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A Distributed Approach to Global Feedback Control of Multi-Agent Systems

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Abstract— This paper presents a distributed approach to global state feedback control of for multi-agent systems. The proposed solution is obtained from the construction of decentralised functional observers. The required local control signal is generated asymptotically using only corresponding local output information. It is shown that a complete distribution of the control of a decentralised system having $N$ agents can be achieved upon the satisfaction of some necessary conditions, and that the resulting controller can emulate the performance of a global state feedback controller. A step-by-step design algorithm is given. Its simplicity and correctness are illustrated through a numerical example.

Keywords- multi-agent systems, functional observers, global feedback, decentralised information structure.

I. INTRODUCTION

It is well-known that the development of efficient distributed controllers for large-scale systems can lead to considerable savings in the information transfer network cost, increased reliability of the overall system, and practical feasibility of control implementation. A considerable amount of research work has appeared in the literature to resolve the decentralised control problem of complex systems where the control task is shared and distributed among a number of local controllers (see, for example, [1-8], [13-18] and references therein).

As it is impractical, in most of the cases, to implement a full state feedback controller for large-scale and complex systems, a linear functional observer scheme may be used to generate asymptotically the global control signal. The design of linear functional observers has been the focus of many researchers over the years. A number of procedures have been proposed for the design of linear functional state observers (see, for example, [9-17]). Of particular interest in industrial process control, mobile robots, flexible manufacturing networks, and water distribution networks is the design of decentralised linear functional observers to implement global state feedback controllers for multi-agent systems under no cross flows of information between the control stations [18]. The application to the control of a group of mobile robots moving in a formation is presented in [19].

This paper deals with the problem of implementing a global feedback controller for multi-agent linear systems under a decentralised information structure where the flows of information between the system control stations are absent. For such a system, the control input for the $i$-th local control station must be obtained from its local input and output information only. The approach proposed here is based on the idea of distributed implementation of a global state feedback controller that has been designed using any existing feedback control design method. The problem of distributed implementation of the global controller is solved through the design of $N$ decentralised linear functional observers. The method has the following features: (i) the observers are completely decentralised with each local control station using only locally available information to generate its control input signal, and (ii) the overall closed-loop system stability and performance is guaranteed upon the satisfaction of certain conditions.

The organisation of the paper is as follows. The problem is stated in the next section. Main results are presented in Section 3. A numerical example is included in Section 4. Finally, Section 5 concludes the paper.

II. PROBLEM STATEMENT

Let us consider the problem of designing decentralised linear functional observers to reconstruct $N$ local control signals of a dynamic system comprising $N$ local agents. The system is described by [1]:

\[
\dot{x}(t) = Ax(t) + \sum_{i=1}^{N} B_i u_i(t) \tag{1a}
\]

\[
\gamma_i(t) = C_i x(t) \tag{1b}
\]

\[
u_i(t) = F_i x(t); \quad i = 1, 2, \ldots, N \tag{1c}
\]
where $x(t) \in \mathbb{R}^n$ is the global state vector, and $u_i(t) \in \mathbb{R}^i$ and $y_i(t) \in \mathbb{R}^i$ ($i = 1, 2, \ldots, N$) are respectively the input and output vectors of the $i$-th control station. Matrix $F_i \in \mathbb{R}^{i \times N}$ ($i = 1, 2, \ldots, N$) is known constant and corresponding to the $i$-th row of a given global state feedback matrix $F \in \mathbb{R}^{N \times N}$. The local control signal $u_i(t) \in \mathbb{R}^i$ is a scalar linear functional state vector to be estimated. The pair $(C_i, A)$ is observable, and matrices $A \in \mathbb{R}^{n \times n}$, $B_j \in \mathbb{R}^{n \times i}$ and $C_i \in \mathbb{R}^{i \times n}$ are real constant. Without loss of generality, it is assumed that matrix $C_i$ take the canonical form

$$C_i = [I_{r_i}, 0],$$

where $I_{r_i}$ is an identity matrix of dimension $r_i$ (if this is not the case, then system (1) can always be transformed by an orthogonal similarity transformation).

The aim of this paper is to design $N$ decentralised observers of the form

$$\dot{z}_i(t) = K_i z_i(t) + W_i y_i(t) \quad (2a)$$

$$\dot{z}_i(t) = E_i z_i(t) + H_i u_i(t) + G_i y_i(t) , \quad (i = 1, 2, \ldots, N) \quad (2b)$$

to reproduce asymptotically the control signals $u_i(t)$ ($i = 1, 2, \ldots, N$). In (2), $z_i(t) \in \mathbb{R}^{r_i}$, and $K_i$, $W_i$, $E_i$, $H_i$, and $G_i$ are constant matrices of appropriate dimensions to be determined.

The problem of observing a scalar functional or multifunctionals of the state vector in systems having cross flows of information between stations has been the subject of numerous papers [9]-[17]. For scalar functional observers, a well-known Luenberger’s classic result [9] provides an upper bound on the order with $p = n-1$, where $n$ is the observability index of the matrix pair $(C, A)$. Reducing the order of multi-dimensional functional observers remains an interesting issue in recent developments of observer design [13-17]. Both necessary and sufficient conditions are provided by Darouach [16,17] for the existence and stability of multi-dimensional functional observers with an order equal to the dimension of the functional vectors to be estimated. It is interesting to notice that the design of decentralised functional observers (2) for systems (1) under no cross flows of information between stations has not been systematically addressed. This paper provides necessary conditions for the existence of decentralised functional observers together with a design procedure for their synthesis.

III. MAIN RESULTS

Let us first verify the following proposition:

**Proposition 1**: $\dot{u}_i(t)$ in (2) is an asymptotic estimate of $u_i(t)$ if and only if the following conditions are satisfied

$$F_i = K_i L_i + W_i$$

(3)

$$G_i C_i - L_i A + E_i L_i = 0, \quad E_i \text{ is Hurwitz}$$

(4)

$$L_i B_i = 0$$

(5)

$$H_i = L_i B_i$$

(6)

where $L_i \in \mathbb{R}^{r_i \times n}$ and $B_r = [B_1, B_{i+1}, \ldots, B_N] \in \mathbb{R}^{n \times (N-1)}$ is the input matrix associated with the remaining $(N-1)$ remote control stations obtained by stacking matrices $B_j$ ($j = 1, 2, \ldots, N; j \neq i$).

**Proof**: Let an error vector $e_i(t)$ be defined as

$$e_i(t) = z_i(t) - L_i x(t); \quad i = 1, 2, \ldots, N.$$ 

Hence

$$\dot{e}_i(t) = \dot{z}_i(t) - \dot{L}_i x(t),$$

$$\dot{e}_i(t) = E_i z_i(t) + H_i u_i(t) + G_i y_i(t) + (H_i - L_i B_i) u_i(t) - L_i B_i u_r(t),$$

where $u_r(t) = [u_1(t) \ldots u_{i-1}(t) u_{i+1}(t) \ldots u_N(t)]$ contains $(N-1)$ input control signals of the remaining $(N-1)$ remote stations. It is clear from the above equation and the conditions of the proposition that $e_i(t) \to 0$ as $t \to \infty$, and consequently $z_i(t) \to L_i x(t)$. This, according to (3), implies $\dot{u}_i(t) \to u_i(t)$ and thus, the state feedback control law (1e) can be implemented by a decentralised dynamical controller (2).

As matrix $E_i$ is chosen to be stable and matrix $H_i$ is derived according to $H_i = L_i B_i$, there are now four sets of unknown matrices ($G_i$, $L_i$, $K_i$, and $W_i$) in (3)-(5) to be solved for. In the following, we first simplify equations (3)-(5) and then derive the existence conditions for observers (2), which in turn will lead to a systematic solution to the unknown matrices.

Partition matrices $F_i$, $L_i$, $B_r$, and $A$ as follows

$$F_i = \begin{bmatrix} F_{i1} & F_{i2} \end{bmatrix}, \quad L_i = \begin{bmatrix} L_{i1} & L_{i2} \end{bmatrix}, \quad B_r = \begin{bmatrix} B_{r1} \\ B_{r2} \end{bmatrix},$$

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix},$$

(7)

where submatrices $F_{i1} \in \mathbb{R}^{r_i \times n}$, $F_{i2} \in \mathbb{R}^{r_i \times (n-r)}$, $L_{i1} \in \mathbb{R}^{r_i \times r_i}$, $L_{i2} \in \mathbb{R}^{r_i \times (N-1)}$, $B_{r1} \in \mathbb{R}^{(N-1) \times r_i}$, $B_{r2} \in \mathbb{R}^{(N-1) \times (N-1)}$, $A_{11} \in \mathbb{R}^{r_i \times r_i}$, $A_{12} \in \mathbb{R}^{r_i \times (n-r)}$, $A_{21} \in \mathbb{R}^{(n-r) \times r_i}$, and $A_{22} \in \mathbb{R}^{(n-r) \times (n-r)}$. 


Incorporate (7) and (1d) into (3)-(6), the following equations are obtained

\[
\begin{bmatrix}
H_i &= L_i B_i \\
G_i &= L_i A_{i1} + L_{i2} A_{i2} - E_i L_i \\
W_i &= F_{i1} - K_i L_{i1},
\end{bmatrix}
\]  
(8)

and

\[
\begin{bmatrix}
F_{i2} &= K_i L_{i2} \\
E_i L_{i2} &= L_i A_{i1} - L_{i2} A_{i2} \\
L_i B_{i1} &+ L_{i2} B_{i2} = 0.
\end{bmatrix}
\]  
(9)

It can be seen that matrices \(H_i, G_i\) and \(W_i\) are directly derived from (8) once the tri-linear problem (9) is solved. In order to avoid a trivial (i.e., zero) solution of (9), let us assume that \(F_{i2} \neq 0\) (note that \(F_{i2} = 0\) results in a static observer).

The remainder of this paper will focus on the exact solution to the tri-linear problem (9). The following theorem presents necessary conditions for such a solution.

**Theorem 1:** A solution to the tri-linear problem (9) exists provided that the following necessary conditions are satisfied:

**Condition 1:** \(\text{rank}(B_{i1}) = (N-1)\).

**Condition 2:** \(\eta_i > (N-1)\).

Furthermore, the order of the observer (2) is predetermined and given by \(p_i \geq \frac{(n-\eta_i)}{(\eta_i + 1 - N)}\).

**Proof:** Choose a stable matrix \(E_i\), and the pair \((K_i, E_i)\) in an observable canonical form

\[
K_i = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
-\alpha_{p_i} & -\alpha_{p_i-1} & -\alpha_{p_i-2} & -\alpha_{p_i-3}
\end{bmatrix},
\]
(10)

where \(\alpha_k (k=1,2,\ldots,p_i)\) are correspondingly chosen constant numbers.

Partition matrices \(L_{i1}\) and \(L_{i2}\) as follows

\[
L_{i1} = \begin{bmatrix}
l_{i1,1} & l_{i1,2} & \ldots & l_{i1,i_1} \\
l_{i1,2} & l_{i1,3} & \ldots & l_{i1,i_2} \\
\vdots & \vdots & \ddots & \vdots \\
l_{i1,p_i} & l_{i2,p_i} \\
\end{bmatrix},
\]
\[
L_{i2} = \begin{bmatrix}
l_{i2,1} & l_{i2,2} & \ldots & l_{i2,i} \\
\end{bmatrix},
\]
where \(l_{i,j,1} \in \mathbb{R}^{i \times \eta_i}\),

\[
l_{i2,j} \in \mathbb{R}^{i \times (n-\eta_i)} (j=1,2,\ldots,p_i).
\]
(11)

Substituting (11) into (9) and after some tedious manipulations, the following equation can be obtained

\[
l_i \Omega_j = \gamma_j,
\]
(12)

where

\[
l_i = [l_{i1,1}, l_{i1,2}, \ldots, l_{i1,i_1}, l_{i1,i_2}, \ldots, l_{i1,p_i}],
\]
\[
\Omega_j = [\alpha_1, \alpha_2, \ldots, \alpha_p],
\]
\[
\gamma_j = [\beta_1, \beta_2, \ldots, \beta_p],
\]
\[
\Psi_i = \begin{bmatrix}
1 & 0 & 0 & \ldots & 0 \\
\alpha_1 & 1 & 0 & \ldots & 0 \\
\alpha_2 & \alpha_1 & 1 & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\alpha_p & \alpha_{p-1} & \alpha_{p-2} & \ldots & 1
\end{bmatrix}
\]
\[
\in \mathbb{R}^{p \times (n-\eta_i)},
\]
\[
\beta_1 = -(F_{i2} A_{i2}^2 + \alpha_1 F_{i2} A_{i2}^{p_i+1}) + \alpha_2 F_{i2} A_{i2}^{p_i+2} + \ldots + \alpha_p F_{i2} A_{i2} + 2 \alpha_p F_{i2} A_{i2}^{p_i+1},
\]
\[
\eta_i = -(F_{i2} B_{i2} + F_{i2} A_{i2} B_{i2} + F_{i2} A_{i2}^{p_i+1} B_{i2}),
\]
\[
\in \mathbb{R}^{p \times (N-1)},
\]
(13)

and

\[
0 & 0 & 0 & \ldots & 0 & B_{i1} \\
0 & 0 & 0 & \ldots & A_{i2} B_{i2} & A_{i2} B_{i2} \\
0 & 0 & 0 & \ldots & A_{i2} B_{i2} & A_{i2} B_{i2} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & B_{i1} & \ldots & A_{i2} B_{i2} & A_{i2} B_{i2} \\
0 & B_{i1} & B_{i2} & \ldots & A_{i2} B_{i2} & A_{i2} B_{i2} \\
B_{i1} & A_{i2} B_{i2} & A_{i2} B_{i2} & \ldots & A_{i2} B_{i2} & A_{i2} B_{i2} \\
A_{i2} B_{i2} & A_{i2} B_{i2} & A_{i2} B_{i2} & \ldots & A_{i2} B_{i2} & A_{i2} B_{i2}
\end{bmatrix}
\]
(13)

It is clear from equations (13a-g) that matrices \(\Theta_i\) and \(\gamma_i\) can be easily obtained from the given system parameters.

Now as equation (12) contains a set of \((n-\eta_i) + p_i (N-1)\) linear algebraic equations with \(p_i\) unknowns, a solution exists if and only if

\[
(i) \quad p_i \eta_i \geq (n-\eta_i) + p_i (N-1), \quad i.e., \quad \eta_i \geq \frac{(n-\eta_i)}{(\eta_i + 1 - N)}
\]

and

\[
(ii) \quad \text{matrix } \Theta_i \text{ has full rank}, \quad \text{i.e.} \quad \text{rank}(\Theta_i) = (n-\eta_i) + p_i (N-1).
\]

From condition (i) above, it is clear that \(\text{rank}(\Psi_i) = (n-\eta_i)\). It is also clear from the structure of matrix \(\Omega_i\) in (13g) that \(\text{rank}(\Omega_i) = p_i (N-1)\) provided that matrix \(B_{i1}\) has full rank (i.e., \(\text{rank}(B_{i1}) = (N-1)\)). Accordingly, conditions 1 and 2 of the theorem ensure the existence of a solution to equation (12). This completes the proof of the theorem.
From theorem 1, a step-by-step design algorithm is now proposed.

**Design Algorithm**

1. Determine a state feedback gain matrix $F$ using any existing control technique.
2. Partition matrices $B_i$ and $A$ according to equation (7).
3. Select the smallest value for the observer order $p_i$ in (2) by using $p_i \geq \frac{(n - r_i)}{(r_i + 1 - N)}$.
4. Choose a stable matrix $E_i$, and the pair $(K_i, E_i)$ according to equation (10).
5. Solve equation (12) for vector $l_i$ (hence $L_{i1}$ and $L_{i2}$).
6. Obtain matrices $H_i, G_i$ and $W_i$ from equation (8).
7. Repeat steps 2-6 for the remaining $(N-1)$ control agents.

**Remark 1:** When there is no interconnection constraint imposed on the $i$-th station, the tri-linear problem (9) is reduced to the following bi-linear problem:

$$
\begin{align*}
F_{i2} - K_i L_{i2} & = 0, \\
E_{i2} L_{i2} - L_{i1} A_{i2} - L_{i2} A_{i2} & = 0,
\end{align*}
$$

and equation (12) is reduced to:

$$
\dot{l}_i \Psi_i = \beta_i,
$$

where $\Psi_i$ and $\beta_i$ have been defined in equations (13d) and (13e), respectively.

From equation (15) it is obvious that a solution for $l_i$ exists if and only if

$$
\begin{bmatrix}
A_{i2} \\
A_{i2} A_{i2} \\
\vdots \\
A_{i2} A_{i2}^{n-2}
\end{bmatrix} = (n - r_i).
$$

Note that the pair $(C_i, A)$ is observable and $\text{rank}(C_i) = r_i$ with an observability index $v_{C_i, A} = \frac{n}{r_i}$. It follows that the pair $(A_{i2}, A_{i2}^{n-1})$ is observable with an observability index of

$$
v_{A_{i2}, A_{i2}^{n-1}} = \frac{n - r_i}{r_i} = \frac{n - 1}{v_{C_i, A} - 1}.
$$

Accordingly, if we choose $p_i = (v_{C_i, A} - 1)$, then $\text{rank}(V_i) = (n - r_i)$ and $V_i$ is non-singular. The above analysis agrees with the theory of linear functional observers [9].

**Remark 2:** It is shown that the desired separation property of the overall closed-loop system can also be verified. In fact, the control signal $u_i(t)$ can be expressed as

$$
u_i(t) = K_i e_i(t) + (K_i L_i + W_i C_i)x(t) = K_i e_i(t) + F_i x(t). 
$$

Accordingly, $Bu(t)$ is rewritten as

$$
Bu(t) = \sum_{i=1}^{N} B_i u_i(t) = \sum_{i=1}^{N} B_i K_i e_i(t) + \sum_{i=1}^{N} B_i F_i x(t) = B F x(t) + B K e(t),
$$

where $K = \text{diag}(K_i); i = 1, 2, \ldots, N$ and $e(t) = \begin{bmatrix} e_1(t) \\ \vdots \\ e_N(t) \end{bmatrix}$.

Hence, the following augmented closed-loop system is obtained

$$
\begin{bmatrix}
\dot{x}(t) \\
\dot{e}(t)
\end{bmatrix} = \begin{bmatrix}
A + BF & BK \\
0 & E
\end{bmatrix} \begin{bmatrix}
x(t) \\
e(t)
\end{bmatrix},
$$

where $E \in \mathbb{R}^{N_c \times N_c} = \text{diag}(E_i); i = 1, 2, \ldots, N$. Equation (20) implies that the separation principle holds.

**Remark 3:** Conditions 1 and 2 of theorem 1 may be made less conservative by relaxing the pattern of the decentralized information structure. To show this, let us assume that the $i$-th control station is now able to receive input and output information from one of the remaining $(N-1)$ control stations. In this case, it is easy to show that Condition 2 can be reduced to $r_i^* > (N - 2)$, where $r_i^*$ is the total number of outputs of the two combined control stations. This also implies that the order of the observer (2) is now reduced to $p_i > \frac{(n - r_i)}{(r_i^* + N - 2)}$.

Similarly, Condition 1 is now reduced to $\text{rank}(B_{i2}^*) = (N - 2)$. The result can therefore be extended to systems with varying degrees of information patterns.

**IV. ILLUSTRATIVE EXAMPLE**

Consider an unstable 5th-order system comprising two control stations, i.e. $N=2$. Each control station has one local control input and two local outputs. The system is described by equation (1), where matrices $A$, $B_i$ and $C_i$ ($i=1,2$) are given by
\[
A = \begin{bmatrix}
-3 & 0 & -0.6 & 1.5 & -0.3 \\
-0.3 & -6 & 0 & 0.6 & 1.5 \\
-1.2 & 1.5 & -9 & 0.3 & -3 \\
-2.25 & -0.6 & -2.4 & 2 & 0 \\
-0.6 & 1.5 & -1.5 & 3.75 & -1
\end{bmatrix}, \quad B_1 = 0.5, \quad \begin{bmatrix}
0 \\
1 \\
1 \\
1 \\
-1
\end{bmatrix}
\]
\[
B_2 = \begin{bmatrix}
0.2 \\
-0.1 \\
1 \\
2 \\
0.3
\end{bmatrix}, \quad C_1 = \begin{bmatrix}
1 & 0.2 & -0.3 & 1 & 2 \\
1 & 0 & 0 & 0 & -0.5
\end{bmatrix}
\]
and \[
C_2 = \begin{bmatrix}
0.5 & 0 & 0.1 & 0.7 & 0.9 \\
0.6 & 0.4 & -0.4 & 0.5 & 0
\end{bmatrix}
\]

with these open-loop eigenvalues: \( \lambda(A) = \{-9.4109, 4.5047, -2.2316, 0.8752, -5.9874\} \).

As the system is controllable, a global linear state feedback control law can be easily derived. Note that the main objective of this paper is to propose decentralised functional observers that are able to generate any acceptably designed set of global state feedback control signals. Therefore, in this example, LQR is used for simplicity with the state and control weighting matrices chosen respectively as \( Q = 2I_5 \) and \( R = I_2 \). The following optimal controller is easily obtained

\[
u(t) = Fx(t) = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} x(t),
\]

where
\[
F_1 = [-0.9320 \quad 0.8682 \quad -1.1046 \quad 1.8187 \quad 7.6367], \quad F_2 = [-1.0132 \quad 0.3520 \quad -1.1734 \quad 3.6866 \quad 4.8109].
\]

Application of the above state feedback controller gives the following closed-loop eigenvalues: \( \lambda(A + BF) = \{-9.4163, -6.0007, -3.4665 \pm j1.0292, -4.5434\} \).

Let us start with the first control station, i.e. \( i = 1 \). Since the output matrix \( C_i \) is not in the canonical form (1d), an orthogonal transformation matrix, \( M_1 \), is needed to transform matrix \( C_i \) into the canonical form, where

\[
M_1 = [C_i^T (C_i C_i^T)^{-1} \text{ null}(C_i)]
\]

Using the above transformation, the system can now be partitioned according to

\[
A = (M_1)^{-1} A_{glob} M_1 = \begin{bmatrix}
2.9886 & -8.2800 & -4.4715 & -0.7352 & 1.2214 \\
-1.0595 & -1.2900 & -1.8252 & 0.7738 & 0.5254 \\
0.2442 & 0.5350 & 0.3700 & -2.3610 & -1.4980 \\
-0.6559 & 0.0270 & -6.4786 & -8.0378 & 0.9827 \\
0.0810 & 0.5654 & 1.5606 & 1.5472 & -6.2807
\end{bmatrix},
\]

\[
B_1 = (M_1)^{-1} B_{i_{glob}} = \begin{bmatrix}
-0.15 \\
15 \\
0.7248 \\
-0.2919 \\
-0.4745
\end{bmatrix}, \quad B_2 = (M_1)^{-1} B_{2_{glob}} = \begin{bmatrix}
1.52 \\
0.05 \\
0.1471 \\
-0.7028 \\
-0.0954
\end{bmatrix},
\]

\[
C_1 = C_{i_{glob}} M_1 = \begin{bmatrix}
1.5 \\
0.05
\end{bmatrix}, \quad F_1 = F_{i_{glob}} M_1 = \begin{bmatrix}
2.7186 \\
-3.8002 \\
0.7795 \\
-0.7028 \\
-0.0954
\end{bmatrix}.
\]

Now, \( \text{rank}(B_1) = \text{rank} \begin{bmatrix}
-1.52 \\
0.05
\end{bmatrix} = 1 \) and \( \eta_1 = 2 > 1 \), therefore Conditions 1 and 2 of theorem 1 are satisfied, and accordingly a third-order \( (p_1 \geq (5-2)/2 = 1.5) \) decentralised functional observer (2) can be used to realise the distributed implementation for the control station 1.

Let the stable matrix \( E_i \) be chosen with eigenvalues of \( \{-4.5,-6\} \), and the pair \((K_1, E_1)\) take the following observable canonical form:

\[
K_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ -120 & -74 & -15 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -120 & -74 & -15 \end{bmatrix}.
\]

The remaining steps 5-6 of the design algorithm can then be easily followed, and the following results are obtained for the control station 1

A) Design parameters for linear functional 1 (control agent 1):

\[
u_1(t) = F_2 x(t) = (K_1 E_1 + W_1 C_1) x(t) = K_2 \dot{x}_1(t) + W_1 y_1(t)
\]

Where

\[
\begin{bmatrix}
0 & 1 & 0 \\
-120 & -74 & -15 \\
1.6179 & -5.2988
\end{bmatrix},
\]

\[
L_4 = \begin{bmatrix} 2.7490 & 0.5446 & -0.6192 & 0.2008 & 1.7515 \\
77.2453 & 20.3327 & -17.5856 & 5.9280 & 53.4191
\end{bmatrix}.
\]
By substituting the above matrices, one can easily confirm that equations (3)-(6) are satisfied. Accordingly, the designed state feedback controller for control station 1 can now be implemented by the above distributed controller using a third-order observer.

By repeating steps 2-7 of the design algorithm, a complete decentralized dynamical output feedback controller for control station 2 can also be obtained as given in the following.

B) Design parameters for linear functional 2 (control agent 2):

\[ u_2(t) = F_2 x(t) = (K_2 L_2 + W_2 C_2) x(t) = K_2 x_2(t) + W_2 y_2(t) \]

\[ x_2(t) = E_2 z_2(t) + H_2 y_2(t) + G_2 y_2(t), \]

where

\[
E_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -120 & -74 & -15 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix},
\]

\[ L_2 = \begin{bmatrix} -0.9743 & 2.5994 & -4.0873 & 1.8306 & -1.1874 \\ -6.7824 & -26.6697 & 35.3551 & -12.0881 & -1.1929 \\ 82.0247 & 196.1149 & -240.5683 & 72.9695 & 34.7101 \end{bmatrix}, \]

\[ W_2 = \begin{bmatrix} 6.6647 & -5.6187 \end{bmatrix}, \]

\[ H_2 = \begin{bmatrix} -8.5596 \\ 60.4838 \\ -379.3008 \end{bmatrix}, \quad \text{and} \quad G_2 = \begin{bmatrix} 14.6597 & 5.1568 \\ -203.5763 & 55.9989 \\ 1568.1339 & -622.9111 \end{bmatrix}. \]

Note that for the third-order controllers, the closed-loop eigenvalues are the union of the eigenvalues of the global state feedback controller and of the two decentralised observers, which is in line with the principle of the separation property.

V. CONCLUSION

We have presented a feasible simple solution to the problem of implementing global feedback control for multi-agent systems in a distributed manner. The proposed approach is based on the development of N decentralised linear functional observers. Necessary conditions for the existence of these observers have been derived. For their design, a step-by-step algorithm has been given, followed by a numerical example to illustrate its simplicity and correctness. It is interesting to note that the design method can be applicable to systems with varying information exchange patterns, and an extension to multi-functionalities is also straightforward.

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