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Robust Optical Flow with Combined Lucas-Kanade/Horn-Schunck and Automatic Neighborhood Selection

Nicholas Bauer, Pubudu Pathirana, and Peter Hodgson

Department of Engineering and Technology
Deakin University
Waurn Ponds, Vic
njbau@deakin.edu.au

I. ABSTRACT

Differential optical flow methods are widely used within the computer vision community. They are classified as being either local, as in the Lucas-Kanade method, or global, such as in the Horn-Schunck technique. Local differential techniques are known to have robustness under noise, whilst global techniques are able to produce dense optical flow fields. We will show that the Horn-Schunck Technique, when combined with Lucas-Kanade, can yield the advantage of having both robust and dense optical flow fields. Selection of neighborhood size is an important tuning parameter for the combined Lucas-Kanade/Horn-Schunck technique. Choosing the optimal neighborhood is a difficult task and greatly effects the performance of optical flow results. We outline a method for the automatic selection of neighborhood size based on Stein's Unbiased Risk Estimator (SURE). Algorithms are derived for a combined Lucas-Kanade/Horn-Schunck technique with automatic neighborhood selection. The performance of SURE neighborhood selection for the combined optical flow technique is simulated via Matlab, providing an illustration of the performance that is attainable.

II. INTRODUCTION

Estimating motion from image data sequences is an important topic in computer vision, and has been studied extensively over the past 20 years, e.g. [9],[12]. The process of determining optical flow is generally carried out through utilizing a brightness constancy constraint equation (BCCE), which makes use of spatiotemporal derivatives of image intensity, [9], [12]. Determining optical flow using the BCCE is an ill-posed problem. This arises from the fact that when a straight moving edge is viewed through a narrow aperture, the only motion component that can determined is perpendicular to that edge, [13]. There are several methods to overcome the ill-posedness of differential techniques, two of which are the Lucas-Kanade and Horn-Schunck techniques. Methods utilizing the BCCE are referred to as differential techniques. Differential techniques can be classified as either local or global. Local techniques involve the optimization of a local energy functional, as in the Lucas and Kanade method [12], or the the frequency based minimization methods found in [2] and [3]. The global category refers to methods that determine optical flow through minimization of a global energy functional, as in Horn and Schunck [9], and numerous other discontinuity preserving approaches, [11], [7], [4], [6], [8], [14], [17]. Differential techniques are widely used due their high level of performance, [10]. Local and global methods each have their own advantages and disadvantages. Local methods offer a robustness to noise, but lack the ability to produce dense optical flow fields. Global techniques produce provide 100 percent dense flow fields, but have a much larger sensitivity to noise, [10], [1].

The selection of tuning parameters is an important issue when calculating optical flow e.g. [15], [16], [18]. None of these approaches involve developing neighborhood selection for combined Lucas-Kanade/Horn-Schunck robust optical flow.

This paper involves combining local and global methods of Lucas-Kanade and Horn-Schunck, to obtain a method which generates dense optical flow under noisy image conditions. Neighborhood selection is though the minimization of mean

III. BACKGROUND: OPTICAL FLOW

A. Brightness Constancy Constraint Equation

There are a variety of optical flow techniques that utilize spatiotemporal derivatives of image intensity e.g. [9],[12]. Assuming that the intensity of part of an object within an image is conserved, the first-order derivatives based on image translation are given by

$$I(z,t) = I(z-vt,0)$$  (1)

where $v = (u,v)^T$. Assuming the motion of the object is small, a brightness constancy constraint equation (BCCE) is formed through use of a Taylor series expansion of (1). The process is as follows.

$$I(z,t) = I(z-vt,0) + \frac{\partial I}{\partial x} I(z-vt,0) dx \quad (2)$$

$$+ \frac{\partial I}{\partial y} I(z-vt,0) dy + \frac{\partial I}{\partial t} I(z-vt,0) dt + ...$$

Excluding the higher-order terms and applying condition (1) we have

$$\frac{\partial I}{\partial x} I(z-vt,0) dx + \frac{\partial I}{\partial y} I(z-vt,0) dy \quad (3)$$

$$+ \frac{\partial I}{\partial t} I(z-vt,0) dt = 0,$$
and forming velocities, 
\[
\frac{\partial I}{\partial x} I(z - vt, 0) \frac{dx}{dt} + \frac{\partial I}{\partial y} I(z - vt, 0) \frac{dy}{dt} + \frac{\partial I}{\partial t} I(z - vt, 0) \approx 0,
\]
(4)
The equation becomes exact in the limit as \(dt\) tends to zero. Hence the BCCE is
\[
I_x u + I_y v + I_t = 0,
\]
(5)
where \(u = dx/dt\), \(v = dy/dt\), \(I_x = \partial I/\partial x\), \(I_y = \partial I/\partial y\), and \(I_t = \partial I/\partial t\).
\(I_x\) and \(I_y\) are the spatial rates of change of intensity, i.e., how fast the intensity changes across the image, and it represents the speed at which the intensity is changing with time at a given pixel. The BCCE is the fundamental constraint equation on which differential optical flow techniques are based.

B. Differential Techniques

Differential techniques [9, 12], determine optical flow through the use of image intensity gradients. The problem is ill-posed in the sense that small intensity perturbations in the image can create large variations in the derivative. The aperture problem also adds to the ill-posedness of optical flow calculations which use derivative techniques [5]. The ill-posedness of the optical flow problem is rectified by first filtering the image for differentiation followed by introducing additional constraints to work in conjunction with the BCCE. The idea being to develop another equation with the same unknowns as (5), then solve the equation for the optical flow components \(u\) and \(v\).

1) Lucas and Kanade: The Lucas and Kanade algorithm [12] is an intensity-based differential technique used to align a set of images. This method assumes that the optical flow field \(v = (u, v)^T\) is constant within a neighborhood region of pixels. The optical flow is calculated through non-linear optimization, applying a weighted least-squares fit of local constraints (5) to a constant model for \(v\) for each spatial neighborhood \(N\). The velocity estimate is given by minimizing
\[
C(v) = \sum_{z \in N} W^2(z) |\nabla I(z, t) \cdot v + I_t(z, t)|^2
\]
(6)
For a neighborhood region of \(n\) pixels \(z_i \in N\), at time \(t\),
\[
A = [\nabla I(z_1), \ldots, \nabla I(z_n)],
\]
(8)
\[
W = \text{diag}[W(z_1), \ldots, W(z_n)],
\]
(9)
\[
b = [I_t(z_1), \ldots, I_t(z_n)].
\]
(10)
Expanding the cost function, \(C(v)\), gives
\[
\hat{v} = \arg\min(v^T A^T W^2 A v - v^T A^T W^2 b - b^T W^2 b)
\]
(13)
Setting \(dC/dv = 0\) identifies the minimum of the cost function \(C(v)\), i.e., fits the two scalar velocities, \(u\) and \(v\), to a curve representing the BCCE. Hence,
\[
\frac{dC}{dv} = (A^T W^2 A v - A^T W^2 b) = 0
\]
(14)
Therefore,
\[
A^T W^2 A v = A^T W^2 b
\]
(15)
The velocity estimate, \(\hat{v}\), is given by,
\[
\hat{v} = [A^T W^2 A]^{-1} A^T W^2 b
\]
(16)
In order to solve (16) \(A^T W^2 A\) must be invertible i.e., it must not be singular and be of full-rank. The eigenvalues of \(A^T W^2 A\) should not be too small (causes noise issue) and the ratio of the largest eigenvalue to the smallest should not be too large, i.e., matrix must be well-conditioned.

Selecting the size of the local estimation window (neighborhood \(N\)) is a critical tuning parameter when implementing the Lucas-Kanade method. Automatic neighborhood selection has been proposed by [18], based on minimizing an estimator of intensity based mean squared error.

2) Horn and Schunck: Horn and Schunck [9] devised an alternative way to determine the optical flow parameters \(u\) and \(v\). As in [12], the method involves evaluating first-order intensity differentials, but rather than evaluating in a local window \(N\) [12], Horn and Schunck introduce a global smoothness constraint which assumes that brightness patterns within an image vary smoothly everywhere. The smoothness constraint is given by minimizing
\[
\sum_{(\partial u/\partial x)^2 + (\partial u/\partial y)^2 + (\partial v/\partial x)^2 + (\partial v/\partial y)^2,}
\]
(17)
which can also be expressed as,
\[
\|\nabla u\|_2^2 + \|\nabla v\|_2^2.
\]
(18)
The optical flow velocity \(v\) is then determined by minimizing the squared error quantity of the BCCE (5) and smoothness constraint (17). The error to be minimized is
\[
E^2(z, t) = \int_D ((\nabla I \cdot v + I_t)^2 + \alpha^2(\|\nabla u\|_2^2 + \|\nabla v\|_2^2)) \, dx \, dy
\]
(19)
The variable \(\alpha^2\) is a weighting term that identifies the influence of smoothness constraint in the optimization. Horn and Schunck [9] selected the magnitude of \(\alpha^2\) to be approximately proportional to the estimated noise in \(I_x^2 + I_y^2\). The minimization problem in (19) can be reduced to solving a set of differential equations. The equations are generated using calculus of variation, giving,
\[
(\alpha^2 + I_x^2)u + I_x I_y = \alpha^2 u - I_x I_t
\]
(20)
\[
I_x I_y u + (\alpha^2 + I_y^2) v = \alpha^2 v - I_y I_t
\]
(21)
A solution to equations (20) and (21) is
\[ u = \hat{u} - \frac{I_x(\hat{u} + I_y \hat{v} + I_t)}{\alpha^2 + I_x^2 + I_y^2} \]
\[ v = \hat{v} - \frac{I_y(\hat{u} + I_y \hat{v} + I_t)}{\alpha^2 + I_x^2 + I_y^2} \]  
(22)
(23)
In order to minimize equations (22) and (23), [9] suggest an iterative solution process, giving,
\[ u^{n+1} = \hat{u} - \frac{I_x(\hat{u} + I_y \hat{v} + I_t)}{\alpha^2 + I_x^2 + I_y^2}, \]
\[ v^{n+1} = \hat{v} - \frac{I_y(\hat{u} + I_y \hat{v} + I_t)}{\alpha^2 + I_x^2 + I_y^2}, \]  
(24)
(25)
where \( \hat{u} \) and \( \hat{v} \) are the neighborhood velocity averages of \( u \) and \( v \) at iteration \( n \).

The Horn and Schunck method assumes that there is no errors in the spatial derivatives during the capturing process. They also assume that the temporal derivative is affected by uncorrelated Gaussian noise.

C. Combining Lucas/Kanade and Horn/Schunck

As discussed earlier, the Lucas Kanade technique involves applying a locally implemented, weighted least squares fit of local constraints (BCCE) to a constant model for \( v \). (6). Combining (6) with the global smoothness constraint proposed by Horn and Schunck (17) the following cost function is derived.

\[ E^2(z,t) = \int_D (W_N^2 (\nabla I \cdot v + I_t)^2 \]
\[ + \alpha^2 (\|\nabla u\|^2 + \|\nabla v\|^2)) dx dy \]
(26)

where \( W_N \) specifies a weighting parameter for neighborhood size \( 'N' \).

Minimization is performed by finding suitable values for \( u \) and \( v \). Using the calculus of variations the following Euler-Lagrange equations are derived.

\[ W_N^2 I_x (I_x u + I_y v + I_t) - \alpha^2 \frac{d}{dt} \left( \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \right) = 0 \]  
(27)
\[ W_N^2 I_y (I_x u + I_y v + I_t) - \alpha^2 \frac{d}{dt} \left( \frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} \right) = 0 \]  
(28)
Introducing a Laplacian into (27)and (28)gives

\[ (\alpha^2 - \alpha^2) \nabla^2 u = 0 \]
\[ (\alpha^2 - \alpha^2) \nabla^2 v = 0 \]  
(29)
(30)
where the Laplacian of \( u \) and \( v \) is defined as

\[ \nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \]
\[ \nabla^2 v = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \]  
(31)
(32)
An estimate of the Laplacian of \( u \) and \( v \) is

\[ \nabla^2 u \approx \hat{u} - u \]
\[ \nabla^2 v \approx \hat{v} - v \]  
(33)
(34)
Approximating the Laplacian, (29) and (30) are expressed as

\[ u(\alpha^2 + W_N^2 I_x^2) = \alpha^2 \hat{u} - W_N^2 I_x I_y v - W_N^2 I_x I_t \]
\[ v(\alpha^2 + W_N^2 I_y^2) = \alpha^2 \hat{v} - W_N^2 I_x I_y u - W_N^2 I_x I_t \]  
(35)
(36)
The determinant of the coefficient matrix is \( \alpha^2 (\alpha^2 + W_N^2 I_x^2 + W_N^2 I_y^2) \). Solving for \( u \) and \( v \) yields

\[ (\alpha^2 + W_N^2 I_x^2 + W_N^2 I_y^2) u = \hat{u}(\alpha^2 + W_N^2 I_x^2) - W_N^2 I_x I_y v - W_N^2 I_x I_t \]
\[ (\alpha^2 + W_N^2 I_x^2 + W_N^2 I_y^2) v = \hat{v}(\alpha^2 + W_N^2 I_y^2) - W_N^2 I_x I_y u - W_N^2 I_x I_t \]  
(37)
(38)
There now exists equations to solve for optical flow vectors \( u \) and \( v \) for every point in an image. These equations can be solved through an iterative process, such as

\[ u^{n+1} = u^n - \frac{W_N^2 I_x (I_x u + I_y v + I_t)}{(\alpha^2 + W_N^2 I_x^2 + W_N^2 I_y^2)} \]
\[ v^{n+1} = v^n - \frac{W_N^2 I_y (I_x u + I_y v + I_t)}{(\alpha^2 + W_N^2 I_x^2 + W_N^2 I_y^2)} \]  
(39)
(40)
where the average of previous velocity estimates \((\hat{u}, \hat{v})\) are used along with derivative estimates to obtain a new velocity estimates \((u^{n+1}, v^{n+1})\).

IV. AUTOMATIC NEIGHBORHOOD SELECTION

Neighborhood selection is a key tuning parameter for the combined Lucas-Kanade/Horn-Schunck technique. The optimal neighborhood size differs between image data sets and greatly effects the accuracy of optical flow calculations. Combined Lucas-Kanade and Horn-Schunck was suggested in [5]. However, neighborhood selection criteria for the combined approach was not investigated. This section outlines a method for the automatic selection of the neighborhood size. The method involves using Stein's unbiased risk estimator (SURE), [19]. The optimal neighborhood size is given by minimising the risk in the following functional.

\[ R = E[\rho - \hat{\rho}]^2 \]
\[ R = E[A^T(v - \hat{v})] \]  
(41)
(42)
where \( \hat{\rho} \) is the estimated image intensity; \( \rho \) is the true image intensity; and \( E \) is an expectation operator. It would be idealistic to solve (41) directly, but since \( v \) is unknown, an unbiased risk estimator, \( \hat{R} \), is constructed and minimized instead.

\[ \hat{R} = \|\epsilon\|^2 + \sigma^2 - 2\sigma^2 \text{trace} \left( \frac{\partial e}{\partial y} \right) \]
(43)
Isolating the terms that are dependent on the neighborhood size leaves

\[ \hat{R} = \|\hat{\rho}\|^2 + 2\hat{\rho} \hat{b} + 2\sigma^2 \hat{\rho} \frac{\partial \hat{\rho}}{\partial y} \]  
(44)
Applying the combined optical flow estimator to (44) gives
\[ \hat{R} = \Sigma((A^T)v) + 2v(A^T + 2v^2(A^T W^T A W)) \] (45)
\( \hat{R} \) is known as Stein's Unbiased Risk Estimator (SURE).

The risk is evaluated for a range of neighborhood sizes, upon which the neighborhood size corresponding to a minimum risk is selected. The results are as follows.

V. EXPERIMENTS

The first of the experiments involved evaluating various optical flow techniques on the Rubik's cube sequence; figure 1. This sequence involves a Rubik's cube rotating counterclockwise on a turntable. The optical flow calculations were performed on frames 14 and 15. There is sufficient flow within this frame set to illustrate the achieved results. Optical flow was first calculated using the global Horn and Schunck technique; figure 2. A dense optical flow is produced where the smoothness operator \( \alpha \) of the Horn and Schunck algorithm (19) regularizes the image in areas of large intensity fluctuations, allowing for regions of uncertainty to be filled-in from data obtained in neighboring areas. In figure 3, the Lucas-Kanade method was implemented on the image sequence. The optical flow shows errors in surrounding regions to the cube, where the image gradient vanishes. This can be explained by the need for the Lucas-Kanade algorithm (16) to be invertible to produce reliable optical flow results. In regions where the small eigenvalue of the system matrix is approaching zero, the aperture problem still exists. The combined Lucas-Kanade/Horn-Schunck method 26, as shown in figure 4, produces similar results to that of Horn and Schunck's technique. This is to be expected due to the image sequences lack of noise.

Optical flow tests were conducted on the Rubik's cube sequence with random noise added to the image data; figure 5. The Horn and Schunck technique shows an inability to cope under noisy conditions; figure 6. This issue is brought about by the noise producing high image gradients. These high gradients weight the smoothness data term in the Horn and Schunck algorithm (19), reducing the importance of \( \alpha \), and hence reducing regularization. The Lucas-Kanade technique, as seen in figure 7, performs quite well under these noisy conditions. Although, there still exists a randomness in the areas surrounding the cube and turntable region. The optical flow produced by the combined Lucas-Kanade/Horn-Schunck techniques, figure 8, shows robust, regularized optical flow under noisy conditions. The positive qualities of both local and global differential techniques are evident in the combined approach.

Automatic neighborhood selection was conducted on frames 14 and 15 of the Rubik's cube sequence. The unbiased statistical risk estimate was generated for a neighborhood region of 9x9 up to a 25x25, see figure 9. The minimum risk for frame 14 was shown to be associated with a neighborhood of 13x13 pixels. In figure 4, the optimal neighborhood size was implemented on the combined Lucas-Kanade/Horn-Schunck techniques. The optimal neighborhood generated a nice optical flow field without any erratic random vectors. On the other hand, when non-optimal neighborhood size is used with the combined technique, see figure 10, the optical flow tends to be random in certain areas of the image.

VI. DISCUSSION

Local and global differential techniques each have their own advantages. Local methods are well known to have the ability to be robust under noisy conditions, whereas global techniques are able to produce dense flow fields. Many computer vision applications require dense flow fields. The source image data to be examined may contain noise, therefore there is a need to provide both accurate and dense results under these conditions.

In this paper we have successfully derived and implemented a combined Lucas-Kanade/Horn-Schunck optical flow technique. The results show that this approach blends the positive qualities of local and global differential techniques. Experiments with the standard Rubik's cube sequence show's that all the tested methods work quite well. Although the Lucas-Kanade method does have issues with producing dense optical flow fields. Once noise is introduced into the image set, there exists issues with the Horn and Schunck technique.

We have designed and implemented a procedure for automatic neighborhood selection of the combined Lucas-Kanade/Horn-Schunck optical flow technique. This SURE based technique proved to be effective. The optimal neighborhood window is directly associated with a minimum risk. Desirable optical flow results were clearly shown using this automatic selection.

Fig. 1. Frame 14 of Rubik cube sequence.

REFERENCES


Fig. 2. Optical flow computed using original Horn and Schunck method.

Fig. 3. Optical flow computed using Lucas and Kanade Method.

Fig. 4. Optical flow using combined Lucas-Kanade and Horn-Schunck algorithm.

Fig. 5. Frame 14 of Rubik's cube sequence with random noise.

Fig. 6. Optical flow of noisy image computed with Horn and Schunck technique.

Fig. 7. Optical flow of noisy image computed with Lucas and Kanade technique.
Fig. 8. Optical flow of noisy image using combined Lucas-Kanade and Horn-Schunck algorithm.

Fig. 9. SURE for frame 14 of Rubik cube sequence.

Fig. 10. Non-optimal neighborhood size of 5x5 pixels for combined Lucas-Kanade/Horn-Schunck.


