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Simultaneous State and Input Estimation with Application to a Two-Link Robotic System

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Abstract
This paper addresses the problem of estimating simultaneously the state and input of a nonlinear system with application to a two link robotic manipulator - the Pendubot. The system nonlinearity comprises a Lipschitz function with respect to the state, and a nonlinear term which is a function of both the state and input. It is shown that under some conditions, an observer can be designed to estimate simultaneously the system's state and input. Simulation and experimental results, obtained around the inverted equilibrium position, are presented to demonstrate the validity of the approach.

1 Introduction
The design of observer for nonlinear systems has recently attracted a great deal of research. For the class of Lipschitz nonlinear systems existence conditions have been established for full-order observers [1], and also for reduced-order observers [2]. The design method is based on the solution of a Riccati equation. For the class of systems with monotonic nonlinearities, an observer design approach is proposed in [3] where robustness analysis is conducted using the circle criterion evaluated by LMI computations.

While most of existing techniques are devoted to estimation of the system state, there has been also an interest in estimation of both the state and input of nonlinear systems. The problem is motivated in part by machine tool and manipulator applications, where an input observer is required to estimate the cutting force of a machine tool or exerting force/torque of a robotic system. For linear systems, a model error compensator based on the output estimation error has been proposed to estimate the input and to be incorporated with an extended Kalman filter to estimate the state [4]. For nonlinear systems, the problem of asymptotically estimating the system state and input has been addressed in [5], where the nonlinear part, expressed as a state-dependent and time varying function, is also the unknown input. Here, exact asymptotic estimation is not achieved, the system state and input can however be estimated to any desired degree of accuracy. More recently, nonlinear state space observer for simultaneously estimating the system state and input has been proposed in [6]. In their work, the nonlinear function is assumed to be Lipschitz with respect to the state and input.

This paper is concerned with the design of an asymptotic observer to estimate both the state and input of a class of nonlinear systems where the nonlinear function is a combination of a state-dependent unknown function including additive disturbance as well as uncertain time-varying terms, and a nonlinear function, Lipschitz with respect to the state and input. The problem, originated from the need to estimate the state and input of a two link robotic manipulator from its perturbed encoder measurements, is formulated generally for a multi-input multi-output system. The paper is organized as follows: After the introduction, section 2 presents the system description and formulates the problem. The main results are given in section 3. It is shown that exact asymptotic convergence of the estimate can be achieved upon the satisfaction of some conditions. In section 4 simulation and experimental results for the Pendubot are included. Finally section 5 concludes the paper.

2 System description and problem statement
2.1 Pendubot modelling:
The Pendubot [7] is a two-link robotic manipulator, shown in Figure 1.

![Figure 1: Schematic of the Pendubot](image-url)
Its dynamics can be obtained using Lagrangian equations of motion [8]:
\[ \tau = J(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q), \]
where \( \tau \) is the vector of the torque applied to the links, \( q = [q_1, q_2]^\top \) is the vector of joint angle positions, and
\[
\begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix},
\]
\[
d_{11} = m_1 l_1^2 + m_2 (l_1^2 + l_2^2 + 2l_1 l_2 \cos q_2) + J_1 + J_2,
\]
\[
d_{12} = d_{21} = m_2 (l_2^2 + l_1 l_2 \cos q_2) + J_2, d_{22} = m_2 l_2^2 + J_2,
\]
\[ C(q,\dot{q}) = \begin{bmatrix} h \dot{q}_2 & h \dot{q}_2 + h \dot{q}_1 \\ -h \dot{q}_1 & 0 \end{bmatrix}, h = -m_2 l_1 l_2 \sin q_2, \]
and
\[ g(q) = \begin{bmatrix} (m_2 l_2 l_1 + m_1 l_1) g \cos q_1 + m_2 l_1 l_2 \cos q_1 + g_2 \\ m_2 l_2 l_1 \cos q_1 + g_2 \end{bmatrix}.
\]
The system parameters \( m_1, l_1, l_2, J_1, m_2, l_2, m_2, J_2 \), and \( g \) are respectively the total mass of link one, the length of link one, the distance to the centre of mass of link one, the moment of inertia of link one about its centroid, the total mass of link two, the moment of inertia of link two about its centroid, and the gravitational acceleration. They can be grouped together into a new parameter set as:
\[ \theta_1 = m_1 l_1^2 + m_2 l_1^2 + J_1, \quad \theta_2 = m_2 l_2^2 + J_2, \]
\[ \theta_3 = m_1 l_1 l_2, \quad \theta_4 = m_1 l_1 + m_2 l_1, \quad \theta_5 = m_2 l_2.
\]
For a control design that neglects friction, these five parameters are all that are needed. They can be identified using the energy theorem to form equations that can be solved for the unknown parameter by a least squares problem [9]. Substituting these parameters into (1) yields the following matrices:
\[
D(q) = \begin{bmatrix} \theta_1 + \theta_2 + 2\theta_3 \cos q_2 + \theta_2 + \theta_3 \cos q_2 \\ \theta_1 + \theta_2 \cos q_2 + \theta_3 \cos q_2 \end{bmatrix},
\]
\[
C(q,\dot{q}) = \begin{bmatrix} -\theta_3 \sin(q_1) \dot{q}_2 - \theta_3 \sin(q_1) \dot{q}_2 - \theta_3 \sin(q_2) \dot{q}_1 \\ \theta_3 \sin(q_2) \dot{q}_1 \end{bmatrix},
\]
\[ g(q) = \begin{bmatrix} \theta_4 g \cos q_1 + \theta_5 g \cos(q_1 + q_2) \\ \theta_5 g \cos q_1 + g_2 \end{bmatrix}.
\]
By selecting \( x_1 = q_1, x_2 = \dot{q}_1, x_3 = q_2, x_4 = \dot{q}_2 \), the state equations are given by
\[ \dot{x}_1 = x_2, \quad \dot{x}_2 = \ddot{q}_1, \quad \dot{x}_3 = x_4, \quad \dot{x}_4 = \ddot{q}_2,
\]
where
\[ [\ddot{q}_1, \ddot{q}_2]^\top = D(q)^{-1} \tau - D(q)^{-1} C(q,\dot{q})\dot{q} - D(q)^{-1} g(q). \]
Denoting \( \tau = [u \ 0]^\top \) and ignoring small terms \( x_3 x_4, x_3^2 \) and \( x_4^2 \), the motion equation of the pendubot around its equilibrium points can be brought into the form:
\[
\dot{x} = Ax(t) + Bu(t) + f(x,u,y)
\]
\[ y(t) = Cx(t) + Du(t), \]
where \( x = [x_1, x_2, x_3, x_4]^\top \) is the state vector, \( u(t) \) is the torque applied to the first link, \( y = [y_1, y_2]^\top \) is the output vector, and \( f(x,u,y) \) is the nonlinear vector field. The system matrices are
\[
A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & 1 \end{bmatrix}, D = \begin{bmatrix} \alpha \end{bmatrix},
\]
where \( \mu = (\theta_2 - \theta_1)^2 \); and \( \alpha \) and \( \beta \) are coefficients representing the level of perturbations in the output measurements. The nonlinear function \( f(x,u,y) \) can be expressed as
\[
f(x,u,y) = f_L(x,u,y) + Wf_2(x,u,y),
\]
where
\[
f_L = \mu [u \theta_1 \cos x_3 + g \theta_2 \theta_3 \cos x_3 \cos(x_3 + x_3) - \theta_3 \theta_2 \cos(x_3 + x_3)].
\]
It is clear that \( f_1(x,u,y) \) is a global Lipschitz function with a Lipschitz constant \( \gamma = \| g' \| \mu (\theta_1 \theta_2 - \theta_2 \theta_1) \), and the other component, \( Wf_2(x,u,y) \), is not Lipschitz in the input variable \( u \).

2.2 Problem Statement

Let us now consider generally a class of nonlinear systems described by equation (5), where for the multi-input multi-output case \( x(t) \in \mathbb{R}^n \), \( u(t) \in \mathbb{R}^m \) and \( y(t) \in \mathbb{R}^p \) are respectively the state, unknown input and measured output. Matrices \( A, B, C \) and \( D \) are real constant and of appropriate dimensions; and \( f(x,u,y) \) is a real nonlinear vector function on \( \mathbb{R}^n \). The nonlinear vector function \( f(x,u,y) \) is assumed to comprise two portions as shown in (6). The nonlinear function \( f_L(x,u,y) \in \mathbb{R}^d \) is an unknown vector field and \( W \) is a known real constant matrix. The following assumptions are made:
Assumption 1: The term \( fL(X,U,y) \in R^m \) is a known nonlinear vector field, Lipschitz in its first two arguments with a Lipschitz constant \( \gamma \):

\[
\| fL(\xi(t),u(t),y(t)) - fL(\hat{\xi}(t),u(t),y(t)) \| \leq \gamma \| \xi(t) - \hat{\xi}(t) \| , \quad \forall \, t \in J,
\]

where \( \xi(t) = [x(t)] \in R^{(m+n)} \), \( \gamma \) is a positive real scalar, and \( \| . \| \) denotes the norm symbol.

Assumption 2: Matrix \( [D \quad CW] \) has full column rank, i.e.

\[
\text{rank}[D \quad CW] = m + d .
\]

Our objective is to design an asymptotic observer from the measured output signals \( y(t) \) to estimate both the state \( x(t) \) and the input \( u(t) \).

### 3 Observer design

For simplicity of presentation, the system described by (5-8) can then be expressed as

\[
E \dot{\hat{\xi}}(t) = \tilde{A} \hat{\xi}(t) + fL(\hat{\xi}(t),y(t)) + WfL(\hat{\xi}(t),y(t)) \quad (9a)
\]
\[
y(t) = H\hat{\xi}(t) ,
\]

where

\[
E = [I_n \quad 0], \quad \tilde{A} = [A \quad B], \quad H = [C \quad D],
\]

and \( I_n \) denotes the \( n \)-dimensional unitary matrix.

Our design objective is included in the problem of designing an observer for the generalized system (9) such that \( \hat{\xi}(t) \) converges asymptotically to \( \xi(t) \). For this, consider the following state observer:

\[
\dot{\hat{\xi}}(t) = N\xi(t) + Ly(t) + TF(\xi(t),y(t)) \quad (11a)
\]
\[
\dot{\hat{\xi}}(t) = a(t) + Q(\xi(t)) ,
\]

where \( \hat{\xi}(t) \) denotes the state estimation vector of \( \xi(t) \). Matrices \( N, L, T \) and \( Q \) are to be determined such that estimate \( \hat{\xi}(t) \) converges asymptotically to \( \xi(t) \).

Define \( e(t) \) as the error between \( \xi(t) \) and its estimate \( \hat{\xi}(t) \) as

\[
e(t) = \hat{\xi}(t) - \xi(t) ,
\]

Substituting (11b) and (9b) into (12a) gives

\[
e(t) = a(t) + (2H - T_{xw})\xi(t) .
\]

Let \( T \) be an \((m+n) \times m\) matrix such that

\[
T_E + QH = I_{nw} ,
\]

then (12b) becomes

\[
e(t) = a(t) - TE\hat{\xi}(t) .
\]

One can easily verify the following proposition:

**Proposition 1:** The system (6) can be employed as an observer for the system (5) if there exists a matrix \( T \in R^{(m+n)\times n} \) such that the following two conditions hold:

\[
\begin{align*}
NT_E + LH - T\tilde{A} & = 0, \\
TW & = 0
\end{align*}
\]

Condition 1: \( T_E + QH = I_{nw} \)

Condition 2: The error \( e(t) \) determined by the observer error system:

\[
\dot{e}(t) = Ne(t) + TfL(e(t) + \xi(t),y(t)) - fL(\xi(t),y(t))
\]

converges asymptotically to zero for all \( \xi(.) \) and \( y(.) \).

From Proposition 1, the design of the observer (11) is reduced to the problem of finding the matrices \( T, N, L \) and \( Q \) so that conditions 1 and 2 are satisfied. This is addressed in the following theorem, which is based on the solution for an unknown matrix using a matrix generalised inverse [10].

**Theorem 1:** For the observer (6), the estimate \( \hat{\xi}(t) \) converges asymptotically to \( \xi(t) \) if there exist matrices \( Z \in R^{(m+n)\times(n+m)} \) and \( F \in R^{(m+n)\times n} \) such that the following condition holds:

\[
\text{Condition 3: The error } e(t) \text{ in the observer error system}
\]

\[
\dot{e}(t) = (\Phi + 2E - FH)e(t) + (\Lambda + Z\Omega)(fL(e(t) + \xi(t),y(t)) - fL(\xi(t),y(t))]
\]

converges asymptotically to zero for all \( \xi(.) \) and \( y(.) \),

where \( \Phi = \Lambda\tilde{A}, \quad \Lambda = [I_{nw} \quad 0] \Sigma^{+T} \begin{bmatrix} I_w \ 0 \end{bmatrix}, \quad \Sigma = \begin{bmatrix} E & W \end{bmatrix}, \quad \Omega = \Omega\tilde{A}, \quad \Omega = (I_{nw} - \Sigma\Sigma^T) \begin{bmatrix} I_w \ 0 \end{bmatrix}, \quad \Sigma^T = (\Sigma^T\Sigma)^{-1}\Sigma^T \text{ is a generalised inverse of } \Sigma.

**Proof:** to be given in [11].

Since matrices \( \Phi, \Psi, H, \Lambda \) and \( \Omega \) are all known, the design of observer (11) is now reduced to the search for the two matrices \( Z \) and \( F \) as suggested in Theorem 1. Once they are found, \( T, Q, N \) and \( L \) can be obtained respectively by the following design equations:

\[
T = \Lambda + Z\hat{\Omega} ,
\]
\[
\Lambda = [I_{nw} \quad 0]\Sigma^T \begin{bmatrix} I_w \ 0 \end{bmatrix}, \quad \Omega = (I_{nw} - \Sigma\Sigma^T) \begin{bmatrix} I_w \ 0 \end{bmatrix}, \quad \Omega\tilde{A}, \quad \Omega = (I_{nw} - \Sigma\Sigma^T) \begin{bmatrix} I_w \ 0 \end{bmatrix}, \quad \Sigma^T = (\Sigma^T\Sigma)^{-1}\Sigma^T \text{ is a generalised inverse of } \Sigma.
\]

**Proof:** to be given in [11].
\[
\Xi = \begin{bmatrix}
I_{n+m} & 0
\end{bmatrix} \Sigma \begin{bmatrix}
0 \\
I_r
\end{bmatrix},
\]
\[
\Omega = (I_{n+m} - \Sigma \Sigma^T) \begin{bmatrix}
0 \\
I_r
\end{bmatrix},
\]
\[
N = \Phi + Z \Psi - F H,
\]
\[
\Phi = \Xi A, \quad \Psi = \Omega A,
\]
\[
L = F + N Q
\]  

(19)

A computationally-efficient way to obtain matrices \(Z\) and \(F\) is given in the following theorem.

**Theorem 2:** The estimation error \(e(t)\) of observer (11) converges asymptotically to zero if there exist matrices \(P = P^T > 0\), \(Z\), and \(F\); and positive scalars \(\delta_1\) and \(\delta_2\) such that the following Riccati inequality is satisfied

\[
\begin{align*}
\Phi^T P + P \Phi + \Psi^T Z^T P + P Z \Psi - H^T X^T P - PF H \\
+ \frac{1}{\delta_1} P A P^T + \frac{1}{\delta_2} \Psi \Omega \Psi^T Z^T P + \gamma^2 (\delta_1 + \delta_2) I < 0,
\end{align*}
\]

(20)

where \(\gamma\) is the Lipschitz constant defined in (7).

**Proof:** to be given in [II].

We now propose that the Riccati inequality of the form

\[
\begin{align*}
\Phi^T P + P \Phi + \Psi^T Z^T P + P Z \Psi - H^T X^T P - PF H \\
+ \frac{1}{\delta_1} P A P^T + \frac{1}{\delta_2} \Psi \Omega \Psi^T Z^T P + \gamma^2 (\delta_1 + \delta_2) I < 0,
\end{align*}
\]

(21)

Using the Schur complement result [13], the Riccati inequality (21) can be converted to the following linear matrix inequality with respect to \(P, X, Y, \delta_1\) and \(\delta_2\):

\[
\begin{bmatrix}
\Phi^T P + P \Phi + \Psi^T Z^T P + P Z \Psi - H^T X^T P - PF H \\
A^T P + \Omega \Psi \Omega^T X^T P - \delta_1 I & 0
\end{bmatrix}
\]

\[
< 0.
\]

(22)

**Remark 1:**

When \(\Omega = 0\) (hence, \(\Psi = 0\)), one can choose \(Z = 0\) (hence, \(Y = 0\)) and the linear matrix inequality (22) is reduced simply to

\[
\begin{bmatrix}
\Phi^T P + P \Phi - H^T X^T - X H + \gamma^2 \delta_1 I & \Phi A \\
A^T P - \delta_1 I
\end{bmatrix}
\]

\[
< 0.
\]

(23)

**Remark 2:**

In our approach the estimation error dynamics can be proved to be affected only by the Lipschitz nonlinearity. As noted in [I], asymptotic convergence of the estimation error will then necessitate that the Lipschitz constant \(\gamma\) should be less than some limit, in addition to the Hurwitz property of the observer error system's matrix, which is \(N = (\Phi + Z \Psi - F H)\) in this paper. A smaller value of \(\gamma\) will therefore enable a better chance for the linear matrix inequality (22) to be feasible. A computational algorithm is stated as follows:

**Design Algorithm**

For a given constant \(\gamma\):

**Step 1:** Solve the LMI problem (22) by using the LMI toolbox.

**Step 2:** If positive constants \(\delta_1\) and \(\delta_2\), and matrices \(P, X\) and \(Y\) satisfying (22) are found, then go to step 3. Otherwise, the value of \(\gamma\) has to be reduced.

**Step 3:** Obtain \(Z = P^{-1} Y\) and \(F = P^{-1} X\). Matrices \(T, Q, N\) and \(L\) are then given respectively by (14-19). The observer design is completed.

4 Simulation results

The Pendubot described in Section 2 is used for illustrations of the observer design. It is identified with the following parameters (in SI units) [9]:

\[
\begin{bmatrix}
8_1 \\
8_2 \\
8_3 \\
8_4 \\
8_5
\end{bmatrix} =
\begin{bmatrix}
0.0761 \\
0.0662 \\
0.0316 \\
0.9790 \\
0.3830
\end{bmatrix}
\]

With \(g = 9.8 m/s^2\) and \(\alpha = \beta = 0.5\), the Lipschitz constant is found to be \(\gamma = 129.2\). As a result from Theorem 1, one can derive the following matrices

\[
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
\Lambda =
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
\Phi =
\begin{bmatrix}
0 & 0 & 0 & 16.55 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
\Omega =
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

By choosing \(Z = 0\) (hence \(Y = 0\)), according to Remark 1, the LMI problem (23) can be solved to yield the following results:

\[
\begin{bmatrix}
79092 & 32.987 & 0 & 0 & 0 & 39432 \\
32.987 & 26.97 & 0 & 0 & 0 & -65.975 \\
0 & 0 & 80494 & 36628 & 0 & 0 \\
0 & 0 & 36628 & 25551 & 0 & 0 \\
39432 & -65.975 & 0 & 0 & 0 & 19944
\end{bmatrix}
\]
From the design equations, an asymptotic observer is obtained with the following matrices, calculated according to (18-19):

\[
X = \begin{bmatrix}
46418 & 0 \\
138.82 & 0 \\
0 & 49095 \\
0 & 17310 \\
22663 & 0 \\
\end{bmatrix}, \quad \delta_1 = 1, \quad Z = 0,
\]

\[
F = \begin{bmatrix}
0.4676 & 0 & 0 & -0.2338 \\
6.62 & 0 & 0 & 0 \\
0 & 0.8676 & 0 & 0 \\
0 & -0.5662 & 0 & 0 \\
0.2338 & 0 & 0 & 0 \\
\end{bmatrix}, \quad T = A = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 2 \\
-2 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
N = \begin{bmatrix}
-0.4676 & 1 & 0 & 0 & -0.2338 \\
-6.62 & 0 & 0 & 0 & 13.24 \\
0 & 0 & -0.8676 & 0.5662 & 0 \\
0 & 0 & 0.5662 & -1.7169 & 0 \\
-0.2338 & -2 & 0 & 0 & -0.1169 \\
\end{bmatrix}
\]

\[
L = \begin{bmatrix}
0 & 2 \\
33.1 & 0 \\
0 & -4 \\
0 & 0 \\
\end{bmatrix}, \quad Q = \frac{1}{A} = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 2 \\
0 & 2 \\
\end{bmatrix}
\]

and the nonlinear vector field:

\[
Tf_1(\xi, \psi) =
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & -2 & 0 \\
-2 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
0 \\
-129.2 \cos(\dot{x}_1) \\
0 \\
0 \\
-129.2 \cos(\dot{x}_1) \\
\end{bmatrix} =
\begin{bmatrix}
0 \\
-129.2 \cos(\dot{x}_1) \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]

For the purpose of simulation let us impose a signal \(u(t) = 3 \sin 8t\) around the mid point equilibrium point \((x_1 = -\pi/2, \quad x_3 = \pi)\) of the Pendubot.

Superimposed in Figure 2 are the time responses of the input signal \(u(t)\) and its estimate, \(\hat{u}(t)\), shown respectively in solid and broken lines. Figures 3-4 show respectively the responses of the system states, \(x_i(t)\), and their estimates \(\hat{x}_i(t)\), \(i = 1-2\).

It is clear from the simulation that all the estimates of both the input and system state converge asymptotically to the actual signals.
5 Experimental results

Data logged from the experiment of swinging up and balancing the Pendubot to both the top-position \((x_1 = \pi/2, \ x_3 = 0)\) and mid-position \((x_1 = -\pi/2, \ x_3 = \pi)\), as pictured in Figure 5, are used to test the observer. Estimates of the input and state of the system around the equilibrium are compared with the actual responses.

For the top position, Figure 6 depicts the time responses of the input signal \(u(t)\) and its estimate, \(\hat{u}(t)\). Figures 7-8 show respectively the responses of the system states, \(x_i(t)\), and their estimates \(\hat{x}_i(t)\), \(i=3-4\) at this position. Again, the estimates are shown in broken lines and the actual signals in solid lines.

The results for the mid-position are shown in Figures 9, 10 and 11 respectively for the actual responses and estimates of the control input, the second joint angle \(x_3(t) = q_2\) and its time-derivative \(x_4(t) = \dot{q}_2\).

It is clear from the experimental results that the reproduced signals by using the proposed observer are quite close to the actual responses in both cases.

6 Conclusion

We have presented an approach to the design of observers for estimating simultaneously the state and unknown input for a class of nonlinear multi-input multi-output systems. The system nonlinearity comprises a Lipschitz nonlinear function with respect to the state and input, and a state-dependent unknown function including additive disturbance as well as uncertain time-varying terms. The observer design can be completed in a computationally efficient manner via the use of an LMI-based algorithm.
The design procedure and performance of the proposed method are illustrated through simulation and experimental results for a two-link robotic system.

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