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DESIGN OF A DISTRIBUTED POWER SYSTEM STABILISER

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Abstract

A new design method for a distributed power system stabiliser for interconnected power systems is introduced in this paper. The stabiliser is of a low order, dynamic and robust. To generate the required local control signals, each local stabiliser requires information about either the rotor speed or the load angle of the other subsystems. A simple MATLAB based design algorithm is given and used on a three-machine unstable power system. The resulting stabiliser is simulated and sample results are presented.

1. INTRODUCTION

Excitation-based power system stabilisers (PSSs) have been extensively used for improving the small disturbance oscillatory performance of power systems [1-9]. A commonly used PSS comprises a wash out circuit and a cascade of two phase lead networks [1,5,7,8]. A number of stabilising input signals, such as terminal voltage, load angle, rotor speed, accelerating power, electric power input etc, and/or linear combinations of these have been investigated, and recommendations regarding their use have been reported in the literature [3,5,6].

The problem of tuning the PSS parameters for single and multi-machine cases has been the subject of much research over the past a few decades. Due to the complex nature of this problem, researchers have tended to simplify it by making a series of intuitive assumptions based on experience and physical appreciation of the power system. As a result of these assumptions, the multi-parameter tuning problem becomes that of tuning only one or two for each machine [7,8]. Then a search procedure is used to determine the best possible values for these parameters with respect to defined performance criteria. Both sequential and simultaneous tuning approaches have been devised and have been reported to produce satisfactory results. However most of these approaches are application specific and require prior practical experience with the system considered. In some cases the outcome of the devised parameter tuning approach may not be considered to be the best possible.

In this paper, we offer an alternative approach to the design of PSS with the following advantages:

(i) The designed PSS emulates the performance of any full state feedback controller.
(ii) The order of the stabiliser could be as low as the number of those machines for which a stabiliser is necessary.
(iii) The input to the stabiliser may be any set of plant measurement for which the system is observable.

A three machine unstable system is considered for the application of the design method. Simulation results on the closed-loop system are presented.

2. POWER SYSTEM INVESTIGATED

A sample power system shown in Figure (1) is considered (see reference [9] for a detailed model). This system consists of three machines with IEEE type 1 excitation system. The three machines are interconnected through a network of six buses and three load centres. For excitation control design, the plant is represented by a third order synchronous machine equipped with a first order exciter. This representation is often used in small disturbance stability studies and is quite adequate for it.

The power system is represented in a state-space form with 12 state variables and 3 control inputs. The output measurements used in this paper are load angles. However, rotor speed or any other measurement for which the system is observable may also be used, as appropriate.

![Figure (1): A diagram of a three-machine unstable power system](image-url)
3. PROBLEM STATEMENT

Let an $N$-station interconnected power system be described as

$$
\dot{x}(t) = Ax(t) + Bu(t) \quad (1a)
$$

$$
y(t) = Cx(t) \quad (1b)
$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ and $y(t) \in \mathbb{R}^q$ are the state, input and output vectors, respectively. Matrices $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$ and $C \in \mathbb{R}^{q \times n}$ are real constant.

Assume that the triplet $(A,B,C)$ is controllable and observable. Now consider the following state feedback stabilizer for the power system

$$
u(t) = Fx(t) \quad (2)
$$

where $F \in \mathbb{R}^{m \times n}$. The theme of this paper is to replace the state feedback dynamic stabiliser (2) with an output feedback dynamic stabiliser of the form

$$
\dot{x}_i(t) = Ax_i(t) + B_i y(t)
$$

$$
y_i(t) = u(t) = C_i x_i(t) \quad (3)
$$

so that the closed-loop performance of the power system under the output feedback stabiliser (3) is comparable to that of the state feedback stabiliser (2).

4. DEVELOPMENT OF METHOD

Let the control signal be decomposed into its basic $N$-station components as

$$
u(t) = F_i x_i(t) \quad (4a)
$$

or

$$
u_i(t) = F_i x(t); \quad i = 1, 2, ..., N \quad (4b)
$$

where $u_i(t) \in \mathbb{R}^m$ ($i = 1, 2, ..., N$) is the input vector of the $i$th control station and $F_i \in \mathbb{R}^{m \times n}$.

Now introduce the following $p_i$th-order dynamic controller for the $i$th control station

$$
u_i(t) = F_i x(t) = (K_i L_i + W_i C_i) x(t) \quad (5a)
$$

and

$$
\dot{z}_i(t) = E_i z_i(t) + L_i B_i u_i(t) + G_i y_i(t), \quad i = 1, 2, ..., N \quad (5b)
$$

where $y_i(t) = C_i x(t) \in \mathbb{R}^{q \times n}$ is the measurement required at the $i$th control station, $C_i \in \mathbb{R}^{q \times n}$ is the output matrix, and $B_i \in \mathbb{R}^{n \times m}$ is the local input matrix.

The controller of (5) has a similar structure to that of (3). This implies that the control signal $u_i(t)$ for the $i$th station can now be generated by using the local output, $y_i(t)$, and a linear combination of the global state vector, $z_i(t) = L_i x(t)$.

The remaining part of this section shows how the controller parameters $K_i$, $L_i$, $W_i$, $E_i$ and $G_i$ can be found to generate the required local stabilising signals.

Let an error vector $e_i(t) \in \mathbb{R}^n$ be defined as

$$
e_i(t) = z_i(t) - L_i x(t); \quad i = 1, 2, ..., N \quad (6)
$$

By some simple manipulations, the following error equation is obtained

$$
\dot{e}_i(t) = \dot{z}_i(t) - L_i \dot{x}(t) = E_i z_i(t) + L_i B_i u_i(t) + G_i y_i(t) - L_i (A x(t) + B_i u_i(t) + G_i y(t)) = E_i e_i(t) + (G_i C_i - L_i A + E_i L_i) x(t) - L_i B_i u_i(t) \quad (7)
$$

where the term $B_i u_i(t)$ in equation (1a) is decomposed according to $B_i u_i(t) = \begin{bmatrix} B_i & B_i \end{bmatrix} \begin{bmatrix} u_i(t) \\ u_i(t) \end{bmatrix}$, $B_i \in \mathbb{R}^{n \times (m \times n)}$ and $u_i \in \mathbb{R}^{m \times n}$.

Equation (7) implies that the dynamic system (5b) can act as a distributed controller for system (1), provided that matrix $E_i$ is chosen to be asymptotically stable and matrices $G_i$ and $L_i$ fulfil the following constraints

$$
\begin{align}
G_i C_i - L_i A + E_i L_i &= 0 \quad (8) \\
L_i B_i &= 0 \quad (9) \\
F_i &= K_i L_i + W_i C_i \quad (10)
\end{align}
$$

In the following, equations (8)-(10) are solved for the controller parameters $K_i$, $L_i$, $W_i$ and $G_i$.

For simplicity and without loss of generality, let us assume that matrix $C_i$ has full row rank, i.e. $\text{rank}(C_i) = r_i$, and takes the following canonical form

$$
C_i = \begin{bmatrix} I_i & 0 \end{bmatrix} \quad (11)
$$
where \( I_r \) is an identity matrix of dimension \( r \). For the case where matrix \( C_i \) is not in the form (11) then the following orthogonal transformation matrix can be used to transform it

\[
M_i = [C_i^T (C_i C_i^T)^{-1} Q_i] \tag{12}
\]

where \( Q_i = \text{null}(C_i) \in R^{m(n-r)} \) is the null-space of \( C_i \).

Accordingly, by using equation (11), equations (8) and (10) can be expressed as

\[
G_i = (L_i A - E_i L_i) \begin{bmatrix} I_r \\ 0 \end{bmatrix} \tag{13a}
\]

\[
(L_i A - E_i L_i) \begin{bmatrix} 0 \\ I_{(n-r)} \end{bmatrix} = 0 \tag{13b}
\]

and

\[
W_i = (F_i - K_i L_i) \begin{bmatrix} I_r \\ 0 \end{bmatrix} \tag{14a}
\]

\[
(F_i - K_i L_i) \begin{bmatrix} 0 \\ I_{(n-r)} \end{bmatrix} = 0 . \tag{14b}
\]

Matrix \( E_i \) can be chosen according to the desired dynamics of the controller to be constructed. It is also clear from equations (13a) and (14a) that matrices \( G_i \) and \( W_i \) are easily derived, once matrices \( K_i \) and \( L_0 \) are obtained. It, therefore, remains to solve equations (9), (13b) and (14b) for matrices \( K_i \) and \( L_i \).

Let matrices \( F_i \) and \( L_i \) be partitioned as follows

\[
F_i = \begin{bmatrix} f_1 & f_2 & \ldots & f_s & f_{s+1} & f_{s+2} & \ldots & f_s \\ \end{bmatrix} \tag{15}
\]

and

\[
L_i = \begin{bmatrix} l_1 & l_2 & \ldots & l_s & l_{s+1} & l_{s+2} & \ldots & l_s \\ \end{bmatrix} \tag{16}
\]

where \( f_j = f_{j,i} \in R^n \) and \( l_j = l_{j,i} \in R^n \) are the \( j \)-th column \((j = 1,2,\ldots,n)\) of matrices \( F_i \) and \( L_i \), respectively.

Incorporating equations (15)-(16) into equation (14b), and after some rearranging, the following matrix-vector equation is obtained

\[
\Phi l = f \tag{17}
\]

where

\[
\Phi = \begin{bmatrix} \Phi_{i,j} \end{bmatrix} \in R^{m(n-r) \times p(r,r)};
\]

\[
\Omega = \text{diag} \{ \Omega_i \} \in R^{m \times (n-r) \times p \times (n-r)} ;
\]

\[
l = \begin{bmatrix} l_1^T & l_2^T & \ldots & l_s^T \end{bmatrix} \in R^{n \times s} ;
\]

\[
f = \begin{bmatrix} f_1^T & f_2^T & \ldots & f_s^T \end{bmatrix} \in R^{n \times s}.
\]

and \( O_{(m(n-r)) \times (p,r)} \) is a zero matrix of dimension \( m(n-r) \times p \times r \).

Let us now consider equation (13b). Using the fact that \( E_i \in R^{n \times p} \) can be chosen to be any stable matrix, equation (13b) can be expressed in a matrix-vector form as

\[
\Psi l = 0 ; \ \Psi \in R^{p \times (n-r) \times p \times n} \tag{18a}
\]

where

\[
\Psi = \begin{bmatrix} a_{s+1,I_{p,n}} & \ldots & (a_{s+1,s+1,I_{p,n}} - E_i) & a_{s+1,I_{p,n}} \\ a_{s+2,I_{p,n}} & \ldots & \ldots & a_{s+2,I_{p,n}} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n-1,I_{p,n}} & \ldots & \ldots & a_{n-1,I_{p,n}} \\ a_{n,I_{p,n}} & \ldots & a_{n,n,I_{p,n}} & (a_{n,n,I_{p,n}} - E_i) \\ \end{bmatrix} \tag{18b}
\]

\( a_{j,k} \) is the \((j,k)\) element of \( A \). Similarly, equation (9) can be rearranged in a matrix-vector form as

\[
\Theta l = 0 ; \ \Theta \in R^{m \times (m-n) \times (p,r)} \tag{19a}
\]

where

\[
\Theta = \begin{bmatrix} b_{1,1,I_{p,n}} & b_{1,2,I_{p,n}} & \ldots & b_{1,n,I_{p,n}} \\ b_{2,1,I_{p,n}} & b_{2,2,I_{p,n}} & \ldots & b_{2,n,I_{p,n}} \\ \ldots & \ldots & \ldots & \ldots \\ b_{n,1,I_{p,n}} & b_{n,2,I_{p,n}} & \ldots & b_{n,n,I_{p,n}} \\ \end{bmatrix} \tag{19b}
\]

where \( b_{j,k} \) is the \((j,k)\) element of \( B_i \). From equation (19b) it is clear that \( \text{rank}(\Theta) = p(m-m_{ij}) \), if \( \text{rank}(B_i) = (m-m_{ij}) \). Thus, there exists an orthogonal connection matrix \( T_i \in R^{m \times m_p} \), where \( T_i = (T_i)^{-1} \), such that equation (19a) can be transformed into

\[
\Theta l = 0 ; \ \Theta = \Theta T_i \ ; \ l = T_i l \tag{20}
\]

where matrix \( \Theta \in R^{(p \times (m-n)) \times (p \times (m-n))} \) is invertible.

Equation (20) can be rearranged as
\[ I = \alpha \bar{I}_z \quad : \alpha = - (\bar{\Omega}_1)^{-1} \bar{\Omega}_2 \]  

(21)

Equations (17) and (18a) may be rearranged according to (20) as follows

\[ \Phi l = (\Phi T_l)T_l = \Psi l = \left[ \begin{array}{c} \Phi_1 \\ \Phi_2 \end{array} \right] \left[ \begin{array}{c} l_1 \\ l_2 \end{array} \right] = f \]  

(22)

and

\[ \Psi l = (\Psi T_l)T_l = \Psi l = \left[ \begin{array}{c} \Psi_1 \\ \Psi_2 \end{array} \right] \left[ \begin{array}{c} l_1 \\ l_2 \end{array} \right] = 0 \]  

(23)

Substituting equation (21) into equations (22)-(23), the following algebraic equation is obtained

\[ \beta \bar{I}_z = \gamma ; \quad \beta = \left[ \begin{array}{c} \bar{\Phi}_1 \alpha + \bar{\Phi}_2 \\ \Psi_1 \alpha + \Psi_2 \end{array} \right] \in \mathbb{R}^{(n+1)+(n+1) \times \{m(n+m)\}} ; \quad \gamma = \left[ \begin{array}{c} f \\ 0 \end{array} \right] \in \mathbb{R}^{n+1 \times \{m(n+m)\}} . \]  

(24)

Equation (24) has \([m(n-r) + p_r(n-r)]\) linear simultaneous equations but \([p_r(n-m+m)]\) unknowns. It is therefore clear that, in general, an exact solution does not always exist. However, as matrix \(\beta\) contains \([m \times p_r]\) elements of matrix \(K_r\), then, provided that it has a full column rank, equation (24) can be solved to minimize the following error norm

\[ \delta(K_r) = \| \beta \beta^\dagger \gamma - \gamma \| \]  

(25)

where \(\beta^\dagger\) is the Moore-Penrose pseudo-inverse of \(\beta\) and vector \(\bar{I}_z\) is derived from

\[ \bar{I}_z = \beta^\dagger \gamma \]  

(26)

The above minimization problem (25) can be solved by searching for the \((m \times p_r)\) elements of matrix \(K_r\), where matrix \(K_r\) has full row rank and its \((m \times p_r)\) elements are within a bounded range. The search only involves a small number of parameters and it can be done by using MATLAB Optimization Toolbox. Once the problem is solved, matrix \(K_r\) and vector \(\bar{I}_z\) are obtained, \(\bar{I}_z\) is derived from (21) and hence \(l\) is obtained from \(l = T_l \bar{I}_z\). Consequently, matrices \(L_r\), \(G_r\) and \(W_r\) can be easily derived from equation (16), (13a) and (14a), respectively.

**Remark 1:** It can be shown that the error \(\delta(K_r)\) of the minimisation (25) determines the overall closed-loop stability of the system. However, due to space limitation, an in-depth analysis can not be given here. In general the smaller the value of the error \(\delta(K_r)\), the better the chance that the closed-loop is stable. For the case where equation (24) is exactly solved, the error norm \(\delta(K_r) = 0\), and the distributed controller (5) exactly emulates the global state feedback controller (4). In this case, closed-loop stability and the principle of the separation property hold.

**Remark 2:** A close study of the minimisation problem (25) reveals that \(\delta(K_r)\) can be made successively smaller and smaller by successively increasing the order of the observer, \(p_r\). This is clear from the theory of matrix pseudo-inverse. Since increasing the order of the observer has the effect of making matrix \(\beta\) squarer. Therefore provided that \(\beta\) has full column rank, the error norm, \(\delta(K_r)\), will always get smaller and smaller. Accordingly, the closed-loop stability and performance will be met by successively increasing the order of the controller (5b). This forms the basis for the following design algorithm.

**Design Algorithm**

1. Design a suitable global state feedback controller \(F\) by using any existing robust state-feedback controller design method.

   \[ \text{Set } j = 0. \]

   \[ \text{For } i = 1, 2, \ldots, N \]

2. Set the order of the controller (5b) as \(p_r = 1 + j\).

3. Partition matrices \(F_i\) and \(L_i\) according to equations (15) and (16), respectively.

4. Select stable matrix \(E_i\).

5. Solve the optimization problem (25) for \(K_i\) and \(\bar{I}_i\). Derive \(\bar{I}_i\) from equation (21), and hence vector \(l\) from \(l = T_l \bar{I}_i\). Derive \(L_i\) from equation (16).

6. Derive matrix \(G_i\) and \(W_i\) from equation (13a) and (14a), respectively.

7. Check the closed-loop stability of system, if satisfied, stop; else set \(j \rightarrow j + 1\) and go to step 2.

**Remark 3:** In the above design algorithm, a lowest order \((p_r = 1)\) is first assigned for the controller (5b). Then the process of finding a distributed controller that would generate as close control signals as possible to the global state controller starts by
partitioning the standard state controller and choosing a stable local controller matrix for each subsystem. Then the optimisation problem, as formulated in equation (25), is solved. From the obtained results, the closed-loop stability is tested. If this condition is satisfied then the design process is completed. Otherwise the order of the controller is increased by one and the procedure is repeated. As discussed in remark 2, when the order of the controller is increased, the error term $\delta(K_i)$ becomes smaller and therefore closed-loop stability will be meet.

**Remark 4:** The design procedure outlined above yields a lowest possible distributed controller. The performance of it is, however, dependent on three factors: (1) the order of the resulting local controller, (2) the robustness quality of the adopted standard state feedback controller, $F$, and (3) the choice of the local controller matrices $E_i$.

5. STABILISER DESIGN

In this section a stabiliser is designed for the three machine unstable power system described in section (2). The design is based on the use of the rotor angles as the stabilising signals. Thus the output measurement is defined as

$$y_i(t) = [\Delta \delta_i, \Delta \delta_i, \Delta \delta_i], \ i=1,2,3$$

To start off the design process, we first need to find a stabilising state controller, $F$, which would satisfy some pre-specified stability and performance criteria. Let us assume that, for the sake of illustration, such a state feedback controller has been found by invoking the MATLAB function $lqr$, as shown below

$$F = -lqr2(A + eye(12), B, eye(12), eye(3)).$$

Using the design method presented in section 4, the following stabilisers are obtained.

**Machine 1:**

$$u_i(t) = K_i \zeta_i(t) + W_i y_i(t)$$

$$\dot{z}_i(t) = E_i \zeta_i(t) + L_i B_i u_i(t) + G_i y_i(t)$$

where

$$K_i = \begin{bmatrix} 100 & 100 \end{bmatrix}, \quad W_i = \begin{bmatrix} -0.7015 & 0.9335 & 2.2105 \end{bmatrix}, \quad E_i = \begin{bmatrix} -5 & 0 \\ 0 & -6 \end{bmatrix},$$

$$G_i = \begin{bmatrix} 0.0590 & -0.0284 & -0.1534 \\ -0.1672 & -0.0431 & 0.0899 \end{bmatrix}, \quad L_i = \begin{bmatrix} 0.0006 & -0.0008 & -0.0118 & -0.1241 & 0.0432 \\ 0.5310 & -0.0163 & -0.0011 & -0.0010 & 0 & -0.0016 \end{bmatrix}.$$

-0.0083 -0.0053 -0.0006 0.5846 0.3466 0.3130 0.0140 0.0010 0.0007 0 0.0013 0.

**Machine 2:**

$$u_i(t) = K_i \zeta_i(t) + W_i y_i(t)$$

$$\dot{z}_i(t) = E_i \zeta_i(t) + L_i B_i u_i(t) + G_i y_i(t)$$

where

$$K_i = \begin{bmatrix} 100 & 100 \end{bmatrix}, \quad W_i = \begin{bmatrix} 0.1199 & -0.6937 & 1.8092 \end{bmatrix}, \quad E_i = \begin{bmatrix} -5 & 0 \\ 0 & -6 \end{bmatrix},$$

$$G_i = \begin{bmatrix} -0.5273 & 0.9546 & -0.4021 \\ 0.6308 & -1.1575 & 0.3781 \end{bmatrix}, \quad L_i = \begin{bmatrix} -0.0326 & 0.0573 & -0.0171 & 2.4467 & -4.3698 & 1.2329 \\ -0.0005 & 0 & -0.0105 & -0.0007 & -0.0012 & 0; \end{bmatrix}.$$

0.0410 -0.0740 0.0121 -2.5653 4.6799 -0.7175 0.0003 0 0.0088 0.0006 0.0010 0.

**Machine 3:**

$$u_i(t) = K_i \zeta_i(t) + W_i y_i(t)$$

$$\dot{z}_i(t) = E_i \zeta_i(t) + L_i B_i u_i(t) + G_i y_i(t)$$

where

$$K_i = \begin{bmatrix} 100 & 100 \end{bmatrix}, \quad W_i = \begin{bmatrix} -0.1322 & -0.0666 & 2.3996 \end{bmatrix}, \quad E_i = \begin{bmatrix} -5 & 0 \\ 0 & -6 \end{bmatrix},$$

$$G_i = \begin{bmatrix} -0.2421 & -0.1101 & 0.2816 \\ 0.3069 & 0.1643 & -0.5842 \end{bmatrix}, \quad L_i = \begin{bmatrix} -0.0140 & -0.0054 & 0.0287 & 1.0504 & 0.3844 & -2.4430 & -0.0005 & 0 & 0.0001 & 0 & -0.0220 & -0.0015; \end{bmatrix}.$$

0.0191 0.0079 -0.0581 -1.1902 -0.4741 3.8701 0.0003 0 -0.0002 0 0.0195 0.0014 [].

The above stabiliser shift the closed-loop eigenvalues to the following stable locations:

$$eig(A_i) = -14.0555 \pm j8.1544, -18.1514, -12.2906, -10.6406 \pm j4.1321, -2.8171 \pm j7.9095, -2.3654 \pm j7.2041, -2.2211 \pm j6.1418, -5.8251 \pm j2.3173, -2.7021 \pm j0.3400, -4.1583, -3.8594.$$

6. SIMULATION RESULTS

To simulate the dynamic responses of the open and closed loop system, a step change in $\Delta T_{m1}$ is applied to the prime mover torque of machine 1. The speed and load angle responses of machine 1 are shown in Figures (2) and (3); space limitation preclude the inclusion of the simulation results for the other machines. Machine 1 is chosen because it is the machine most affected by the disturbance, and
therefore its response should be worse than the other two machines. The responses shown in the figures are those of the system under a full state feedback controller and its distributed equivalence designed by the method proposed in this paper. It is clear that the two responses are comparable to each other and that the power system has been stabilised with a sufficient degree of stability margin.

The same design method was followed but with the output measurement being the rotor speed. Space limitation does not allow for the simulation results to be shown here, but the results were quite comparable to those for the rotor angle.

The power system stabilisers shown in this paper use the load angle as the stabilising signal. However, speed or any other measurement for which the system is observable can be equally used.

8. REFERENCES