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Two-mode overconstrained three-DOFs rotational-translational linear-motor-based parallel-kinematics mechanism for machine tool applications

Sameh Refaat†, Jacques M. Hervé‡, Saeid Nahavandi†* and Hieu Trinh†

†*Intelligent Systems Research Laboratory, Deakin University, Geelong, VIC 3217, Australia.*
‡*Ecole Centrale Paris, 92295 Châtenay-Malabry, France.*

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SUMMARY
The paper introduces a family of three-DOFs translational-rotational Parallel-Kinematics Mechanisms (PKMs) as well as the mobility analysis of such family using Lie-group theory. Each member of this family has two-rotational one-translational DOFs. A novel mechanism is presented and analyzed as a representative of that family. The use and the practical value of that modular mechanism are emphasized.

KEYWORDS: Overconstrained mechanisms; Symmetric parallel-kinematics mechanisms; Rotational-translational mechanisms; Three-DOFs PKMs; Linear motor; High-speed; High-acceleration; Five-axis machine-tools.

1. Introduction

Parallel-Kinematics Mechanisms (PKMs) have many advantages.[24,28] Six-DOFs PKMs might be thought of as PKMs in the extreme where the drawbacks of such PKMs are extremely pronounced. These drawbacks are mainly limited to workspace and poor manipulability. To utilize the benefits of the concept of parallel kinematics while avoiding its drawbacks, there is a trend of relying on PKMs with less than six DOFs.

Three-DOFs PKMs attract decent amount of interest for this reason. A large number of these three-DOFs PKMs have been proposed. See refs.[24,28] for an early survey. After that, synthesis and enumeration of possible three-DOFs PKMs that can provide either translational or rotational DOFs have not only been extensively studied[1,2] but also been analyzed based on screw theory[18,19] and Lie-group theory.[4,13,14,15,16,17]

PKMs with three DOFs that are a combination of rotational and translational DOFs have also been studied. These were symmetrical nonoverconstrained, i.e., each limb has five-DOFs. A PKM that utilizes three R-P-S (i.e., Revolute-Prismatic-Spherical) limbs and provides one-translational and two-rotational DOFs has been proposed.[11,20] A similar PKM that utilizes three P-R-S limbs has also been reported.[23] Replacing the prismatic joints by revolute ones would maintain the mechanism's three-DOFs.[12] The R-S-R PKMs have been implemented as well.[3,7] An overconstrained asymmetrical PKM with one-rotational and two-translational DOFs has been proposed as well.[22]

Apart from these mechanism-specific efforts, screw theory was presented as a tool for synthesis of lower-mobility PKMs.[8,9,25] There was no specific focus on three-DOFs rotational-translational PKMs though.

In particular, three-DOFs PKMs that can provide two-rotational and one-translational DOFs have special importance in machine tools as these PKMs can potentially replace the problematic two rotational DOFs that always reduce the speed of response and rigidity of five-axis machine tools. Two asymmetrical overconstrained families of these PKMs have been proposed and are on their way for implementation.[26]

Symmetrical PKMs are those that have limbs of identical architectures. Symmetry represents one of the main advantages of PKMs that allows their modularity and reduces their cost. However, overconstrained PKMs are those with limbs that provide similar constraint(s).[10] That is, the motion constraint provided by one limb is also provided by the other limb. These overconstrained mechanisms do move despite the fact that Grübler/Kutzbach criterion in its original form[10,24,28] concludes that they should not, and they (overconstrained mechanisms) are mobile only when certain geometrical condition is satisfied. The main advantage of these overconstrained mechanisms is the fact that they use less joints and links, resulting in a simpler mechanism. The price is the need for strict manufacturing tolerance and the excessive loads on some links and/or joints.

2. Lie-Group Theory

Screw theory is increasingly utilized to synthesize PKMs. Many reviews of the basics of screw theory can be found in the literature.[28] However, screw theory natively deals with the instantaneous (or local) mobility of a mechanism. Lie-group, however, is a useful tool for full-cycle or finite mobility. In other words, screw-theory represents the differential aspect of Lie-group theory. Background about Lie-group theory can be found in the literature.[5]

It may be worth recalling that a Lie group is a set endowed with the algebraic structure of a group together with the algebraic structure of an analytic manifold. In the special case of the displacement Lie group, the product of displacements, which can be represented by a matrix product in any frame

*Corresponding author. E-mail: nahavand@deakin.edu.au

of reference, is the closed binary operation of the group. As a matter of fact, the product of two displacements is still a displacement; the identical transformation is the neutral displacement $\boldsymbol{E}$; and any displacement $\boldsymbol{D}$ has an inverse denoted $\boldsymbol{D}^{-1}$; $\boldsymbol{D}\,\boldsymbol{D}^{-1} = \boldsymbol{D}^{-1}\,\boldsymbol{D} = \boldsymbol{E}$. Moreover, differential calculation is allowed with displacements. An infinitesimal displacement is a transformation, which can be represented by the addition of the identity $\boldsymbol{E}$ and a twist. This can be further explained using matrix operators. Point O being any origin, the point M transformation, $\mathrm{M} \longrightarrow \mathrm{M}' = \boldsymbol{D}\,\mathrm{M}$, can be represented by the position vector transformation $(\mathbf{OM}) \longrightarrow (\mathbf{OM}')$ such as

$$\begin{pmatrix} \mathbf{OM}' \\ 1 \end{pmatrix} = \begin{pmatrix} \exp(\mathbf{R}\mathrm{x}) & \mathbf{T} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{OM} \\ 1 \end{pmatrix}$$

where $\mathbf{R}\times$ is the skew-symmetric linear operator of the vector product by $\mathbf{R}$. The vector $\mathbf{R} = \theta\,\mathbf{R}/\|\mathbf{R}\|$ characterizes the rotation of angle θ around an axis parallel to the unit vector $\mathbf{r} = \mathbf{R}/\|\mathbf{R}\|$. The vector $\mathbf{T}$ is the translation of the point that coincides with O before the displacement. The formula

$$\begin{aligned}(\mathbf{OM}') &= \exp(\theta\,\mathbf{r}\times)\,(\mathbf{OM}) \\ &= (\mathbf{OM}) + \sin\theta\,\mathbf{r} \times (\mathbf{OM}) + (1-\cos\theta) r \\ &\quad \times [r \times (\mathbf{OM})]\end{aligned}$$

is known as the Rodrigues formula for rotation. If θ has the infinitesimal value $\mathrm{d}\theta$, then $exp(\theta\,\mathbf{r}\times) = 1 + \mathrm{d}\theta\,\mathbf{r}\times$.

An infinitesimal displacement results from an infinitesimal rotation $\mathrm{d}\mathbf{R} = \mathrm{d}\theta\,\mathbf{r}$ together with an infinitesimal translation $\mathrm{d}\mathbf{T}$, which yields

$$\begin{aligned}\begin{pmatrix} \mathbf{OM} + \mathbf{dM} \\ 1 \end{pmatrix} &= \begin{pmatrix} \exp(\mathbf{R}\times) & \mathbf{dT} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{OM} \\ 1 \end{pmatrix} \\ &= \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} \mathbf{dR}\times & \mathbf{dT} \\ 0 & 0 \end{pmatrix}\right] \begin{pmatrix} \mathbf{OM} \\ 1 \end{pmatrix}.\end{aligned}$$

The linear operator $\$ = \left(\begin{smallmatrix} \mathbf{dR}\times & \mathbf{dT} \\ 0 & 0 \end{smallmatrix}\right)$ is a twist expressed in a frame-free (or intrinsic) geometric manner. Using symbolical notations, the set D^* of infinitesimal displacements is the sum $\boldsymbol{E} + \mathrm{d}$, d denoting the set of all twists, which is a six-dimensional (6-d.) Lie algebra.

The following are the notations of Lie subgroups of displacements;

- $\mathbf{G}$ ($\mathbf{x}$): Planar gliding perpendicular to unit vector $\mathbf{x}$.
- $\mathbf{R}$ (N, $\mathbf{x}$): Set of rotations around the axis having a frame of reference (N, $\mathbf{x}$).
- $\mathbf{T}$: Set of 3-dof translations.
- $\mathbf{T}_2$ ($\perp \mathbf{x}$): Set of 2-dof translations perpendicular to $\mathbf{x}$.
- $\mathbf{T}$ ($\mathbf{x}$): Linear translations parallel to $\mathbf{x}$.
- $\mathbf{S}$ (O): Spherical motion about the point O.
- $\mathbf{X}$ ($\mathbf{x}$): Set of Schoenflies (Schönflies) motions of direction $\mathbf{x}$, (it is a 4-d. Lie subgroup).

Vectors are bold-faced characters, unit vectors are bold lower-case characters, points are capital letters, and calligraphic letters indicate subgroups and subsets.

From Mozzi–Chasles theorem, any finite or infinitesimal displacement is a screw motion. Hence, the subgroups $\mathbf{H}$ (N, $\mathbf{u}$, p) of helical motions of given axes (N, $\mathbf{u}$) and pitches p cannot be ignored. So is the 3-d subgroup $\mathbf{Y}$ ($\mathbf{u}$, p).[6] However, mechanisms implementing screw pairs are out of the scope of this article. The improper subgroups of $\mathbf{D}$ are the zero-dimensional group $\boldsymbol{E}$, which contains only the identity $\boldsymbol{E}$, and $\mathbf{D}$, which is the six-dimensional group of displacements.

Arthur Schoenflies (also spelt Schönflies) is a mathematician who wrote a full book chapter about a special 4-dof motion type. A Schoenflies motion can be defined as the commutative product $\mathbf{T}\,\mathbf{R}$(N, $\mathbf{x}$)**,** the point N being any one. The $\mathbf{X}$ ($\mathbf{x}$) subgroup contains an infinity of 1-d. subgroups $\mathbf{R}$ (N, $\mathbf{x}$) of rotations, which have the same direction of axis. Hence, without loss of information, a X-motion can be called a 3-translational 1-rotational motion that constrains two rotational DOFs. However, the constraint space is not endowed directly with an algebraic structure; moreover, the constraints are actually wrenches that are only locally defined through the expression of the power. The power is the invariant Klein form of general Lie's theory. Any serial array of H, R, P pairs producing 4-dof motion between the distal bodies of the chain, is a generator of *X*-motion, provided that the H or R pairs have parallel axes. The set of Schoenflies motion is endowed with the algebraic structure of a Lie group. Hence, the product of Schoenflies motion subsets is a Schoenflies motion subset because of the product closure in any subgroup. For instance, the serial array of two H pairs with parallel axes produces a 2-d manifold of displacements included in a X-subgroup. This 4-dof motion type generalizes the 3-dof planar motion. As a matter of fact, it can be obtained also as the commutative product of planar motion subgroup by a linear translation subgroup, namely $\mathbf{X}(\mathbf{x}) = \mathbf{G}(\mathbf{x})\,\mathbf{T}(\mathbf{x}) = \mathbf{T}(\mathbf{x})\,\mathbf{G}(\mathbf{x})$.

The set of twists of a displacement Lie subgroup is endowed with the algebraic structure of a Lie algebra and is called a Lie subalgebra. Algebra in its general meaning is an aspect of mathematics. In our context, algebra is the name of a particular algebraic structure. An algebra is a vector space endowed with a closed product. The Lie bracket defines a closed product in the twist space. Hunt who ignored Lie's theory of continuous groups of transformations found the Lie subalgebras of twists as "screw systems that guarantee full-cycle mobility." Moreover, in Lie's theory, the exponential map of a Lie subalgebra provides the corresponding Lie subgroup.[27] Hence, there is a one-to-one mapping between Lie subalgebras of twists and Lie subgroups of finite displacements. The Lie subalgebras of twists are denoted with small letters: $\mathbf{g}$ ($\mathbf{x}$) is the Lie algebra of the Lie group $\mathbf{G}(\mathbf{x})$, and so on.

For instance, if the canonical parameters of $\mathbf{G}(\mathbf{x})$ have only infinitesimal values, then the set $\mathbf{G}^*(\mathbf{x})$ of the corresponding infinitesimal planar motions is $\mathbf{G}^*(\mathbf{x}) = \boldsymbol{E} + \mathbf{g}(\mathbf{x})$.

The main useful algebraic property is the closure of the product in any displacement subgroups. From a long time, everybody knows that the products of translations are translations, the products of planar displacements are planar displacements along the same plane, and the products of spherical displacements are spherical displacements around

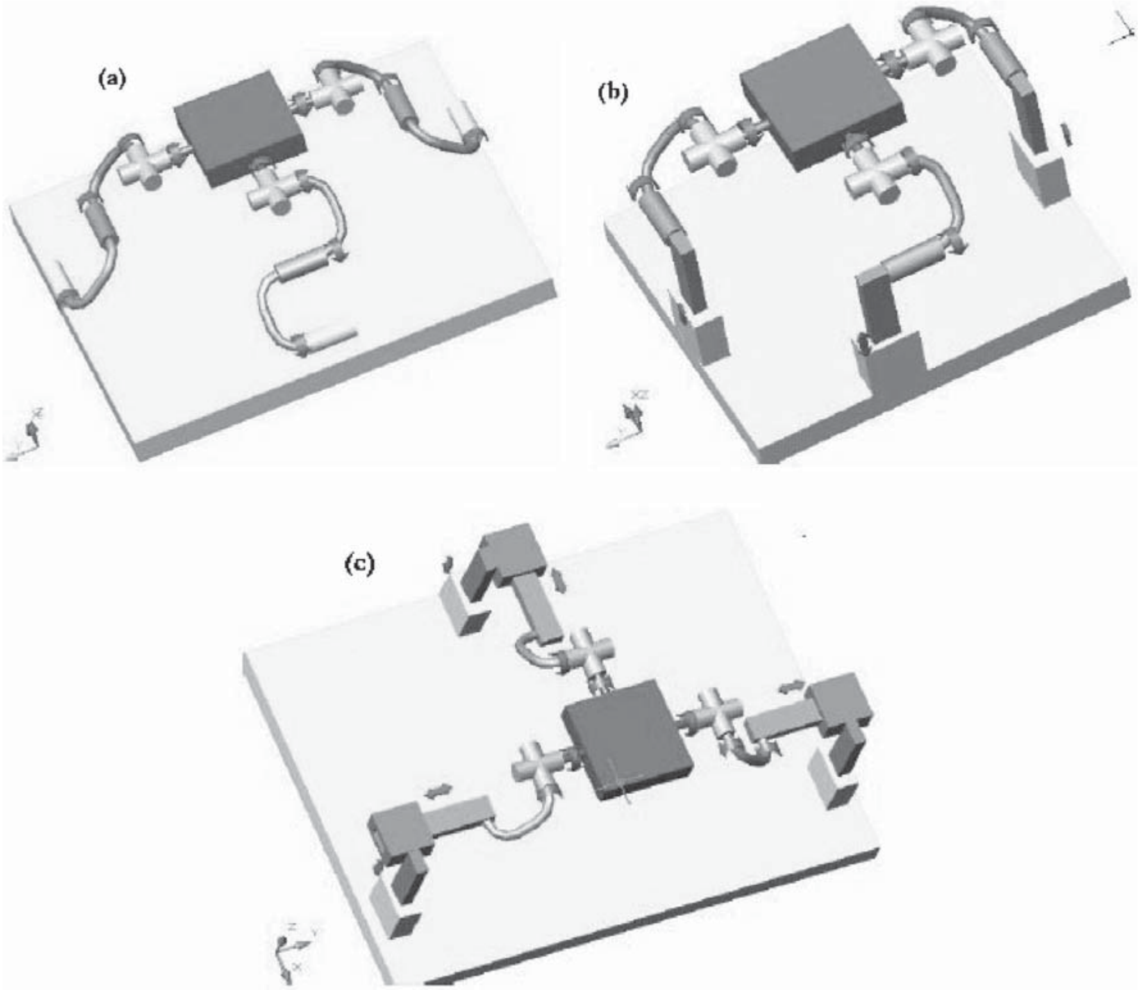

Fig. 1. Members of the two-rotational and one-translational DOFs two-mode symmetrical PKM family.

the same point. Group theory formalizes and generalizes that intuitive approach.

3. Symmetrical Overconstrained 3-DOFs Family of PKMs

Here, a family of rotational-translational DOFs PKMs is presented. Each of the mechanisms provides its platform with one-translational-two-rotational DOFs. All the PKMs presented here have three limbs to allow the actuators to be placed on the machine base. The members of the family are shown in Fig. 1. An orthonormal vector base (**x**, **y**, **z**) is used for referencing the Euclidean space.

Figure 2 shows two of the limbs of the mechanism of Fig. 1(a). Using Lie-Group theory as discussed earlier, one can see that a serial array of four revolute R pairs in each of these limbs generates:

R (A, **x**) **R** (B, **x**) **R** (C, **x**) **R** (N, **y**). The axes (A, **x**), (B, **x**), and (C, **x**) must not be in a plane; else the three twists in the joints are linearly dependent and, consequently, the RRR array is singular.

Further, when the singularity is avoided, **R** (A, **x**) **R** (B, **x**) **R** (C, **x**) is a three-dimensional manifold of displacements. The identity ***E*** belongs to all displacement manifolds and, therefore, this manifold is a 3-d neighborhood of the identity. **R** (A, **x**), **R** (B, **x**), and **R** (C, **x**) are contained in **G**(**x**) where **G**(**x**) stands for subgroup of planar gliding displacements,

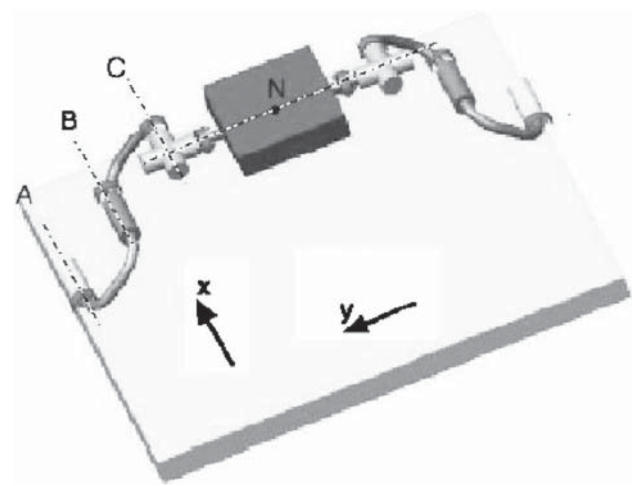

Fig. 2. Two limbs of the mechanism of Fig. 1(a) that generate the same motion type.

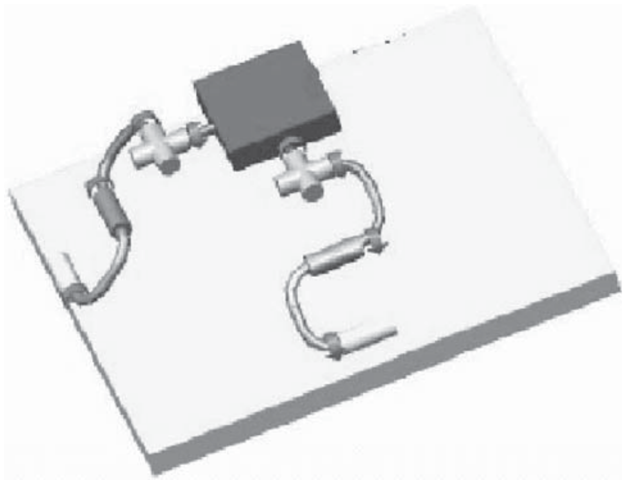

Fig. 3. Two limbs of the mechanism of Fig. 1(a) that generate different motions.

the plane being perpendicular to **x**. Because of the product closure in the subgroup **G**(**x**), the 3-d product **R** (A, **x**) **R** (B, **x**) **R** (C, **x**) is contained in the 3-d group **G**(**x**): **R** (A, **x**) **R** (B, **x**) **R** (C, **x**) ⊆ **G**(**x**). When dealing with motion types, one can ignore the boundary of the foregoing neighborhood and, therefore, can state the equality

$$\mathbf{R}(\mathrm{A},\ \mathbf{x})\mathbf{R}(\mathrm{B},\ \mathbf{x})\mathbf{R}(\mathrm{C},\ \mathbf{x}) = \mathbf{G}(\mathbf{x}).$$

Hence, each of the two limbs shown in Fig. 2 generates **G**(**x**) **R** (N, **y**). The dimension of this product is

$$\dim \mathbf{G}(\mathbf{x})\,\mathbf{R}(\mathrm{N},\mathbf{y}) = \dim \mathbf{G}(\mathbf{x}) + \dim \mathbf{R}(\mathrm{N},\ \mathbf{y}) - \dim \mathbf{G}(\mathbf{x}) \cap \mathbf{R}(\mathrm{N},\ \mathbf{y}) = 3 + 1 - 0 = 4.$$

G(**x**) **R** (N, **y**) is a 4-d manifold of displacements.

Notice that the motion of the platform depicted in Fig. 1 can be produced using only the two limbs shown in Fig. 3. One limb generates **G**(**x**) **R** (N, **y**) and the other limb generates **G**(**y**) **R**(N, **x**). Consequently, the platform motion is expressed by **G**(**x**) **R** (N, **y**) ∩ **G**(**y**) **R** (N, **x**). The foregoing set intersection is not a smooth manifold of displacements as will be explained in what follows.

First, we remark that if the R pairs generating **R** (N, **y**) and **R** (N, **x**) are locked, then we can write **R** (N, **y**) = **R** (N, **x**) = ***E***; ***E*** = identity displacement. **G**(**x**) ***E*** ∩ **G**(**y**) ***E*** = **G**(**x**) ∩ **G**(**y**). By the general study of the displacement group, the intersection is **G**(**x**) ∩ **G**(**y**) = **T** (**z**). The set **T** (**z**) is the 1-d subgroup of linear translations parallel to **z**, **z** being perpendicular to **x** and **y**. That is, the platform can undergo 1-DOF linear translation.

Furthermore, if the R pair generating **R** (N, **y**) is the only one locked, then the allowed motion of the platform is expressed by **G**(**x**) ***E*** ∩ **G**(**y**) **R** (N, **x**) = **G**(**x**) ∩ **G**(**y**) **R** (N, **x**).

One can write: **G**(**x**) ∩ **G**(**y**) **R**(N,**x**) ⊇ **G**(**x**) ∩ **G**(**y**) ***E*** = **G**(**x**) ∩ **G**(**y**) = **T** (**z**) because Ebelongs to all displacement manifolds, **R** (N, **x**), for instance, and, therefore, the platform can undergo 1-DOF linear translation **T** (**z**). But **G**(**x**) ∩ **G**(**y**) **R** (N, **x**) ⊇ **G**(**x**) ∩ ***E*** **R** (N, **x**) = **G**(**x**) ∩ **R** (N, x) = **R** (N, **x**) because **R** (N, **x**) ⊂ **G**(**x**). Hence, the platform can also rotate around the axis (N, **x**). Conversely, using general theorems of group theory one can write;

$$\mathbf{T}(\mathbf{z})\,\mathbf{R}(N,\mathbf{x}) = [\mathbf{G}(\mathbf{x}) \cap \mathbf{G}(\mathbf{y})]\,\mathbf{R}(\mathrm{N},\mathbf{x}) \subseteq [\mathbf{G}(\mathbf{x})\,\mathbf{R}(\mathrm{N},\mathbf{x})] \cap [\mathbf{G}(y)\,\mathbf{R}(\mathrm{N},\mathbf{x})] = \mathbf{G}(\mathbf{x}) \cap [\mathbf{G}(\mathbf{y})\,\mathbf{R}(\mathrm{N},\mathbf{x})].$$

Moreover, by a more lengthy reasoning, one can prove the set equality

$$\mathbf{T}(\mathbf{z})\,\mathbf{R}(\mathrm{N},\mathbf{x}) = \mathbf{G}(\mathbf{x}) \cap [\mathbf{G}(\mathbf{y})\,\mathbf{R}(\mathrm{N},\ \mathbf{x})]$$

T (**z**) **R** (N, **x**) is a 2-d manifold of displacements.

By the same token, one can establish

$$\mathbf{T}(\mathbf{z})\,\mathbf{R}(\mathrm{N},\mathbf{y}) = [\mathbf{G}(\mathbf{x})\,\mathbf{R}(\mathrm{N},\mathbf{y})] \cap \mathbf{G}(\mathbf{y}).$$

Consequently, the set **G**(**x**) **R** (N, **y**) ∩ **G**(**y**) **R** (N, **x**) contains both **T** (**z**) **R** (N, **x**) and **T** (**z**) **R** (N, **y**).

Therefore, the mechanism has two working modes. Locking the pair generating **R** (N, **y**) at the given posture of the mechanism, the platform can undergo 2-DOF motion expressed by **T**(**z**) **R** (N, **x**). This corresponds to a first working mode of the mechanism. The same way, one can show that locking the pair generating **R** (N, **x**), the platform can undergo 2-DOF motion expressed by **T** (**z**) **R** (N, **y**). This corresponds to a second working mode of the mechanism.

Further, the second working mode is prohibited after any operation in the first mode and vice versa. As a matter of fact, by operating in the first mode, the generator of **G** (**y**) remains a generator of **G** (**y**) but the axis (N, **y**) becomes (N', **y**') with **y**' ≠ **y**, and thus **R** (N, **y**) ⊂ **G** (**y**) is no longer available, which conditions the second working mode.

As a conclusion, the platform motion is either **T** (**z**) **R** (N, **x**) or **T** (**z**) **R** (N, **x**). Using set theory notations, the platform motion is expressed by

$$\mathbf{T}(\mathbf{z})\,\mathbf{R}(\mathrm{N},\mathbf{x}) \cup \mathbf{T}(\mathbf{z})\,\mathbf{R}(\mathrm{N},\mathbf{x}) = \mathbf{T}(\mathbf{z})[\mathbf{R}(\mathrm{N},\mathbf{x}) \cup \mathbf{R}(\mathrm{N},\mathbf{x})]$$

which is not a smooth manifold.

Employing **G**(**x**) = $\mathbf{T}_2(\perp\mathbf{x})$ **R** (N, **x**) and **G**(**y**) = $\mathbf{T}_2(\perp\mathbf{y})$ **R** (N, **y**), which also express the two planar motions, one can notice that the set of feasible displacements **G**(**x**) **R** (N, **y**) ∩ **G**(**y**) **R** (N, **x**) can be equated to $\mathbf{T}_2(\perp\mathbf{x})$ **R** (N, **x**) **R** (N, **y**) ∩ $\mathbf{T}_2(\perp\mathbf{y})$ **R** (N, **y**) **R** (N, **x**).

The product of finite rotations generally does not commute: **R** (N, **x**) **R** (N, **y**) ≠ **R** (N, **y**) **R** (N, **x**). However, $\mathbf{R}^*(\mathrm{N},\mathbf{x})$ and $\mathbf{R}^*(\mathrm{N},\mathbf{y})$ representing rotations with infinitesimal angles, the factor commutation is valid, namely $\mathbf{R}^*(\mathrm{N},x)\ \mathbf{R}^*(\mathrm{N},\mathbf{y}) = \mathbf{R}^*(\mathrm{N},\mathbf{y})\ \mathbf{R}^*(\mathrm{N},\mathbf{x})$. As a matter of fact, $\mathbf{R}^*(\mathrm{N},\mathbf{x})\ \mathbf{R}^*(\mathrm{N},\mathbf{y}) = [\boldsymbol{E} + \mathbf{r}(\mathrm{N},\mathbf{x})]\ [\boldsymbol{E} + \mathbf{r}(\mathrm{N},\mathbf{y})] \approx \boldsymbol{E} + \mathbf{r}(\mathrm{N},\mathbf{x}) + \mathbf{r}(\mathrm{N},\mathbf{y})$.

Hence, $$\mathbf{T}_2(\perp\mathbf{x})\mathbf{R}^*(\mathrm{N},\mathbf{x})\mathbf{R}^*(\mathrm{N},\mathbf{y}) \cap \mathbf{T}_2(\perp\mathbf{y})\mathbf{R}^*(\mathrm{N},\mathbf{y})\mathbf{R}^*(\mathrm{N},\mathbf{x}) = [\mathbf{T}_2(\perp\mathbf{x}) \cap \mathbf{T}_2(\perp\mathbf{y})][\mathbf{R}*(\mathrm{N},\mathbf{x})\mathbf{R}*(\mathrm{N},\ \mathbf{y})] = \mathbf{T}(\mathbf{z})[\mathbf{R}*(\mathrm{N},\mathbf{x})\mathrm{R}*(\mathrm{N},\ \mathbf{y})].$$

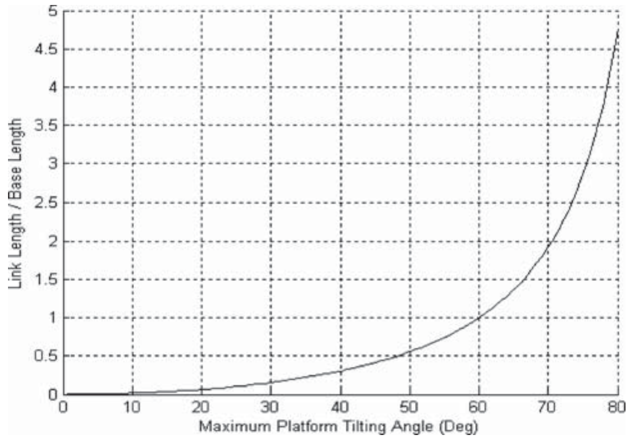

Fig. 4. Link-length against operating range.

From the previous expression, locally in the singular pose, the platform motion has 3 DOFs, namely one translation with finite amplitude and two rotations with infinitesimal amplitude. After any translation, a bifurcation toward two working modes can still be achieved.

It is worth remarking that for mechanisms belonging to the family disclosed in this paper, the usual acceptance of the concept of degree of freedom is questionable.

The authors do not believe that the corresponding nonoverconstrained mechanism or family of mechanisms exists.

Kinematics, singularities, dynamics, experimental control results of one member of the proposed family and application of such mechanism are the subject of other work.[29] We here state the important result that singularities of that mechanism do not limit the workspace making the mechanism practically useful. Figure 4 shows how the working space can usefully be increased by simply increasing the length of the limbs

Figure 5 demonstrates the small servo positioning error of one of the joints even when this joint is loaded.

4. Conclusions

Using the equivalencies that are proven by the closure of the product in any algebraic subgroup, a family of PKMs has been conceived.

Each of the limbs of the family discussed simply provides a planar motion (two-translations and one rotation about an axis normal to the translation plane) as well as one rotation about an axis that lies within that plane. By each proper arrangement of two of these limbs, the mechanism can utilize two limbs to realize a two-mode motion. The first working mode is translation in the one direction (say the **z** direction) and rotation about one of the other Cartesian directions (say the **x** direction), while the second working mode is translation in the same **z** direction and rotation about the third Cartesian direction (in this case, it would be the **y** direction). Further, if two limbs that generate the same motion are placed opposite to each other, as shown in Fig. 4, this will enlarge the singularity-free workspace. This is because if one limb is in its singular position, its supporting limb will not be. In fact, this is kinematics redundancy that has been used previously to avoid singularity.

A mechanism that provides two rotational DOFs and one translational DOF has very large number of applications in manufacturing. Every single expensive five-axis machine-tool does utilize a two-rotational-one-translational DOF mechanism as a subsystem. The importance and application of the proposed mechanism was the subject of a separate work.[29,30]

An industrial-scale version of the family member of Fig. 1(b) has been built and is shown in Fig. 6 The translational stroke is 500 mm, the rotation angle is 80° when a supporting (i.e., redundant) limb is used and 50° when the

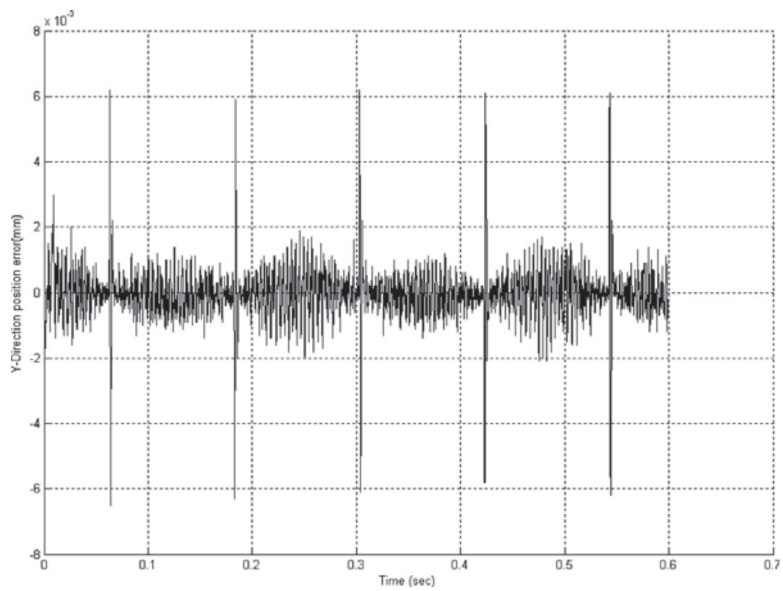

Fig. 5. Joint servo error.

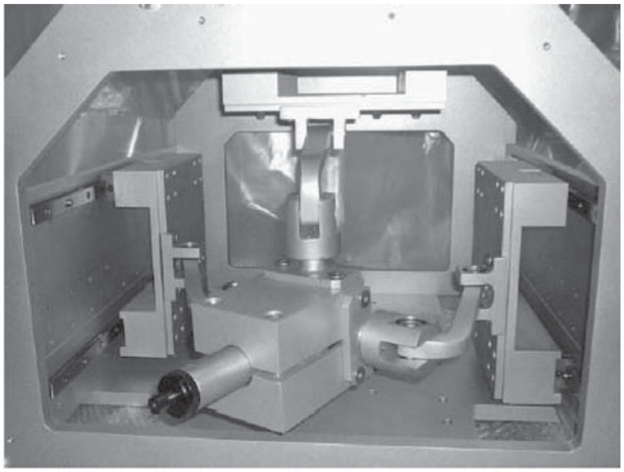

Fig. 6. Industrial-scale model of one member of the family.

limb generating the second rotational mode is not supported by a redundant limb.

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