus student the following lesson. The validity of evidence that relied heavily on interview reconstruction has been justified in descriptions of how the interview was undertaken (see above). The student in this case (Eden) was visible on the student camera when he moved across to view Darius’ screen. At other times he was visible on the whole class camera.

As three students undertook spontaneous exploratory activity on more than one occasion, eight spontaneous explorations were identified in total. Each of the five students was found to have an optimistic explanatory style (Williams, 2003a). Curiosity (2 students), a quest for elegance (2 students), or operational deficiency—lack of adequate resources to perform the teacher task— (4 students) contributed to their impetus to explore. Operational deficiencies differed in nature: absence of appropriate cognitive artefacts (3 students including Eden, e.g., see Williams, 2003b) or absence of physical resources (1 student, see Williams, 2005a).

Eden attended an Australian inner-suburban government school. The students in this class had all achieved high results in mathematics the previous year. The student population was drawn from across the city because the school had a high academic reputation. Students’ socio-economic and cultural backgrounds varied markedly.

Eden sat quietly in class, listened, and undertook the required work. He did not engage in the types of disruptive classroom activity that were often instigated by one of his out-of-class friends. He was a conscientious independent thinker who scored in
the top 5% for mathematical problem solving on a national test. His teacher perceived him as an average student in mathematics and had decided not to recommend him for the advanced stream the following year. Eden displayed indicators of optimism (Seligman, 1995) on all three dimensions in his post-lesson interview and no indicators of lack of optimism (see Table 1). Table 1 lists the three dimensions of optimism [Column 1] and the indicators of optimism [Columns 4, 5] and indicators of lack of optimism [Columns 2, 3] in relation to both successes and failures.

Eden perceived prior learning as able to help in “a similar circumstance” (Success as Permanent), that “work[ing] everything out for yourself … you will be able to think clearer” (Failure as Temporary), and that individual effort would lead to success (Success as Personal). Eden’s teacher’s perception that he had average ability in mathematics was contrary to the evidence from the problem solving questions on a national test. Rather than perceiving ‘failure’ (his teacher’s perception of him as average) as a personal attribute (Failure as Pervasive), Eden constrained his use of the term average to the classroom in which that assessment was made. “In the class [researcher’s emphasis]” he was average and there was “no way to explain it” (Failure as Specific to the class; Failure as External, it was the assessment of another). Eden preferred problem solving to work on basic skills.

Problem solving's pretty good to work out and stuff ... you've gotta (pause) use your mind a little bit more than just (pause) know how to add up sums and stuff [Eden, post-lesson interview].

In the atypical lesson under study (Lesson 6, where the teacher did not present a rule and expect students to practise it), students were seated side by side at with their own computers (Eden next to Darius). The teacher stated the general equation for a linear function without discussing the role of the constants, and students commenced the game ‘Green Globs’. Green Globs is a computer game that displays a Cartesian Axes System with 13 randomly placed ‘globs’ on integer co-ordinates. Trajectories of linear graphs are used to hit globs. Students input linear equations and obtain larger scores when they hit more globs with a single line (one for the first glob hit, two for the second, four for the third and so on). Although the class had not studied linear functions previously, Eden’s Year 7 teacher had accelerated some students: “last year we did a bit on this stuff except I had forgotten most of it” [Eden’s interview]. Eden was initially unaware of the general equation for a linear function, did not know that real number laws applied within an algebraic equation, and did not know what a gradient was. Students chose their own directions of exploration, and discussed ideas with those around them. When asked questions, the teacher provided assistance with the computer program but did not give directions or hints on how to proceed with the game. Darius was intent upon manipulating numbers to hit globs and gain a high score. Eden wanted to know why equations positioned the lines as they did: “I didn't exactly know why it always happened like that” [Eden’s post-lesson interview].

Transcript notation used in this paper: … transcript omitted without altering meaning; - change in flow of talk; [text ] researcher explanations within transcript
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* Transcript containing multiple indicators
ANALYSIS AND RESULTS

Eden spent 15 minutes trying to find how to generate sloping lines that hit globs. For a start (12:50 Mins into the lesson), he asked Darius; “What's the rule for that [sloping line on Darius’ screen]? That's the sort of angle” and at 27:54 Mins he still did not know “[Eden to Darius generating Figure 1] I don’t know how you get that”.

During those 15 minutes, Eden worked out how to generate horizontal lines and that real number operations apply within equations. At 27:54 Mins, when Darius did not respond, Eden remained motionless, watching the dynamic display Darius generated [27:58-28:15 Mins] before returning to his own computer and working intently for seven minutes [captured on the whole class camera] then exclaiming: “y … crosses over with x” [35:12 Mins]. He described what he had realised in his interview: “Well (pause) the graph's drawn up already (pause) for you to look at- that's the only help you get to answer”. He pointed to the graph and later a table to aid his explanation:

You have to work out (pause) what y was (pause) which was … minus three (pause) minus two (pause) minus one (pause) and zero … y is all the time it is always one behind it … Then the rule (pause) is ah (pause) would be (pause) um (pause) y (pause) equals (pause) x (pause) minus one”

Eden focused simultaneously on the y value and x value of each co-ordinate (‘synthetic-analysis’, a subcategory of ‘building-with’, see Dreyfus, Hershkowitz, & Schwarz, 2001; Williams, 2005a) and described the relationship (when y is minus three, x is minus two, when y is minus one, x is zero). After ‘recognizing’ this relationship (the y value is always one less than the x value), Eden formulated the equation (‘constructing’, Dreyfus, Hershkowitz, & Schwarz, 2001). Focusing his attention idiosyncratically on Darius’ screen, Eden identified a complexity (the relationship between the y and x values) and spontaneously explored its potential.

DISCUSSION AND CONCLUSIONS

Eden lacked the necessary cognitive artefacts to proceed with the Green Globs game using the general equation given, so he found a way to proceed using relationships between x and y values of ordered pairs instead. Kerri (Williams, 2005a) forgot her graph paper in a test so used a sketch and the Cartesian Axis System in a way that she
had not been taught. Dean (Williams, 2003b) could not recognize angles in polygons so drew upon the ‘180 degrees in each triangle’ rule instead (a connection not made in class). In each case, the student encountered an operational deficiency (absence of cognitive or physical resources) that denied them access to the mathematics the teacher required. Undeterred by this ‘failure’ these optimistic students searched for a way to circumvent the difficulty they encountered. Thus, they identified an unfamiliar mathematical complexity that appeared potentially productive and impetus to explore resulted. Operational deficiency, optimism, and identifying a relevant complexity, together contributed to the impetus to explore.

For example, Eden persevered when he could not generate sloping lines, because he perceived failure as temporary and able to be overcome through personal effort. He did not consider his failure as confirming a negative attribute of himself but rather perceived the failure as specific to the situation. Thus, he changed aspects of his exploration to find a way to overcome his difficulties (e.g., changed the type of line studied, considered legitimate operations with symbols, changed his focus from the equation to the co-ordinates) (Failure as Specific not Pervasive). Eden’s perception that failure was specific was crucial to his finding a mathematical complexity that focused his spontaneous exploration. Eden displayed indicators of optimism in his interview and enacted optimism in his response to failure in class. This supports Sfard and Prusak’s (2005) position: “[we can] accept the discourse as such … Words are taken seriously and shape one’s actions” (Sfard & Prusak, 2005, p. 51).

Both Eden’s optimism, and the complexity he identified through his optimistic activity contributed to his impetus to explore. Without the failure arising from his operational deficiency, his optimistic response to failure may not have been activated and the complexity that led to new insight may not have been identified. Eden created a flow situation when he spontaneously decided to explore an identified complexity to developed new knowledge (Williams, 2002). Theoretically, success with this activity should strengthen his optimistic orientation (Seligman, 1995). Longitudinal studies are required collect empirical evidence of this theorised link. Tasks like Green Globs, with many opportunities for students with varying mathematical backgrounds to encounter operational deficiencies, would assist in creating opportunities for students to experience flow and researchers to study optimism-building activity.

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References

Williams


