Primary Numeracy

A mapping, review and analysis of Australian research in numeracy learning at the primary school level

Report

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<th>Description</th>
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<tbody>
<tr>
<td>AAMT</td>
<td>Australian Association of Mathematics Teachers, Inc</td>
</tr>
<tr>
<td>ACER</td>
<td>Australian Council for Educational Research</td>
</tr>
<tr>
<td>AEI</td>
<td>Australian Education Index (database)</td>
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<td>AESOC</td>
<td>Australian Education Systems Officials Committee</td>
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<tr>
<td>AMT</td>
<td>Australian Mathematics Trust</td>
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<tr>
<td>ARC</td>
<td>Australian Research Council</td>
</tr>
<tr>
<td>ATSI</td>
<td>Aboriginal and Torres Strait Islander</td>
</tr>
<tr>
<td>AUSTROM</td>
<td>AUSTROM is a compilation of databases (RMIT Publishing)</td>
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<tr>
<td>DEST</td>
<td>Department of Education, Science and Training</td>
</tr>
<tr>
<td>DETYA</td>
<td>Department of Education Training and Youth Affairs</td>
</tr>
<tr>
<td>DfEE</td>
<td>Department for Education and Employment, UK</td>
</tr>
<tr>
<td>ERIC</td>
<td>Educational Resources Information Center</td>
</tr>
<tr>
<td>ESL</td>
<td>English as a second language</td>
</tr>
<tr>
<td>LBOTE</td>
<td>Language background other than English</td>
</tr>
<tr>
<td>MCEETYA</td>
<td>Ministerial Council on Education, Employment, Training and Youth Affairs</td>
</tr>
<tr>
<td>MERGA</td>
<td>Mathematics Education Research Group of Australasia</td>
</tr>
<tr>
<td>NCTM</td>
<td>National Council of Teachers of Mathematics</td>
</tr>
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<td>NCISA</td>
<td>National Council of Independent Schools Associations</td>
</tr>
<tr>
<td>PME</td>
<td>International Group for the Psychology of Mathematics Education</td>
</tr>
<tr>
<td>SES</td>
<td>Socio-economic status</td>
</tr>
<tr>
<td>SPIRT</td>
<td>Strategic Partnership with Industry Research and Training</td>
</tr>
<tr>
<td>TIMSS</td>
<td>Third International Mathematics and Science Study</td>
</tr>
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</table>
Acknowledgements

We would like to thank the many people who have made this project possible.

Our thanks go firstly to the people who have provided us with information about their own and other people’s research into primary numeracy, in particular the Australian and International Consultants for the project; the academics and the personnel from the Government, Catholic and Independent education sectors in all States and Territories with whom we held discussions during our interstate visits; the members of the Mathematics Education Research Group of Australasia (MERGA) who responded so generously to our requests for information through the internet; and our research librarian Brenda O’Donnell. We would also like to thank Angie Bloomer for her highly efficient project management and Ian Lowe for his research support.

Special thanks go to the members of the Advisory Committee for their continuing commitment to the project, constant support, timely advice and invaluable critique.

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We would also like to thank the teachers and academics who took part in the focus groups, offering advice on the CD and electronic database, and particularly Robyn Zevenbergen and Debbie Efthymiades for conducting the focus groups in Queensland and the Northern Territory. Special thanks also go to Brian Doig for his expert advice and critique.

Finally, we would like to thank Kathy Savige for her untiring work in the design and construction of the database, website and CD, and her assistance in many other aspects of the project.

Timeframe of the data

The projects and publications included in the database and report are indicative of the range of mathematics education research and development reports from the period from 1990 to 2003.
# Primary Numeracy Concept Map

## Assessment
- Achievement
- Assessment techniques
- Assessment programs
- Diagnostic assessment
- School entry assessment
- International assessment
- National assessment
- State-wide assessment

## Broader contexts
- Literature reviews
- Major reports
- System initiatives

## Classroom practice
- Grouping
- Intervention
- Pedagogy
- Resources
- Teaching strategies
- Teaching aids
- Technology
- Textbooks
- Calculators
- Computers
- Motivating students
- Problem solving and investigations
- Questioning and discussion
- Real world contexts

## Students
- Gifted
- Informal learning
- Learning styles
- Student attitudes
- Students at risk

## Teachers
- Pre-service
- Professional development
- Teacher beliefs
- Teacher change
- Teacher effects
- Teacher knowledge
<table>
<thead>
<tr>
<th>Curriculum and processes</th>
<th>Concept development</th>
<th>Number</th>
<th>Space</th>
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<td>Curriculum issues</td>
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<td>Developmental frameworks</td>
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<td>Mathematical thinking</td>
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<td>Using mathematics</td>
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<th>Chance &amp; data</th>
<th>Measurement</th>
<th>Computation</th>
<th>Counting</th>
<th>Decimals</th>
<th>Estimation</th>
<th>Fractions</th>
<th>Number sense</th>
<th>Percentages</th>
<th>Place value and the Number System</th>
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<tr>
<th>Equity</th>
<th>Disability</th>
<th>Ethnicity</th>
<th>Gender</th>
<th>Indigenous</th>
<th>Language factors</th>
<th>LBOTE</th>
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<th>School community</th>
<th>Community</th>
<th>Parents</th>
<th>Primary-secondary transition</th>
<th>School factors</th>
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*Figure 1:* Themes and sub-themes used to organise the Primary Numeracy database.
Executive Summary

This project, *Primary Numeracy: A Mapping, Review and Analysis of Australian Research in Numeracy Learning at the Primary School Level*, has been funded by the Australian Government Department of Education, Science and Training under the *National Strand of the Numeracy Research and Development Initiative*.

The project has mapped the extensive body of Australian primary numeracy research carried out during the last decade and has reviewed this research in the international context. The mapping makes research summaries and findings easily accessible to a range of potential users, and provides a foundation and direction for further research.

The report of the project is available in 3 formats: (a) a written report; (b) a shorter compendium with accompanying CD-ROM; and (c) a website. The CD-ROM and website include an indexed electronic database with details of 185 projects and 726 publications.

Data collection included the use of national and international consultants, library collections and databases, interviews with university and systems personnel, and professional discussion lists. National data and policy sources, records of funding agencies, State and Territory reports, and research summaries were used. Themes and sub-themes were developed to provide a structure for the electronic database and the report. Focus groups of experts provided advice on the best way to present material on the CD-ROM and website.

**Strengths identified**

The most striking feature of Australian research into numeracy at the primary school level during the past decade has been its quantity and diversity. Significant strengths in the research reviewed include:

- a rich, coherent and continuing body of research into children’s early number learning;
- a significant body of Australian and collaborative international research into number sense, estimation and mental computation;
- the extent of numeracy research in areas other than *Number*, particularly in *Chance* and *Data*, and, to a lesser extent, *Measurement*;
- a substantial body of research into the use of calculators as teaching aids, with consistent positive findings relating to children’s development of number sense, but continuing evidence that they are not being used as often or as effectively as possible, especially in the early years;
- large-scale research investigating effective teaching of numeracy in the early and middle years of schooling, using developmental frameworks and rich assessment tasks to provide teachers with opportunities and tools to engage with their students’ understandings of primary mathematics;
- the construction of developmental frameworks based on extensive research, particularly in the early years, where the use by teachers of interviews has been a powerful professional development tool; and
Primary Numeracy

- a significant amount of exploratory research into areas such as the use of open questions, children’s reasoning, and inquiry-based approaches to teaching.

**Gaps identified**

Nevertheless, there are significant gaps in Australian primary numeracy research.

**Disability and learning difficulties**

There is little recent research into the numeracy development of students with physical and/or learning disabilities, with much research focused on literacy rather than numeracy. Projects funded under the *National Literacy and Numeracy Strategies and Projects Programme* are likely to assist in this area.

**Indigenous students**

Much of the research into factors that enhance or impede achievement has been action research involving small numbers of students. There is a need for nation-wide, co-ordinated research studies, involving members of Indigenous communities. The Australian Government’s current *National Indigenous English Literacy and Numeracy Strategy* will help partially to fill this gap.

**Rural and regional students**

The research reviewed revealed little about the specific numeracy needs of students in rural and/or isolated areas of the country. Issues associated with multi-age learning settings, accessing and using technology, catering for special needs, and teacher development are areas for investigation.

**Equity factors: gender, SES, and ethnicity (including language background)**

There is evidence that gender interacts with socio-economic background and ethnicity in shaping learning outcomes. There is a need to determine the interactions of these factors. Teachers’ attitudes, beliefs, and practices are also important factors.

**Gifted students**

The paucity of research on how to develop the full mathematical potential of gifted and talented youngsters needs to be addressed on the grounds of both equity and its potential benefit to the community.

**Students at risk**

There has been considerable research and many programmes to identify very young students who are at risk of not learning mathematics successfully, but little on identification of children in middle and upper primary years who are at risk. The longer-term effects of intervention programmes have not been investigated.

**Grouping**

Substantial differences have been found in ways that effective and ineffective groups work, with no clear evidence about the most effective group structures. The effectiveness of group work should be compared not only with individual work, but also with varied models of whole class teaching.
Executive summary

Teaching aids
There is evidence that over use of teaching aids interferes with learning. This project found very little research into which aids are most useful in promoting children’s numeracy learning, when and how such aids are best used, and how to make sure that use results in mathematical abstraction and generalisation.

Technology
There is a need to identify effective pedagogical strategies and software that enrich the use of computers. The research indicates limited use of calculators, minimal use of research findings about using calculators and computers effectively, and the need for related professional development projects.

Problem solving and open-ended tasks
There has been little empirical, large-scale research on whether problem solving and the use of open-ended tasks improve learning outcomes, and whether all children benefit. This project found no research into children’s use of mathematics in everyday situations and non-mathematical contexts.

Effective teaching of numeracy
While there is considerable convergence in the research reviewed, effective teachers are not easily characterised. No single method of teaching assures high levels of student achievement. Most comparative, international research has been carried out at the secondary level.

Concept development
Gaps identified here were in research about children’s understandings in the area of graphical literacy (interpreting graphs and other representations); ways in which connections between measurement and our decimal system can be used to support students’ acquisition of numeracy skills; pedagogical approaches to develop children’s number sense; models and materials for instruction in mental and written computation; and tasks and teacher actions that enhance learning in the content area of Space.

Developmental frameworks
Further research is needed at both the pre-school and primary-secondary transition levels to provide a common language that enables clear communication of children’s capabilities and needs between pre-school staff, primary and secondary teachers, and parents.

Mathematical thinking
The use of tasks with low cognitive demand is seen as one reason for the frequent absence of purposeful mathematical dialogue in Australian classrooms. This suggests a need for the development of conceptually focused, robust tasks and new teaching approaches to support the development of sophisticated mathematical thinking.

Assessment
All States and Territories now have large-scale assessment programmes in place. The advantages, limitations, and effects of alternative forms of system-wide assessment have not received research attention; e.g. use of Item Response Theory, Rasch measurement to devise numeracy scales, developmental assessment frameworks, and “rich” assessment tasks. There has been insufficient work on appropriate tools for profiling pre-
school children’s mathematical development in order to smooth the transition between pre-school and school.

**Pre-service teacher education**

Pre-service teachers’ beliefs, levels of self-confidence, and lack of suitable past experiences, act as constraints on their ability to support high-level mathematics learning. Many pre-service teachers feel insufficiently prepared in mathematics content knowledge and pedagogical content knowledge. While innovative practices in pre-service education, such as the use of interactive multimedia resource, have been found effective in small studies, further research is needed into benefits for pre-service teachers of analysing carefully selected video and other records of learning and teaching to improve their teaching practice.

**Professional development and teacher change**

There has been a substantial amount of research into professional development, but few longitudinal studies of teacher change and its effects on students’ numeracy outcomes.

**Teacher beliefs and knowledge**

Research on teachers’ beliefs appears fragmented, and little is known about how beliefs relate to effective numeracy teaching. No Australian research reviewed looked specifically at the mathematical content knowledge of practising teachers. Research appears to be needed into ways in which such knowledge can not only be assessed but also improved.

In summary, while there is an impressive array of Australian research into primary numeracy education as outlined in the CD database accompanying this project report, there are still many gaps evident in research and development. In order to achieve the goals agreed by State, Territory, and Australian Government Ministers for Education, that all students should attain the skills to be numerate, it is important that numeracy research remains a high priority for Australian research.

*Susie Groves, Judith Mousley, and Helen Forgasz*
Chapter 1

Introduction

A major policy objective of the Australian Government is to provide all young people in Australia with strong foundations in numeracy and English literacy skills.

This project, *Primary Numeracy: A Mapping, Review and Analysis of Australian Research in Numeracy Learning at the Primary School Level*, has been funded by the Australian Government Department of Education, Science and Training under the National Strand of the Numeracy Research and Development Initiative.

The project seeks to provide an information base on key Australian research into numeracy at the primary school level for a range of stakeholders. This will assist in the further development and implementation of programmes and policies designed to achieve the national numeracy goal. It will also assist in providing a base for future research that is targeted at effectively improving students' numeracy.

Background

In 1999, all Education Ministers endorsed new national goals for schooling in the twenty-first century (MCEETYA, 1999a) — known as the *Adelaide Declaration*. The numeracy and English literacy goal agreed by Ministers is that students should have “attained the skills of numeracy and English literacy; such that, every student should be numerate, able to read, write, spell and communicate at an appropriate level” (MCEETYA, 1999a, p. 4).

The National Literacy and Numeracy Plan, agreed by all Education Ministers in 1997, provides a coherent framework for working towards the achievement of the national literacy and numeracy goal, through a coordinated approach by the Australian Government, States and Territories to improving students’ literacy and numeracy outcomes (see, for example, Department of Education, Training and Youth Affairs, 2000, p. 17).

The National Plan has a strong focus on ensuring that all students, including the most educationally disadvantaged, attain sound literacy and numeracy skills to equip them for effective participation in schooling and in society. This goal sets high expectations for numeracy and English literacy achievement in Australian schools. Disadvantaged groups with lower numeracy and English literacy achievement, including Indigenous Australians, have been targeted for special attention.

The importance of sound numeracy skills for young children has also been widely acknowledged elsewhere (see, for example, Australian Association of Mathematics Teachers, 1997; Numeracy Task Force, 1998).

State and Territory government and non-government education authorities have adopted a range of approaches to numeracy education in schools, reflected in a variety of initiatives. In recognition of this fact, this project takes a broad view of numeracy that includes the following aspects:
• the development of students’ mathematical knowledge, skills and understandings to provide sound underpinnings for numeracy; and
• the fostering of students’ capacities and disposition to be able to make effective use of their learning.

The latter aspect recognises that current approaches to numeracy tend to emphasise the provision of support for further learning and the need to enable students to deal effectively with the general demands of their lives.

Purpose of the project

The purpose of this project is to build a comprehensive picture of current key issues as well as to provide a basis for future planning in the arena of numeracy. By surveying existing research data and projects, the project provides programme developers and policy makers with a knowledge base to assist in further program and policy development.

The project maps and provides information on key Australian research in the area of numeracy teaching and learning, with a focus on the primary school level, and reviews this research in the international context.

In particular, the project seeks to provide a mapping of the extensive body of Australian research, undertaken in the last decade, which has examined numeracy learning with the goal of improving student outcomes in the primary years.

The mapping indicates the major research studies that have been conducted in key areas with a view to:
• making research summaries and findings easily accessible to a range of potential users;
• providing a strong foundation and direction for further research that prevents unnecessary duplication of previous studies; and
• identifying directions for future development.

A critical review of recent and current Australian research in the area of numeracy teaching and learning at the primary school level has been developed, contextualised within the framework of key international work in the field. The review focuses on identifying research findings of most relevance for achieving the national numeracy goal. Implications of the research findings and gaps in the research have been identified.

The resulting synthesis and interpretation provides an information base for a range of numeracy education stakeholders, to assist (i) policy development; (ii) planning of numeracy teaching programs; and (iii) research initiatives that avoid duplication and have clearly targeted outcomes.

Methodology

The project sought to produce a report based on a high-level interpretation of existing data and research literature. Information on research conducted from 1990 onwards in the area of primary numeracy was gathered from a wide range of enterprises, including:
Introduction

- State and Territory government and non-government agencies;
- universities and other organisations that have undertaken relevant research;
- libraries and databases; and
- State and national mathematics education organisations.

The work of the project team was supported by an Advisory Committee, which met on three occasions during the course of the project. Details of the membership of the Advisory Committee are given in Appendix B.

This section describes the methodology adopted by the project.

**Scope**

Given the extensive body of Australian and international research in numeracy, a major issue has been what constitutes relevant data for the project.

In terms of what constitutes significant Australian research on primary numeracy, the first issue relates to the definition of numeracy itself, with many definitions available (see, for example, Australian Association of Mathematics Teachers Inc, 1997; Brown, 2001; Hogan & Kemp, 1999; Primary Mathematics Association, 1997; Scott, 1999; Siemon, 2000; Willis, 1990; 1998).

This project has adopted a broad view of numeracy and includes research, projects and publications that are clearly relevant to all aspects of primary teaching and learning aimed at enhancing students’ capacities in mathematics, and the disposition to make effective use of their mathematical learning.

The primary years include the year before Year 1 up to Years 6 or 7, depending on the State or Territory. Research in middle years of schooling often includes both primary and secondary students or deals with issues such as primary to secondary transition. Thus, not all the research reported here is confined to the primary years of schooling.

The research reported here has mostly been carried out by Australian researchers, within Australia, from 1990 onwards. Significant international research which contextualises the Australian research, and a small amount of research prior to 1990, however, have also been included when it was deemed appropriate.

An issue throughout the project has been what constitutes research — as opposed to curriculum development, assessment, and professional development projects — for the purpose of inclusion in the data collection and analysis and in the database included on the CD-ROM version of this report (see later sections of this chapter). A further issue has been what constitutes significant research for the purpose of producing this report.

The term research is used in its widest sense for the purposes of the project. Not only formal research projects have been included in the database, but also curriculum projects, professional development projects, and innovative teacher-as-researcher projects.

Similarly, not all publications that have been included in the database describe research activity in the formal sense.

The report, however, focuses mainly on significant research as well as other projects and developments that have shown numeracy improvement for the children involved. While such work may not have involved large sample populations, traditional formal research processes, or quantitative measurement of the outcomes, we have included projects and
publications that have findings, or have raised issues, that appear to be important for furthering the improvement of numeracy outcomes for Australian children.

**Data collection**

The project’s data collection included using national and international consultants, searching of library collections and electronic databases, interviews with university and education system personnel, and the use of electronic networks and professional discussion lists.

Consultants with expertise in mathematics education research in each Australian State and Territory, as well as the United States, the United Kingdom, Canada and New Zealand, were engaged to provide information on any projects (current or recently completed) that they knew of in their State, Territory or country. The Australian consultants also provided local information about useful resources, associations, and people working on numeracy in their geographic area. A full list of the Australian and international consultants is given in Appendix C.

Project team members used this information to assist in planning two-day visits to each State and Territory during the period May to August 2001. Acting on the advice of the Australian consultants, as well as our own knowledge of the field, we conducted e-mail, telephone, and face-to-face interviews with personnel from universities, research centres and research organisations, higher-degree project supervisors and co-ordinators, and government and non-government education system personnel. Systems personnel responsible for numeracy, educational researchers in the field of numeracy, and relevant educational consultants were interviewed during these visits and libraries were checked, for doctoral and other theses, curriculum documents, and other materials not usually available via interlibrary loans. A full list of people consulted by members of the project team appears in Appendix D.

Written data were also collected from sources including:

- relevant national data and policy sources generated by DEST, ACER, etc.;
- records of funding agencies, including ARC and SPIRT grants;
- data and reports collected by education providers in each of the States and Territories;
- other electronic databases (for example, AUSTROM, AEI, ERIC) and relevant e-journals;
- State and National library collections;
- published reports, reviews and summaries of research such as the four-yearly MERGA review; and
- university library collections.

A web-based data collection instrument was developed and used in the second half of 2001 to obtain information from members of the *Mathematics Education Research Group of Australasia* (MERGA) — see Appendix E for details.

**Construction of the database**

Web-based technology was used to develop an extensive electronic database with standardised fields, for current and recently completed projects and relevant publications.
The same technology was used to compile data from the library searches, interviews and interstate visits. A working web site created for the project was used to collect appropriate data and to summarise the project and its progress. The working web site was publicised through electronic education networks and discussion lists, and used to encourage researchers to provide data about their own research (see Appendix E). The MERGA list was also used in early 2002 to request members to check their own entries in order to see if any were missing, the wording used was appropriate, the descriptors seemed correct, and if any further useful details could be added.

The database is current to July 2003. There may be some discrepancies in the data presented, but it is as accurate as we were able to ascertain.

Within the database, the publications and projects are linked, so that selection of any publication that is a product of a project shows the project’s title (and provides an electronic link in the CD-ROM). Similarly, the details of any project include reference to relevant publications.

**Analysis of the data**

Initially a list of 60 descriptors was developed specifically for the project, based on relevant descriptors from the standard thesauruses for databases such as ERIC and AEI, and those used by major mathematics education research groups in Australia and elsewhere. As the projects and publications were categorised using these descriptors, others were added as needed, while some were deleted if they were found to be repetitious or otherwise inappropriate. A series of iterations was required as more data became available and additional criteria for inclusion emerged.

As part of the analysis of the data, the descriptors were grouped under themes and sub-themes. Lists of articles classified under each sub-theme were checked to establish the robustness of the classification system. The final classification system has been represented on the *Primary Numeracy Concept Map* (see page viii), while Appendix G shows a breakdown of the numbers of projects and publications under each of the themes and sub-themes that were used to organise the database. As most projects and publications in the database were allocated more than one descriptor, entries at each level of the table in Appendix G cannot be summed to give the total number at the next level.

These themes have been used to structure the four chapters that form the core of this report, as well as to provide a structure for searching the database on the CD-ROM. The four core chapters, which summarise research on *Equity and the School Community*; *Teachers, Students, and Classroom Practice*; *Curriculum and Processes*; and *Assessment*, provide the basis for the analysis and final synthesis of the research.

**Use of focus groups**

Focus groups — arranged and run by interstate co-ordinators with telephone linkups — were held in Melbourne, the Gold Coast, and Darwin in order to get feedback from teachers and mathematics education specialists. Advice was sought about the organisation of the database and the usefulness of the types of information being entered as well as ideas for the indexing system (see Appendix F). The focus groups made many suggestions regarding the content, nature, and structure of the database, most of which
have been incorporated in the final version. Details of the focus group meetings are provided in Appendix H.

The report

The report of the project is available in three formats: (a) this written report; (b) a printed compendium with accompanying CD-ROM; and (c) a website. The CD-ROM contains a summary of the project and its findings together with an indexed electronic database with details of approximately two hundred projects and more than seven hundred publications arising from Australian research, curriculum development, and education system initiatives; the full report in html format with active links to appropriate sections of the database; and the full report in pdf format for downloading and printing.

The website can be accessed from the list of DEST publications at www.dest.gov.au/schools/publications/.

Structure of the written report

The full written report includes an executive summary that summarises the project and its methods, major findings and implications for future research and development. This executive summary is likely to be most useful for education systems personnel, as well as for professional associations and other bodies responsible for the dissemination of findings arising from key national projects.

This chapter, Chapter 1: Introduction, presents the aims, scope, methodology and the products of the project, while Chapter 2: International and National Developments provides the broad contextual framework.

The central component of this report comprises a critical review of recent and current Australian research. This central component is organised into four sections:

• Chapter 3: Research on Equity and the School Community
• Chapter 4: Research on Teachers, Students, and Classroom Practice
• Chapter 5: Research on Curriculum and Processes
• Chapter 6: Research on Assessment

These four chapters include the analysis of research findings of most relevance for meeting the Australian Government’s aim of supporting enhanced numeracy outcomes for all students, and especially those who may be disadvantaged in terms of their numeracy outcomes. Our aim has been not only to summarise Australian work in the field, but also to provide an interpretation of existing and current research on Australian primary school numeracy learning.

In the final two chapters, Chapter 7: Key Findings, and Chapter 8: Directions for Future Research, we summarise common themes and issues, key findings, and the strengths and gaps that were identified as we reviewed the body of relevant projects and publications. These were also used as a basis for drawing implications for further research and development, consistent with efforts to achieve the national numeracy goal.
The CD-ROM and Compendium

The executive summary, compendium, and full report are also available on a CD-ROM disc and via the DEST web site, together with details of over two hundred projects and more than seven hundred publications arising from Australian research, curriculum development, and education system initiatives.

The summaries of these projects and publications (i.e., the individual entries in the electronic database) can be accessed directly from the links in the electronic report and the index systems of the CD-ROM and web site. This database of projects and publications comprises a bibliography of major Australian and international numeracy projects that were included in our critical review of the literature, as well as a wider selection of publications accessed during the data collection process.

The CD-ROM disc is packaged together with the printed Compendium. The Compendium contains a summary of the main points from each chapter of the report. It is likely to be most useful for teachers, researchers, and administrators; and users can seek more information about areas of interest from the enclosed CD.
Chapter 2

International and National Developments

The increased emphasis on numeracy development that was evident in Australian political and educational circles during the latter half of the 1990s was part of a movement that was apparent in many countries. These emphases include:

- a growing expectation that at least public schools, if not all schools, would follow broad-based (national or state) curriculum guidelines;
- “numeracy for all” being seen as essential for a country’s economic and social viability;
- clear expectations that all students can become numerate;
- detailed articulation of minimum standards to be reached, by most children, by particular year levels;
- increased emphasis on individual children’s conceptual development, particularly the development of relational or connected understanding;
- the construction of models for concept development, in the form of developmental frameworks;
- increased emphasis on children’s counting and strategies for carrying out basic operations in number, especially in the early years of schooling, with an associated emphasis on funding for early assessment and intervention programmes;
- a greater focus on children’s thinking and classroom discussion, together with reduced expectations that children will follow set methods;
- increased exploration of how to use a range of electronic technologies for the teaching and learning of mathematical concepts;
- a greater focus on assessment and reporting, with an accompanying emphasis on teacher accountability and an associated impact on curriculum content; and
- articulation of standards for teachers and teaching.

This chapter sets out to contextualise our review of Australian research into primary numeracy within the framework of key international policy and research, and major national numeracy initiatives.

The international context

Expectations that all children should become numerate citizens have been the driving force behind initiatives around the world, with a significant focus on the early years of school. In Canada, England, Germany, Israel, Malaysia, New Zealand and South Africa — among other countries — there has recently been increasing emphasis on early years outcomes-based curriculum frameworks, evaluation, and intervention as needed (see, for example, England’s National Numeracy Strategy website, http://www.standards.dfee.gov.uk/numeracy).
In this section, we will provide a brief review of major aspects of international policy developments in the area of numeracy, together with a discussion of some of the international research into the effective teaching of numeracy and approaches to improving the teaching of numeracy.

**Policy developments**

A significant milestone in policy development in England and Wales was the report of the Committee of Inquiry into the Teaching of Mathematics in Schools, *Mathematics Counts* (Cockcroft, 1982). This three-year inquiry made numerous recommendations about curriculum and pedagogy, including recommendations that: mathematical facts and concepts should not be committed to memory without a proper understanding of the mathematics to which they relate; there should be more emphasis on practical work; consideration should be given to the use of calculators as an aid to teaching and learning and, as a consequence, the extent to which arithmetical aspects of the curriculum may need to be modified; and the inclusion of realistic problem solving, including the application of mathematics to “real-life” contexts.

This was followed in 1989 by the development of a national curriculum specifying statutory content for each “key stage” of education (Department of Education and Science, 1989). This national curriculum was quickly revised in 1991, and again in 1995 and 2000, with statutory national testing at ages 7, 11 and 14 introduced (Brown, 2001a).

The report of the Numeracy Task Force (Department for Education and Employment, 1998), drawing on work in the National Numeracy Project, proposed that classes should provide opportunities for pupils to talk; be listened to; receive feedback; explain their knowledge, thinking and methods; and suggest alternative ways of tackling problems (Brown, 2001b). Key features of the resulting National Numeracy Strategy, implemented in the 1999–2000 school year are: an emphasis on calculation – especially mental calculation; 45–60 minutes of mathematics daily; a three-part template for the daily mathematics lessons based on a 10–15 minute oral introduction, followed by direct teaching of the whole class and groups, and a 10 minute plenary review; detailed planning using a suggested week-by-week framework; and a national professional development program (Brown et al., 2001, p. 1).

At the time that the national curriculum was introduced in England, the National Council of Teachers of Mathematics (NCTM) co-ordinated the development of national standards for curriculum, assessment and professional development in the United States. The *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics, 1989) and the *Professional Standards for Teaching Mathematics* (National Council of Teachers of Mathematics, 1991) both took a socially-situated view of learning, while stressing the importance of children developing their own mathematical meanings (see, for example, National Council of Teachers of Mathematics, 1989, p. 10; 1991, p. 25). Since then, the National Council of Teachers of Mathematics has continued to produce documents and teacher-support materials with this emphasis. For example, the 1996 report of the President of the NCTM (Price, 1996) focused on “building bridges of mathematical understanding“ (p. 3); and the recent “reform” movement in the United States focuses on deepening children’s ability to reason numerically (McLaughlin, Shepard, & Day, 1995).
In New Zealand, the key curriculum document, *Mathematics in the New Zealand Curriculum*, gives the same message:

> Mathematical understanding and skills contribute to people’s sense of self-worth and ability to control aspects of their lives. Everyone needs to develop mathematical concepts and skills to help them understand and play a responsible role in our democratic society. Mathematics education aims to provide students with those skills and understandings. (Education Review Office, Ministry of Education, New Zealand, 1994, p. 7)

New school guidelines that emphasise literacy and numeracy were gazetted by the New Zealand government (Higgins, 1999). New Zealand’s Curriculum Framework outlines not only expected curriculum content, but also ways of developing essential skills. These skills include communication, numeracy, information, problem solving, self-management and competitiveness, as well as social, co-operative, physical, work and study skills. The emphasis on numeracy development in early childhood can be seen in the New Zealand School Entry Assessment (SEA), which includes a numeracy task called “Check Out” based on a shopping game.

In both England and the United States, there has been considerable debate regarding:

- the wisdom of having national curriculum documents;
- who should be involved in writing them;
- the tension between traditional curriculum content and content which emphasises problem solving, mathematical reasoning, and making connections between different areas of mathematics and everyday contexts;
- the extent to which expectations expressed in curriculum documents challenge most children; and
- whether outlining (as well as testing or reporting against) minimum content results in teachers lowering their expectations of children who may be capable of higher achievement.

In the United States the extent of contestation over the curriculum (and to a lesser extent pedagogy) has reached the stage where it has been referred to commonly as the *Math Wars*. There is, however, often a gap between expected standards and classroom results (Olson, 2002), with evidence that support for standards-driven improvement is dwindling in the United States. “Polls conducted by the [Albert Shank] Institute have found that teachers’ support of standards-based school improvement dropped from about 73 percent in 1999 to just over 50 percent in 2001” (Feldman, cited in Olson, 2002, p. 1).

**Effective teaching of numeracy**

Reynolds and Muijs (1999), in a review of American and British research on the effective teaching of mathematics, state that there is a general consensus that whole-class interactive teaching, together with differentiated groups and some individual work, with calculators being used alongside mental calculation and mental strategies, forms the basis for effective practice in terms of mathematics achievement. According to Reynolds and Muijs (1999, pp. 274–276), American research on effective mathematics teaching identifies the following factors as positively affecting mathematics achievement:

- high opportunity to learn;
- an academic orientation from the teacher;
• effective classroom management;
• high teacher expectations of students;
• a high proportion of whole-class teaching;
• heavily interactive teaching that involves students in class discussion; and
• the use of questions calling for student explanations.

Moreover, the positive effect of a high proportion of teacher time spent on questioning and student involvement in class discussion found in American research was paralleled by findings from the UK study *Observational Research and Classroom Learning Evaluation* (ORACLE), which showed that students in classes of teachers labelled as “Class Enquirers” made greater progress in mathematics than those in “Individual Monitoring” classes (Galton & Croll, 1980; Croll, 1996, cited in Reynolds & Muijs, 1999).

According to Reynolds and Muijs (1999), a further UK study (Office for Standards in Education, 1996) found that characteristics of classrooms in which achievement in basic mathematics skills was low included:
• too much emphasis on repetitive number work;
• too much individualisation of work; and
• too little fluency in mental calculation.

By way of contrast, characteristics of high-achieving classrooms in this study included:
• a clear lesson structure, making good use of time, maintaining challenge, pace and motivation;
• sessions of direct teaching with the teacher being involved pro-actively;
• regular teacher-pupil interaction, the use of perceptive questioning, careful attention to misconceptions, and constructive help;
• the rehearsal of existing knowledge and skills, and quick recall of number facts; and
• the use of a variety of activities on a topic to consolidate and extend understanding. (Reynolds & Muijs, 1999)

Reynolds and Muijs (1999) stress, however, that, in order to enhance higher order thinking as well as basic skills, it is also necessary to focus on meaning and understanding, the teaching of higher level cognitive strategies and problem solving, and the use of cooperative group work, with the opportunities this provides for developing children’s thinking skills through reflection and verbalisation. While international research suggests that developing problem-solving skills through cooperative group work leads to enhanced conceptual understanding, effective group work requires planning and effort on the part of teachers.

Askew (2001), however, questions whether attempts by studies such as ORACLE to cluster teachers into distinct teaching styles and relate these to attainment in mathematics have succeeded, claiming that not only did such studies find it hard to identify clusters of teaching styles, but that findings about whole-class teaching were ambivalent, with questioning at a high cognitive level, rather than classroom organisation, appearing to be the key factor. Askew further claims that Galton, Hargreaves, Comber, Wall and Pell (1990), following up on the ORACLE study twenty years later, regarded the attention paid to examining organisational strategies as having “diverted attention from important differences in ‘tactics’ used by teachers within, rather than across, differing organisational
styles” (Askew, 2001, p. 45). Moreover, according to Askew, much of the research in the UK and Holland (for example, Creemers, 1994) attempting to link classroom practice with learning outcomes has been based on Rosenshine’s (1987) assumption that “direct instruction is at the heart of effective teaching” (Askew, 2001, p. 46).

The UK study *Effective Teachers of Numeracy* (Askew, Brown, Rhodes, Johnson, & Wiliam, 1997) examined the effect of grouping and other classroom practices on students’ gains on a test of numeracy and found no association between such aspects of pedagogy and student achievement. Instead, Askew et al. found an association between student achievement and teachers’ beliefs about how best to teach mathematics. Students in classrooms where teachers had “connectionist” orientations had relatively high mean achievement gains, while those whose teachers had “transmission” or “discovery” orientations had relatively low mean gains. In particular, high gains were associated with teachers who focused on:

- children’s mathematical learning, rather than on provision of pleasant classroom experiences;
- providing a challenging curriculum, rather than a comforting experience; and
- having high expectations of initially lower attaining pupils.

These factors were found to be more important than differences in overall teaching style. Further details of the *Effective Teachers of Numeracy* project can be found in Chapter 4 of this report.

The report on the first three years of the implementation of the National Numeracy Strategy (Office for Standards in Education, 2002) claims that the strategy has brought about radical change in the way mathematics is taught in English primary schools and that it has had a positive impact on attainment, although the biggest gains were in the first year (p. 25).

Brown et al. (2001), however, question the research evidence for the pedagogy adopted by the National Numeracy Strategy, pointing out that the gains in attainment in science have exceeded those in mathematics in the period since the adoption of the National Numeracy Strategy, and that the biggest gains in mathematics were made in the year before the implementation of the strategy. Commenting on the two major pedagogic features of the National Numeracy Strategy — *whole class teaching* and *direct interactive teaching* — Brown et al. firstly point out that Creemers (1997), in a review of Dutch studies of effective schools and effective teachers, found only 3 of 29 studies showing a significant correlation between whole class teaching and attainment — although all three showed a positive correlation. Regarding interactive teaching, Brown et al. claim that while the term is intended to include the use of high-level questioning, the emphasis on fast-paced lessons in the National Numeracy Strategy leads teachers to reduce classroom dialogue to low-level question and answer sequences.

Brown et al. (2001) further point to international and English research studies, which suggest that aspects of teaching correlating with high attainment include the use of higher-order questioning, using dialogue to make connections both within mathematics and between mathematics and contexts, collaborative problem solving, and student autonomy (see, for example, Askew, Brown, Rhodes, Johnson, & Wiliam, 1997; Cobb & Bauersfeld, 1995; Stigler & Hiebert, 1997; Yackel & Cobb, 1996). While the National Numeracy Strategy may espouse these aspects, the crowded curriculum and emphasis on pace impose serious obstacles to teachers working in these ways.
As part of the *Leverhulme Numeracy Research Programme*, a five-year longitudinal study of the teaching and learning of numeracy, an adapted version of the teacher questionnaire from the *Third International Mathematics and Science Study* (TIMSS) (Mullis et al., 1997) was used in an attempt to identify factors related to effective teaching of numeracy in Years 4 and 5 as measured by class mean gains in numeracy attainment. No pedagogical variables were found to have a statistically significant effect on gains in attainment (Brown et al., 2001). Based on a qualitative analysis of records of lessons of teachers whose Year 4 class mean gains fell in the lowest and highest 15 of the 74 classes, a list of 18 characteristics of effective teachers was developed. According to Brown et al., the most effective teachers:

- challenge pupils to think mathematically;
- expose and relate to children’s existing knowledge;
- develop significant mathematics e.g. strategies, generalisations;
- stimulate pupils’ interest, curiosity, and excitement and sustain engagement;
- don’t set artificial ceilings;
- permit access to mathematics and task for all pupils;
- have integrity of mathematics and context;
- have consistency between task and objectives;
- use symbols, diagrams and apparatus not for window dressing or as objects in themselves but to communicate, represent, and/or provide good models for thinking;
- involve a range of models of expression;
- encourage development of more sophisticated strategies;
- focus on mathematics rather than work, or getting answers;
- allow sharing of methods and value contributions of children;
- show teacher working with children (use of “we”);
- recognise multiple meanings;
- focus on reasoning rather than answers (not “cued elicitation”); and
- accept and work with children’s errors. (Brown et al., 2001, pp. 8–9).

However, the effect of these factors was found to be small and Brown et al. conclude that

We have considered what can be the cause of our difficulty in describing reliably the characteristics of effective teaching. The problem could be that our instruments for assessment of pupils and/or of lessons are not sufficiently reliable … [but] we have been working to refine our tests over a period of about 10 years and our evaluation instruments over about 5 years. … We are therefore left with the perhaps rather happy conclusion that the behaviour of effective teachers and less effective teachers are not easily characterised; much depends on the particular way that teachers and classes as people relate together. … Although there is some evidence that certain behaviours are effective in teaching mathematics their effect seems to be small and variable. (pp. 15–16)
So while there has been considerable research into what constitutes effective teaching of numeracy, with many similar and some conflicting findings, the relatively difficulty of capturing the effects of teachers’ approaches has important implications for policy and research regarding the improvement of mathematics achievement in schools.

**Approaches to improving the teaching of numeracy**

A common feature of recent international research has been an emphasis on children’s thinking and the construction of models of conceptual development, in the form of so-called developmental frameworks. Fuson and her colleagues in Illinois (Fuson, Smith & Cicero, 1997; Fuson 1982), Jones and his colleagues in Australia and Illinois (1996), Simon in Pennsylvania (see, for example, Simon, 1995), Steffe and Cobb in Georgia (see, for example, Steffe, von Glasersfeld, Richards, & Cobb, 1983), and de Lange and Streefland (see, for example, Streefland, 1988) in The Netherlands, have all focused on primary students’ conceptual development. The foci of their studies have covered counting, place value, early number operations, and fractions, and have often included the modelling of realistic problems. Examples of some of the major programs issuing from the work of some of these, and other, researchers are provided below.

**Cognitively Guided Instruction**

In the United States, *Cognitively Guided Instruction* (commonly known as CGI), was an approach developed by Carpenter, Fennema, Peterson and others (see, for example, Carey, Fennema, Carpenter, & Franke, 1995). Their extensive research into children’s mathematical thinking has led to the development of a knowledge base upon which teachers can draw to better understand, and capitalise on, their students’ thinking. Here “teachers can use their knowledge of students’ thinking to select and design appropriate tasks and to use them wisely” (Hiebert et al., 1997, p. 35). Research, investigating the implementation of CGI as the core of a professional development programme, suggests that when teachers are presented with detailed research-based information on how children conceive of, and solve, problems, they are better able to anticipate the development of their students to create, or choose appropriate problems, and to interpret the children’s solutions (Carpenter et al., 1989, Carpenter & Fennema, 1992). The CGI professional development courses appear to have produced positive outcomes. For example, Vacc and Bright (1999), and Vacc, Bright and Bowman (1998), reporting on the introduction of CGI to elementary teachers in a two-year in-service course, note changes in teachers’ beliefs about teaching and learning mathematics, as well as in their ability to base teaching on children’s understandings and patterns of thinking. Bright, Bowman and Vacc (1998) also showed that teachers’ ways of perceiving the curriculum influence how they interpret children’s mathematical thinking. Training in CGI led to teachers placing more importance on students’ cognitive developmental frameworks and on children’s solution strategies.

The more this knowledge is used to gain an understanding of individual children’s thinking and ability, the more important it becomes to teachers. They increasingly ask questions that elicit children’s thinking, listen to what the children report, and build their instruction on what is heard. (Bright, Bowman, & Vacc, 1998, pp. 102–103)

Increasingly, CGI is being recognised as a viable pedagogical approach, with the influence of CGI evident in the number of Australian teachers and researchers focusing on children’s thinking and strategies (see also Chapter 5 of this report).
Connected mathematics

Conceptual knowledge is characterised most clearly as knowledge that is rich in relationships. It can be thought of as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information. Relationships pervade the individual facts and propositions so that all pieces of information are linked to some network. (Hiebert & Lefevre, 1986, p. 3)

The emphasis on the connected nature of mathematical knowledge recaptures Skemp’s (1976) notion of relational understanding, involving an understanding of the underlying relationships between concepts and processes, linked to the wider network of mathematical ideas — for example, multiplication as repeated addition, and the relationship between decimals and metric measurement. Skemp contrasted this with instrumental understanding, where procedures are used in isolation, divorced from an understanding of the relevant concepts.

As discussed earlier, the findings of the Effective Teachers of Numeracy project (Askew, Brown, Rhodes, Johnson, & Wiliam, 1997) showed an association between student achievement and teachers’ beliefs, with students in classrooms of teachers having “connectionist” orientations showing relatively high mean achievement gains. It is therefore not surprising that the notion of connected mathematics has underpinned a number of research and development projects.

Ball (1993) outlined an example of practice typical of the mathematically rich environment of a “reform” classroom in the United States, showing how children’s comprehension of concepts grows from raw beginnings to incorporate a wider range of conceptual structures, words and symbols, and thus, becomes refined into more general and generalisable understandings. “This picture implies that understanding is an ongoing process, deepening as conceptions expand and the number and solidity of connections among them develop” (Ball, 1993, p. 3). Thus Ball claims that teaching for understanding does not merely involve teaching a concept or process as if it is an entity — “there are always new ways to look at it and more connections to be made” (p. 4).

There has been significant research into what happens when teachers aim for the development of relational understanding, as opposed to mere instrumental understanding. For example, Pesek and Kirshner (2000) found that teachers who aimed for both relational and instrumental understandings were less effective than those who focused on relational understanding only. They concluded that “relational units appended to the existing classroom regimen may effectively be blocked from achieving the relational understanding sought” (p. 536).

A second interpretation of connected mathematics draws on the notion of situated cognition, where understandings are developed in context, or in relation to everyday situations. Here, multiplication might be studied through solving problems with money, and decimals might be investigated in relation to petrol prices. The Everyday Mathematics curriculum (Fraivillig, Murphy, & Fuson, 1999) is typical of this emphasis on contextualised mathematics in many Western countries.

Making connections through problem-based learning in everyday contexts has been an important component of this movement, and as a result of their evaluation of the Connected Mathematics Project, Ben-Chaim, Fey, Fitzgerald, Benedetto and Miller (1997) concluded that such forms of instruction can be useful in helping students construct effective personal networks of understanding and skills. This project, based at Michigan
State University, had a middle school focus, and collected data for two years (see, for example, Zawojewski & Hoover, 1996). Despite using a problem-based approach for this period, the students performed well on the traditional, decontextualised test items that comprised the standardised state-mandated tests. The longer the students had been working in the connected mathematics programme, the greater was their superiority over students not involved in the programme. The students in the programme performed better in problem solving and open-ended tasks, just as they had with basic skills. However, it should not be assumed that a “real-world” connection is necessarily meaningful for children, because “the culture of each child is used to structure the learning environment so that he or she is able to construct relationships and learn mathematics with understanding” (Carey, Fennema, Carpenter, & Franke, 1995, p. 98).

In the Netherlands, a significant long-term research and development programme, usually referred to as Realistic Mathematics Education, has used realistic contexts and problems as a basis to enable students to grow mathematically. Sequences of carefully crafted, sequential problems have been developed, using as a starting point the “realistic mathematics” research of Freudenthal from as early as the 1960s (for example, Freudenthal, 1973). The problems are designed to bring particular ideas to the fore, to cause cognitive conflict, and to assist children to develop abstract ideas from practical situations. As part of the continuing research and development programme in the Netherlands, the empty number line has been introduced as a model for children’s addition and subtraction strategies. The empty number line has been used by researchers such as Paul Cobb and Erna Yackel in the United States as a tool to support the development of children’s mathematical thinking, and is increasingly being seen as a valuable tool for supporting mental computation in England, as well as in Australia in New South Wales’ Count Me In Too project (see the Computation section of Chapter 5 of this report for further details).

Many major research projects in the United States in recent years have focused on developing children’s strategies for solving problems in familiar contexts, and using these with appropriate materials, to deepen children’s understanding of the “big ideas” of number (for example, Ball, 1993; Ball & Cohen, 1996; Cobb, 1997). An example is the use of the idea of packing candy into rolls of ten, and then packets of ten tens, to provide a realistic context for learning about place value.

In summary, significant bodies of international research have focused on making “connections” — connections between realistic contexts and mathematical ideas as well as connections within a network of mathematical ideas. The “big ideas” (such as part-whole relationships, and unitising ten things as one “ten”) are providing structure for this work, while contexts are facilitating meaningful interpretations, with concrete materials and strategies for representation being used as tools to support thinking. More detail about some of the features of numeracy development that encourage children to think mathematically, rather than merely rely on concrete materials to carry out operations, is included in the Number section of Chapter 5 of this report.

**Lesson study**

A completely different approach to focusing on children’s thinking, “lesson study”, has been the subject of research and development in Japan (see, for example, Stigler & Hiebert, 1999; Mousley, 2000).
The report on the *Third International Mathematics and Science Study* (TIMSS) (Mullis et al., 1997) provided valuable information on mathematics classroom practice around the world, based on data from teacher questionnaires. While a common Western view of Japanese teaching is that it consists mainly of rote learning, there is extensive research evidence to show that it in fact involves a considerable amount of whole class, teacher orchestrated discussion, building on students’ ideas (see, for example, Stevenson & Stigler, 1992). Moreover Stigler’s TIMMS study of video data from 100 German, 81 United States and 50 Japanese Year 8 classrooms led him to conclude that “Japanese teachers come closer to implementing the spirit of current ideas advanced by American reformers than do American teachers” (Stigler, 1996, p. 7). Pedagogical advice to Japanese teachers does not come from curriculum documents, as these do not include the finer details of what to teach and are in fact relatively short. Moreover, the small numbers of nationally-approved textbooks include only the intensive study of relatively few concepts, and do not include large numbers of practice examples, while worksheets prepared by commercial publishers are non-existent.

Nevertheless, teaching in Japan is generally more prescribed than in Australia, with teachers having access to detailed advice on how and what to teach and when to teach it. The detail of how to teach particular concepts comes from teachers’ “lesson study”. Typically, groups of teachers in a school undertake self-guided study of particular lessons (*kenkyuu jugyou*, or lesson research) where they study ways of teaching one particular lesson for several hours a week over many weeks. Interestingly, the lessons studied are called *jugyou kenkyuu* (research lesson). Lesson study focuses on the ideas that children need, and common patterns of children’s thinking and talking about the particular concepts and their applications. Teachers, in small groups, discuss in detail the language, materials and activities that can be used to convey underlying mathematical concepts. Also discussed are the appropriate ordering of ideas and useful connections that can be made, common misconceptions and mistakes that children make, and how best to make the most of these as part of the normal teaching and learning process. The detail in these discussions even includes which numbers should be used in particular problems, with teachers raising competing ideas about such details as they study the lesson (usually videotaped, sometimes with a transcription available). Teachers earn recognition for publishing a description of the lesson and a summary of different opinions about points arising during their discussions. The focus in planning lessons is the consideration of the best ways for students to move from current development and expectations for where they will be at the end of the teaching period.

The “lesson research” form of well-supported professional development is so common that a Japanese teacher quoted by Stigler and Hiebert (1999) claimed “You won’t find a school without research lessons” (p. 111).

Japanese lesson study (also used in many schools in Korea and Taiwan) echoes many of the characteristics of the Japanese mathematics classroom, including:

- intensive, long-term focus on the objects of study;
- co-operative efforts to share knowledge and ideas;
- willingness to put forward thoughts in the knowledge that all can learn from their analysis;
- recognition that even the less experienced can make worthwhile contributions and that mistakes are a good source of learning;
• a sense of wanting the whole group to move forward; and, most importantly,
• intrinsic motivation and relatively autonomous work on a shared problem.

It can be seen that the mathematics lessons and this form of teacher development share a focus on the development of deep understanding about the object of study. The notion of Lesson Study has received considerable international attention, as can be seen for example from the fact that a US-Japan joint seminar *The Professionalisation of Teachers Through Lesson Study* was sponsored by the US National Academy of Sciences in July 2002, with many well-known United States and Japanese mathematics educators as participants (for further details see http://www7.nationalacademies.org/usnc-mi-seminar/AGENDA.html, accessed 6 June 2003).

Thus, contrary to popular Western opinion, there is a focus in Japanese primary mathematics classrooms on children’s thinking about “how” and “why” particular strategies, patterns and generalisations can be used, together with the high expectations for performance and knowledge of basic skills for which Eastern countries are usually well-known.

**MathWings**

A strong focus on concepts is also an underpinning of Robert Slavin’s *MathWings*. Slavin, a long-time proponent of co-operative learning, has used research on co-operative learning in literacy, the *Success for All* literacy programme, in mathematics. The mathematics programme, *MathWings*, is based on a two-part lesson framework: an *action mathematics* unit, which is a whole class focus on a major concept, and a *power mathematics* unit, which is individualised and focuses on computation and application. These two unit types use co-operative learning with curriculum content and objectives based on the National Council of Teachers of Mathematics standards.

The programme has been implemented in hundreds of US schools, as well as internationally (Slavin, 1999). This is possibly due, in part, to the fact that the programme does not require much additional funding, although Slavin insists that schools using the programme must make a long-term commitment to co-operative learning.

**The West Sussex Numeracy Project**

In England, the *West Sussex Numeracy Project* (WSNP) looked at “how teachers can integrate the cognitive aspects of teaching mathematics with pedagogy and translate this into a unified practical teaching proposition” (Ahmed & Williams, 1997, p. 361). Preliminary findings from the project suggest that mathematical understanding is improved by practices that include an emphasis on students’ experiences and the relevance of mathematics in everyday life, the encouragement of the use of appropriate language to gain meaning from mathematical experiences, and a focus on relationships as well as patterns (Ahmed & Williams, 1997, p. 370).

**New Zealand’s Numeracy Development Project**

The *Numeracy Development Project 2001* was part of the New Zealand Ministry of Education’s Literacy and Numeracy Strategy. It included, as two of its four projects, the *Early Numeracy Project* and the *Advanced Numeracy Project* (Thomas & Ward, 2002). The *Early Numeracy Project* is based on the successful implementation in 2000 of a pilot project derived from Australia’s *Count Me In Too* project, with a Number Framework forming the core of the professional development programme. An increased interest in developmental frameworks, with identifiable growth points, is identified as stemming from
American research, starting with Bruner’s (1986) notion of “scaffolding”. According to Thomas and Ward (2002), findings from the evaluation of the *Early Numeracy Project 2001* include:

- teachers reporting increased understanding of mathematics content and pedagogy, based on their increased understanding of the Number Framework;
- teachers asking students to explain their thinking and using questioning and explanations of other students to support student thinking being identified as central to effective teaching of early number; and
- planning practices underlying effective teaching being found to include the use of clearly defined objectives to focus learning and the selection of appropriate learning activities, with the objectives being made explicit to students (Thomas & Ward, 2002, p. iii).

Similarly, in evaluating the *Advanced Numeracy Project 2001*, Higgins (2002, p. iii) found that diagnostic interviews can be “an important factor in enhancing teacher content and pedagogical knowledge”.

### A Canadian study

A current Canadian study, the *Early Numeracy Project* (ENP), is being carried out at the University of British Columbia. This project, which is working with practising teachers to explore changes in teaching practices related to their assessment of students’ early numeracy, is developing:

- strategies for assessing levels of numeracy for Kindergarten to Year 2 children;
- strategies for recognising learners at risk;
- instructional strategies for supporting the numeracy development of at risk students in these grade levels.


In summary, much of the recent international research and development focusing on improving numeracy outcomes for children has focused on building on previous research into children’s conceptual development to produce developmental frameworks, which can be used to inform teaching practice. The connected nature of mathematical learning, both within mathematics and to real-life situations, as well as the need for children to think mathematically and to be able to explain their thinking, have also been emphasised.

### The Australian context

This section provides a review of major national policy developments in the area of numeracy, together with a brief discussion of major current and recently completed national and state research projects, many of which are described in more detail in later sections of this report.

Details of Australian research into the effective teaching of numeracy — other than that carried out since 2000 as part of major Australian Government initiatives such as the *Numeracy Research and Development Initiative* — are not included in this chapter, but can be found in the later chapters of this report.
Policy developments

A major policy objective of the Australian Government is to provide all young people in Australia with strong foundations in numeracy and English literacy skills.

While the delivery of school education, including curriculum, is a State and Territory responsibility, the Australian Government provides major support — including targeted support for literacy and numeracy — for education authorities and schools to attain these goals and to strive for quality schooling outcomes (Department of Education, Training and Youth Affairs, 2000, pp. 4–5).

In 1999, all Education Ministers endorsed new national goals for schooling in the twenty-first century, known as the Adelaide Declaration (MCEETYA, 1999). The numeracy and English literacy goal agreed by Ministers is that students should have “attained the skills of numeracy and English literacy; such that, every student should be numerate, able to read, write, spell and communicate at an appropriate level” (MCEETYA, 1999, p. 4). The 1999 National Goals replaced the Common and Agreed Goals for Schooling in Australia, which had been endorsed in 1989 as the Hobart Declaration.

The National Literacy and Numeracy Plan, agreed by the Australian Government Minister and all State and Territory Ministers in 1997, provides a coherent framework for working towards the achievement of the national literacy and numeracy goal, through a coordinated approach by the Australian Government, States and Territories to improving students’ literacy and numeracy outcomes (see, for example, Department of Education, Training and Youth Affairs, 2000, p. 17).

The key elements of the National Plan are:

- early identification of students at risk of not making adequate progress in numeracy through assessment of all students as early as possible in their first years of schooling;
- the use of effective early intervention strategies for students identified as being at risk;
- the development of agreed literacy and numeracy benchmarks against which all students’ achievement can be measured; and
- professional development for teachers to support the key elements of the National Plan.

The National Plan has a strong focus on ensuring that all students, including the most educationally disadvantaged, attain sound literacy and numeracy skills to equip them for effective participation in schooling and in society. This goal sets high expectations for numeracy and English literacy achievement in Australian schools. Disadvantaged groups with lower numeracy and English literacy achievement, including Indigenous Australians, have been targeted for special attention.

Australian Government initiatives

As part of the Australian Government’s commitment to improving literacy and numeracy outcomes, significant funding from 2000 has been made available for new initiatives to extend and support the work already undertaken by education authorities and schools. A brief summary of three of the major initiatives follows. This is followed by a separate
section on the Numeracy Research and Development Initiative, of which this project is a part.

National Indigenous English Literacy and Numeracy Strategy
The National Indigenous English Literacy and Numeracy Strategy involves all Australian States and Territories in supporting, during 2000–2004, the identification and dissemination of effective practice models and teaching methods drawn from pilot projects undertaken in 1998 and 1999. The objective of the Strategy is to achieve English literacy and numeracy and school attendance outcomes for Indigenous students at levels comparable to those achieved by other young Australians. Specific goals of the Strategy are:

• involvement of Aboriginal people in educational decision-making;
• equality of access to educational services;
• equity of educational participation; and
• equitable and appropriate educational outcomes.

For further details of this Strategy, see Chapter 3 of this report.

Literacy and Numeracy Development in the Middle Years of Schooling (Beyond the Middle)
The purpose of Literacy and Numeracy Development in the Middle Years of Schooling project was to provide information on current teaching and learning strategies, special programs, organisational reforms and resources used in Australian schools and systems in all States and Territories which are effective in improving literacy and numeracy learning outcomes of educationally disadvantaged students in the middle years of schooling. The aim was to investigate the perceived efficacy of middle years programs in all States and Territories.

Among the extensive findings, comments in the report of the project Beyond the Middle (Luke et al., 2003) are the following:

• there is a dominance of literacy as a policy priority across all sectors, with this sometimes being at the expense of numeracy (p. 5);
• where interventions take the form of withdrawal programs — as encountered in traditional high schools and primary schools where there is a strong “test score driven” state mandate — student gains are difficult to sustain unless the interventions are “linked and articulated back into mainstream classroom pedagogy and curriculum reform efforts in the school” (p. 6);
• there is a need to focus on renewing mainstream pedagogy in the middle years, including the need for a “much fuller, research-based understanding of what is going on every day in school classrooms … [and] more systematic emphasis on intellectual demand and student engagement … that moves beyond … increased participation rates and basic skills development” (p. 8);
• there is a need to align innovations in pedagogy and assessment to focus on student outcomes, with teachers reflecting together on pedagogy and the quality of student work having been shown to lead to improved learning outcomes (p. 9);
• caution is necessary before making mathematics part of an integrated curriculum (p. 46);
• concern that, when mathematics classes in middle school are taught by teachers with no specific mathematics teacher training or limited mathematical understanding, rather than expert teachers of mathematics who have the relevant
pedagogic content knowledge, the first years of secondary school pose little intellectual challenge and contain little dialogue (p. 46);

• mathematics in the middle years, rather than “marking time through revision” should strengthen students’ mathematical understanding, leading to the development of “inter-connected, linked and meaningful” mathematical knowledge (p. 47); and

• exemplary practice in middle years numeracy was evident in “classrooms where higher-order thinking about mathematical topics was encouraged, where teacher’s subject-matter knowledge was strong, and where mathematical topics were linked to real situations and situations relevant to students” (p. 129).

**Quality Teacher Programme (QTP)**

This Australian Government program focuses on the renewal of teacher skills and understandings in a range of areas including literacy and numeracy. The program includes State and Territory activities involving professional development and strategic Australian Government initiatives including research into teacher issues. According to Stephens and Steinle (2003), as of April 2002, there were a total of 32 numeracy or mathematics sub-projects in QTP.

**The Numeracy Research and Development Initiative**

A major program to support the Australian Government’s National Literacy and Numeracy Plan is the *Numeracy Research and Development Initiative*. Based on consultation with key stakeholders, the Australian Government has identified the following priorities for strategic work in the area of primary numeracy: national coordination and dissemination; early numeracy; effective teaching practice; equity; home, school and community partnerships; technology; and professional development (Department of Education, Training and Youth Affairs, 2000, p. 44).

National coordination and dissemination of the *Numeracy Research and Development Initiative* are being provided via a National Coordinator engaged by the Australian Government to support its numeracy projects and the Clearinghouse for Literacy and Numeracy Research, based at Griffith University.

The other aspects of the *Numeracy Research and Development Initiative* fall under two strands: the *National Numeracy Research and Development Projects* and the *Strategic Numeracy Research and Development Projects*.

This project — *A Mapping, Review and Analysis of Australian Research in Numeracy at the Primary School Level* — has been funded as one of the *National Numeracy Research and Development Projects*, as has *Project Good Start* and the *Home, School and Community Partnerships* project— see below.

A further ten *Strategic Numeracy Research and Development Projects* have also been funded after State and Territory government and non-government school authorities were invited to submit research and development proposals. A brief summary of these ten major projects is also included below.

**Project Good Start: Effective Numeracy Practices in the Year Before and the First Year of School**

The purpose of *Project Good Start: Effective Numeracy Practices in the Year Before and the First Year of School* is to improve the numeracy outcomes of school age children by
providing information on the practices and learning experiences that support the early numeracy development of a sample of children in the year before school and the first year of schooling.

The project, which commenced in 2001, includes a quantitative study of a nationally representative sample group of students to examine and measure students’ development in numeracy from the year before school through to the first year of formal schooling. The sample group includes children from geographically isolated, rural, low socio-economic urban areas and areas with high Indigenous populations. The project will report its findings in 2004.

**Home, School and Community Partnerships project**
The aim of this project was to provide information on Home, School and Community Partnerships which supported children’s numeracy in the one to two years prior to school and in the primary years of schooling. The main focus was on partnerships that extended beyond schools and included other important contexts in which children live, develop, and learn. Data collection included a questionnaire survey mailed to peak body organisations representing education and child care providers, professional associations, research organisations, and parent and community groups; and an email survey of a representative sample of primary schools throughout Australia. Interviews were conducted with key personnel responsible for numeracy and/or mathematics in all State and Territory Education Departments, Catholic Education Commissions/Offices, and Associations of Independent Schools to identify specific programs or initiatives that connected schools with families and/or communities to support children’s numeracy learning. Seven case studies of exemplary, sustained programs included two large State/Territory funded programs (the NT Mobile Pre-school Project and the Victorian Early Years Numeracy Parent Pack), a longstanding parent-school partnership program (Family Maths Project Australia), two primary schools, and a commercial tutoring agency.

**Assessing Numeracy in Primary Schools**
Assessing Numeracy in Primary Schools is an Australian Capital Territory cross-sectoral project seeking to improve student numeracy outcomes through the development of a whole-school approach by linking a comprehensive assessment regime with numeracy teaching and learning. The research, which is being carried out during 2001–2003, will provide guidance on how to develop structures that make best use of large-scale assessment procedures and classroom assessments to inform numeracy teaching and learning within the school. Project outcomes will include: guidelines on ways in which teachers can use the results of system and school assessment procedures to improve their classroom practice; identification of classroom assessment practices that complement system and school assessment procedures; and development of teaching approaches that make effective use of assessment to support student learning, particularly for at risk students (see also Chapters 4 and 6 of this report).

**What’s ‘Making the Difference’ in Achieving Outstanding Primary School Learning Outcomes in Numeracy?**
The purpose of this New South Wales cross-sectoral project, What’s ‘Making the Difference’ in Achieving Outstanding Primary School Learning Outcomes in Numeracy?,
being carried out during 2001–2003, has been to identify educational policies, strategies, practices and programs that contribute to outstanding numeracy outcomes for students. An achievement scale has been developed to assess and monitor growth in numeracy development over the life of the project. Through a series of intensive case studies, the project has identified three sets of strategies (within the classroom, throughout the school and beyond the school) that appear to have a strong influence on numeracy achievement. For further details of this project, see Chapter 4 of this report.

**Supporting Indigenous Students’ Achievement in Numeracy**

The cross-sectoral Northern Territory research project, *Supporting Indigenous Students’ Achievement in Numeracy* is investigating appropriate methods of assessment and recording of student achievement in numeracy in the middle years among Indigenous students in remote non-urban communities. A key feature of the project, which commenced in late 2002, will be the development of authentic (rich) assessment tasks suitable for use by teachers of these students. The tasks will be attuned to the cultural backgrounds and conceptual numeracy needs of Indigenous students who are in remote, non-urban settings and whose first language is not English.

**What Elements of Learning Environments Promote Enhanced Student Numeracy Outcomes?**

The research project *What Elements of Learning Environments Promote Enhanced Student Numeracy Outcomes?* aims to identify key elements of effective learning and teaching practice through research in eight primary schools across a range of settings and within the three schooling systems in Queensland. The project focuses in particular on “effective teaching practice” that leads to enhanced student numeracy outcomes. The project uses both quantitative and qualitative research methodologies, with State-based test results from 2000 forming baseline data as part of profiling a school’s achievement. Extensive qualitative data has also been collected on practices, knowledge, beliefs, and attitudes of teachers, students, parents, and administrators. Measurement of “enhanced” student numeracy outcomes is being achieved by comparing numeracy achievement profiles for each participating school at the project’s commencement and completion. For further details of this project, see Chapter 4 of this report.

**Profiling High Numeracy Achievement**

The South Australian Department of Education, Training and Employment project *Profiling High Numeracy Achievement* aims to identify and document effective teaching, learning and school practices that will support improved student numeracy outcomes, in order to inform the development of a coordinated and strategic plan for numeracy improvement at the system, school and classroom level. Data from schools that have consistently shown improvement in numeracy results from Years 3 – 5 has been used to develop a profile of the programs, practices and strategies deemed most effective in improving student numeracy outcomes. Other schools in the second phase of the project are trialing the profile through an action research model (see also Chapter 4 of this report).

**Making Sense of the Complexity of the Constructivist Mathematics Classroom**

The South Australian Catholic Education Office project *Making Sense of the Complexity of the Constructivist Mathematics Classroom* aims to identify effective teaching strategies that support all students to improve their numeracy outcomes through the construction of meaningful mathematical understandings. Rasch modelling is being used in the analysis
of data from pre- and post-tests of mathematical understandings of 250-300 students in ten classrooms in nine schools in order to determine the degree of growth in students’ learning. Qualitative data are being analysed to identify factors that appear most significant in achieving high levels of growth (see also Chapter 3 of this report).

**Understanding Place Value: A Case Study of the Base 10 Game**
The South Australian Association of Independent Schools project *Understanding Place Value: A case study of the Base 10 game* has adopted an action research methodology to investigate approaches to improve primary children’s learning of the base 10 number system through the use of base ten games and other learning activities. Pre- and post-test student data have been collected, together with teacher reflections on student learning. The project findings will be used to inform and enhance numeracy teaching and learning within the Independent sector and beyond.

**Developing Computation**
The purpose of the Tasmanian cross-sectoral project *Developing Computation* is to support the development of informal written methods in Years 2 to 4 in nine participating schools, in order to investigate the effects of using informal written calculation methods on students’ number sense and computational ability, and analyse critical features of effective teaching strategies. Data collection and analysis focuses on student performance as measured by tests and interviews, together with student and teacher attitudes (see also Chapters 4 and 5 of this report).

**Researching Numeracy Teaching Approaches in Primary Schools**
This Victorian cross-sectoral project *Researching Numeracy Teaching Approaches in Primary Schools* has identified and investigated the effectiveness of a set of generic teaching approaches that teachers can consistently apply to the teaching of mathematics. The project has identified, through the analysis of extensive quantitative and qualitative data, effective classroom teaching approaches in mathematics for students in a range of primary school settings, including a special school, and investigated their potential for improving student numeracy outcomes (see also Chapter 4 of this report).

**Research to Establish the Nature and Extent of the Relationship Between a Student’s Mathematical Knowledge and Skills, and the Capacity to Use Mathematical Ideas and Techniques in Other Contexts**
This Western Australian project is also cross-sectoral. Its researchers are examining the relationship between students’ mathematical skills and their capacity to use these skills in situations other than in a mathematics class. Approximately 1000 students in Years 5 and 7 are taking part in a quantitative study investigating the relationship between numeracy test results and achievement on a range of numeracy-rich situated tasks, while a qualitative study is looking at the numeracy work across the curriculum of two classes of students in nine schools (see also Chapter 4 of this report).

In summary, the National Literacy and Numeracy Plan has provided a framework within which major Australian Government initiatives, such as the *Numeracy Research and Development Initiative*, have led to the establishment of major numeracy research projects. These projects, together with those that have been, and are currently being, funded by the States and Territories, provide a solid foundation for the improvement of numeracy outcomes for Australian primary students.
The remainder of this report provides a critical review of the extensive body of Australian research, undertaken in the last decade, which has examined numeracy learning with the goal of improving student outcomes in the primary years.
Australian children have diverse educational requirements and numeracy development is influenced by different social and cultural contexts. It is important for numeracy teaching and learning to recognise the diversity of students, communities, and educational settings. In particular, students from disadvantaged backgrounds, such as those from low socio-economic backgrounds, Indigenous students, and students with disabilities, including those with learning difficulties, present particular challenges and may require different responses from teachers and schools for them to gain appropriate numeracy skills.

This chapter is divided into two major sections: the first is *Equity*, and the second is *The School Community*. Findings from pertinent Australian numeracy projects and research studies are presented and discussed.

**Equity**

Starting in the 1970s, a time in Australia in which policy and research on gender equity was strong, the 1980s and 1990s saw growing awareness of, and attention to, a range of equity issues. This is evidenced in the literature. For instance, in 1994, the Australian Government and the Australian Association of Mathematics Teachers funded the development of *Maths Works: Equity and Social Justice* (Rice & Mousley, 1994), a professional development workshop program. The materials were aimed at introducing key aspects of *A National Statement of Mathematics for Australian Schools*, and were developed after an extensive review of current literature. The publications and workshop materials were designed to increase teachers’ and curriculum document developers’ awareness, knowledge and skills in the area. They were based on the following principles:

- that equity and social justice are reflected in policy and action at all levels;
- that policy is what is practised, not merely what is espoused;
- that curriculum is the total school experience, in that it encompasses all that is learned and how it is learned in schools;
- that teachers’ beliefs about mathematics influence students’ experiences of school mathematics;
- that social justice in mathematics education goes beyond catering for groups of mathematically disadvantaged students to providing a more just experience of school mathematics; and
- that providing social justice through mathematics education needs broad practical, attitudinal, and structural changes.

Internationally, Lubienski and Bowen (2000) undertook an analysis of articles in the field of mathematics education that were included in the ERIC database and published between 1982 and 1988. They reported that *disability* and *ethnicity* were each the focus of 2.3% of
the citations, that 0.7% of the studies were related to class (socio-economic status), and that gender issues were examined in a larger proportion, 8.5%, of the studies. In the Australian context, the proportions of the research literature that we found on these topics at the primary level were even smaller.

The first three points of Section 3 of the national goals for schooling in the Twenty-first century of the Adelaide Declaration (MCEETYA, 1999a) read as follows:

3. Schooling should be socially just, so that:

3.1 students’ outcomes from schooling are free from the effects of negative forms of discrimination based on sex, language, culture and ethnicity, religion or disability; and of differences arising from students’ socio-economic background or geographic location

3.2 the learning outcomes of educationally disadvantaged students improve and, over time, match those of other students

3.3 Aboriginal and Torres Strait Islander students have equitable access to, and opportunities in, schooling so that their learning outcomes improve and, over time, match those of other students. (Appendix 1, p. 7)

The dimensions of equity in Australian education noted in the Adelaide Declaration are relevant to the numeracy outcomes of primary students. They form the basis of the ensuing discussion.

In the sections that follow, research findings on the following aspects of equity are discussed in turn: disabilities and learning difficulties, ethnicity and LBOTE, socio-economic status, gender, and Indigenous students.

**Disabilities and learning difficulties**

The Australian Government (DETYA, 1999) has defined a student with a disability as one who is attending a government or non-government school and who has been assessed by a person with relevant qualifications as having intellectual, sensory, physical, social/emotional or multiple impairments to a degree that satisfies the criteria for enrolment in special education services or programs provided by the government of the State or Territory in which the school or centre is located. (p. 1)

Within the literature, a plethora of definitions has been used to describe children experiencing learning difficulties that are not physically based. According to Louden et al. (2000), these include

‘learning difficulties’, ‘learning disabilities’, ‘at educational risk’, ‘special needs’, [and] ‘needing support’. All these terms and others are used in Australian schools to describe children who have difficulties with literacy and numeracy learning. What the terms mean, which children they are applied to, and what consequences these labels have for children varies from State to State and from school to school.

‘Learning disabilities’ was the term used in the Department of Education, Training and Youth Affairs brief for this research project. In the brief, this term was used to describe ‘a heterogeneous group of students who have significant difficulties in the acquisition of literacy and numeracy and who are not covered by the Australian Government’s definition of a student/child with a disability’.

In this section, research on learning difficulties that may result from social, or psychological factors – that is, learning difficulties that are not physically-based – and research on children with physical disabilities that may make full participation in
mainstream classroom activities difficult, are presented and discussed. It should be noted that research findings on the numeracy learning of primary children attending special schools in which their physical impairments are catered for directly are not discussed here.

**Learning difficulties**

*Mapping the Territory: Primary Students with Learning Difficulties: Literacy and Numeracy is a major project in this area, funded by the Australian Government.* Difficulties with numeracy were reported to have had a relatively lower priority than difficulties with literacy in Australian primary schools. In the identification of children with learning difficulties, most schools were reported to use the test of whether children were one or two years behind their age group. It was less common for numeracy difficulties than literacy difficulties to be identified and the procedures for identification were less comprehensive. In the national school survey conducted as part of this project, the mean estimate of children experiencing learning difficulties was 16%, a figure consistent with previously published Australian estimates. The conflation of literacy and numeracy problems was considered likely to be the most serious barrier to improved support for children experiencing numeracy difficulties in the early years. Some early years teachers were reported to view numeracy as a form of literacy, since much of the content in mathematics in the early years focuses on language concepts. It was reported that the mathematics background of early years teachers, in particular, was seen as questionable. More than 80% of schools in the national survey supported children with learning difficulties. Schools used multi-focused approaches, including in-class support, small group withdrawal, and individual withdrawal programs. More than half of the survey schools reported using parents in classroom support roles. That teachers regard numeracy as a form of literacy is an issue needing further investigation.

Given that many primary teachers (and particularly teachers in the early years of schooling) are not confident about teaching numeracy, it may be that teachers and others, including materials developers and parents, who believe that numeracy is a form of literacy, will focus on learning difficulties and curriculum developments that fall within their comfort zones. If numeracy is subsumed under literacy there is a danger that the pertinent issues will not surface. The relationship between literacy (usually reading comprehension ability) and numeracy has been explored in several Australian research studies. In others, the particular learning difficulties experienced by children who fall into various different equity categories (for example, LBOTE) have been examined. Findings from many of these studies are discussed in detail later in this chapter.

As discussed elsewhere in this report, the findings from recent Australian studies seem to support the common claim that many primary teachers have relatively low levels of confidence and/or competence in mathematics – see, for example the section on *Teacher knowledge* in the chapter on *Teachers, Students, and Classroom practice*. This is of concern in relation to helping children with numeracy difficulties because teachers need a strong background in mathematical content as well as in how to teach the content (pedagogical content knowledge) in order to identify underlying misconceptions and design appropriate mathematical tasks to address the difficulties. This is a problem recognised worldwide, as evidenced by the number of times it is mentioned in mathematics education publications and at international conferences.
Using parents and/or teacher aides in classrooms may do little to assist children with numeracy difficulties, given that parents’ and aides’ confidence levels, knowledge of the primary mathematics curriculum and pedagogical content knowledge are unlikely to be as good as those of the average teacher. The *Family Mathematics Project of Australia* (FAMPA) studied by Horne (1993) and Hollingsworth (1999) did not focus on developing parental knowledge of mathematics or on using parents to help tutor children with learning difficulties, but on developing parents’ knowledge of the types of activities that encourage mathematical thinking. Horne made the point that for parental and teacher aide involvement to be effective, an appropriate “in-service” program developed specifically for them — perhaps with some informal certification — is needed.

A different outcome resulted from the project *Numeracy Skills Across the Learning Areas: Assisting Students with Learning Difficulties to Put Meaning into Maths* conducted by the Learning Difficulties Support Team, Department of Education, Training and Employment, South Australia. Mentors visited twelve teachers each week; and story-based and activity-based numeracy packs were developed for parents to use at home. The results are worth noting, and suggest that it would be fruitful to explore ways that this project could be extended into other Australian schools. It was found that:

- student enthusiasm and confidence increased;
- teachers used a more problem-solving, activity-based approach;
- a wider range of assessment procedures were used by the teachers;
- inclusive programming that provided detailed and visible accountability provided a powerful planning tool for teachers;
- the use of a range of relevant learning activities and concrete experiences provided for deeper knowledge and understanding; and
- many parents commented on their children’s increased enthusiasm about mathematics lessons, and on how they were talking about using mathematics in different ways.

In many Australian States and Territories, there have been attempts to prevent learning difficulties with what are sometimes called “first wave” programs (for example, the *Flying Start* program in Tasmania, or the *Early School Assessment* project in New South Wales). Other programs, such as the *Year 2 Diagnostic Net* in Queensland, focus more on remediation. However, as shown by the *Mapping the Territory: Primary Students with Learning Difficulties: Literacy and Numeracy Project*, multi-focused approaches are needed and the problems are complex. The authors of the report concluded that it was “not possible to determine accurately the level of prevalence of learning difficulties in literacy and numeracy” (p. 23). One reason was a lack of agreement on definitions. They claimed that there was a need for professional development about learning difficulties, that partnerships between schools, parents and providers were essential for success in addressing children’s learning difficulties, and that there was a need for a continuing focus on assisting such children throughout their schooling. It was noted that more attention had been given to literacy than to numeracy in the past decade and, with respect to numeracy, Louden et al. (2000) recommended that

More attention should to be paid to identification of children who encounter learning difficulties in numeracy, to quality numeracy teaching in the early years, to early intervention programs, and to support for children who continue to encounter numeracy difficulties in the later years of schooling (p. 25).
Louden et al. (2000) noted that teachers in their case study schools “were reluctant to construct children [with learning difficulties] as having ‘deficits’ and some parents expressed concerns about the consequences of labelling children” (p. 23). As an alternative to a “deficit-based” analysis of learning disabilities in mathematics, Munro (1995) used a “cognitive style” model in which the focus was on learner-instruction processes and the mismatch between students’ preferred learning styles and the demands of the teaching approaches used. Implications for teaching, and especially the management of learning disabilities in mathematics were presented. Among other implications from his observations, Munro claimed that it was important to focus on the conditions under which children learn most effectively when identifying needs, and that the relationship between teaching styles and children’s preferred ways of learning mathematics should be monitored.

Zammit, Meiers, and Frigo (2000) maintained that “good assessment materials and procedures are those which are meaningful, accessible, challenging and appropriate for a diverse range of students” (p. 46). They called for care in assessing and reporting the results of benchmark testing, as well as for research into the development of new assessment instruments to deal with a range of equity issues. We report on further research on this issue in the Assessment Techniques section of this report.

From the findings of the Mapping the Territory: Primary Students with Learning Difficulties: Literacy and Numeracy Project report and from the other research cited here, a co-ordinated series of national research projects in this area appears to be required. More needs to be known about the identification of children with numeracy learning difficulties and the means by which to address their needs. Related professional development programs are also needed. The co-ordination of researchers, Education Department personnel, regional officers, and teachers in the New South Wales Count Me in Too program is a noteworthy model.

**Physical disabilities**

Children with physical disabilities may or may not exhibit learning difficulties in mathematics. However, in many cases, they may have very special needs that vary according to the particular physical disability. The needs of deaf children for example, are likely to differ from those of children whose mobility is restricted. For children with disabilities, the authors of the Literacy, Numeracy and Students with Disabilities Project concluded that:

- there had been greater emphasis on literacy than on numeracy;
- there was a dearth of information for all groups of students with disabilities;
- there is a shortage of literature on students’ numeracy achievement; and
- the research found was often out of date.

It was noted that there was an abundance of teacher aides in classrooms, many with little numeracy training, who took on much of the numeracy instruction. Two factors were considered important in the provision of literacy and numeracy for students with disabilities: growing inequalities among schools (for example, based on SES and social and cultural differences), and the increasing cultural and linguistic diversity of school populations.
The Report on Braille and the Acquisition of Literacy and Numeracy for Students who are Blind or Vision Impaired has not yet been published. Focusing particularly on the current status and use of Braille, the goal of the project is to articulate issues, impediments, and practical approaches to literacy and numeracy acquisition of blind or vision-impaired students.

Supporting the conclusions of the authors of The Literacy, Numeracy and Students with Disabilities Project, we found very few studies examining numeracy issues with respect to children with physical disabilities. In one of these, 77 Queensland students from Years 1 to 12 — students who had been identified as requiring special educational assistance because of their hearing loss — completed surveys that consisted of 24 word problems involving additive or subtractive strategies (Hyde, Power & Zevenbergen, 1999). Their performance levels were consistent with earlier research and confirmed “that deaf students are somewhat delayed in their performance in mathematics” (p. 281). Trigger words are used by students as cues to the meaning of problems. The patterns of responses of the deaf and hearing-impaired students revealed that some trigger words were particularly difficult for them, for example, twice and less.

Of course, research involving children with disabilities often has much to say to teachers of all children. For example, Markey (1997), in experimental work with deaf children learning about fractions, found that the children responded well to the use of games that focused on concept and language development — features of pedagogy that all children respond to, as shown in the Fractions section of this report.

There are many other physical disabilities of greater and lesser degrees - for example, specific medical conditions and mobility constraints – that are found among Australian children. Over the last ten years, it appears that no research has been published about the numeracy learning of these children.

In summary, based on the research over the past decade discussed above, it would appear that issues associated with the numeracy learning of primary-aged children with various physical disabilities have received very limited research attention. Little is known about the capabilities, limitations, and needs of many of these children nor how best to address their needs. This is clearly an area in which greater research attention and funding would be beneficial.

The Australian Government has committed funding of $4.5 million under the National Literacy and Numeracy Strategies and Projects Programme to assist in equipping teachers to better meet the needs of students with disabilities and learning difficulties. Projects will be supported at the national and state levels. The projects will focus on more effective teaching and learning practices for students with disabilities and learning difficulties in the early and middle years of schooling. State and Territory government and non-government education authorities are developing cross-sectoral proposals for funding consideration. Several projects commenced in 2003 and the findings and outcomes are likely to be of great value.

Ethnicity and LBOTE

Findings from two closely related topics are discussed in this section: ethnicity and language background other than English (LBOTE). Studies that examined language issues associated with LBOTE students have also been included. As discussed by Yates
and Leder (1996), it should be noted that there have been many definitions used for LBOTE in the Australian literature. As appropriate, the definitions adopted in the projects and research studies discussed here are clarified.

Cumming (2000) identified ethnicity as one dimension associated with research on computational numeracy. Considering that over 20% of Australian citizens were born outside Australia (see Australian Bureau of Statistics: http://www.abs.gov.au/), it was somewhat surprising to find only a few Australian primary numeracy research studies in which the specific focus was on issues of ethnicity or LBOTE students. More often, ethnicity was included as one of a number of other variables of interest in the research study.

There was a time when it was widely believed that mathematics was culture free, that is, unaffected by the society and the people in which it was used. This perspective has been challenged by research in the field of ethnomathematics and is no longer the current view.

Recent research has negated the notion that mathematics education is a universal and culture free entity and its findings have moved to the notion that appropriate mathematics education requires an interactive relationship with its cultural environment (Wotley, 2000, p. 664)

Researchers have examined Australian mathematics textbooks for ethnic inclusiveness. Clarkson (1993) noted that only 8% of people depicted were not Anglo-Australian.Forgasz (1997), considering the issue of images portrayed in mathematics textbooks, commented that compared with the past, Australia’s multi-cultural profile was in evidence in the graphics and photographs. Thomas (1997) argued that textbook publishers should go beyond the trivial, superficial aspects of inclusiveness, and provide genuine tools for learning. Appropriate language and contextual settings were required to meet the mathematics learning needs of all students. O’Toole’s (1997) book on children’s mathematical thinking entitled Mapping Children’s Mathematical Thinking: Measurement — Time includes teaching strategies that provide opportunities for success, with a focus on English as a Second Language (ESL) students.

Background data collected in the Third International Mathematics and Science Study (TIMSS) included the main language spoken at home (English or other) and students’ place of birth (English or non-English speaking countries) (Lokan, Ford & Greenwood, 1997). Population 1 (nine-year-olds) data were analysed by the combination of these two factors. Of the four resulting groups in Australia, Lokan, Ford and Greenwood (1997) found that the lowest achievers were those born in Australia whose families used a language other than English at home (about 4% of the cohort). The highest achievers were children born in non-English speaking countries whose families had adopted English as their main language at home. Haung (2000) confirmed these findings with an analysis of the TIMSS findings of students from five English-speaking countries: England, the United States, Canada, Australia, and New Zealand. The performance levels of students whose home language was not English were substantially lower, whether or not the students were immigrants.

These findings appear to be consistent with those of Young-Loveridge (2000) in New Zealand who examined Year 3 children’s understanding of the number system. Children from four schools — three of low and one of average socio-economic status (SES) — participated. About one third of the children in the sample were Maori. The children’s understanding of the number system was found to vary as a function of ethnicity and
socio-economic status. Children from the average SES school generally had better understanding than those at the low SES schools. Non-Maori children were found to have better understanding than Maori children. Although there has been much research on Australian children’s understanding of Number (found elsewhere in this report), there would not appear to be any research equivalent to Young-Loveridge’s work with respect to a breakdown and comparison by ethnicity and SES.

Lack of English proficiency, MacGregor (1993) argued, was the main barrier to mathematical success in multilingual classrooms. She suggested classroom strategies that would develop the required English proficiency without sacrificing children’s first language. Thomas (1997) discussed a range of principles and issues associated with teaching strategies that would aid in the development of the relevant language skills needed by LBOTE students for the learning of mathematics. Poor English skills can also result in difficulties for adult immigrants taking mathematics education courses (Stacey & MacGregor, 1991). These findings appear to conflict with those from the TIMSS data which revealed that the highest achievers were children born in non-English speaking countries whose families had adopted English as their main language at home (described in the previous section above). Once Australian LBOTE students have acquired English language proficiency, it must be assumed that various other factors contribute to their high levels of mathematics achievement.

There has been a longstanding debate in the general education literature on whether bilingualism has effects on student learning. Clarkson and Thomas (1993) concluded that bilingualism of itself did not necessarily impede learning. Two studies, however, appear to lend support to the view that for numeracy learning the effects may be adverse. In a school with a partial French immersion program, the mathematics performance of students in Year 5 was tested over two years, 1995 and 1996 (de Courcy, Burston, Warren & Young, 1997). Some did the test in English, others in French. In 1995 no performance differences were found. However, in 1996, those taking the test in English performed better than those taking the French version. The authors claimed that they were able to identify information on how children process mathematics problems in a second language. Their error analyses revealed that comprehension problems are among the most common causes of errors made in solving mathematics word problems. Clarkson (1991) found high levels of comprehension errors among bilingual students in Papua New Guinea. It was claimed that the frequency of such errors was related to language competency in both the mother tongue and the language of instruction, English.

The aims of Clarkson and Dawe’s Australian Research Council project, entitled Problem Solving in Two Languages: A Longitudinal Study of Bilingual Students in Melbourne and Sydney, included exploring (a) how bilingual children’s level of competence in each language affects their performance on mathematical tests; and (b) how students switch languages as they process mathematical problems. Of interest were Italian, Arabic, Vietnamese and Cambodian students in Years 4 to 8 (Clarkson & Dawe, 1994). Clarkson & Dawe (1996) reported on the patterns of language switching of bilingual Vietnamese Year 4 children. They found that there were likely to be several overlapping factors that may be involved when children switched languages. Factors found included: difficulty, particularly if the meaning was unclear; and memory of being helped with a similar problem before — if this was in their first language, then a language swap may occur.
As shown in the research described here, numeracy issues related to ethnicity and second language learners have received some research attention in the past decade. It would appear that the more blatant inequities of the past, for example the earlier absence of ethnic diversity in textbooks, have been addressed. The Australian TIMSS data revealed that the achievement of students born in non-English speaking countries from homes in which English is spoken at home were the top achievers; children from homes in which the non-English language was spoken at home were the lowest achievers. In other research it has been shown that lack of English language proficiency, and comprehension problems in particular, appears to be a major barrier to mathematical success. Other factors, such as socio-economic status, may interact with language backgrounds and affect numeracy outcomes.

**Socio-economic status**

It was rare to find Australian primary numeracy research studies or projects with socio-economic status as a primary focus, although in many projects this factor was considered to be one of a set of relevant aspects attended to in the data analysis and reporting.

An Australian Research Council grant which commenced in 2002 was awarded to Hill, Yelland and Thelning for the project *Children of the New Millennium: Using Information and Communication Technologies for Playing and Learning in the Information Age*. The researchers aim to examine how very young children from diverse socio-economic areas use information and communications technologies (ICTs), and to find out the extent to which technology relates to play as well as to literacy and numeracy learning. The researchers are exploring:

- the development of young children’s expertise with ICT from preschool to the second year of school;
- where young children in diverse socio-economic areas use ICT;
- how many forms of ICT children use;
- to what extent technology relates to other forms of play and learning; and
- how children’s knowledge, understanding, and use of technology changes over time.

In meeting these goals, the findings of the research study may provide valuable insights that inform policy and curriculum directions in numeracy for the early years of education.

The *High Performance in Literacy and Numeracy in Disadvantaged Schools* project, conducted by the Primary Principals Association of South Australia, began in 2001. The aim was to identify classroom practices in literacy and numeracy across the primary grade levels and, with a focus on whole school change, to support disadvantaged schools to improve literacy and numeracy outcomes. In the first stage of the project, schools worked with a researcher to identify and document school and classroom practices in literacy and numeracy. The second phase involves professional development, school visits, and continuing support.

Aspects of numeracy learning among children in disadvantaged schools have been studied. Hill (1999) argued that expectations for children from disadvantaged backgrounds are often set too low. It would appear that the achievements and understandings of some of these children are indeed below those of children from higher socio-economic circumstances. Wright (1991c), for example, found a wider range of ability levels among
kindergarten children from lower rather than higher socio-economic backgrounds. It was suggested that the provision of appropriate mathematical experiences by parents had the greatest potential to advance children's number knowledge prior to beginning school. The studies described above indicate that socio-economic status is a factor that can impact on children’s numeracy learning outcomes. Further work is needed to determine how best to address the effects of lower socio-economic status in particular. This dimension of inequity needs to be monitored constantly.

Rural students

The *Children, On-line Learning and Authentic Teaching Skills in Primary Education* project relates to rural students. The findings are discussed in the section on computers in the *Teachers, Students, and Classroom Practice* chapter of this report. As on-line access becomes more common for students and teachers in rural areas, the findings of this project should prove useful Australia-wide. The researchers are studying teaching and learning behaviours in primary education, with the aim of identifying authentic skills and strategies used by teachers engaged in on-line programs in rural and urban settings. They are seeking to match best practice in teaching with quality on-line learning outcomes. The remaining projects and publications exploring rural issues were related to Indigenous Australians and are discussed later in this chapter.

Gender

Reviews of Australasian research on gender issues and mathematics learning have been published for the periods 1988–1991 (Leder & Forgasz, 1992), 1992–1995 (Barnes & Home, 1996) and 1996–1999 (Forgasz, Leder & Vale, 2000). From the 1970s to the 1990s, there was considerable concern about girls’ achievements in and attitudes towards mathematics. There has also been a vast body of research on gender issues associated with secondary and tertiary level mathematics students. However, in the past decade there appear to have been no major projects at the primary level with gender as the main focus, although there were many smaller studies in which gender issues were explored. In the majority of these, gender was a variable of interest among a range of other factors, including other equity considerations (for example, Young, 1994). Several of these publications are discussed in detail elsewhere in this report.

In the sections that follow, the research discussed is related to previously identified potential contributors to gender differences in some outcomes of numeracy learning – perceptions of ability, and performance levels. In the subsequent section, findings are presented from research studies focusing on various numeracy issues for which the researchers have included gender as a variable of interest.

Forgasz (1992) examined the gender differences in Year 2 students’ perceptions of their achievements in mathematics. It was found that in all five participating classes, males rated themselves higher than females, although the class teachers had rated girls higher in some classes and boys higher in others. Hays (1994) found that teachers’ ratings of girls’ mathematics achievements and their test results were higher than boys’. However, these did not translate into higher mathematics self-perceptions. In a study that focused on the transition from primary to secondary school, Trent (1993) found that in Year 6, girls’ perceptions of mathematical ability surpassed those of boys but that this pattern was
reversed in Year 7. In both year levels, there were no gender differences in achievement levels on mathematics tests. In one of the three studies reported by McIntosh (1996), Year 3, 5 and 7 boys were found to be more sure of their ability to calculate mentally and girls were more likely to be wrongly diffident of their ability. Yates (1999) gathered data on the optimism, pessimism, and mathematics achievement of students in Years 5 to 9 in 1993 and 1995. In both years, boys were found to be significantly more pessimistic than girls. It was interesting that year level, but not gender, was found to be a significant factor in the mathematics achievement of the students.

Barnes (1997) provided a detailed analysis by gender of the NSW Basic Skills Testing Program (BSTP) results from 1990 for children in Years 3 and 6. With respect to overall performance, Barnes claimed that the differences found were small and not educationally significant. There were, however, interesting patterns in the differences noted when the data were examined by topic area. At the Year 3 level, girls, on average, achieved better than boys in all three strands – measurement, space, and number. However, at the Year 6 level, girls did better than boys on the number items and boys did slightly better than girls on the measurement and space items. The gender differences in the Year 6 findings were small, but were consistent with the gender-stereotyped patterns that had been reported in earlier times. Ensuring that boys and girls have equal opportunity to learn mathematics was seen as a major contributor to the similarity in performance of boys and girls that was found. The ways that the home, out-of-school, and classroom experiences of boys and girls can differ and thus affect performance are very subtle.

In the chapter on Assessment, findings by gender from other large scale assessment projects are presented and discussed.

A wide range of related issues relevant to numeracy learning was evident in the many studies reviewed in which gender was a variable of interest. The numeracy topic or issue of major interest, and the gender similarities and differences found are summarised in Table 1.

### Table 1.
Findings from studies in which gender was a variable of interest.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Topic/issue</th>
<th>Findings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bana &amp; Korbosky (1995)</td>
<td>Number facts</td>
<td>No gender differences for automatic recall, or in levels of understanding.</td>
</tr>
<tr>
<td>Bishop &amp; Clements (1994)</td>
<td>Predictions of gender differences in the known performance of Year 5&amp;6 students on a selection of items used in a mathematics competition for primary children</td>
<td>Females were more likely than males to be correct in their predictions of gender differences. Mathematics education researchers and primary teachers were likely to be more accurate than trainee teachers.</td>
</tr>
<tr>
<td>Clarkson (1993)</td>
<td>Analysis of 18 Australian mathematics textbooks</td>
<td>45% of people depicted were male, 39% were female</td>
</tr>
<tr>
<td>Forgasz (1997)</td>
<td>Australian textbook analysis</td>
<td>Vestiges of gender stereotyping favouring males were evident, but a vast improvement on the past</td>
</tr>
</tbody>
</table>
The findings reported in Table 1 are generally consistent with persistent perceptions over time of mathematics as a “male domain”, that is, beliefs that boys are more capable mathematically and that mathematics is more suited to boys than to girls. It would have to be said that, compared with earlier times, the level of gender stereotyping appears to have diminished in recent publications and that the research findings are not entirely consistent. It is also worth noting that there is little research evidence at the primary level — either in Australia or overseas — to support the perceptions and claims of some that boys, rather than girls, are now the disadvantaged group with respect to numeracy learning.

It is critically important that gender issues do not fall off the numeracy education agenda. Clearly there are areas in which gender inequities persist, albeit more obvious at the secondary level of education and beyond. There are, for example, more boys than girls studying the most challenging mathematics subjects at the Year 12 level across the nation (Forgasz, Leder, & Vale, 2000). It is not exactly clear why this remains the case. It is therefore important to continue to monitor gender-related patterns and trends in mathematics education research. Researchers not already doing so should include gender as a variable of analysis, and research studies focusing on gender issues in numeracy education at the primary level should be supported.

It is worth noting here that the Australian Association of Mathematics Teachers published *A National Statement on Girls and Mathematics*, in 1990 (AAMT, 1990). In the 1980s and 1990s there was much research effort, extensive national and state policy development – for example, the *National Policy for the Education of Girls in Australian Schools* (Australian Government Schools Commission, 1987) – funding support, and extensive intervention programs to promote girls’ participation in mathematics (see Leder & Forgasz,
Equity and the School Community

1992). Since that time there have been significant gains in teachers’ consciousness about the need to cater equally well for girls and boys in mathematics classrooms.

From the findings presented in this section of the report, it is evident that much progress has been made in overcoming previous practices that worked against females and in favour of males with respect to mathematics learning outcomes. If this state of affairs is to persist, it seems important to continue to monitor students’ and teachers’ attitudes and beliefs, classroom practices, and the outcomes of assessment programs.

**Indigenous students**

Regardless of whether Indigenous issues were the primary focus or factors within a range of other variables, all research projects in which Indigenous issues were examined with respect to numeracy learning are discussed in this section.

With respect to the numeracy learning of Indigenous students, it was reported that in 1999 “little progress overall has been made in improving the numeracy outcomes of Indigenous students and, in many cases, outcomes for 1999 were below those of previous years” (MCEETYA, 1999b). One of the national goals for schooling in the twenty-first century (MCEETYA, 1999a) involved the achievement of equity for Indigenous students. To this end, initiatives have been undertaken in each state and territory. The initiatives were summarised in the papers prepared for each state/territory and presented at the 2000 World-Class Curriculum conference. These are available from http://www.curriculum.edu.au/conference/conf2000.htm, the conference website. Many of these initiatives, as they pertain to primary numeracy education, are described below.

When Aboriginal children come to school to learn mathematics, Bishop (1994) claimed, teachers are faced with special difficulties, mainly due to the children having been brought up in environments that can seem impoverished in relation to formal school mathematics. Howard (1998) found that teachers believed this to be the case. Indeed, Howard’s findings on teachers’ and Aboriginal educators’ and parents’ beliefs revealed an incompatibility between some school and classroom practices and the students’ cultural backgrounds and experiences. Some published work has focused on bridging this perceived gap. For example, Bishop (1994) put forward suggestions on how the home experiences of these children might be enriched to facilitate the development of appropriate school mathematical skills and attitudes.

Identifying factors that affect Indigenous students’ mathematics learning was central to the study reported by Howard (1995; 1997). An ethnographic study in a rural community of New South Wales was conducted and Aboriginal parents, educators, children and their teachers were interviewed on their beliefs about learning and teaching mathematics. Teacher-student relationships, teacher consistency, expectations, and learning styles emerged as factors believed to influence the children’s learning. One of the teaching strategies suggested was particular to Indigenous students: recognising that Indigenous students faced conflict associated with language use, learning styles, teaching and questioning styles (Howard, 1995). In a paper aimed at teachers, Howard (1998) outlined eight principles on which he based his teaching of mathematics to Aboriginal children:

- when Aboriginal children say “learn me” that is exactly what they are asking us to do;
• Aboriginal children have their own worldview and, for many, this view concerns the inter-relatedness of everything that exists;
• Aboriginal people live in union with the land;
• kinship systems provide all Aboriginal people with complex ordering patterns;
• there are varying ways to conceptualise learning;
• the Dreaming is the foundation of Aboriginal Identity;
• language and kinship are critical elements of Aboriginal identity; and
• mathematics is but a part of the curriculum and but a part of one’s identity.

These principles are vital and need to underpin both development and delivery of curriculum. Too much focus on elements of difference, however, can also lead to lowering of expectations and limited and limiting methods of teaching, especially in cases where teachers come from different cultures or social-economic groups. Thus we also need to consider the limitations of mainstream curriculum and dominant pedagogical styles. In fact, Robinson and Nichol (1998) described the education introduced to Indigenous Australians by non-Indigenous people as generally discriminatory and inappropriate. Creating an improved mathematics learning environment was the goal of a mathematics workshop conducted through the Anangu Teacher Education Program (ANTEP) for teachers preparing to teach in Aboriginal communities (Patrick-Rolf, 1990). The strategies adopted focused on planning, lesson delivery, and record keeping. Similarly, Nichol and Robinson (2000) described characteristics of Aboriginal learners and explored pedagogical strategies that could assist students’ learning and teachers’ delivery. The characteristics of their view of Indigenous pedagogy included:

• holistic learning: complete, co-operative, integrated and all encompassing — strongly linked to notions of identity and Aboriginality;
• imaginal learning: relatively unstructured, consisting of thoughts, images and experiences of learning — also strongly linked to notions of identity and Aboriginality;
• kinaesthetic learning — tactile learning through manipulation and movement within the learning environment;
• co-operative learning;
• contextual learning; and
• person-orientated learning.

General pedagogical strategies were recommended and were grouped under four headings: social aspects, environmental aspects, assessment, and general teaching. With respect to mathematics teaching, the authors maintained that mathematics was a problematic aspect of Aboriginal education and that

Teachers of mathematics to Aboriginal students need to both examine and appreciate the cultural constraints on learning faced by their students within the context of a mainstream curriculum and to build on the large pool of knowledge and pedagogy that the Aboriginal society bequeaths to Indigenous students. (Nichol & Robinson, 2000, p. 503)

In order to help students with their mathematical knowledge so that it will be appropriate and relevant to their cultural background and heritage and also equip them for life and work in the wider Australian society, the following strategies were put forward:
• provide opportunities to learn by doing, with as much community involvement and
teaching by Aboriginal people as possible;
• emphasise ‘showing’ or modelling rather than explaining;
• use models and examples to demonstrate concepts, in particular from the local
environment and resources;
• use multimedia resources including computers and video to demonstrate concepts;
• incorporate the manipulation of materials into the lessons; and
• use geometric shapes to provide concrete understanding of fractions before
proceeding to written work; and
• demonstrate the meaning of terms and concepts.
(Nichol & Robinson, 2000, p. 502)

Cultural differences and the knowledge base of the Indigenous community could be used
as a force to promote learning (Nichol & Robinson, 2000). The Garma Mathematics
Program was cited as one that exemplified the authors’ recommendations. In this
program, the Yolngu (Aboriginal) Community’s knowledge and the curriculum of the
Northern Territory are integrated (see, for example, Grenfell, 1998; Stanton, 1991;
Watson-Verran, 1992). Appropriate kin relationship is used in the presentation as a
starting point for developing children’s understanding of the way that our number system
is organised. This is an example of how the program “involves the reactivation of
traditional, conceptual and practical thought and the incorporation and adaptation of new
ideas” (Nichol and Robinson (2000). Nichol and Robinson advocated that Aboriginal
community organisations and teachers develop appropriate programs to meet the specific
needs of the students. It was also noted that Indigenous students had many needs that
were no different to those of others: positive relationships with teachers, a sense of
ownership of knowledge, appreciation of their cultural background, and knowing that
school is a relevant and productive environment.

Richards (1997) reported findings from a study in which the classroom teacher revealed
sensitivity to Aboriginal socio-linguistic behaviours and outlined the identified pedagogical
behaviours that had served to enhance the students’ access to mathematical ideas. In her
thesis, Richards showed how sensitivity to alternative behaviours can enhance the
establishment of good relationships and this, in conjunction with the intriguing activities
provided, brought about a high level of involvement by the Aboriginal children.

Positive self-identity is regarded as a factor contributing to Indigenous students' attachment to school and to their positive school outcomes. With the aim of raising the
positive self-identity of Indigenous students, one of the recommendations of the project
Positive Self-identity for Indigenous Students and Its Relationship to School Outcomes
was that teacher education institutions develop specialised modules or units that focus on
Indigenous education and that might include methods for the successful teaching of
subjects such as mathematics. The other recommendations of this project were not
specific to mathematics but were similar to those put forward by Nichol and Robinson
(2000), reported above.

Generally, the Australian Indigenous population remains educationally disadvantaged
(Buckskin, 2000). Based on a number of factors, Indigenous students’ mathematical
performance levels are lower than those of other groups. Young (1994) found that,
regardless of location or school size, Aboriginal students — and also students attending
schools in lower socio-economic environments — were likely to have lower levels of mathematics performance. This was also found to be true for Aboriginal students in urban settings (Northern Territory Department of Education, 1993). Some reasons for these differences have been identified. For example, Buckskin (2000) identified a range of issues impeding the achievement of educational equality.

Lokan, Doig and Underwood (2000), in discussing the results from the Third International Mathematics and Science Study (TIMSS) presented a summary of test results that demonstrate how Indigenous Australian children are generally lagging behind others. They wrote that

The report of the 1996 Western Australian MSE testing comments that “the performance of Aboriginal and Torres Strait Islander students continues to be a concern. In general terms, their performance at each year level was almost a full outcome level lower than the performance of the rest of the population” (p. 6).... The report of the 1995 to 1997 testing in Queensland commented that the performance of Aboriginal and Torres Strait Islanders was “more than extremely below” that of the rest of the population (Queensland School Curriculum Council 1998, p. 18) (p. 24)

However, New South Wales testing in 1997 showed that Aboriginal and Torres Strait Islander children demonstrated “more growth in numeracy from Year 3 to Year 5, using longitudinal data, than any other group” (Lokan, Doig & Underwood, 2000, p. 24). This suggests that improvements in early years numeracy — including pre-school developments — are essential. Also worthy of note, and likewise suggestive of where curriculum emphasis could be placed, were the percentages of Indigenous students overall

who did not attempt to respond to the items which required answers to be written rather than selected. The results may suggest that some kinds of assessment are culturally specific and that other forms of assessment should be used to ascertain what these students know and can do. (Lokan, Doig & Underwood, 2000, p. 25)

It is clear that bringing about improvements is not merely a matter of increasing emphasis on mathematics teaching for all children, as shown by Currie (1991) and Currie, Kissane and Pears (1992). Focusing on Aboriginal children who did not attend western-style government schools, they reported findings from a Pilbara region school in which an enriched mathematics program had been introduced. Pre-program and post-program data were gathered by interview, and Aboriginal children in five other schools were also interviewed in order to make comparisons. A wide range of mathematical performance was found within individual schools and across schools, but the target children’s mathematical performance level over the year of the study changed very little. School comparisons revealed that the target children’s performance levels did not fall further behind the others’, nor did they reduce any of the substantial gap measured originally.

Projects targeting Indigenous students

It is anticipated that the National Indigenous English Literacy and Numeracy Strategy (2000–2004) will enable greater progress towards equity. Through a range of co-ordinated Australian Government initiatives, the objective is for Indigenous students to achieve numeracy (and literacy) levels comparable to those of other young Australians. This will be achieved for Indigenous students through more effective use of State, Territory, and Australian Government education funding programs to:
• lift school attendance rates to national levels;
• address hearing and other health problems that undermine learning;
• provide pre-schooling opportunities where possible;
• train enough teachers in the skills and cultural awareness necessary to be effective in Indigenous communities and schools, and to encourage them to remain for reasonable periods of time;
• ensure that teaching methods known to be most effective are employed; and
• institute transparent measures of success as a basis for accountability for schools and teachers.

There are a number of common elements in the goals and in the outcomes of the projects described in this section of the report. These include:

• the need to take into consideration the cultural and language backgrounds of Indigenous students in the development of appropriate curricula, materials, and learning experiences;
• the real world experience contextualisation of learning activities, and
• focused professional development of teachers.

The goal of the English Language and Numeracy (ELAN) program, established in 1991 in Western Australia, was to assist young Aboriginal children to reach the same skill levels as other students. ELAN was based on First Steps (see Developmental frameworks section of this report for details), a curriculum project aimed at improving children’s numeracy and literacy development (Jarred, 1994). Appointed within selected schools (32 schools participated), ELAN teachers became a resource to other teachers, to help them meet the special needs of Aboriginal students. ELAN teachers received professional development in First Steps, Aboriginal learning styles, and in teaching English as a second language. They met twice a year to share resources and discuss effective teaching practices. A similar program, Changing Places operates in Tasmania. Based on Tasmania’s successful Improving Numeracy for Indigenous Students in Secondary Schools (INISSS) program (see Callingham, Griffin & Corenille, 1999) at the secondary level, a major purpose of Changing Places is to improve primary level Indigenous children’s proficiency in literacy and numeracy. Workshops and associated school-based activities are the medium for investigations into what comprises inclusive practice. Improvements in students’ outcomes are measured through state monitoring programs. A task-based assessment process, developed in consultation with program participants, is the means of measuring student improvement in outcomes.

Findings from the Indigenous Students Achieving Numeracy (ISAN) project and the Northern Territory Numeracy in Schools (NISP) project were summarised by Efthymiades, Roberts, and Morony (2000). ISAN was conducted at five schools with the goal of demonstrating that improved learning outcomes for Indigenous students could be achieved quickly. In each school, projects that built on understanding the local context and needs were designed and implemented. Mainstream assessment and reporting frameworks were used to measure student achievements. Issues that emerged included the important roles of: language, making connections with students’ lives, affective dimensions, having programs which allow for breaks in children’s education, and having high and consistent expectations of students and their potential to achieve. McRae et al. (2000) provide details of the work at one of the five participating schools.
Structurally and conceptually similar to ISAN, the Northern Territory Numeracy in Schools (NISP) project focused on all Northern Territory students, and many remote Indigenous community schools that had bilingual programs were involved (Efthymiades, Roberts & Morony, 2000). The projects developed were “needs based” and varied greatly. The necessity for the numeracy development of teachers was an additional element that emerged from this project. Both projects, the authors argued, highlight the need for assessment to be meaningful. For Indigenous children in remote communities who come to school with understandings linked to their culture and first language there is a need for the development of “tools” to assess these understandings in first language and then base learning programs on an appropriate combination of first language, ESL and English-only experiences. (Efthymiades, Roberts & Morony, 2000, p. 30)

In the Contextualising Mathematics Focus Schools project, conducted by the Department of Education, Training and Employment (South Australia) in 1998, seven units of work were developed to be used in schools with high proportions of Aboriginal children to illustrate the basic approach of the contextual model. The model was described as one in which the learning should be placed in a real context that is known to students, that there be agreement on the tasks, processes and the end point of the activity, and that explicit teaching be used to assist children to reach the end point.

The Count Me In Too: Indigenous project is aimed at producing Count Me In Too (see Intervention) materials and teaching strategies that are more culturally appropriate for Aboriginal students. Professional development of teachers to identify and address children’s needs is a focus of the program. Currently, children’s progress is being monitored.

In summary, the numeracy outcomes of Australia’s Indigenous primary children are lower than for the rest of the population. At the same time, there is increasing recognition that Indigenous children have different learning needs, and that Indigenous culture and pedagogy are strong and valid (Nichol & Robinson, 2000). Researchers have identified factors that impede and factors that can enhance the learning opportunities of Indigenous students. The Indigenous community has been found to be a powerful influence. The findings reveal that the strengths of the Indigenous culture need to be drawn upon, that the children’s preferred learning styles should be exploited, and that the teachers who work with Indigenous children need appropriate professional development.

The school and its community

Factors beyond the four walls of the classroom, both within and beyond the school perimeter, contribute to the complex web that forms the educational context in which numeracy learning occurs. In our explorations of the literature on primary numeracy education, three inter-related themes related to these factors emerged: parents, the community, and school factors. The findings are presented in this section. It should be noted that all research on Indigenous students that involve these themes was included in the earlier discussion of findings in this chapter under the heading Indigenous.
**School factors**

Various school factors and their effects on the numeracy learning of students are discussed under many different headings in various sections of this report — for example, teachers, parents, ethnicity, Indigenous, socio-economic status and others.

In the Victorian *Early Literacy Research Project*, Hill and Crévola (1998) developed a model to improve learning outcomes that they believed applied to literacy, numeracy, and other curriculum areas. Nine elements comprised the model’s design including: affective factors as well as home and school-related factors (e.g., leadership, teaching programs, school and class organisation, and special needs). In several numeracy projects, including the *Early Numeracy Research Project* in Victoria, the Hill-Crévola model was used as a basis.

One project, however, stood out as having the identification of school factors in relation to numeracy learning as a major focus. The *What’s ‘Making the Difference’ in Achieving Outstanding Primary School Learning Outcomes in Numeracy?: Strategic Numeracy Research and Development Project, NSW* commenced in 2001. The main aim of the project was to identify effective school teaching and learning practices that lead to measurably improved student numeracy outcomes. Based on the Year 3 and Year 5 Basic Skills Tests, schools with outstanding results were identified from across the three educational sectors — Government, Catholic, and Independent. In 2001, case studies of 25 schools, representative of SES, ATSI, LBOTE, rural, isolated, and metropolitan contexts, were undertaken. The goal in 2001 was to identify the features that make these schools exemplary. The features of interest included classroom factors and external factors such as parental influences. In 2002, the project undertook case studies of a further 20 schools, where outstanding programs appeared to be in place. In 2002 and 2003, schools concerned about the numeracy attainment levels of their students were supported in a trial intervention using the strategies identified by both sets of case studies. As there has been very little specific research in Australia on school factors that may contribute to effective numeracy learning, the findings of this project are likely to be extremely useful.

The project also constructed a criterion-referenced achievement scale to track the numeracy development of students in the project (Mulligan, 2001). It includes 280 numeracy tasks (Busatto, 2001).

**Parents and the school community**

There have been several studies and projects in which the relationships between school, the school community, and home have been examined.

Family involvement in mathematics learning was the focus of the Australia-wide *Family Maths Project of Australia* (FAMPA). The rationale for FAMPA was to involve parents in the types of mathematical activities undertaken in schools. The program was flexible and a working team from each participating school community decided on the details. Workshops, evening meetings, outside speakers, and newsletters were included (Knox, 1993). Horne (1993, 1998) examined the effects on parents’ attitudes of their involvement in the FAMPA project. The parents were from four schools using FAMPA for the first time. Parents’ personal image in relation to mathematics, particularly of the mothers, was found to have improved. Fathers were able to experience the enthusiasm and confidence of their
children. However, parents whose negative feelings about mathematics related to formal tests were not assisted because testing was not dealt with in the program. Ways in which parents could get involved in their children’s numeracy learning were suggested and included: becoming involved in curriculum committees, assisting in classrooms, helping in the preparation of materials, and participating in mathematics activity days. Vasey (1990), who looked at the attitudes of parents’ (30 families) and Year 3 and 4 students, before and after involvement in the FAMPA program, also reported improvements.

Different forms of parental involvement in their children’s mathematics learning have been examined. Leder (1992) found that parents and pre-school teachers were more encouraging and gave more independence to a boy classified as “bright”, than to a boy classified as “average”. As happens with other discipline areas, Hay and Eddy (1998) advocated a take-home book program in mathematics where children select books to share with their families on a nightly basis. Kibble (1998) found that parents engaged with their children in home activities related to mathematics, were aware that their children’s mathematics learning was different from their own, and felt they did not have the knowledge to help their children. Keeping parents informed about current teaching approaches and generally open communication between school and home were suggested. Newsletters to home have been shown to be beneficial, although mothers of low achievers tended to use them as means of assessing progress rather than as stimuli for home experiences with mathematics (Savell, 2000). Roles that parents, the community, and policy makers can take in enhancing students’ numeracy outcomes were outlined by Hogan and Kemp (1999). Raising community awareness of how mathematics is connected to everyday life and recognition of the importance of numeracy education was considered to be a critical element. Costello, Horne, and Munro (1991) provide information for parents wanting to help their children learn mathematics, and present materials that enable families to learn, think and talk about mathematics.

Hawkins (1991), in a *Parents as Tutors Programme (Mathematics)*, reported success with a program that spanned ten two hour workshop sessions. Parents welcomed the opportunity to learn more about mathematics so that they could more confidently help their children. In another program, parents were trained to use particular techniques for teaching multiplication facts — “constant time delay” and “sequenced count-by” — to tutor their children (Wilson & Robinson, 1997). The decrease in error rates for the children in both of these groups was greater than for a control group given supervised home-learning tasks over the same time period.

In the Australian Capital Territory, the *Supportive Practices for the Enhancement of Literacy Learning Project (ACT SPELL)* aimed to improve both literacy and numeracy outcomes for students in the middle years of schooling. The project focused on three broad research questions whose exploration involved the nexus between in-school and out-of-school teaching and learning:

- how schools can foster a culture of communication that recognises the needs and abilities of all of the members of their communities as resources;
- how teachers can rethink their approaches to designing homework to overcome the problems and inequities prevalent in current approaches to homework design and practice; and
• how teachers can recognise the literacy and numeracy skills and experiences that children acquire and develop in their daily lives and integrate them into an innovative approach to literacy and numeracy teaching.

The specific focus of the project was on bringing about a closer integration of home and school numeracy practices. The research recommendations included the development of a kit addressing communication, homework, and integrated practices together with an in-service professional development program, and the examination by all schools of their existing practices of communication and develop communication policies and processes in order to support children’s learning. It was suggested that all schools develop approaches to homework that recognise issues of access and equity in the design and practice of homework. The success of the pilot project also led to recommendations about the use of approaches to teaching and learning that recognise and further develop the interests and experiences that children have in all areas of their lives, and especially drawing on children’s out-of-school experiences with new technologies.

Parental involvement has been a component of some state-based numeracy projects. With the assistance of parents and teachers, Maths Matters was an early numeracy strategy of the Victorian State Board of Education aimed at raising community awareness of the importance of mathematics. In the Opening Doors Initiative, run by the Victorian Directorate of School Education, materials were produced for use by teachers with the children’s parents, in order to give parents the skills and confidence to assist their children’s learning in both literacy and numeracy. Numeracy packs for families to borrow and use with their children were developed as part of the South Australian Numeracy Skills Across the Learning Areas: Assisting Students With Learning Difficulties to Put Meaning Into Maths project.

Project Good Start: Effective Numeracy Practices in the Year Before and the First Year of School commenced in 2001. The focus is to examine the possible links between before-school numeracy experiences — home, preschool and childcare — and those in the first year of schooling. The goal is to provide information about effective learning environments and strategies for successful transition to school. The findings from two projects commencing in 2002 — the Department of Education, Science and Training-funded project, A Project on Home, School and Community Partnerships to Support Children’s Numeracy and the Department of Family and Community Services (FACS)-funded project, Longitudinal Study of Australian Children — should also provide more valuable information on the links between children’s numeracy learning at home and at school.

The research evidence generally supports the view that children’s learning will be enriched if there are strong educational partnerships between school, the school community, and home. Although there was no direct evidence that parental involvement in their children’s numeracy learning improved outcomes, there was no evidence to the contrary either. Certainly parents gained much from programs aimed at them, and there were clear advantages for the schools in having parents interested and more appreciative of what their children were learning. Research into the longer term effects of parental involvement on children’s numeracy learning outcomes should be considered.
Chapter overview

In this chapter, primary numeracy research on equity factors, and research focusing on community and school factors have been reviewed.

In summary, the research on equity factors reveals the following:

- little is known about the needs of students with physical learning disabilities and how to address them;
- efforts have been made to identify and provide remediation for children with numeracy difficulties in the early years of schooling. Less, however, has been done to sustain this effort into the later primary years;
- language proficiency, whether English is spoken in the home or not, and other factors such as socio-economic backgrounds all affect numeracy achievement levels;
- gender differences in overall numeracy achievement are virtually non-existent; however, there are patterns of small (non-significant) differences by content area that are consistent with those of the past. However, gender-stereotyped attitudes and perceptions of girls’ and boys’ capabilities persist; and
- Indigenous children’s numeracy achievements are lower than for the rest of the population. It has been found that Indigenous children have different learning needs and factors that impede and factors that can enhance their learning opportunities have been identified. These include drawing on the strengths of the Indigenous culture, focusing on the children’s preferred learning styles, and providing teachers with the appropriate professional development.

Research on school factors and the community reveals that:

- parents and schools gain much from parental involvement in their children’s numeracy learning. However, little is known about which school factors in particular will contribute to improved student numeracy outcomes, and there is no evidence of improved numeracy learning outcomes for children as a consequence of parental participation in mathematics classrooms.
Chapter 4

Research on Teachers, Students and Classroom Practice

More than a decade ago, Willis (1990) called for an investigation into “conditions that are most likely to facilitate changes in school mathematics curriculum and pedagogy” (p. viii). She discussed the implications of redefining numeracy for teaching practice, and the importance of supporting teachers while they introduce reforms in pedagogy. Much of the research in the last decade has focused on teachers’ everyday practices.

Teachers

In examining a focus on the teacher, one should take note of the influence of the British Effective Teachers of Numeracy study (Askew et al., 1997), which led to the funding of the five-year initiative, the Leverhulme Numeracy Research Programme. While the initial, smaller study focused on teachers’ general approaches to teaching mathematics and showed relatively high mean achievement gains if teachers had “connectionist” orientations and relatively low mean gains for children of teachers with “transmission” or “discovery” orientations, the researchers concluded that effective teachers focus on:

- children’s mathematical learning, rather than on provision of pleasant classroom experiences;
- providing a challenging curriculum, rather than a comforting experience; and
- having high expectations of initially lower attaining pupils.

These factors were found to be more important than differences in overall teaching style. As the authors of the reports of both of these projects noted, the relatively low impact of the teacher’s approach has important implications for policy and research regarding the improvement of mathematics achievement in schools.

A significant amount of Australian numeracy research in the past decade, and much current research, has focused on teachers and their classroom activity, with the most common theme being characteristics of effective teachers. However, due to the range of classroom compositions and organisational factors that contribute to children’s learning, the difference that a teacher can make can be overestimated. For example, Lamb and Fullarton (2000), commenting on results from the Third International Mathematics and Science Study (TIMSS), reported that although classroom differences account for an identifiable portion of variation in children’s achievement, not all of this was due to the teachers themselves.

In the Leverhulme Numeracy Research Programme, this caveat was supported, leading researchers to conclude, *inter alia*, that:

- easily measurable teacher characteristics seem to have little association with pupils’ achievement gains; and
the range of attainment within each class is generally much larger than the differences between classes, pointing to the secondary effect of the teacher.

Teaching standards

As noted in the chapter on *International and National Developments*, the last decade has seen a clearer articulation of teaching standards and more accountability against these. One aspect of this trend has been the development of professional standards for teachers developed by professional bodies, while a second trend is seen in the increasing amount of research on the characteristics of effective teachers and quality teaching.

The United States was the first country to produce standards for the teaching profession. The National Council of Teachers of Mathematics (NCTM) produced the *Professional Standards for Teaching Mathematics* (NCTM, 1991). The Australian Association of Mathematics Teachers (AAMT) has followed this lead. An Australian Research Council-funded Strategic Partnerships in Industry Research and Training grant supported the joint work of Monash University and the AAMT to develop standards for teaching mathematics. The research team of the *Research and Development of National Professional Standards for Excellence in Teaching Mathematics* gathered information through a study of the literature and a survey of teachers Australia-wide. The team developed a framework that articulated “best practice”, and specified protocols for performance management and certification of levels of “excellence” for mathematics teaching at all levels. Through higher-level certification of qualified mathematics teachers, the aim is to improve the professional standards and status of teachers. The Standards are organised into three “domains”:

- Professional knowledge: encompassing knowledge of children, knowledge of mathematics, and knowledge of children’s learning of mathematics;
- Professional attributes: encompassing personal attributes, personal professional development, and community responsibilities; and

Major international studies into successful mathematics education have provided a strong foundation for both pre-service teacher education and professional development in Australia. The two-year *Effective Teachers of Numeracy* project, funded by the Teacher Training Agency in England, focused on teachers of children aged 4–11 years, and explored the characteristics of “highly effective” teachers as measured by their children’s higher-than-expected gains in numeracy achievement. The study found that these teachers had a particular set of coherent beliefs and understandings that underpinned their teaching. The effective teachers:

- believed that all children can achieve in numeracy;
- believed that being numerate requires a rich network of connections between mathematical ideas;
- saw class discussion as an important factor in developing the ‘connections’ children need;
- intervened to assist children to work more efficiently;
- used strategies that challenged all children;
- built upon children’s existing mental strategies;
Teachers, students, and classroom practice

- encouraged purposeful discussion in whole classes, small groups and with individuals;
- expected children to explain and listen;
- engaged children in solving realistic problems; and
- were more likely to have undertaken extensive discipline-based professional development.

Teachers who were found not to be very effective also had some characteristics in common. They:
- emphasised standard arithmetical methods over establishing understanding and connection;
- dealt with areas of mathematics discretely;
- frequently referred to children’s ability to remember what was taught;
- used assessment to check that taught methods had been learned rather than to inform subsequent teaching;
- gave priority to the use of practical equipment rather than developing effective methods; and
- delayed introduction of more abstract ideas until they felt a child was ready for them.

The research team noted that, while classroom practices were influential in children’s success, the teachers’ beliefs and understandings of the mathematical and pedagogical purposes behind those particular classroom practices seemed more important than the forms of practice themselves (Askew, Brown, Rhodes, Johnson & Wiliam, 1997). The researchers summarised what the project seems to suggest is quality teaching, calling it “direct teaching”. Here, the teacher:
- instructs and demonstrates, explains and illustrates mathematics;
- sets work in contexts and links it to previous work;
- maximises opportunities to interact with children so that they can talk and be listened to;
- gives feedback that helps children to develop their mathematical knowledge, skills and understanding; and
- allows children to show what they know, explain their thinking and methods, and suggest alternative ways of tackling problems.

In the United States, the National Council of Teachers of Mathematics Professional Standards for Teaching Mathematics (1991) presented six standards for the teaching of mathematics. Again, these did not focus on specific methods of organising for learning, but put an emphasis on teachers’ and children’s roles in classroom discourse, and tools for enhancing it. Also featured was advice about the learning environment, the nature of worthwhile mathematical tasks, and the importance of continuing analysis of teaching and learning. NCTM’s (2000) more recent Six Principles for School Mathematics include the underpinning maxim that “Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well” (p. 16).

Since the mixed results for the United States in the Third International Mathematics and Science Study, there have been calls for educators to change their beliefs about how
children learn mathematics and for more direction on what constitutes effective teaching (Sellers & Ahern, 2000), and major studies are under way. For instance, in Texas, a study to identify the components of an effective mathematics and science middle school (Adams, Brower, Hill & Marshall, 2000) involved 100 selected schools and a sample population of 350 teachers. This study identified the following practices as being necessary to enhance mathematics and science middle school reform:

• a curriculum that demands depth of a significant core of skills;
• an emphasis on the development of the children’s reasoning and problem-solving abilities;
• hands-on activities and use of the technology; and
• assessment that involves interdisciplinary tools.

Using data from a 50-state survey of policies in the United States, Darling-Hammond (1999) correlated data from the 1993–94 Schools and Staffing Surveys and the National Assessment of Educational Progress. She examined ways in which teacher qualifications and other school inputs related to children’s achievement across states. Both her qualitative and quantitative analyses suggested that policy investments in the quality of teachers were related to improvements in children’s performance. Measures of teacher preparation and certification were by far the strongest correlates of children’s achievement in mathematics, both before and after controlling for child poverty and language status. Policies adopted by states, regarding teacher education, licensing, hiring, and professional development, were shown to make important differences in the capacities that teachers brought to their work.

No similar extensive studies of quality teaching, or the effects of teaching qualifications, have been undertaken in the Australian context to date. Yet aspects of this research focus are currently echoed in every Australian state with research efforts attempting to identify the key components of mathematics programs and practices that lead to improved learning outcomes. In these studies, research approaches that recognise the complexity and richness of educational contexts are being used. No findings have been reported to date. A representative sample of current research of this kind is described in the sections that follow.

As intervention programs are implemented in schools and “recontextualised” (Bernstein, 1990), action research approaches have put teachers at the centre of the research process. This does not mean, however, that prior research and expert personnel do not support the classroom innovations. Many action research projects have demonstrated productive partnership between teachers, academics, and system personnel. In South Australia, for example, the Profiling High Numeracy Achievement: Strategic Numeracy Research and Development Project, SA has explored the following two questions in Department of Education, Training and Employment schools:

• What are the variables that impact on numeracy learning in schools that have consistently shown improvement in individual children’s numeracy achievement from Years 3 to 5?
• What practices and programs supported this improvement?

This project has used a literature search to identify effective teaching and learning strategies, school structures and programs that are claimed to improve children’s numeracy outcomes. Action research methodology has informed the project by undertaking whole school and, or, classroom-based action research. Teachers’ reflective
journals, and the preparation of school reports, have described the processes adopted, and the changes that occur.

The fact that there were few major empirical Australian studies completed in the past ten years is being partially remedied by current and recent major projects.

The cross-sectoral Victorian project Researching Numeracy Teaching Approaches in Primary Schools: Strategic Numeracy Research and Development Project, Vic aimed to identify and investigate the effectiveness of a set of generic teaching approaches for the teaching of mathematics in the early and middle years. Mathematics teaching approaches for children, in a range of primary school settings, have been investigated. The research questions in this project are:

- What are the key components of teaching approaches that lead to improved learning outcomes for numeracy in the primary years of schooling?
- If teachers implement a defined suite of teaching approaches, does this result in improved learning outcomes for children?
- How can these teaching approaches in numeracy be described, so as to support teachers to implement them effectively in their primary school classrooms and improve children’s learning?

Twelve scaffolding practices, or communicative acts, that teachers use to support their students’ mathematics learning have been identified, described and exemplified.

The New South Wales project, What’s ‘Making the Difference’ in Achieving Outstanding Primary School Learning Outcomes in Numeracy?: Strategic Numeracy Research and Development Project, NSW, has a self-explanatory title and sought to answer the questions:

- What are the educational policies, strategies, processes and practices which seem to be ‘making the difference’ in the achievement of outstanding numeracy outcomes?
- To what extent, and in what ways, can such educational practices, identified in one or more specific contexts, be successfully applied more generally to other educational contexts?

This cross-sectoral project has involved fifty-five schools. The case study schools were identified as having constantly good results, or noticeably improved results, on the annual New South Wales Basic Skills Tests or were considered to have outstanding programs in place. The focus is broader than classrooms; the project has investigated the educational policies, strategies, practices and processes, as well as external factors such as parental influence, that contribute to numeracy education in these successful schools.

To date, through a series of intensive case studies, the project has identified three sets of strategies that appear to have a strong influence on numeracy achievement. These are:

**What’s making the difference within the classroom?**
- constructive classroom interactions;
- purposeful pedagogy;
- accommodation of difference; and
- dynamic teaching.

**What’s making the difference throughout the school?**
- a school commitment to numeracy;
- school policies that support numeracy; and
- specialised programs that support numeracy.

What’s making the difference beyond the school?
- a shared vision;
- communicating about learning; and
- mathematics at home.

Each of these strategies has been described in greater detail by the project. Some of these strategies were trialled in schools that were keen to improve their numeracy profile. Improved outcomes were noted in the second year of trialling.

In 2003, the project documented a sample of lessons where outstanding teaching practices were occurring, analysing in depth the role of the teacher and student in these lessons, and then collating and reporting these classroom “snapshots” as contributions to outstanding numeracy outcomes via a CD-ROM.

Through research in schools across a range of settings and within the three school sectors in Queensland, the Australian Government-funded What Elements of Learning Environments Promote Enhanced Student Numeracy Outcomes?: Strategic Numeracy Research and Development Project, QLD, also aims to identify key elements of effective learning and teaching practice. Researchers have interacted with children, teachers and administrators in collaborative partnerships at each research site, and both quantitative and qualitative research methodologies have been used in case studies of schools. Action research has also been used, with the aim of supporting teachers’ and administrators’ reflective practice, enhancement of numeracy classrooms, and whole school and community environments. The effects of these changes on school programs and plans, teachers’ classroom practices, and on partnerships, including those formed between teachers and children and between the schools and families, has been evaluated.

Measurement of outcomes has included comparison of each school’s numeracy achievement profiles at the commencement (2000) and completion (2002) of the project. It is anticipated that this project will inform future directions for numeracy education, staff professional development, and school resourcing and accountability in Queensland school systems.

It is interesting that there are many projects with similar foci being supported at present, while there was a dearth of literature and advice on the features of quality mathematics teaching five years earlier. This development is the result of funding provided by DEST as part of the Australian Government’s Numeracy Research and Development Initiative.

In the research report that summarised their project Features of Quality Teaching, Sullivan and Mousley (1997) commented that prior to their project there had been no major projects conducted and little relevant literature available. They found that, in theory at least, there was some consensus about features of quality teaching. Teacher educators and well-qualified, experienced, mathematics teachers in Australia, the United States, and the United Kingdom, were surveyed to establish features of quality mathematics teaching.

Six major components of quality teaching were identified: building understanding, communication, nurturing, classroom organisation, use of materials, and engaging children in learning.

Subcategories and exemplars were also developed (see Sullivan & Mousley, 1994). While the six identified groups of features have proved very useful for the description and
analysis of classroom teaching, and useful for structuring both pre-service and in-service teacher development programs in Australia and overseas, Mousley and Sullivan (1992) found that there was not a strong consensus about practice, and there were some alarming contradictions. The researchers found that judging practice required knowledge of the teacher’s approach and rationale, and that experienced and knowledgeable people interpret specific terms and practices very differently. This project only outlined perceptions of quality teaching and no attempt was made to collect data on measurable outcomes of the teaching strategies.

A comprehensive Australian study relevant to this topic is Victoria’s Early Numeracy Research Project (ENRP). Of the many teachers involved in this project, six were selected for intensive study, using lesson observations, interviews, questionnaires, and responses to specific questions. On the basis of their children’s mathematical growth (see Developmental frameworks, below), these teachers had been identified as particularly effective (Clarke et al., 2002) in the project’s learning, teaching, and assessment framework domains. In the final report of this project, common themes for the case study teachers were as shown in Figure 2.

<table>
<thead>
<tr>
<th>Effective early numeracy teachers</th>
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<tbody>
<tr>
<td><strong>Mathematical focus</strong></td>
</tr>
<tr>
<td>• focus on important mathematical ideas</td>
</tr>
<tr>
<td>• make the mathematical focus clear to the children</td>
</tr>
<tr>
<td><strong>Features of tasks</strong></td>
</tr>
<tr>
<td>• structure purposeful tasks that enable different possibilities, strategies and products to emerge</td>
</tr>
<tr>
<td>• choose tasks that engage children and maintain involvement</td>
</tr>
<tr>
<td><strong>Materials, tools and representations</strong></td>
</tr>
<tr>
<td>• use a range of materials/representations/contexts for the same concept</td>
</tr>
<tr>
<td><strong>Connections/ links</strong></td>
</tr>
<tr>
<td>• use teachable moments as they occur</td>
</tr>
<tr>
<td>• make connections to mathematical ideas from previous lessons or experiences</td>
</tr>
<tr>
<td><strong>Organisational style(s), teaching approaches</strong></td>
</tr>
<tr>
<td>• engage and focus children’s mathematical thinking through an introductory, whole group activity</td>
</tr>
<tr>
<td>• choose from a variety of individual and group structures and teacher roles within the major part of the lesson</td>
</tr>
<tr>
<td><strong>Learning community and classroom interaction</strong></td>
</tr>
<tr>
<td>• use a range of question types to probe and challenge children’s thinking and reasoning</td>
</tr>
<tr>
<td>• hold back from telling children everything</td>
</tr>
<tr>
<td>• encourage children to explain their mathematical thinking/ideas</td>
</tr>
<tr>
<td>• encourage children to listen and evaluate others’ mathematical thinking/ideas, and help with methods and understanding</td>
</tr>
<tr>
<td>• listen attentively to individual children</td>
</tr>
<tr>
<td>• build on children’s mathematical ideas and strategies</td>
</tr>
<tr>
<td><strong>Expectations</strong></td>
</tr>
<tr>
<td>• have high but realistic mathematical expectations of all children</td>
</tr>
<tr>
<td>• promote and value effort, persistence and concentration</td>
</tr>
</tbody>
</table>
Reflection • draw out key mathematical ideas during and/or towards the end of the lesson
• after the lesson, reflect on children’s responses and learning, together with activities and lesson content

Assessment methods • collect data by observation and/or listening to children, taking notes as appropriate
• use a variety of assessment methods
• modify planning as a result of assessment

Personal attributes of the teacher • believe that mathematics learning can and should be enjoyable
• are confident in their own knowledge of mathematics at the level they are teaching
• show pride and pleasure in individuals’ success

Figure 2. Common themes emerging from six individual ENRP case studies.
(Clarke et al., 2002, p. 18).

Another Victorian project, the Middle Years Numeracy Research Project, had as its aim to determine what was effective in improving numeracy teaching in the middle years, particularly in relation to those students who fall behind.

Initial data collection from a structured sample of Year 5 to 9 students indicated that a significant number of students in Years 5 to 9 have difficulty with tasks involving explaining and justifying their mathematical thinking; working with formulae and solving multiple step problems; and connecting the results of calculations to the realities of the situation, interpreting results in context and checking the meaningfulness of conclusions.

Among the extensive conclusions reported by Siemon, Virgona, and Corneille (2001) are the following:
• there is as much difference between classes at the same school as there is between schools, which suggests that teachers do make a difference;
• opportunity to learn is as much a factor in explaining differences in performance as so-called ability;
• there is a significant ‘dip’ in numeracy performance from Year 6 to Year 7 that students do not appear to recover from until they reach Year 9, suggesting the need for a reappraisal of how transition from primary to secondary school is managed;
• early diagnosis and intervention are critical, with a need for key numeracy-related growth points and the scaffolding needed to help students move from one growth point to be identified and elaborated, with teachers needing to be supported to work with the ‘big ideas’;,
• teaching approaches and strategies for dealing with difference that maximise engagement, opportunity to learn and provide students with the means to access and connect new learning to prior learning are needed to support more effective practice.
• teachers and targeted programs make a difference to numeracy outcomes, particularly where they share a common set of beliefs and understandings and are
supported by a whole-school approach to planning, with effective professional leadership being an essential feature; and

- teachers and students need time to elaborate and explore ideas, to connect, generalise and conjecture, which in turn requires a shift in expectations and targets from a large range of relatively disconnected ideas to a smaller, more connected set of ‘big ideas’ and the scaffolding needed to acquire and use those ideas with confidence (pp. 3–5).

Other projects identifying specific characteristics of teaching are attempting to measure the impact that these can have in practical contexts. However, whether teachers are well-prepared to implement new teaching practices is yet to be determined (see, for example, Elmore, Peterson & McCarthey, 1996).

In summary, it would appear that there are several on-going studies in Australia from which useful findings related to factors associated with effective teachers are likely to be identified. As described above (for example, see Figure 2), some of these factors already appear to have emerged from several other studies.

**Teacher education and development**

There are several avenues for teacher development. They include reading reports of research, teacher education programs (both pre-service and in-service), and school-based projects. The first of these, reading published research, is unreliable in both its take-up and its effect. Many schools subscribe to professional journals and teachers read them. However, professional journals are not common avenues for the publication of research. On the other hand

Mathematics education researchers ... undertake their research with the ultimate goal of affecting what happens in mathematics classrooms. The results of their research appear in a variety of research articles and book chapters. Yet teachers rarely access original research reports, perhaps because researchers tend to write in a style that is often not teacher-friendly. Few teachers ever open (research journals) unless they are assigned to do so for professional development or for a graduate class. (Sowder & Schappelle, 2002)

A broader avenue for conveying recent developments in numeracy to teachers is through teacher education programs. The research in this area has focused on the development of pre-service teachers’ knowledge of mathematics, mathematical pedagogy, and children’s development of numeracy concepts and skills, and on the pre-service teachers’ beliefs and confidence.

**Pre-service teacher education**

The National Commission on Teaching and America’s Future (1997) stated that the most important features for ensuring high-quality outcomes for teacher education, both pre-service and professional development, regardless of the length of the programs, are:

- a common vision of good teaching that is apparent in all coursework and clinical experience;
- well-defined standards of practice and performance that guide and measure courses and clinical work;
- a rigorous core mathematics curriculum;
• extensive use of problem-based methods, including case studies, research on
teaching issues, performance assessments, and portfolio evaluation; and
• strong relationships with reform-minded schools that support the development of
common knowledge and shared beliefs among school and university faculty.

A key factor in successful numeracy teaching is the teacher’s own understanding of
mathematics, as it is not just the ability to do primary mathematics that is required, but a
deep understanding of concepts and processes, as well as an ability to explain these
logically, with the use of appropriate and meaningful language and examples.

Wardlaw (1994) investigated these aspects when he interviewed pre-service teachers
using a diagnostic protocol. Their ability to provide multiple representations for tasks, and
to demonstrate connections between mathematical ideas were monitored, and this was
compared with the confidence that the subjects appeared to exhibit. Wardlaw noted a
strong correlation between the level of self-confidence shown by pre-service teachers and
their demonstrated mathematics performance. Past, successful, mathematical
experiences, self-confidence in one’s mathematical ability, and the likelihood that
individuals will embrace the opportunity to reconstruct their knowledge, were seen as
positive features of higher-performing pre-service teachers. Wardlaw pointed out that this
project demonstrated why pre-service teachers must have opportunities to develop a deep
understanding of mathematics, prior to beginning teaching methodology classes,
regardless of marks gained on traditional competency tests.

Pre-service teachers bring to their teacher education courses a range of ideas about
mathematics and numeracy. Schuck (1999) reported in her paper “Driving a mathematics
education reform with unwilling passengers”, that many of her own (Australian) student
teachers held beliefs about mathematics, and about learning, that constrained their
access to rich and powerful ways of learning mathematics. She realised that her task was
to make her students more aware of their beliefs, as well as the implications of these
beliefs for teaching and learning. However, Schuck also became aware that her own
practice as a teacher educator revealed obstacles to reform, and reported how she had
changed her course activities and tools as a result. The researcher’s conclusion was that
deep, continuing, self-study of teacher education practices is essential if teacher
educators are to lead reform practices. To assist in this process, Tracey, Perry, and
Howard (1998) designed a questionnaire that could be used to help teachers, both pre-
service and in-service, to know their own teaching styles.

Another study with implications for primary teacher educators was that undertaken by
Gervasoni (1998). This researcher asked teachers to identify a dilemma that could form a
useful basis of further professional development. Two-thirds of the dilemmas raised
related to the need for teachers to develop a deeper understanding of how young children
learn mathematics, and in knowing which teaching strategies could most effectively help
children to develop numeracy skills and understandings.

An important aspect of teacher preparation is the practicum, which provides opportunities
to observe and practice teaching. Because teachers or schools are generally paid to have
pre-service teachers in their classrooms, this is an expensive aspect of teacher education.
The aim of the Features of Quality Teaching project, mentioned earlier in this chapter, was
to address this problem by developing a CD that would help prepare pre-service teachers
for observation, by developing their skills of analysis as well as their understanding of
some of the finer points of teaching. Trials in several settings found the CD to be effective,
with pre-service teachers demonstrating increased observation skills as well as improved ability to discuss teachers’ work. This software, based on a close analysis of only one lesson, made a positive contribution to post-practicum discussions.

The notion of studying one lesson in depth is the core of professional development practices in Japan (see, for example, Mousley, 2000). Typically, groups of Japanese teachers in a school, or in a teacher preparation course, undertake close study of a particular lesson for several hours a week over many weeks (known as “kenkyuu jugyou” or “lesson research”). There is potential for research to see whether this approach could be translated into Australian contexts.

In summary, the research on pre-service teachers reveals the importance of a sound understanding of mathematics, and the links with beliefs and confidence levels. The potential implications for teaching were highlighted.

**Professional development**

A typical feature of Australian, longer-term, school-based, action research produces what Rice (1993) called the “empowerment potential when teachers are committed to action” (p. 77). In this style of professional development, teachers have helped to define directions for change, in contrast with the top-down “delivery” style of short-term professional development that seems typical in other countries. Rice found that successful professional development, having long-term and school-wide effects and leading to improved outcomes for all, relies on:

- teachers being given time and appropriate resources to enable them to reflect on their teaching and make changes as and when they see fit;
- teachers working collaboratively in groups of three or more, assisting each other in the process of reflection and change;
- providing continuing support and encouragement while teachers are exploring possibilities and trialling new strategies in their classrooms;
- changes being introduced gradually and not requiring major restructuring of programs; and
- opportunities for teachers to meet and share their ideas and experiences with colleagues from other schools.

Researchers and professionals at a two-day conference in the United States, *Bridging the Gap between State Standards and Classroom Achievement*, made similar points. They agreed on features of effective professional development for teachers.

> Among other characteristics, participants said, such practices should be content-focused, linked to correcting a well-defined problem, sustained, situated in or near classrooms where teachers work, and rooted in the curriculum they teach. Talking about professional development independent of academic content, independent of curriculum, really doesn’t make much sense. (Olson, 2002, p. 2).

Many teachers see the need for professional development (Darling-Hammond & McLaughlin, 1995) and, since the continual building of skills and knowledge is the keystone of any profession, the question to be asked is, “What forms of professional development have been shown to be the most successful?”

A key Australian study of the processes of professional development was that of Hollingsworth (1999, p. 530), who investigated whether professional development
programs can act as appropriate vehicles for the professional growth of teachers of primary mathematics. Her longitudinal case studies focused on primary teachers involved in the Exploring Mathematics in Classrooms (EMIC) program. A theoretical model of teacher professional growth was used to represent the teachers' growth. The data provided evidence of a strong link between the content and outcomes of professional development programs. The study also provided insight into the processes involved in teacher professional growth and into factors associated with the way professional development programs influence this growth.

Various forms of action research have been used in many professional development projects and programs undertaken by practising teachers. As Pearn, Hunting, Merrifield and Mihalic (1997) noted in Research Informing Practice and Practice Reflecting Research, this puts teachers at the core of the research process. It also melds professional development with everyday practice, and does not overload teachers with more courses held outside, or interrupting, their working day. Sparrow and McIntosh (1998) identified this as a major issue, commenting on the impediments, and challenges, faced by the teachers when trying to implement change in their classrooms.

The Assessing Numeracy in Primary Schools: Strategic Numeracy Research and Development Project, ACT project involves Independent, Government, and Catholic schools in the Australian Capital Territory. It is an example of the style of project that seeks to improve children's numeracy outcomes through the use of whole-school approaches to improving numeracy teaching and learning. Information has been gathered about classroom teaching and assessment practices, children’s performance, and child and teacher attitudes. A range of data collection methods (for example, questionnaires, teacher self documentation, semi-structured interviews, and classroom observation) document whether changed classroom practice improves student numeracy outcomes. The Australian Capital Territory Assessment Programme (ACTAP) was used to assess the numeracy outcomes of the children. Teachers were helped to make links between the large-scale, detailed assessment regime and specific aspects of classroom interactions. The teachers involved focused on how improved classroom practices can lead to improved numeracy outcomes, and vice versa.

Other projects have focused on networking schools with researchers and consultants. The Victorian Numeracy Strategy Project brought together fifty teachers with six university academics and several Catholic Education Office consultants from across Victoria, with the aim of providing professional development for teachers who might take on leadership roles in future years. The current Year 3–8 Numeracy Project will also draw together key Year 3 to 8 teachers to work together to explore the notion of numeracy, become more confident in their knowledge and strategies, and develop useful structures that will support and challenge children’s existing numeracy. It is expected that this three-year study will increase experienced primary teachers’ knowledge and understanding in the broader study of mathematics, and well as increase their understanding about how children construct mathematical knowledge and develop numeracy skills. Gervasoni (1999) described another project that provided strong networking for teachers. She discussed the effectiveness of the professional development model used by the Catholic Education Commission of Victoria in its Numeracy Strategy Project that has subsequently developed into the Success in Numeracy Education (SINE) project.
Some projects have more defined professional development sessions and materials. For example, the Changing Places project in Tasmania has used a Reading, Writing, and Mathematics inventory to monitor students’ attitudes and growth of understanding. This inventory describes typical behaviours on learning tasks that are arranged into an expected learning sequence. The program is for selected primary schools with relatively large numbers of Indigenous children, and the aim is to decrease the gap in achievement between them and non-Indigenous children. The Calculators in Primary Mathematics project, reported on in detail in the Technology section of this report, involved teachers in school-based and wider networks during the four years of the project, and used newsletters to encourage teachers to communicate about their own learning as well as their classroom activities and successes (see Groves, 1997).

Time span and contact hours have been shown to be key elements of successful professional development (Garet, Porter, Desimone, Birman, & Yoon, 2001) and it has long been recognised that even experienced teachers need continuing support in order to implement significant changes to their current practices (Willis, 1990). A project attending to this need is the Early Literacy and Numeracy Partnerships. The aim of this project is to improve learning outcomes of educationally disadvantaged children in the pre-school year. A sense of the project’s emphasis on partnerships is provided by its objectives:

- development of a professional development program, strategies, and resources to support intervention in the preschool sector;
- provision of professional support for pre-school teachers and other relevant professionals;
- establishment of a network of trained personnel, pre-school teachers, and families;
- training of early childhood co-ordinators to advise and support teachers;
- conducting professional training sessions with network groups; and
- conducting parent information sessions.

Some projects are focusing on the development of children’s mental strategies. One example is the Enhancing Numeracy Outcomes project in which university researchers and classroom teachers work together. Classroom-based research processes involve teachers trialling mental computation strategies in their classrooms and providing feedback to the research team. The team then decides on the development of subsequent strategies according to this feedback. Such activities bring teachers into close contact with people who can support teachers’ own expertise with knowledge of related research and a wide range of pedagogical approaches and strategies. Similarly, in the South Australian High Performance in Literacy and Numeracy in Disadvantaged Schools project, schools work with a researcher to identify and document school and classroom practices in literacy and numeracy. In the second stage of the project, initial project schools support other disadvantaged schools to improve literacy and numeracy outcomes through professional development, school visits, and continuing support.

In the early 1990s, some States and Territories ran intensive professional development projects that involved teachers in school-based or regional-centre-based “tutorials” once a week. Teachers worked on aspects of their classroom practice during intervening times. For example, the Victorian middle-school program, Continuing Maths, grew out of, and complemented, Exploring Mathematics in Classrooms (EMIC), a successful and popular program for primary teachers. The EMIC and Continuing Maths professional development programs were very successful in introducing new approaches to teaching and learning,
such as problem solving, the use of real-world contexts, group work, and ways of catering for a range of individual differences.

A more extensive professional development program undertaken more recently in New South Wales, the *Count Me In Too* early numeracy program, has built on the theory and methods of *Mathematics Recovery* program – entitled analogously to the *Reading Recovery Program* (Clay, 1987) – and based on the work of Steffe and colleagues in the United States. All educational sectors have been involved, and many regions are supported to elaborate this work further with regional and school projects.

As stated in *Count Me In Too* descriptions, the aim of the program is to improve numeracy outcomes in the early years of school (K-3) by providing teachers with support, in the use of developmental or learning frameworks for assessment, to guide instruction in *counting* and *number*. The original *Count Me In* development used the *Learning Framework for Number* and the *Schedule for Early Number Assessment (SENA)* in one-to-one interviews with young children, to learn more about their ways of thinking, strategies, strengths, and needs, so that these can be considered in planning for teaching. The placement of children’s numeracy achievements on the learning framework also helps inform decisions about appropriate future learning activity. This state-wide program commenced in 1996, and now has off-shoot developments *Count Me In Too: Indigenous*, and *Count Me In Too: Measurement*; and *Count Me In Too: Angle*. It has been extended to the primary-secondary transition years with *Counting On* and *Counting On: Transition 6–7*. It has been implemented in Tasmania, the Australian Capital Territory, and New Zealand. Most importantly, the project has been successful in improving children’s numeracy outcomes (Bobis & Gould, 1999). As stated in a Department of Education and Training (NSW) pamphlet, “Evaluations of the project provide clear evidence of the positive impact that *Count Me In Too* can have on the mathematical performance of children. As well, the project is having an impact on the beliefs and professional knowledge of teachers and the result has been a tremendous increase in teacher confidence in teaching mathematics”.

Every one of these numeracy projects has significant professional development elements. Some key projects funded by various state and territory systems that include professional development in their stated objectives are presented in Table 2.

The projects reported in Table 2 have some elements in common. First, most focus on *number* more than general numeracy. This is an issue that must be addressed by States and Territories, as well as in regions and schools. Second, most of the initiatives centre on identifying children who already seem to be behind their peers’ achievement levels, either when they enter school or after about two years of schooling. Such judgements are based on observations of the children as well as more formal evaluations. The aim is to identify and remediate problems before they affect further progress or become insurmountable. A third common feature is the use of a curriculum framework that is taken as a sensible way of scaffolding learning and assessment. Some of these frameworks are closely tied to state curriculum documents and others to empirical research findings; more is said about this in the *Developmental frameworks* section of this report.
Table 2.
Some current and recent early years numeracy projects, with professional development as a key objective.

<table>
<thead>
<tr>
<th>Project</th>
<th>Professional development dimension</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Count Me In Too (Tasmanian implementation) Tas.</td>
<td>Aims to assist teachers to broaden their knowledge of how children learn mathematics, and to support teachers in making links between assessment, planning, and use of effective teaching strategies.</td>
<td>Callingham (1999)</td>
</tr>
<tr>
<td>Early Years Numeracy Project, SA: Catholic Education</td>
<td>Addresses issues in mathematics education and numeracy in the early years. The project will focus on current research findings and their possible implications for classrooms.</td>
<td></td>
</tr>
<tr>
<td>English Language and Numeracy Program for Aboriginal Students, WA</td>
<td>The ELAN teacher, from within the school, has professional development in <em>First Steps</em>, Aboriginal learning styles and teaching English as a second language. This person becomes a resource for other teachers.</td>
<td>Jarred (1994)</td>
</tr>
<tr>
<td>First Steps in Mathematics, WA</td>
<td>Aims to improve teachers’ understandings of teaching and learning mathematics within a developmental framework. The first part of the program focuses on “students at educational risk”.</td>
<td></td>
</tr>
<tr>
<td>Numeracy Strategy K-6, NSW</td>
<td>A numeracy plan, a diagnostic assessments tool, a learning framework, and professional development are inter-related elements. A “train the trainer model” is in use.</td>
<td></td>
</tr>
<tr>
<td>School Entry Assessment: Numeracy Research Project, SA</td>
<td>Extends teachers’ repertoires of effective assessment techniques, to adapt their teaching strategies to be more responsive to children’s needs</td>
<td>Bleckly, Papps &amp; Hugo (2001, October)</td>
</tr>
<tr>
<td>Success in Numeracy Education, CECV.</td>
<td>Clinical interviews with children extend teachers’ understandings of children’s strengths and needs as well as necessary curriculum developments.</td>
<td></td>
</tr>
<tr>
<td>Year 2 Diagnostic Net, Qld</td>
<td>Teachers are involved in on-going appraisement and reporting of children’s progress, defined with indicators, explanations of these, and examples.</td>
<td>Kable (1996) Grieshaber (1997)</td>
</tr>
</tbody>
</table>

A fourth commonality is the use of individual interviews with children that enable teachers to identify each child’s level of understanding, hence enabling children’s development to be placed on a learning framework. This activity informs planning and, as one teacher noted after being involved in trialling Queensland’s “Net”:

I believe that the main benefit of the Net has been to focus teachers’ attention on children’s development. It has forced many teachers to do some ‘kid-watching’, to listen to children and consider a broader range of information as valid evidence of development. It has also forced teachers to look at how children develop and assess
their understandings of concepts rather than the rote learning of facts. The Net has encouraged teachers to look carefully at their practice. (Kable, 1996, p. 52)

Clarke (2001), who led Victoria’s *Early Numeracy Research Project* (see Clarke, 2000) gave examples of research being a powerful tool for professional growth, and stressed the need for teachers to be treated as co-researchers rather than mere recipients of professional development courses. The professional development program aspects of the *Early Numeracy Research Project* included professional development days, after-school regional cluster meetings, and visits to schools by cluster co-ordinators. Aside from teachers’ knowledge of mathematics, other changes noted by the research team were:

- more focused teaching (in relation to growth points);
- greater use of open-ended questions;
- provision of more time to explore concepts;
- greater opportunities for children to share strategies used in solving problems;
- provision of greater challenges to children, as a consequence of higher expectations;
- greater emphasis on “pulling it together” at the end of a lesson, as part of a whole-small-whole approach;
- more emphasis on links and connections between mathematical ideas and between classroom mathematics and “real life mathematics”; and
- less emphasis on formal recording and algorithms; allowing a variety of recording styles.

Clarke (2001) claimed that powerful professional development resulted from teachers interviewing their own pupils, becoming deeply involved in researching their mathematical understandings, and thus furthering their own understanding of how children learn mathematics and developing their repertoire of teaching approaches accordingly. These principles are now embedded in many of the previously cited projects. For example, teachers participating in *Success in Numeracy Education* are required to carry out interviews of children and develop teaching plans for individuals or whole groups as a result of the interviews.

The power of research as a stimulus for professional growth has already been noted by earlier researchers (for example, Carroll, 1997; Mulligan & Thomas, 1995; and Wright, Stanger, Cowper & Dyson, 1996). Bobis and Gould (1998; 2000) and Mulligan, Bobis and Francis (1999) report how concept mapping exercises were used to determine whether changes to teacher knowledge had occurred as a result of their involvement in *Count Me In Too* in New South Wales. However, the authors acknowledged that while there was significant change in teachers’ knowledge of how children learn early mathematical concepts, and some change in pedagogical knowledge, there was little change to the teachers’ knowledge of mathematics itself. This links to the findings of Jacobson and Lehrer (2000) in the United States who found that specific subject-matter knowledge is needed to support effective teaching, and that pedagogical content knowledge needs to be based on typical patterns of children’s thinking and not simply general knowledge about children and their thinking.

An impetus for professional development in Australia has been increased teacher accountability and the introduction of assessable standards for teachers and teaching. As mentioned in the chapter on *International and National Developments*, over the past
decade there has been an increasing emphasis on being able to demonstrate improved teaching and learning in a tangible way (for example, with a professional portfolio for teachers, with improved learning outcomes for children). A major Victorian professional evaluation and development exercise, *Research and Development of National Professional Standards for Excellence in Teaching Mathematics*, was an Australian Research Council-funded project to develop detailed, specific standards for teaching mathematics. Using the ideas of researchers and mathematics teachers Australia-wide, a framework of “best practice” has been constructed, and specific protocols have been developed for performance management and certification of teachers’ levels of “excellence”. It is proposed that this will lead to higher-level certification of mathematics teachers, and hence to significant professional development activity that draws on the framework, and its illustrative and analytical content.

A further force has been the “numeracy across the curriculum” movement, again part of a wider international trend. The fact that many key learning contexts have underpinning basic mathematical ideas (such as maps in Studies of Society, measures in Physical Education, pattern in Dance, and note values in Music) implies that it is not just mathematics teachers who need professional development in how to develop children’s numeracy. This point is expanded further in *Making the Links. Numeracy R-3: Identifying Numeracy Across the Curriculum* by Costello and Walter (1998).

Another force for professional development, and particularly new approaches such as problem-based mathematics pedagogy, has been the wish for greater involvement of parents in their children’s education, either as helpers in classrooms, or supportive family members who will give the children the right messages about school mathematics. Horne (1993) studied the effects on parents’ attitudes of a program involving families in mathematics. She reported that the program was not only beneficial for families, to varying degrees, but was also a major professional development stimulus for teachers.

As can be gauged from the projects and research studies discussed here, teacher professional growth has occurred in a variety of forms. Action research projects appear to have been very successful with individual teachers. On a larger scale, there have been programs that focus on the professional development of groups of teachers; some have also involved parents. It was recognised, however, that individuals could face impediments when attempting to implement change. Whole school approaches appear to hold greater potential for success. Another avenue for professional development was teacher involvement in large scale projects centred on developmental frameworks associated with children’s numeracy learning. These have proven to be powerful avenues for teacher professional growth and change; knowledge of mathematics and of how children learn mathematics were enhanced, and classroom practices were found to reflect these new understandings.

**Teachers and change**

Any change to practices associated with curriculum content, pedagogy, assessment or reporting methods, is bound to cause disquiet and discomfort. One significant change in recent times has been the expectation that all children will progress well beyond basic levels of numeracy. Major projects that are reinforcing this expectation and supporting teacher change were reviewed earlier. For example, the Learning Difficulties Support Team (Department of Education, Training and Employment, South Australia) organised
the Numeracy Skills Across the Learning Areas: Assisting Students with Learning Difficulties to Put Meaning Into Maths project, which sought to introduce a play-based approach for teaching children with learning difficulties. Weekly visits to teachers and the provision of numeracy packs for families to borrow and use with their children were among supporting components of this project.

Clarke (1999) reported on the extent to which teaching changes over time. He reported on a small study that is nonetheless significant, especially in light of the fact that there are no larger empirical projects to provide useful information on this topic. Two middle-school teachers were observed and interviewed during a seven-month period when innovative materials were introduced, and again for a brief time five years later. The author reported that the greatest long term changes in the teachers’ roles related to increasing comfort with posing non-routine problems to children and allowing them to struggle together, and the provision of structured opportunities for children to reflect upon activities and learning. However, he noted that little change was evident over the five-year period in the teachers’ use of assessment practices or in their articulation of the “big ideas” of mathematics.

Jacob (1996) studied the effects of global changes, such as the “new maths” reforms, as well as local changes, such as those in assessment, content, and teaching methodologies. Secondary mathematics teachers in the Northern Territory were surveyed, and the teachers felt that children were not being prepared well in primary schools for the problem-solving approaches used in many secondary schools, or for the sensible use of calculators.

There has not been much research on teacher change during the past ten years. Knowledge is limited about the longer term effects of teacher change on students’ numeracy outcomes.

**Teachers’ beliefs**

It is widely known that attitudes and beliefs affect behaviour. Although Australian research on teachers’ beliefs is not extensive, it covers a range of avenues of inquiry that are also reported in other chapters of this report. In other sections, teachers’ beliefs about the following are examined:

- electronic technology use for the teaching and learning of numeracy (Baturo, Cooper, McRobbie, Campbell & Kidman, 1999; Bramald & Higgins, 1999; Groves & Cheeseman, 1993; Stacey & Groves, 1994; Swan & Sparrow, 1997);
- the efficacy of small group work (White, 1999);
- children’s ability to learn (**Effective Teachers of Numeracy**);
- the effects of teaching for numeracy given changing patterns in Australia’s migration (Wotley, 2000); and
- use of informal methods as opposed to traditional algorithms (Buzeika, 1999: *Developing Computation: Strategic Numeracy Research and Development Project, Tasmania*).

From the list above, it is clear that much of the research on teachers’ beliefs was related to the use of calculators and, to a lesser extent, computers. While this is understandable, given the importance of electronic technologies, it is surprising that there has not been at least equal interest shown in teachers’ beliefs about how children learn mathematics and develop numeracy knowledge and skills. Findings from a few such studies follow.
White (2000) developed what he called “The theory of planned behaviour”, which he used as a framework for analysing teacher action theories in relation to the use of stencils, group work, and children’s use of calculators. Teachers’ prior beliefs were found to be a dominant factor in whether, how much, and how well, teachers used these theories, as well as in the teachers’ evaluation of learning and their self-evaluation of the lessons. Warren and Nisbet (2000) identified factors associated with changes in teachers’ beliefs. They found that beliefs about assessment were highly influential, and beliefs about mathematics itself were important. Archer (2000) reported on variations between the beliefs of primary and secondary teachers in relation to the nature of mathematics and its place within the school curriculum. He reported that primary teachers tended to see mathematics as tied to children’s everyday lives and to other aspects of the curriculum, while secondary mathematics teachers tended to see mathematics as self contained, with their role being to guide students through its orderly, logical structure. Primary and secondary teachers of English made no similar distinctions.

Australian research in the last ten years on teachers’ beliefs and about teacher change is not extensive and appears somewhat fragmented. Little seems to be known about how teachers’ beliefs are related to effective numeracy teaching, although the findings related appear consistent with the more general findings, that is, that beliefs do affect behaviours.

**Teacher knowledge**

Shulman’s (1986) categories of teacher knowledge, pedagogical, content, and pedagogical content knowledge (knowing how to teach specific concepts and topics), are evident in the focus of much of the Australian numeracy research literature.

Content knowledge was a centre of attention of Watson and Collis’ *Cognitive Functioning in Probability and Statistics and its Relationship to the School Curriculum* project. The concern of the researchers was that *Chance and Data* comprised 20% of the 1991 Australian statement on curriculum content, but that no research had been conducted to assess teachers’ knowledge in the area. Teachers’ knowledge in other aspects of numeracy is also vital, as Watson (1991) pointed out when she described the potential traps in number work, for example, that are not well known by teachers.

Ball (2000) notes that one distinguishing feature of knowledge for teaching is to be able to deconstruct it so as to be able to see it from the learner’s perspective. Ball writes that “knowing for teaching requires a transcendence of tacit understanding” (2000, p. 245), so that the critical components central to, but often invisible in one’s own compressed mature knowledge, are revealed. A related aspect is knowledge of students’ thinking — incorporating what makes specific topics easy or hard for students to learn; having strategies for overcoming students’ misconceptions; knowing the capabilities of students at different ages; knowing common difficulties, misconceptions and obstacles; and understanding how individual students in a teacher’s own class are likely to think.

Shulman also identifies *curricular knowledge* of curriculum programs and instructional materials, which is of less interest in this current discussion. Researchers have found it helpful to use the phrase *Pedagogical Content Knowledge*, or more precisely *Mathematics-specific Pedagogical Content Knowledge*, to refer to an amalgam of all these aspects of teacher knowledge for mathematics.

It has been claimed that teachers’ knowledge of mathematics (i.e., their own numeracy) can be improved through their involvement in projects that focus on children’s thinking.
For example, 80% of the teachers interviewed by Mulligan after participation in the *Count Me in Too* project, reported that their own knowledge of mathematics had improved. However, Bobis and Gould (2000) reported that the most significant change in teacher knowledge, in this same project, occurred in an aspect of pedagogical content knowledge — how children learn mathematics — and there was little change in their content knowledge. Similarly, many of the teachers involved in the *Early Numeracy Research Project* commented on ways that their own mathematical knowledge was enhanced as they concentrated on children’s mathematical thinking.

In recent years, the impact of components of mathematics-specific pedagogical content knowledge on student achievement has been examined. In an influential book, Ma (1999) measures teachers’ “profound understanding of fundamental mathematics” and contrasts the deep conceptual knowledge of Chinese primary school mathematics teachers with the relatively shallow procedural understandings of their United States counterparts. She also reports how the knowledge of Chinese teachers develops during their careers; asserting that differences in teachers’ ability to make connections among mathematical ideas are largely responsible for the difference in performance between Chinese and U.S. students. Ma presents some large-scale quantitative data to support such a link.

In considering teachers’ characteristics and their association with children’s numeracy performance in Britain, Askew et al. (1997) identified teachers’ recognition of deep connections between mathematical ideas as one of the few predictors of high learning gains by children. This project saw mathematics as richly connected and adopted classroom strategies that helped children to make links.

Mathematics-specific pedagogical content knowledge influences student achievement through the way it is enacted in certain classroom practices. The case-based work of Deborah Ball has emphasised a previously unappreciated level of connection between mathematics-specific pedagogical content knowledge and practice. She asserts that teachers’ understanding of mathematical content affects many teaching decisions, such as what questions to ask, what test items to set, what examples to choose, which child to bring to the front of the class to explain a method, and so on. She comments that the “analysis and preparation of a single math[ematics] problem reveals how much core tasks of teaching involve significant mathematical reasoning in the context of practice” (Ball, 2000, p. 243).

Kanes and Nisbet (1996) argue that understanding the common strengths and limitations of the knowledge bases of mathematics teachers is an important task in constructing adequate models for teacher education and teacher classroom practice. However, as Kanes and Nisbet also note, there has been little systematic research focused on this task. They studied both primary and secondary student teachers’ mathematics knowledge relevant to their teaching levels, as well as their content specific teaching knowledge and curriculum knowledge. Fewer than half the pre-service teachers believed that they were sufficiently prepared in mathematics content, and almost two-thirds of the sample believed that their level of knowledge in contemporary teaching methodologies was insufficient for their role as teachers. The authors noted a high level of “No response” to this question, and stated that this may indicate a hesitancy to disclose information, suggesting that it might be related to feelings of inadequacy and embarrassment. In relation to pedagogical content knowledge, almost two-thirds of the teachers felt insufficiently prepared. The authors suggested that this has implications for both pre-service and in-service teacher
education. In relation to mathematics curriculum knowledge, the findings were more heartening for the primary teachers who completed the survey.

In contrast to the Kanes and Nisbet study, most of the research into pre-service teachers’ knowledge has centred on specific aspects of teachers’ knowledge of, and competence with, primary mathematics, some of which has been reported elsewhere in this report. Taplin (1998), for example, studied pre-service teachers’ problem-solving processes in order to identify common difficulties, and Schuck (1995) researched attitudes towards the use of calculators.

In his doctoral thesis, Kaminski (1996b) documented the development, implementation, and partial evaluation of a “number sense” program for primary pre-service teachers. What was explored in the study were participants’ use of number sense in their understanding of mathematics, the contribution of their previous experiences to their views of mathematics, how a number sense program could assist them in their understanding and use of mathematics, and how such a program could promote reflective practice. Kaminski found that these student teachers appeared to have had little experience with activities that might promote number sense or reflective practice in mathematics, and had seldom had their views and beliefs about mathematics challenged. He concluded that their experiences in a program focusing on number sense, using socio-cognitive, constructive, and reflective approaches, assisted them in moving beyond their initial levels of understanding. In summary, Kaminski seems to be saying that trainee teachers are yet to develop extensive pedagogical content knowledge in mathematics.

Kaminski (1997) concluded that the pre-service teacher education students in the study displayed an underdeveloped sense of Number, and also found that pre-service teachers very infrequently:

- drew on their intuitive mathematical knowledge;
- used notions of numbers as quantities rather than numbers as formal, abstract entities;
- drew upon, or used, relationships or understandings from mathematical areas much beyond the specific solution given;
- used mental computation with confidence;
- reviewed the appropriateness of the strategies employed, or the reasonableness of results obtained;
- demonstrated flexibility in decomposing and recomposing number, or relating number and operations in meaningful ways;
- showed confidence in their ability to interpret and provide effective mathematical explanations; and
- demonstrated understanding of multiple relationships in the domains of number and operations.

The recently announced national Project to Investigate the Preparation of Teachers to Teach Literacy and Numeracy in Primary And Secondary Schools is expected to throw light on teacher education practices across Australia. This project should provide information on effective practices in pre-service preparation of teachers to teach numeracy to all students up to Years 9 and 10 and, in particular, to educationally disadvantaged students. The project involves:

- the compilation of a review of relevant literature;
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- a mapping of pre-service teacher education courses; and
- a survey of strategies that pre-service teachers have learnt in their teacher education courses, in terms of discipline knowledge, pedagogical understandings, and beliefs.

A similar study, in a school-based program, was reported by Hill (2000). It was a collaborative effort between primary schools and university lecturers. The project enabled student teachers to practice what they were learning in theory in a continuing, integrated, and consistent manner, and to reflect on their experiences. The findings from test, questionnaire and interview data, indicated that the weekly cycle of theory, practice, and reflection, in which the pre-service teachers engaged, enhanced their capacity with numeracy as well as their knowledge of children and school contexts. The lecturers played a vital role in helping students make sense of, critically analyse, and adapt what they had seen in classrooms, and to consider how best to meet the numeracy needs of their pupils.

Reports of growth in general pedagogical knowledge (ways of structuring classrooms to enhance learning) are numerous (for example, Bobis & Gould, 1998; Pearn, Hunting, Merrifield & Mihalic, 1997; Sparrow, 2000).

The research discussed above reveals that there has not been any research in which primary teachers’ levels of mathematics content knowledge have been assessed. However, findings on pre-service teachers revealed that many felt that their mathematical backgrounds, as well as their knowledge of how to teach mathematics, were inadequate. By involving teachers in research on children’s thinking, it has been shown that their knowledge of mathematics as well as their knowledge of how to teach mathematics effectively can be improved.

Students

Research on children and their numeracy learning is the focus of this section. Many of the studies about children include issues or factors that have been discussed elsewhere in this report. For example, research on Indigenous students is reviewed in the chapter on Equity and the School Community, and research on children’s learning in different content areas is reviewed in the chapter on Curriculum and Processes. In this section, research in the following areas are discussed: gifted children, children’s learning of mathematics in informal settings, children’s attitudes towards mathematics, children’s mathematical self-concepts, motivation, students at risk of not meeting expectations, and language issues.

Informal learning

Recognising and identifying children’s pre-existing understandings are considered essential starting points in building and developing those understandings. Throughout their schooling, children encounter, experience, and learn about mathematics both within and beyond school. Research on the links between primary-aged children’s informal learning and their classroom-based experiences is discussed next.

The contrast between some children’s rich out-of-school mathematics knowledge and the difficulties they experience in the classroom has been recognised, and several researchers claim that mathematical understanding will be enhanced if teachers link
children’s mathematical experiences out-of-school with what is taught in the classroom. Ellerton and Clements (1994) reported on studies in which it was found that many children from Papua New Guinea, Southeast Asia, New Zealand, and Australia were unable to handle fractions. This, they claimed, was due to the children’s perception that classroom fraction work was associated with rules and terminology that had no meaning outside the classroom. Assisting children to build on their informal knowledge, by helping them make mental links between real-life situations involving fractions, formal language, and written symbols, was advocated.

Grier’s (1993) study involved children in a Year 1 class. Data on the children’s knowledge of mathematics were gathered, the children were observed in class, and their parents completed questionnaires about numerical learning opportunities in the home. Grier concluded that the ways in which children learn mathematics out-of-school were more open-ended than school methods. Grier claimed that it was necessary to recognise and build on children’s prior knowledge, and bring the more social methods of learning that children experience out-of-school into the classroom.

It might be assumed that children with more limited non-mathematical knowledge will have weaker foundation knowledge, and hence may not respond to schooling so well. However, Price (1997) found just the opposite. Prior knowledge of mathematical concepts, generalised reading ability, and motivation and attitude towards mathematics, were factors examined in relation to progress in one mathematics classroom. While it was expected that individual differences would increase over time, with children who possessed high prior knowledge scoring higher on delayed retention tests, this was not found to be the case. Individual differences decreased during the research period and after delayed testing.

The findings from the narrow range of Australian research on children’s mathematical knowledge gained in informal settings indicate that there appear to be no detrimental effects on children’s school learning of mathematics and, if capitalised upon, there were indications of positive benefits and less likelihood of the children experiencing cognitive conflict.

### Attitudes, self-concept and motivation

A very broad definition of *attitudes* has been adopted in the grouping of research studies under this heading. Beliefs and self-concept measures have also been included as has work on motivation.

Collecting data on children’s attitudes towards, or beliefs about, mathematics has been included in the research design of several projects including: *Assessing Numeracy in Primary Schools: Strategic Numeracy Research and Development Project, ACT* and *The Third International Mathematics and Science Study* (TIMSS). Investigating whether there are changes in children’s attitudes towards, and feelings about, mathematics was a component of the *Thinking and Working Mathematically* project. Many researchers have gathered data on children’s beliefs and attitudes to mathematics content, or mathematical tasks or activities (for example, Watson & Chick, 2001a).

Data from many studies in which children’s attitudes and beliefs were tapped and examined for gender differences were discussed in the chapter on *Equity and the School Community*. Discussed below are findings from other studies in which attitudes (as
defined above) towards mathematics or towards dimensions associated with the learning of mathematics, were the focus.

**Attitudes and pedagogical approaches**

Herrington (1992) found a positive correlation between Year 6 children’s performance on a standardised achievement test and their beliefs about themselves and others as learners of mathematics, beliefs about mathematics, and beliefs about the ways mathematics can be learnt. An instructional program was devised to teach appropriate strategies and beliefs about learning mathematics. Compared with children who were taught traditionally, children in this program had higher performance scores, were more confident, used a wider range of strategies, and used higher quality approaches.

In another action research study, involving four schools and children in Years 5 to 8, teaching strategies were trialled that were aimed at improving children’s engagement, attitudes, and achievement (Vale, 1999). Teachers reported that the children were positive and open to the new mathematics teaching strategies. Vale recommended that mathematics needs to be meaningful for young adolescents. The teachers in the study believed that the teaching of numeracy in the middle years ought to cater for individual differences and ways of knowing, promote the explaining and writing of mathematical thinking, make use of team learning and develop team skills, engage students through the use of appropriate concrete materials (manipulatives) and tools, and use assessment to identify student needs and modify their teaching program. Vale noted that these beliefs informed their choice of teaching and learning strategies in their classrooms. The ideas for teaching and learning practices of particular interest to the teachers in the project were integrated learning, problem solving, good questions, real life learning, autonomous learning, tools, concrete materials, investigations, games, open ended activities and personal experiences.

**Attitudes associated with technology**

Researchers have focused on primary-aged children’s attitudes towards and beliefs about calculators. Arvonen and Bobis (1995) reported findings from a study that included administration of an *Attitude towards calculators* questionnaire to children in three upper primary school classes. All of the children who used calculators thought them to be useful. Many enjoyed using calculators, and considered:

- mathematics learning more enjoyable with a calculator,
- calculator use to be relatively easy, and
- mathematics to be easier with a calculator.

Most of the children who did not use calculators in their mathematics learning felt that mathematics would be easier with a calculator. Many of these were found to be unsure about the legitimacy of using calculators for mathematics learning.

Doig (1993) also gathered data on Year 3 children’s views about calculators. For some of the children calculators were part of their everyday experiences of school mathematics; for others they were not. Concerning teaching or learning with, or from, calculators, Doig claimed that the Year 3 children “equate(d) teaching with ‘telling the answer’ and learning with ‘memorizing the answer’” (p. 233). There were no responses indicating that
calculators could be used for exploring numbers, patterns, or operations, even from children who had experienced such uses in class.

McRobbie, Baturo, and Cooper (2000) measured the effectiveness of an integrated learning system (ILS) for low achieving Years 5 to 9 children. The children’s attitudes towards computers became less positive over time, although they liked the ILS and felt that it had helped them learn.

In summary, it would appear that less traditional pedagogical approaches can have beneficial effects on students’ attitudes, engagement, and mathematics achievement. While children appear to respond positively towards technology use for mathematics learning, the findings are mixed with respect to their beliefs about its effects on their learning. The findings reported on student attitudes support the broad generalisation that attitudes, beliefs, levels of self-concept, motivation, and achievement are intrinsically inter-related. Classroom practices and teachers’ attitudes are implicated in the direction of these attitudes.

**Mathematics self-concept and motivation**

Craven, Marsh and Debus (1991) reported that primary children’s mathematics (and reading) self-concept could be improved in a relatively short time by providing feedback based on positive ability and on performance. In the transition from Year 6 to Year 7, Trent, Russell, Cooney, and Robertson (1994) found that the factors accounting for mathematics self-concept were:

- at the end of Year 6, perceptions of performance; and
- at the beginning of Year 7, mainly perceptions of performance, but also (marginally) teacher and classmate support.

Changes in perception of mathematical ability were independent of changes in performance, and there were no gender, time, or type of school effects. Martin (1996) found that ego concerns (including competence-valuation) were positively associated with mathematical achievement and motivation.

In an examination of children’s perceived levels of control in the learning process as they progressed from primary to secondary schools, Fullarton (1998) identified four distinct groups of children:

- a highly successful group who took transition in its stride;
- a poorly engaged and unmotivated group who continued to struggle;
- a group whose members were uncertain and not highly engaged in primary school, who seemed to show higher levels of engagement in secondary school; and
- a group, unlikely to have been identified by their primary teachers as potentially having problems at secondary school, who suffered declines in perceived control, engagement, coping skills, self-regulation and self-perception.

The fourth group was by far the largest, and Fullarton (1998) claimed that these students were at particular risk of disaffection in mathematics.

Middleton and Spanias (1999) carried out a review of United States research literature on student motivation. Based on this work, the authors drew conclusions about contextual factors, cognitive processes, and the types of interventions that affect both children’s and teachers’ attitudes. In summary, the authors noted that it seems that:
Primary Numeracy

Student motivation in mathematics is highly influenced by teachers’ instructional practices. If appropriate practices are consistent over a long period of time, children can and do learn to enjoy and value mathematics. Moreover, even if children’s histories have been consistently poor over a long period of time, the research reviewed ... indicates that classroom practice can be positively reinvented so that the culture of the classroom can become conducive for students to learn and enjoy mathematics. (p. 75)

Similarly, Australian studies have shown that teachers’ actions, and particularly their encouragement, can have a positive influence on children’s motivation and self-concept. Burnett (1999) studied the “self talk” of 269 Australian children, in Years 3 to 7, in reaction to the frequency of their teachers’ positive statements, and found these to be more influential than parents’ or peers’ comments. Positive statements made by teachers were more influential than negative statements.

Findings on children’s motivation in mathematics classrooms have frequently been included in studies with other major aims. Findings from such studies, and their specific foci, are summarised in Table 3.

Table 3.
Reports that include the question of children’s motivation in mathematics classrooms.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Children</th>
<th>Approach</th>
</tr>
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<tbody>
<tr>
<td>Changing Places, Tas.</td>
<td>Aboriginal children, Years 4 &amp; 6</td>
<td>Professional development sessions used a Reading, Writing, and Mathematics inventory to monitor children's attitudes and growth of understanding.</td>
</tr>
<tr>
<td>Making a difference: Challenging and enthusing children for mathematics in the early years</td>
<td>Early years</td>
<td>Clarke (1998): Position paper rather than research report, which outlines some key principles for making mathematics challenging but enjoyable in the early years, drawing upon classroom examples that put these principles into practice.</td>
</tr>
</tbody>
</table>

The research on mathematics self-concept and motivation reveals that children’s self concepts are positively affected by feedback on ability and performance. Negative self-concept can have detrimental consequences on attitudes towards mathematics. Children not identified by their primary teachers as likely to have problems in secondary school can suffer declines in engagement, self-regulation, and self-perception.

**Gifted students**

Over the last decade, there has been very little Australian research that has inquired into issues associated with primary aged children who are gifted and talented mathematically. There has been only one pertinent Australian Capital Territory government-funded project, the Talented School Students Mathematics Program. The Australian Mathematics Trust (AMT) has run the Mathematics Challenge for Young Australians for many years. This program is for secondary students and culminates with the selection of an Australian team for the international Mathematics Olympiad. With Australian Government funding, the AMT has embarked on the Talented School Students Mathematics Program aimed at talented children in the upper primary years. Attendance in the program is voluntary; sessions are
run out of school hours; and the children engage with challenging problems across a range of mathematical topics. Although not the major focus of the South Australian Catholic Education Office’s Transition Years 6–10 project, the needs of gifted students are addressed in this professional development project.

There have been some smaller research projects related to mathematics and high ability children. Among high ability Year 3 to 6 children, Thomas and Mulligan (1994) found that children exhibited evidence of a dynamic range of internal representations of the counting sequence from 1–100. The demonstrated awareness of the way the number system is structured and sequenced, were able to describe numbers flexibly. Open-ended questions, Stone (1994) maintained, can be used to encourage potentially gifted mathematics children to think at higher cognitive levels. Lowrie (1996) examined the higher order thinking skills of three talented children who worked co-operatively. Although their preferred strategies differed, Lowrie maintained that the conflicts arising from the problem-solving activities with which they had engaged, promoted higher levels of metacognitive activity. Similarly, Hall (1997) found that working in groups to solve non-routine problems provided opportunities for talented primary children to talk, think, and write mathematically.

Research on gifted and talented primary children and numeracy has been restricted to a narrow range of topics. Based on only a few research studies, however, it would appear that higher-level thinking is promoted among potentially talented children when they engage in problem solving in co-operative small group settings.

**Students “at risk”**

Students “at risk” have been defined as those identified as being in need of remediation in their numeracy learning, that is, their skills and knowledge are below those expected of children of their age or at their year level. These children are in danger of not becoming numerate (Willis, 2000).

Based on “growth points” from the *Early Numeracy Research Project* assessment framework, Gervasoni (2000) discussed data from an assessment interview with Year 1 children that could be used to develop profiles of children in order to identify those at risk.

There have been many funded and unfunded projects with the goal of identifying students at risk, and subsequent intervention programs aimed at assisting these children. There have also been relevant professional development programs associated with many of these projects. Such projects are explored in the *Intervention* section of this report.

In Table 4 a summary of projects that include a focus on children at risk is presented. References to some pertinent research papers have been included. Some of these projects are discussed in more detail in the *Intervention* section found later in this chapter.

*Table 4.*

Summary of projects with a focus on students “at risk”.

<table>
<thead>
<tr>
<th>Project</th>
<th>Children “at risk” dimension</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACT Assessment Program and Special Assistance</td>
<td>Professional development of teachers to address needs of lowest achievers in mathematics.</td>
<td>ACT Assessment Program and Special Assistance</td>
</tr>
<tr>
<td>Project</td>
<td>Children “at risk” dimension</td>
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<tr>
<td><strong>Counting On: Transition 6–7</strong></td>
<td>In order to assist Year 6 children in NSW to attain Stage 3 before moving on to secondary school, the project aims to establish children's most advanced level of thinking with respect to particular concepts. Teachers’ awareness of children’s strengths and weaknesses has developed.</td>
<td>Counting On: Transition 6–7</td>
</tr>
<tr>
<td><strong>Early Literacy and Numeracy Partnerships</strong></td>
<td>The aim is to improve the learning outcomes of educationally disadvantaged Victorian children in the pre-school year. The proposed methods involve: the development of intervention strategies and resources; professional development and evaluation of its effectiveness; informing parents; and studying children’s early numeracy achievements on school entry.</td>
<td></td>
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<tr>
<td><strong>Early School Assessment Project</strong></td>
<td>NSW children in Years K to 3 in need of assistance were identified. Materials based on Count Me In Too were developed for these children. Professional development was offered to identify children at risk, and to encourage the use of appropriate teaching strategies to address their needs.</td>
<td>Department of Education, Tasmania (2000)</td>
</tr>
<tr>
<td><strong>Flying Start</strong></td>
<td>To support all children in P-2 to achieve appropriate, literacy, numeracy, and social skills; additional teachers were provided in classrooms. Early identification and intervention programs for children at risk have been developed.</td>
<td></td>
</tr>
<tr>
<td><strong>Literacy and Numeracy Development in the Middle Years of Schooling</strong></td>
<td>To provide information on current teaching and learning strategies, special programs, organisational reforms and resources used in Australian schools and systems in all States and Territories which are effective in improving literacy and numeracy learning outcomes of educationally disadvantaged students in the middle years of schooling.</td>
<td>Beyond the Middle (Luke et al., 2003)</td>
</tr>
<tr>
<td><strong>Lower Secondary Numeracy Project</strong></td>
<td>Children at risk in Years 6 to 9 in SA Catholic schools were included in this project. An action research model was used to gather data to enable teachers to plan more effectively for children’s numeracy learning. Children at risk were often found to have experienced success in classrooms once specific strategies were adopted.</td>
<td></td>
</tr>
<tr>
<td><strong>Making a Difference: Students at Educational Risk</strong></td>
<td>WA teachers developed child profiles in order to identify strengths and limitations in the mathematical achievements of children at risk.</td>
<td></td>
</tr>
<tr>
<td><strong>Mathematics Intervention</strong></td>
<td>Victorian children at risk of not coping with the mathematics curriculum are identified through a clinical interview. The children then work in small groups with children of similar achievement levels. Teachers are trained to conduct the clinical interviews and the follow-up intervention program.</td>
<td></td>
</tr>
<tr>
<td><strong>Mathematics Recovery</strong></td>
<td>The aims of this NSW project included the development of a program of recovery education in early arithmetic learning for Year 1 children. The program involved a long-term individualised teaching program to advance the children’s knowledge, so that they were likely to learn successfully in regular classrooms.</td>
<td></td>
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</tbody>
</table>

There were many projects, small and large, independent and State-initiated, on *students at risk*. It is evident that there has been extensive work on the identification of such students and in the provision of intervention programs aimed at addressing their needs. There is less known about the effects of these programs on the students’ numeracy learning, particularly in the longer term.

**Students' language ability**

Relevant research literature in the field of language factors and mathematics learning prior to 1990 was reviewed by Ellerton and Clements (1991) and the Australasian literature for the period 1996 to 1999 was reviewed by Ellerton, Clements, and Clarkson (2000). Menon (1998) provided an overview of the international literature, and discussed classroom implications, but Zevenbergen (2000) maintained that little was known in any systematic form about the impact of language on primary children’s numeracy development. Ellerton and Clements (1991) had earlier reported a Pearson product-moment correlation of 0.5 between children’s scores on a reading comprehension test and, six years later, on an ACER mathematics test. They argued that this evidence was strong enough to raise the question of the relationship between early language ability and future mathematical achievement. With respect to the numeracy learning of primary children, they considered that the greatest challenge facing teachers and teacher educators was

not only to develop programs that succeed in ‘dejargonising’ school mathematics... but also to provide learning environments that create links between the personal worlds of children and both the formal language (verbal and written) and applications of the mathematics they study. (p. 143).

In this review of the literature, studies were located in which language factors, other than those associated with lack of English proficiency among LBOTE children, have been identified as affecting the numeracy learning of Australian primary children in various ways.

Support for links between learning difficulties in mathematics and in reading was provided by Pearn (1994). Clinical interviews were conducted with Year 2 children. Each of the small number of children, identified to be in need of a mathematics intervention program, was also found to be involved in a reading recovery program, or to have previously been involved in one.

Very young children’s language-related difficulties in understanding mathematics instruction were discussed by Padula and Stacey (1990). They claimed that children need to learn the words for new, or previously unlabelled, concepts, extend their understandings of familiar words, and learn new ways to combine words to generate
meanings. They advocated that children hear, say, and act on the words they need to learn.

Despite the extensive reviews of earlier literature in the field, there has been little work in the past decade to extend knowledge on the identification of language factors that hinder or promote children’s mathematics learning.

**Classroom practice**

Researching classrooms is not an easy process. Replication of studies or even of research results from one classroom is difficult to achieve because of the varied, complex, and busy nature of these learning environments. In many studies of mathematics classroom practice carried out in the last decade, case studies have been used as the research approach. This approach has advantages. Fine details and differences between cases can be represented, but it means that the findings from one particular context (i.e., one school or classroom or geographical area) are usually not generalisable. Even when phenomena are apparently widespread, any finding will need to be interpreted carefully with respect to its applicability in other settings.

As noted in the chapter on *International and National Developments*, federal funding has been available for numeracy projects, and the States and Territories have organised initiatives that enhance mathematics teaching. Many of these have been research-based in that either there was a significant research focus, or the aim was to implement research findings into classroom-based projects. A representative selection of these projects, with an emphasis on improving mathematics pedagogy, is discussed in the following sections.

Australia has a strong reputation worldwide for its teachers-as-researchers approach to professional development (Zack, Mousley & Breen, 1997). Factors that have stimulated this include:

- the participation of, and contributions by, teachers at professional conferences (for example, AAMT, MANSW, MAV and MAWA);
- a very active Australian Association of Mathematics Teachers;
- low levels of prescriptive directions regarding the structuring of lessons;
- significant state-supported developments of programs that involve teachers as research partners in school-based and classroom-centred projects;
- involvement of teachers as partner-researchers with academics in both funded and unfunded projects;
- university courses that encourage students who are teachers of mathematics to undertake classroom-based research projects; and
- special initiatives at the annual conference of the *Mathematics Education Research Group of Australasia* catering for teachers (for example, teachers’ day).

Genuine school-wide and teacher change result from these types of grass-roots involvement in research. This applies to growth in teachers’ mathematical knowledge as well as to their pedagogical change (see, for example, Clarke, 2001).

Much of the research on teaching and learning in the area of numeracy development is classroom-based, for example, *Numeracy 3–10 Research and Development Project* and *Numeracy Intervention*. Bobis and Gould (1998) studied the impact of a 1996 research
project on the professional development of teachers. They found that the classroom-based model of professional development was a major factor in the success of the project. Gervasoni’s (1999) research throws some light on reasons for this (see Teacher education and development section of this chapter).

Some of this classroom-based research is supported by the various State educational systems. For example, the Lower Secondary Numeracy Project was a three-year project initiated by Catholic Education South Australia. It spanned the divide between primary and secondary schools, with teachers working in an action research mode to:

- develop a shared understanding of numeracy;
- recognise and be confident in their own numeracy;
- develop structures that support and challenge their children’s numeracy, with particular emphasis on children at risk; and
- recognise the need, at both the primary and secondary levels, to develop structures for supporting the development of numeracy in all areas of study.

The research in this project included a compilation of case studies within each teacher’s class of a range of students, including some considered “at risk”. Information from the classrooms was collated to produce a planning and programming document based around growth points, which was to be shared across Catholic schools. The project researchers believed that the findings demonstrated the need for more work in this area, particularly to develop secondary curricula to be more inclusive for students at risk, to develop a flexible framework to support teachers in planning for all students, and to conduct further research on schooling structures for middle school.

In an action research-based project conducted in nine South Australian Catholic primary schools, in which 250-300 Year 3 to 5 children and ten teachers were involved, pre-test and post-test scores were used to determine the degree of growth in children’s numeracy learning. This project, Making sense of the complexity of the constructivist mathematics classroom, involved collaborative enquiry between teacher researchers and numeracy consultants and researchers. Qualitative data collected in the action research component of the study included children’s work samples, observations and reflections of classroom interactions, and individual and collaborative reflections on effective teaching strategies and barriers to learning for individuals and groups of children. In addition to the whole-class data, the case studies of selected children have enabled detailed analyses of the strategies that seem to support improvement in numeracy outcomes. The analyses of significant work samples, field notes on observations, and descriptions of planning documents have also been included in the case studies.

**Intervention**

Over the past ten years, there has been increasing emphasis on early intervention programs, both in Australia and overseas. Intervention differs from remediation in its broad approach to development of underlying skills, understanding, and confidence, rather than focusing on particular errors that a child, or group of children, makes (Booker, 1999).

[Although *intervention* is sometimes taken as a replacement for the more familiar term, *remediation*, it is more than just a change of name for a word that has become tired over time.... *Remediation* has been concerned with remedying faults found in students identified as having incomplete or flawed development at the conclusion of a particular learning sequence. …By then particular ways of viewing the mathematical situation and
carrying out the processes involved were likely to have become ingrained which is hardly appropriate if understanding and sense making are to be the essence of mathematics learning. In contrast, intervention …requires a cycle of activities from observations which lead to insights about the status of an individual’s knowledge that in turn need to be probed so as to be verified or amended before being built on to provide a means to new and deeper understanding… Only then can a process of re-construction be engaged in to build up full and appropriate ways of thinking and acting that will allow for the development of abilities to use this knowledge to solve problems and form further mathematical generalisations. With these steps, intervention can be a pivotal part of everyday classroom life, taking place within the regular learning program as an integral component of ongoing teaching so that difficulties can be attended to as soon as they arise and learning can proceed at a time and pace that is suited to each individual. (p. 19)

In the past, it was assumed that intervention would mean withdrawal from classrooms for individual or small-group attention. However, current broad-based initiatives are having an impact on this perception. In more recent programs teachers examine their own actions (planning, teaching and assessment) in relation to what they can find out about children’s strengths and needs. They then seek to change their practices in order to maximise children’s learning.

With attention paid to children in need of special attention, intervention also differs from broad-based early years programs (sometimes called “first-wave” programs) that aim to help all children achieve a solid foundation in numeracy. The distinction holds even when first-wave programs involve the provision of extra numeracy teachers and resources.

The aim of any intervention program is to identify students who have not achieved expected outcomes by a certain stage, in order to give them extra attention that should result in improvement in broad learning outcomes. It has been shown that intervention can have positive effects not only on academic performance but also on general self-concept, although this is not a straightforward process (Craven, Marsh & Debus, 1991).

Most Australian States have established major intervention projects. These have been described in a major 2000 study (Louden et al., 2000a; 2000b — see also Mapping the territory: Primary students with learning difficulties: Literacy and numeracy project), which found that support for children with numeracy difficulties is increasing as schools respond to the range of intervention programs being implemented by various State and Territory educational systems. Grieshaber (1997) wrote of some different levels of justification for such programs. He acknowledged that most teachers can identify children needing help without an elaborate documentary system, but that the process does not provide a mechanism through which “the educational enterprise can be defined, measured and evaluated in interests of employers, administrators, and policy makers” (Jackson, 1993 in Grieshaber, 1997, p. 35). With reference to the Queensland “Net”, Grieshaber wrote:

Politically, assessment and intervention programs provide evidence that there is a process in place that can ensure all young children will acquire the basics in reading, writing and number in the early years of schooling. In an economic sense, the Net is making teachers of young children more accountable for levels of literacy and numeracy in our society … The monitoring, testing and reporting procedures of the Net invoke certain types of accountability and uniformity of practice, processes more concerned with assessment and efficiency than with improving teaching and learning. (p. 28, p. 31)

Grieshaber pointed out one danger in putting a lot of emphasis on broad-based assessment programs.
Because a top-down approach was used as the mechanism for implementation, assessment procedures have been the focus of classroom teaching and learning. (p. 34)

However, it seems clear from what teachers have written about their involvement in State-supported assessment and intervention programs, that they are invaluable as professional development exercises, as well as beneficial in teacher-child relationships and understandings. For example, Kable (1996), describing her experiences as a teacher in Queensland’s Net program, stated:

I believe that the main benefit of the Net has been to focus teachers’ attention on children’s development. It has forced many teachers to do some ‘kid-watching’, to listen to children and consider a broader range of information as valid evidence of development. It has also forced teachers to look at how children develop and assess their understandings of concepts rather than the rote learning of facts. The Net has encouraged teachers to look carefully at their practice. (p. 50)

State and education system intervention initiatives

In New South Wales, the Early School Assessment Project is used to identify children who require intervention. Progressive support is then provided across Years K to 3 using the Count Me In Too materials. The research and development work of Wright and colleagues (for example, Wright, 1991a; Wright, 1992; Wright, Cowper, Stafford, Stanger & Stewart, 1994; Wright, Martland & Stafford, 2000; Wright, Stanger, Cowper & Dyson, 1996) formed the basis of the Early School Assessment Project and Count Me In Too, as well as other assessment and intervention programs in Australia, New Zealand and the United Kingdom. Wright and colleagues undertook a comprehensive assessment project based on a five-stage learning framework in early number developed by Steffe, Von Glasersfeld, Richards, and Cobb (1983). This framework can be used to analyse the results of the assessments of children, as well as planning for both teaching and the assessment of their learning (see the Developmental frameworks section of this report). Wright, Cowper, Stafford, Stanger, and Stewart (1994) reported that the four year Mathematics Recovery Project that began in 1992, focuses on preparing specialist teachers in the provision of long-term, individualised, mathematics teaching programs for low attaining Year 1 children. Participating children underwent eight-week teaching cycles, consisting of thirty minutes of individualised teaching for four mornings per week. Almost all of the children made major progress. Overall, the progress of participants notably exceeded that of their non-participant counterparts. One of the major objectives in the early years of this project was to provide detailed information about the necessary content of the proposed professional development courses for Mathematics Recovery teachers and teacher leaders.

It is clear that such programs have offered valuable professional development opportunities as teachers work together, or with other people with expertise, to develop and trial appropriate activities, materials, and teaching strategies that address identified needs. For example, in research organised and evaluated by Wright, Stanger, Cowper, and Dyson (1996), each teacher undertook a twenty-week program that included a six-week orientation course. Fifteen children were assessed and teaching sessions for the following eight weeks were planned. Further professional development was undertaken during the intervention period. The group of teachers videotaped the children’s assessment interviews for analysis. The outcomes of this project included a bank of useful
instructional activities. Importantly, the researchers were also able to show the teachers how their previous emphasis on counting was likely to prevent the children at risk from developing more powerful strategies. The researchers noted that the principles and techniques developed have also proven useful for children other than low achievers. The obvious inference here is that good teaching is good for all!

Queensland provides an example of a statewide diagnostic program in which children in need of intervention are identified. Key indicators of literacy and numeracy development, grouped into phases of development on core material, are being used by classroom teachers to assess and chart the development of children identified as being at risk. Critical growth aspects are defined with indicators, explanations of these, and examples. Appraisal takes place in Years 1 to 7, but mid Year 2 is the critical assessment period for the Year 2 Diagnostic Net. At this time, validation tasks are used to confirm teachers' judgements. Children who require intervention support receive additional assistance, and teachers complete a support plan that builds on strengths and meets identified needs of each child in the program. Schools report the results of the Net to the Department, and funding for teacher time and specialist teaching support is provided when required.

Much of the work on intervention is exploratory and there are no clear-cut strategies for successful intervention. A major project tackling this issue is the Literacy and Numeracy Development in the Middle Years of Schooling project. In selected government and non-government schools across Australia, data have been gathered to provide information on current teaching and learning strategies, special programs, organisational reforms, and resources used in Australian schools and systems, that are effective in improving literacy and numeracy learning outcomes of educationally disadvantaged students in the middle years of schooling. The aim of this research was to analyse the effectiveness of the identified initiatives, using standardised achievement data. Methods of assessment and reporting that contribute to the effectiveness of special programs have also be identified.

**Intervention research projects**

A series of Mathematics Intervention programs run in one school from 1993 to 1997 by La Trobe University staff had promising results. The aim of the research-based program was to identify, then assist, children in early primary levels who were at risk of not coping with the mathematics curriculum. The children identified had mathematics difficulties of a significantly different order of magnitude than other children, with many of their problems being related to poor language skills. The children’s emphasis was on following procedures, with little thought as to whether processes or answers made sense. The researchers involved in this project noted the need for the hand-in-hand development of language and mathematics for some children, with child-focused remedial support. After interventionist teaching for ten to twenty weeks, the children were re-assessed and all had made significant progress, although a small number of children appeared to require further intervention in the later primary years.

A further focus of the Mathematics Intervention project was on a research-based professional development program for the teachers. Participating teachers were taught how to use clinical interviews, based on the research of Steffe and his colleagues in the United States, to assess each child’s mathematical understandings. The researchers argued that teachers needed to be confident and competent in mathematics, have well-developed observation skills, and be able to interpret children’s mathematical activity.
Some teachers involved in the project, and in the implementation of the intervention program, became teacher-clinicians and introduced a withdrawal program in the school. Other teachers involved in the research have identified common difficulties that need to be addressed by classroom teachers and parents (see Pearn, Hunting, Merrifield & Mihalic, 1997).

Other projects that have built on research findings from elsewhere include the *QuickSmart Programme* in which findings from the United States serve as a basis for an assessment and intervention program. Detailed profiles of children were developed, along with descriptions of cognitive obstacles that prevented children from achieving acceptable standards of literacy and numeracy. Approaches that facilitated classroom teachers’ identification of specific stumbling blocks to children’s acquisition of appropriate numeracy skills, as well as the ways in which technology could be adapted to assist in overcoming the learning problems identified were documented.

While early-years intervention programs such as the ones described above can result in short-term improvements in children’s competence and confidence, there may be a need for follow-up attention in later years. This was illustrated by Pearn (1998). Fifty-seven children, who had participated in the *Mathematics Intervention* program in Year 1 (see above), were tested in Years 3 or 4. A focus on the same number operations used in the original program was maintained. Pearn found that although all children had shown improvement in their mathematical knowledge and the types of strategies they used since the Year 1 testing, some children had improved to a lesser extent than their peers. However, there were also students who were not achieving at the level predicted by their Year 1 results. Pearn argued for the need for mathematics intervention programs with specially-trained teachers for children in Years 3 and 4. According to Pearn and Merrifield (1998) it is crucial that mathematics intervention teachers share their knowledge of at risk children with the classroom teacher, in order that the classroom teacher can use the knowledge to develop appropriate classroom activities for the children.

In New South Wales, the need for later testing and associated intervention has been recognised. The *Counting On* project is currently in operation in 120 schools, and is intended to be in all secondary schools by 2003. The aim is to identify children who have not achieved Stage 3 (end of primary school) numeracy outcomes when they commence secondary schooling. The objectives are to identify what the children understand, and then to build on this in order to develop the necessary mathematical concepts and skill, to build an atmosphere of trust and respect, and to plan for effective teaching and learning trajectories. Quantitative tests, survey results, and anecdotal evidence have all suggested that the program is successful. It has now been extended into Year 6 in order to provide earlier intervention. This development, the *Counting On: Transition* project, operated in forty secondary and feeder primary schools in 2000. Children were interviewed to establish the most advanced level of thinking of which they were capable. Child assessment interviews were videotaped to give teaching teams time for considered analysis and for discussion about appropriate teaching strategies. There have been improvements in participating teachers’ awareness of children’s strengths and weaknesses, ways of probing these, and responses in terms of teaching activity. Classroom strategies used by the teachers now show greater awareness of learning sequences and of connections between areas of mathematics that are dependent on some key foundational ideas.
When, and how, to implement intensive intervention was a key theme in a study by Gervasoni, *Assisting children who experience difficulty learning mathematics*. During 2000, twenty-one schools already participating in the *Early Numeracy Research Project* in Victoria implemented an intervention program called *Extending Mathematical Understanding* (EMU) for Year 1 and Year 2 children identified as low attaining in mathematics. Specialist teachers developed an individual learning plan for each child in consultation with the classroom teacher. They met with the classroom teacher at least twice per term to discuss each child’s progress both in the classroom and in the specialist program. The EMU project included diagnosis of individual difficulties using the *Extending Mathematical Understanding Assessment Interview* and activities based on Victoria’s *Early Numeracy Research Project* “growth points”. Depending on the progress of children, teachers worked with groups of three or four children or with individual children for ten to twenty weeks. The children were engaged in experiences that required ‘hard’ thinking. The emphasis was on activity and articulation of what and how they had learnt. Typically, each thirty-minute session was structured to include two minutes of focused reflection on the learning from the previous session, eight minutes of activities focusing on counting and place value, fifteen minutes of rich learning activities focusing on problem solving, and five minutes for reflective discussion about the key aspects that had been covered in the session. Small-group and individual program structures were used. Comparisons were made to determine whether variations in progress could be attributed to Year level or program structure. Gervasoni concluded that:

- there were no clear data to suggest that an intervention program in Year 1 was more effective than in Year 2. However, if schools were to focus on one year level only, then Year 1 would be better so that children might benefit from assistance as early as possible; and
- both the individual and group program structures for the intervention program were effective. Effectiveness was dependent on the concept being studied, and the ages of the children.

With respect to technology and intervention, the project of McRobbie, Baturo, and Cooper (2000) demonstrated that more than well-designed content is needed for success. The researchers studied what happened when low achieving Years 5 to 9 mathematics children used an integrated computer learning system. No statistically significant gains in achievement were found using the integrated learning system, and the children’s attitudes to computers became less positive over time. While many of the children felt that the integrated learning system had helped them to learn, this belief was not supported by test results.

In contrast to this computer-delivered teaching, Currie (1991) reported on a more holistic project undertaken in eight Western Australian schools, that was designed to improve Indigenous children’s performance in mathematics, starting from their earliest introduction to number work. The use of a learning environment that was designed not to conflict with traditional Aboriginal learning styles was explored. There was a consistent gain in achievement scores over the course of the first year, and no evidence that the children regressed or dropped further behind as they advanced through school.

In summary, there have been many State and Territory projects and programs focusing on the identification of children with numeracy difficulties in the early years of schooling. Teachers have been directly involved in many of these projects. There have been reports
of significant professional growth as a consequence, particularly in the projects that involved one-on-one interviews with the children. There have also been a number of intervention programs, often accompanying the identification process, with promising indications of improvement in children’s numeracy skills and understanding. From the findings discussed above, it would appear that the timing of intervention programs may be critical and that follow-up may be needed in subsequent primary years. Research findings from studies on the primary-secondary transition and on the numeracy learning of children in the middle years of schooling (next section) highlight a need for more research into numeracy intervention for children at risk as they proceed to make the move from primary to secondary learning settings.

**Intervention in the middle years of schooling**

In recent times there has been growing recognition of the numeracy learning issues associated with the transition from the primary school into the secondary school. There has been some research into the effects of the primary-secondary transition (for example, Fullarton, 1998; Trent, 1993; Trent, Russell, Cooney, & Robertson, 1994). There have also been State government-funded projects specifically centred on numeracy in the middle years of schooling — for example, the NSW *Counting On* and *Counting On: Transition 6–7* projects; and the South Australian *Lower Secondary Numeracy Project*. These studies and projects have been discussed elsewhere in this report, but are relevant to this section because they resulted in a recognition that there is a need for intervention programs to continue into lower secondary schooling.

One project that suggests direction for such intervention in the Middle Years is the Victorian *Middle Years Numeracy Research Project*. The numeracy performance of a large sample of students in Years 5 to 9 was the basis for the selection of 20 schools to participate in this project (see also Siemon, 2000). The aim was to determine what was effective in improving students’ numeracy outcomes. It was found that targeted programs, supported by a whole school approach and effective leadership at the local level, contributed to measurable improvements in performance. In a similar vein, the main aim of the *Literacy and Numeracy Development in the Middle Years of Schooling* project has been to provide information on effective teaching and learning strategies, special intervention programs, organisational reforms, and other practices that improve numeracy outcomes for students in the middle years of schooling.

In the next sections, findings from projects and research studies that focused on specific classroom practices are presented.

**Use of resources**

While resources include human resources, in this section the focus is on research into the use of material resources, such as teaching aids, textbooks, calculators, and computers. Many State and Territory governments have targeted projects that provide general resourcing in some areas. Tasmania, for instance, developed the *Flying Start* program to support all children in Prep to Year 2, to achieve strong literacy, numeracy, and social skills. The program involved an extra allocation of resource teachers for early years classrooms (thus effectively reducing student-staff ratios), targeted professional
development courses, parental programs, and money for equipment and technological aids.

**Teaching aids**

The claim that “Manipulatives [concrete materials] help pupils develop and understand the concepts, procedures, and other aspects of mathematics” (Szendrei, 1996, p.) has long been uncontested. Policy documents, teacher’s journals, textbooks, duplicated worksheets, teacher education, and staffroom conversations all support the use of activities with materials at the lower levels of schooling. Support for their use is also found among researchers elsewhere in the world such as Bauersfeld (1992a), Hiebert et al. (1997), Thompson and Lambdin (1994), and in the reviews by Szendrei (1996), and by Perry and Howard (1994).

However, the relationship between the use of teaching aids and children’s learning is considered more complex now than it was a decade ago. In England, Hart (1989) reported discouraging results from schools that actively encouraged the use of such materials. She interviewed children who were learning a wide range of mathematics topics, progressing through stages of practical structured hands-on activity, and then experiencing subsequent instruction in formalised concepts. Hart summarised the results as “Sums are sums and bricks are bricks” in order to describe the lack of connection between the two types of learning made by the children (1989, pp. 138-139). Other international and Australian authors (for example, Becker & Selter, 1996; Bobis, 1993; Hall, 1995; and Perry & Howard, 1994) have also questioned the idea that concrete materials always add ‘reality’ to mathematics learning. Although the research literature has reported contradictory findings about the effectiveness of concrete materials for many years, it is not uncommon to hear teachers make comments like, “The children enjoy hands-on stuff, but they can’t explain what they have done” (Nichol & Robinson, 2000, p. 495).

Further, there is the possibility that concrete materials may lead to misunderstandings that constrain further learning, or necessitate some un-learning. Bobis (1993) studied the processing demands that can be imposed on children by instructional materials. From some of her six experiments related to the use of aids and diagrams for geometric tasks, Bobis concluded that:

- redundant instructional material (including the use of diagrams) increases cognitive processing load;
- a format in which instructions are embedded in materials that require manipulation facilitates learning by reducing redundant information, and removes the need to split attention between physically separate materials;
- the performance of children who were unaware of a completed model’s appearance was superior to that of children who examined a completed model, and who were explicitly instructed to concentrate on the model or a combination of the model and the instructions; and
- some initial instructional presentation formats may interfere with, rather than facilitate learning.

A question raised in some Australian research has been whether the use of some materials adds unnecessarily to cognitive processing loads. Boulton-Lewis is currently carrying out a major study of this question in the project *An assessment of the information*
processing loads, value and limitations of mathematical representations used by teachers and young children. In a preliminary study, Boulton-Lewis (2000) pre- and post-tested 28 children as they progressed through Years 1 to 3, finding that many regressed in their ability to represent numbers with ‘efficient’ materials such as multi-based attribute blocks (MAB). She attributed this regression to teachers encouraging their pupils to use a variety of materials, and suggested that children “should be encouraged to use a particular representation of sets of tens and units regularly, so that, with practice, that representation becomes well mapped into the place value of numbers” (pp. 86–87). Boulton-Lewis felt that the use of materials confused children as they were trying to cognise mathematical concepts. While most of the children had overcome this confusion by the end of Year 3, only 38 percent used their knowledge when faced with a subtraction algorithm. Boulton-Lewis saw this as a lack of connectedness between use of materials and the need to complete abstract operations. When left to choose their own methods, most of the children chose not to use materials, as they “preferred to use mental procedures as much as possible” (p. 86). In her current research, Boulton-Lewis is exploring the contention that some representations impose a cognitive processing load that is too high and therefore interferes with understanding of concepts. She hypothesises that children’s own representations may be more useful, and she will assess and compare the processing loads of child and teacher representations in order to make recommendations about maximising learning and minimising limitations.

A significant amount of overseas research on the use of the Empty Number Line (see the Number section of this report) supports Boulton-Lewis’ view, suggesting that rather than using concrete materials that represent an adult view of place value, children need to be provided with simple ways of recording number operations as they work abstractly. It is argued that such methods support a shift to abstract and generalisable thinking.

Findings from research over the past decade on concrete teaching materials appear to challenge earlier faith in their beneficial effects on children’s mathematical learning. There is now evidence that they may interfere with, rather than facilitate learning particularly in the early years, and may add to the cognitive demands of numeracy tasks.

Textbooks

Textbooks are used in many primary classrooms, either as the basis for teaching and learning, or as the source of activity sheets for children to complete. It is surprising to find little research into either the effects of textbook use, or ways that they are used.

Due to the subtle effects that everyday classroom interactions and tools such as textbooks have in shaping children’s values, and the children’s views of what mathematics is and what purposes it serves, textbooks must be culturally inclusive with appropriate contexts (Thomas, 1997). In a study of eighteen textbooks, Clarkson (1993) found that most segments of the books did not refer to people, 45% of people depicted were clearly male compared with 39% clearly female, and only 8% of instances showed a person who was not Anglo-Australian. However, Forgasz (1997) found that the situation with regard to gender stereotyping may be improving. She surveyed Year 6 and Year 7 children, focusing on their perceptions of the nondescript cartoon characters that had been used throughout a commonly-used textbook. The children were asked to indicate their perceptions of the ‘gender’ and ‘ethnicity’ of each. The two most frequently appearing cartoon characters were perceived to be ‘male’; the two least frequently occurring to be
‘female’. These findings support those of Clarkson. However, the characters were also perceived to reflect a wide ethnic mix. Forgasz commented that the pictures used appeared to avoid gender stereotypes and were inclusive with respect to gender and ethnicity. While vestiges of earlier biases were still evident, Forgasz concluded that this was a vast improvement on what had been found in earlier mathematics textbook analyses.

Menon (1996) investigated what children feel about the varied purposes of textbook questions. He analysed children’s journal writing in which they responded to questions about their thinking through a variety of problems set by teachers and compared them to those that the children had created by themselves. He found, for example, that questions children posed proved more valuable as an indicator of children’s understanding than questions posed by teachers.

The effect that textbooks have on teaching practice was revealed when data obtained with a questionnaire, entitled Calculator Use in Western Australian Primary Schools, sent to 787 primary schools in Western Australia were analysed by Sparrow and Swan (1997). They found that teachers tended to follow the expectations of textbook series with regard to the use of calculators, for example, despite research and the curriculum framework suggesting that working otherwise would be more appropriate.

**Technology**

Australian research on the use of technology in the past decade points to the importance of well-informed teachers who have a sense of how to use technology to best effect. This area does not appear to have been researched widely, despite findings from some major projects when calculators were first introduced. In the sections that follow, research on the use of calculators and computers with primary aged children are presented.

**Calculators**

A major change in primary schools in the past twenty years has been the availability of simple calculators. In many schools, however, calculators are used only for checking already-completed work, or for special calculator activities. Howard (1991) showed the importance of teachers’ beliefs about calculators. While 85 of the 89 teachers interviewed in the study ‘supported’ the use of calculators in primary mathematics classes, only 32 actually used them. Teachers indicated that calculators were not being used because none were available, there was no school policy on their use, or they were not considered relevant for the lessons being taught. When Howard was writing about the situation in 1991, many schools still had only one set of calculators that could be taken into classrooms. Now, in contrast, sets are readily available for use as the children desire (Mousley & Herbert, 2000). This current position raises the question of whether children will develop basic numeracy skills if they can reach for a calculator instead of making the effort to memorise number facts.

Groves, Welsh, and Stacey carried out an extensive Australian Research Council-funded project in Victorian schools in 1992–1993, Calculators in Primary Mathematics (see, for example, Groves, 1995; Groves & Cheeseman, 1993; and Groves & Cheeseman, 1995). Eighty teachers and one thousand children participated in the project. The researchers studied the potential for calculators to change mathematics curriculum and teaching, and found it to be profound. The specific purpose of this project was to study the long-term
effects of calculator use on the development of children’s mathematical concepts. Data collection included classroom observation, teacher interviews, written tests, interviews with children, and videotaping. The researchers found that giving all children in the early years of schooling free access to affordable hand-held computational power, made a significant difference to both curriculum processes and content. Groves (1994b) reported results of assessment interviews with 58 Year 4 children with long-term experience with calculators in the project. She found that these children performed better on the mental computation interview items, number knowledge items, and estimation items than children not involved in the project. Overall, their performances were better than the other children on 34 of the 39 items, with the greatest differences in performance being for mental computation items. The project children’s patterns of use of standard algorithms and invented methods for mental computation items did not vary greatly from that of the non-calculator children. In summarising children’s progress in the project as a whole, Groves (1995) wrote:

Overall results — based on classroom observations, teachers interviews, a survey of parents and a program of testing and interviews with children — show that project children, having been exposed to large numbers, negative numbers and decimals at a much earlier age through their use of calculators, developed more sophisticated conceptual understandings in these areas than children without such long-term experience of calculator use. (p. 310)

Similarly, Stacey and Groves (1994) tested 225 Years 3 and 4 children to establish that children did understand the number system better after sustained calculator use, and that they were better able to choose an appropriate operation in a word problem. A series of interviews showed that calculator use had assisted children to develop strong number sense as well as skills of mental computation. This project also provided strong and extensive evidence that free access to calculators did not have a detrimental effect on children’s learning of number skills. For example, Stacey (1994a) analysed arithmetic test scores of all of the Years 3 and 4 children who had been in the project, and who used calculators whenever they wished for at least three years. The results were compared with those of a control group, matched for other relevant factors. Stacey found that all children handled whole number calculations equally well, the difficulty being determined only by how many transfers from paper to calculator and vice versa were required. However, project children were better able to handle calculations involving decimals or negative numbers, and were also better able to identify appropriate operations for solving word problems. These children also used more efficient strategies, such as multiplication rather than repeated addition. As the author noted, these very important observations are supported by data from other sources, including the major Calculator-Aware Number (CAN) project, conducted in the late 1980s in England.

By the end of their third year of involvement in the Calculators in Primary Mathematics project, many project teachers had made substantial changes to their teaching of mathematics. One of the major aims of the project was to investigate changes in teachers’ expectations of their children; an extensive written questionnaire and teacher interviews were used for this purpose. All seven teachers who had participated in the project from the Prep level for two or more years significantly changed their expectations of children’s mathematical performance, especially in relation to the children’s ability with counting, large numbers, and negative numbers. Nevertheless, teachers’ predictions remained conservative compared with actual levels of performance. Five of the seven teachers
reported changes in their teaching, the most common changes being the extent of open-endedness of tasks given to children, and the extent to which children were able to lead the learning process.

Based on their project, *Calculator Use in Western Australian Primary Schools*, Sparrow and Swan suggest that research findings on the potential of calculators to help develop children’s numeracy skills and understandings are not reaching teachers, not convincing enough to stimulate technology-rich curricula, or are not being supplemented by policy and practical support for the necessary changes in curriculum processes. Sparrow and Swan sent questionnaires to 787 primary schools in that state, requesting anonymous responses from Years 1, 3, 5 and 7 teachers. The project concentrated on four questions:

- To what extent are calculators being used in primary schools?
- For what purposes are calculators used in the primary classroom?
- What is the attitude of teachers toward the use of calculators in the primary school?
- What impediments are there to the use of calculators in the primary school?

They found that most teachers were in favour of all students using calculators, with the majority wanting them to start below Year 5, and that many schools supplied class sets. However, as Stacey and Groves had found in 1994, actual use falls far behind the support expressed. Sparrow and Swan noted that the influence of the textbook series used in various schools was very strong, that teachers were divided about whether students should use calculators before they master the basic number facts, and that teachers were maintaining clear control over when the calculator could be used. It is clear from the reports of the project (see, for example, Sparrow & Swan, 1997) that calculators are not being used as a teaching aid, because their most frequent uses were to check work and to carry out calculations that were not core lesson content. Teachers provided little evidence of their use in problem solving or mathematical investigations.

Swan and Bana (1998) examined children’s preferences for the use of mental, written and calculator methods. The children often chose to use the calculator in questions involving larger numbers and for calculations involving decimals. As adults do, they made sensible shifts from mental to calculator-assisted work when they realised that they had reached their mental calculation limitations, or realised that they had made a mistake. However, in practice, children do not usually make choices of modes. In the same questionnaire, Swan and Sparrow (1997) also identified many reasons why teachers were reticent to use calculators freely. These included:

- unavailability (not supplied; class sets not available when needed; in need of repair; or insufficient numbers);
- the range of calculators that children brought to class;
- textbooks not requiring their use;
- teachers’ restricted views of the potential of calculators; and
- beliefs about the age or year level of children allowed to use calculators.

Booker (2000), working with teachers in an Aboriginal and Torres Strait Islander school, found that using standard calculators had a motivating effect and proved effective in building pupils’ understanding of whole numbers and decimal fractions, and for skills such as re-naming and rounding. He also reported that the understanding of place value engendered by the activities and calculators was instrumental in overcoming dysfunctional
language used with larger numbers and decimals. Booker also reported that with appropriate teaching, calculators fostered deep mathematical thinking about the conceptual base of number operations, and that this led to meaningful adoption of conventions for the order of operations. The calculators were not doing the ‘thinking’ for children. In order to obtain sensible solutions, the activities used required children to develop an ability to think about the types of numbers in use and likely results prior to using a calculator. Booker acknowledged that this was a small project, but one with great potential for the teaching of number. In order to build understanding and facility with number processes and applications, he recommended that calculators be used routinely in Australian schools with all children.

Sparrow and Swan (1997) suggested several strategies to improve the situation with respect to calculator use:

- develop a whole school policy on calculator use, using the 1996 statement on the use of calculators and computers for mathematics in Australian Schools (AAMT);
- allocating resources to buy calculators, provide professional development for teachers, and purchase support materials;
- develop a dialogue between schools and parents about the use of calculators to avoid misunderstandings; and
- encourage children to develop number sense, i.e., the ability to make a sensible choice as to when it is appropriate to: make an estimate, perform a mental computation, use a pencil and paper algorithm, or make use of a calculator or computer. (p. 287)

The development of numeracy through calculator use in primary schools has implications for curriculum development and curriculum processes, as Stacey and Groves (1996) noted:

Teachers immediately recognised that their curriculum was going to need to change yet again and there would be a flow on to other grade levels as well, as teachers in the lower grades covered areas which had been reserved for higher grades. (p. 223).

There are also related implications for teacher education. Schuck (1995) collated the thoughts of pre-service teachers about calculator use in the primary classroom. Schuck summarised her work by saying that many students are entering teacher education programs with the view that calculators should be avoided in Years K to 6. Thus, there is a strong obligation on the part of teacher education programs to show ways to use calculators as effective teaching aids, and to convince pre-service teachers of the benefits of their use.

The wider community, as well as children themselves, have diverse views about when, and how calculators should be used. Doig (1993) questioned about 200 children from two schools that gave their pupils unlimited access to calculators and two non-calculator schools matched on socio-economic variables. Many of the children felt that calculators are only for ‘sums’, and none of the responses indicated that calculators could be used for activities such as exploring numbers, patterns or operations. Doig proposed that by Year 3, the curriculum had turned from learning about numbers, to operations with them, and the opportunities for exploration with calculators had passed. However, in schools where calculators were freely available, many children did not rely on them, and had well-developed alternative computational methods. A small study of some Sydney teachers and their classrooms by Arvonen and Bobis (1995) had similar findings. These researchers used a questionnaire and interviews to assess children’s attitudes towards
calculators. For children who had easy access to calculators, they found that the majority of the children enjoyed using a calculator, considered mathematics learning to be more enjoyable with a calculator, and perceived calculator use as relatively easy. This suggests that a better attitude toward mathematics could result from calculator use, a finding supported by other studies (Hembree & Dessart, 1986). On the other hand, children for whom a calculator was not a part of their regular mathematics instruction were unsure about the legitimacy of using a calculator, and the researchers felt that this attitude could impede the integration of calculators into classrooms.

**Computers**

In 1995, Fitzgerald and Hughes examined *Computer Assisted Learning Systems in Thirty Victorian Schools*. Government primary, special and secondary schools were involved in the evaluation of Years 5 to 8 literacy and numeracy, and 21 schools were studied in some depth. One program, *SuccessMaker*, was used as an integrated learning system, and two other ‘open learning’ systems, *EduQuest* and *Acorn*, were included in the evaluation to assess the initial reactions of teachers and children to the systems. Both teachers and principals were very positive about the use of computer-aided learning, although many teachers pointed out that it should only be seen as another resource in supporting student learning, rather than a total approach. Teachers, who were experienced users of the software, generally felt that skill development and complex learning could be supported by such a system. However, they were concerned about the lack of professional development and of supporting information on ways to integrate computer-based activities with other learning activities. Fitzgerald and Hughes found that children in schools using *SuccessMaker* outperformed comparable children in control schools on some aspects of mathematics and reading comprehension. However, the children’s use of the integrated learning system was in addition to, and not a substitute for, normal classroom teaching and learning. In the Australian literature, no evaluations of currently available software were found with the aims of identifying how features of computer use affect children’s motivation, learning processes, and achievement outcomes, or of how teachers can best incorporate software to supplement regular teaching without sacrificing the advantages of teacher-child engagement.

Walta (2000) demonstrated that the role of the teacher remains vital when computers are used. Children exposed to teacher intervention activities, as well as experience with computers, outperformed those who were not. Baturo, Cooper, McRobbie, and Kidman (1999) studied teachers of Years 4 to 7 using an integrated learning system. Teachers’ success with the programs and whether they were prepared to endorse the programs as useful relied on the teachers’ computer experience and beliefs about the learning system. Differences in teachers’ beliefs about the pedagogy of the learning system appeared very significant, and their beliefs in relation to benefits for the pupils appeared to be related to their own knowledge of the educational uses of computers in classrooms, and not necessarily to knowledge of their children’s needs or beliefs. McRobbie, Baturo and Cooper (2000) found that, while gains appeared to be made by children when using an integrated learning system, scores on standardised mathematics achievement tests did not support this.
Over the past five years, there has been some development of Australian computer software that attends to specific mathematical content and is aimed at preventing or remediating common problems that children have in these areas.

Detailed information about the development of children’s knowledge of decimals and the way this develops from Years 4 to 10 were gathered by Stacey and Sonenberg in their project, *Improving Learning Outcomes in Numeracy: Building Rich Descriptions of Children’s Thinking into a Computer Based Curriculum Delivery System*. The researchers described the conditions under which children best made the necessary transitions in their thinking. They used this knowledge to develop teaching modules with the potential to advance children’s understanding. The modules were incorporated into a computer-based system with features that enabled diagnosis of individual children’s strengths and weaknesses, as well as features that adapted the program to these.

McIntosh, Stacey, Tromp, and Lightfoot (2000) reported an evaluation of children’s interactions with two computer games designed, from a constructivist perspective, to enhance learning about decimals. The games were effective in challenging children’s misconceptions about decimals. Children assisted by teachers developed strategies for dealing with decimals. The researchers focused on the nature of the teaching assistance that could be programmed into the games to further strengthen their usefulness.

The topic, *Fractions*, has been common content material for software development. For example, Hunting and Davis’ project, *A Longitudinal Study of Children’s Fraction Learning*, demonstrated how software developers can employ user-driven animations to allow children to interact with representations of fractions. The researchers used this software in a three year teaching experiment to study how children’s knowledge of fractions and ratios develop, with a focus on points of transition in learning.

Another area that has been widely reported over the past years is the use of the *Logo* programming language, which is now incorporated into the software package *MicroWorlds*, for the teaching of mathematical thinking, specific numeracy concepts, and the basics of computer programming. *Logo* was a research laboratory-based product but, as Nevile (1992) reported, there has been little formal research on the longer-term effects of its use. Research has still not clarified the role of *Logo* in education. There have been no significant research projects in the area and most of the published work is about how to use specific features of the software to complete particular tasks, or to stimulate particular kinds of thinking. Grieshaber and Yelland, in their project, *Becoming Numerate with Information Technologies in Early Childhood Education*, look at teachers who are aiming to investigate and document ways that computers are used effectively, and hence to create a detailed model of teaching and learning that will be useful for planning educational opportunities for Australia’s young learners. Grieshaber, Yelland, Matters and Cook have a similar aim in their *Beyond Letters, Numbers and Screens: New basics, Technologies, Numeracy and Early Childhood Education* project. Using information technologies, they create detailed models of Queensland’s *New Basics* approach to teaching and learning numeracy. In the research project, *Children of the New Millennium: Using Information and Communication Technologies for Playing and Learning in the Information Age*, Hill, Yelland, and Thelning explore the development of young children’s expertise with information and communication technologies. The project is focused on the early years, from the pre-school to the second year of school. The aims are to examine when young children from diverse socio-economic backgrounds use information and
communication technologies, how many forms of it they use, and to what extent technology relates to other forms of play and learning in literacy and numeracy.

Recent developments in information and communication technologies have raised opportunities for a focus on on-line teaching, either to supplement regular classroom teaching or as a substitute for it in special cases such as remote children, children with disabilities, or adult learners studying basic numeracy and literacy content. This is a new field, so no research projects have been completed to date. Children, On-line Learning and Authentic Teaching Skills in Primary Education is a project undertaken by Robertson, Fluck, and Murray. The aims are to identify the strategies adopted by teachers using on-line learning programs in rural and urban locations, and to match these strategies with learning outcomes. Benchmarked data links with numeracy and literacy standards and with specific on-line teaching strategies.

The wide range of software available, society’s calls for the use of computers in schools, and queries into the effects of children spending larger, or lesser, amounts of classroom time working with computer-based pedagogy have resulted in some investigations of the effectiveness of information and communication technology. In a research project, Mathematics, ICT and Effective Teaching, produced by the Teacher Training Agency, a summary of existing work on identifying and overcoming barriers to using information and communication technology effectively in the classrooms was produced. For year-level groups in which computer use was high, links to high levels of achievement or very positive attitudes were identified; 120 teachers were studied more closely. Little association between the amount of computer use and either achievement level or children’s attitudes to their use was found. However, there was a significant relationship for two of the groups; the small number of classes where children were said to use computers ‘more than once a day’ had very positive average attitude scores; and groups where children used computers ‘less than once a month’ had negative attitude scores.

Barriers identified by teachers and observers fell into categories of time, equipment, and pupil-related issues. The most significant issues were:

- that it is difficult to attend to only two or three pupils who are working on computers;
- the number of computers per classroom is too low;
- opportunities are needed to learn new software;
- technical support is rarely available immediately; and
- there were concerns over the ever growing cost of consumables.

It was concluded that effective teachers use computer-based examples and counter examples as part of their normal teaching, and that the pupils of such teachers tend to give examples during lessons, and to model their work both when talking to the teacher and to their peers. The teachers’ and the pupils' levels of personal confidence also had conspicuous effects upon pupil achievement, and this appeared to be firmly rooted in their personal IT skill levels. Having someone in their classroom to help get pupils over an initial threshold of personal skills, was seen as necessary, so that teachers could concentrate on the mathematical purpose of the lessons.

In summary, there has been considerable research on the use of technology – calculators and computers – for primary numeracy learning, although the range of factors explored has been somewhat limited. There would appear to have been little research attempting to identify effective pedagogical strategies that supplement and enrich the use of particular
technological tools or teachers’ activities with them. There was an initial flurry of research interest when calculators were first used in some primary classrooms and the initial findings were promising. Children using calculators freely developed more sophisticated and flexible number understandings than children without long-term experience of calculators. However, there has been little recent research on the effective use of the technology. The extent to which computers are used for numeracy learning has been linked to teachers’ competence and confidence with the technology. There appears to have been no research on establishing the range of computer software packages commonly used for numeracy learning in primary classrooms. Research on the effects of the development of specific software packages in particular content areas reveals that they can have positive effects on children’s conceptual understandings. There appear to be no clear links between computer use, attitudes and achievement. Recent developments in information and communication technologies have provided opportunities to focus on on-line teaching. In this new field, no research projects have been completed to date.

**The use of specific teaching strategies**

Contextualising mathematics is related to broader classroom activity, and especially activity that encourages children to use mathematical concepts and processes in a variety of situations. In the five-year *Thinking and Working Mathematically* project, researchers are working with teachers at all levels to focus on what it is to ‘work mathematically’. A set of key principles underpins this, including:

- talking mathematics;
- relating mathematical ideas to real life; and
- approaching mathematics in an investigative and problem-solving manner.

These principles fit with a variety of classroom strategies purported to assist the development of numeracy that have been studied during the past decade. The research findings related to the following strategies are presented in this section of the report: problem solving and investigations, open-ended tasks, questioning and discussion, and the use of ‘real world’ contexts.

**Problem solving and investigations**

Problem solving and the use of open-ended tasks are two areas of research that have attracted vast international attention.

A key shift in the teaching and learning of numeracy in recent decades has been the use by most teachers of problems, and by some of mathematical investigations (the latter being more common in secondary schools). The reasons for this are varied, but most primary teachers would refer to the need for children to see mathematics as useful in real contexts. The term ‘real world mathematics’ is frequently used. An equally important justification for this shift is to give children experience at being numerate, that is, able to use basic skills in everyday situations. A further justification is the potential of problems to promote higher-order thinking (see, for example, Lowrie, 1996), and provide opportunities for children to talk mathematically, listen to a range of mathematical ideas and approaches, be listened to, receive feedback on their individual and group solutions, and demonstrate and explain their knowledge, thinking and methods (DfEE, 1998).
Because children often solve problems in small groups speaking aloud, there is potential to audiotape or videotape children working. This gives a glimpse of the ways that they are thinking, their knowledge, and their cognitive functioning. Rather than just imitating the teacher, children use the knowledge and skills that they feel comfortable with to solve the problems themselves (see, for example, Owens, 1994). The feedback that teachers get makes research in this area ideal for them. This is evident in reports of action research in the research literature (for example, the Early Assistance Action Research Project) as well as in teacher resource materials (for example, Booker & Bond, 2001).

There have been many foci in the research on problem solving and investigations. These include the potential for developing specific problems that bring concepts and the potential for cognitive growth to the fore. Other foci have been problem contexts, classroom organisation during problem-solving and inquiry-based lessons, the use of models to represent and to help abstraction and generalised thinking. Setting up the ideal contexts for children to engage seriously and meaningfully with mathematical ideas and processes is a common theme. In their discussion of a range of research and development work in modern classrooms, Fosnot and Dolk (2001) provide a useful guide for developing what they call ‘situations for mathematizing’, saying that such situations must have three major components:

1. The potential to model the situation must be built in (Freudenthal, 1973)....

2. The situation needs to allow children to realise what they are doing....

3. The situation prompts learners to ask questions, notice patterns, wonder, ask why and what. Questions come from interacting with the world around us, from setting up relationships, from trying to solve problems. When the problem is “owned”, it begins to come alive. (p. 22–23)
In Australia, Watson and Collis, in their study *The Assessment of Higher Order Cognitive Functioning During Co-operative Learning in Probability and Statistics*, researched cognitive functioning with the aim of framing a useful assessment hierarchy described in terms of developmental, multi-model, functional, and optimal performance. They used co-operative problem-solving situations appropriate for the higher level objectives of the probability and statistics curriculum. Watson and Chick (2001b) identified factors that influence the effectiveness of collaboration during small-group problem solving, basing their work on a review of recent literature. They identified particular cognitive, interpersonal, and external phenomena that influence cognitive improvement, as well as factors in the interactions that led to no cognitive change and even reduced levels of functioning.

English, with her focus on children’s structural understanding of combinatorial problems, models students’ common understandings so that these can inform teaching as well as assessment. As English points out, success in school mathematics does not necessarily imply a facility to think mathematically in dealing with novel problems. In her project, *The Cognitive Competence of Low, Average and High Achieving Children in Solving Novel Mathematical Problems Involving Combinatorial and Deductive Reasoning*, she examines children’s competence with problems that demand little formal mathematics. The problems draw on general knowledge and reasoning processes, as well as on the ability to transfer learning from one problem context to another, as well as to other contexts. Conclusions reached included that:

- children can construct important mathematical ideas through solving novel problems;
- their level of achievement in school mathematics is not a reliable predictor of ability to solve novel problems;
- bright children’s abilities to generate ideas for themselves can be inhibited by formal mathematical rules; and
- assessment of children’s mathematical competence must include a range of novel problems (English, 1996a).

As well as providing a window into children’s thinking, using problem solving with children also allows observation. When faced with longer tasks, attitudes and patterns of behaviour demanding focused attention, and sequences of logical thinking can be inferred. Taplin (1994a) had individual children talk aloud as they solved problems. She investigated their perseverance with unfamiliar number problems, and their reactions if they did not reach a satisfactory solution. Using task analysis maps, she noted how successful problem-solvers managed their use of strategies, and found that these children were more flexible in their approaches and in their use of strategies. Taplin developed a useful model in which a sequence of strategies used most consistently by successful students is described.

*Open-ended tasks*

The use of open-ended tasks, especially problems that have many possible answers, is a technique that allows children, either in groups or individually, to draw on and demonstrate the range of their knowledge. The way that open-ended tasks, including ‘open questions’, incorporate key aspects of investigative and problem-solving pedagogy as well as directing students’ attention to valued skill and concept development, can be explained by
means of an example. If a conventional classroom task is: "Find the perimeter of a rectangle with sides of 10 cm and 2 cm", the corresponding open-ended task would be: "A rectangle has a perimeter of 24 cm. How long might its sides be?" As well as addressing content and skills explicitly, open-ended tasks are useful for developing numeracy in that they:

- allow students, at various stages of understanding a concept, to make an initial response then investigate, seek patterns and connections, generalise, and identify alternatives; and
- focus children’s attention on key aspects of concepts, as distinct from merely trying to recall a rule or procedure.

Open-ended tasks can be used with classes of children with varying levels of understanding, as children can approach the tasks at different levels and in different ways. Using whole class and target group measures of behaviour and achievement, Bourke (1993) found that children of different abilities are able to become involved in the exploration of open-ended questions and to be challenged by them. As a basis for classroom teaching and learning in mathematics, Bourke suggests that the use of open-ended questions is a viable alternative to traditional practice. However, the traditions of school mathematics teaching, learning, and assessment regimes make it a challenge to keep the problem-solving process open (Stacey, 1995).

Research on the use of open-ended tasks and ‘good questions’ was initiated at the Australian Catholic University (see, for example, Sullivan & Clarke, 1992; Sullivan, Warren, White & Suwarsano, 1998). There is an extensive literature on the effects, advantages and limitations of these questions (see, for example, Bourke, 1993; and Sullivan, Warren & White, 2002), as well as on teacher support materials that have been developed (for example, Beesey, Clarke, Clarke, Stephens & Sullivan, 1998). Open-ended tasks are based on what Sullivan and Clarke defined as “good” questions, and have three main features:

1. They require more than remembering a fact or reproducing a skill.
2. Pupils can learn by answering the questions, and the teacher learns about each pupil from the attempt.
3. There may be several acceptable answers. (Sullivan & Lilburn, 1997, p. 2)

The use of open-ended or “Rich Assessment Tasks” was a feature of the Middle Years Numeracy Project. Open-ended tasks such as those used in this project are by no means novel to Australian teachers. What was important in this study was the development of scoring rubrics for teachers to use in grading the quality of students’ mathematical responses to the tasks, and also the alignment of tasks and associated scoring rubrics with different levels of performance on the Curriculum & Standards Framework (CSF) II: Mathematics (Board of Studies, 2000). These developments enabled open-ended tasks to be used to measure student performance in a large-scale study, and at the same time provided a means of checking consistency of teachers' gradings across schools (Siemon & Stephens, 2001). The use of scoring rubrics also provided clear indications to teachers of different level of mathematical responses that the tasks might elicit, and how the tasks themselves related to different levels of the Victorian CSF. Any further developments in the use of open-ended or rich assessment tasks nationally will need to show the links between possible responses to the tasks and related national benchmarks of performance and/or State/Territory curriculum standards.
As noted in the chapter on International and National developments, related research has taken place internationally. Widely used publications by international and Australian authors (for example, Sullivan & Lilburn, 1997) are numerous. The review of the research literature related to the use of open-ended tasks indicates that there has been little empirical, large-scale research on whether the frequent use of open-ended tasks improves outcomes as measured by test performance, whether all children benefit, and which contexts for problems interest children or lead to greater problem-solving success. Sullivan, Mousley, and Zevenbergen are currently studying the last two questions, using classes with relatively high numbers of lower socio-economic, and Aboriginal, children (for example, Zevenbergen, Sullivan & Mousley, 2001). However, there seems to be little other inquiry into this topic in Australia or overseas, although Wiest (2001) focused her recent research in the United States on the issue of increased problem-solving success.

**Questioning and discussion**

The importance of questioning and classroom talk has been the subject of many position papers and a few research studies.

One recent study explored the issue of when children are working in small groups, how it is sometimes difficult for teachers to know when, and how to intervene. Watson and Chick (2001c) summarised this by asking “Does help help?” They videotaped a Year 5 and 6 classroom to investigate the circumstances surrounding instances when help was sought, offered, or provided in collaborative open-ended problem-solving situations. They focused on questions asked by the children and by teacher, the answers provided, and the outcomes achieved. Other children and teachers provided help for children who asked questions. The researchers compared the levels of questions and responses when the questions were initiated by the children, and by the teachers. They found that children’s questions were generally lower level, and that this had an impact on the responses received. Watson and Chick concluded that children’s soliciting of help through questioning was not as effective as unsolicited help offered through questioning by teachers. Some differences associated with gender, or the gender composition of groups, were also observed.

Groves and Doig (1998) videotaped three teachers conducting mathematics lessons. The teachers were then interviewed in an attempt to establish their beliefs and practices about the nature and role of discussion. For all three teachers there was consistency between their beliefs and practices, and all saw enhancing children’s self-esteem as an important goal. However, the teachers had substantially different ways to achieve this. One had few strategies for children to contribute ideas to discussion; another only allowed a short time, precluding follow-up of ideas. In the third teacher’s classroom, clear norms were established that related to discussion as well as to what counted as explanations and justifications.

Leder (1990) encouraged children in a Year 3 classroom to talk about, and ultimately reflect on, the work they had learnt in mathematics. Lessons were videotaped and key excerpts were subsequently replayed to the children in a one-to-one setting. The information obtained about the ways in which individual children synthesised and made sense of the material taught highlighted a number of unexpected, inappropriate, and idiosyncratic interpretations that could be addressed. Substantial differences in the ways
that the children who were interviewed constructed meaning out of shared mathematical experiences were identified. As with other projects using individual interviews, the investigation confirmed that listening to children talk about their own mathematical experiences provides a rich source of information about children’s learning and error analysis. Leder pointed out that less individually focused, whole-class discussions often do not provide such detailed information.

Groves, Doig, and Splitter investigated the notion of *Mathematics Classrooms Functioning as Communities of Inquiry: Models of Primary Practice*. In this project, they sought to identify the extent to which dominant models of practice support mathematics classrooms functioning as communities of inquiry, as well as the degree of support amongst Victorian educators for mathematics classrooms functioning in this way. Lessons were videotaped and vignettes were used as a stimulus for focus group meetings. A high level of support for the notion of communities of inquiry was found among principals, teachers, and mathematics educators, together with a realisation that current practice falls far short of this goal. A hinderance was the common perception of the curriculum being a fragmented, outcomes-based document. Principals and mathematics educators rated the cognitive demands of typical lessons as low to very low, and not challenging for most children. Teachers saw the cognitive demand as being determined by the tasks (Doig, Groves & Splitter, 2001; Groves, Doig & Splitter, 2000). The researchers later compared a Victorian and a Japanese lesson in terms of the conceptual focus and cognitive demands of the instructional tasks, and the opportunities these afford teachers for advancing students’ conceptual understanding. Groves and Doig (2002) suggest that insufficient attention in Australian classrooms is being paid to the critical role of the development of conceptually focused, robust tasks that can be used to support the development of sophisticated mathematical thinking.

With the large amount of time expended in mathematics classrooms on discussion and on questioning by teachers and children, and all the advice about the importance of this, it was surprising to find how few research studies focused on these issues. This situation contrasts markedly with the 1960s and 1970s, when research on the use of “wait time”, and types of questions, was common. The findings from the studies in which aspects of classroom discussion and questioning were explored revealed the potential for students’ learning of encouraging them to speak, and that teachers need to know how best to use these strategies and capitalise on the outcomes.

**Grouping**

In order to encourage such discussion, and based on the assumption that students talking with each other about their work facilitates the development of understanding, the last fifteen years have seen a strong trend in primary schools to organise children into small groups. It is not surprising that this has been a focus of research projects in the last decade. Advice about using groups to encourage children to talk about mathematics and to explain their thinking is common (for example, Fullerton, 1997; Groves & Stacey, 1990; Stephens, 1993; and Watson & Chick, 2001b). The trend is not only Australian. The final report of the Numeracy Task Force (DfEE, 1998) in England drew on work from the *National Numeracy Project* and recommended a classroom framework emphasising whole class teaching supported by various groupings working on tasks or problems.
Gooding and Stacey (1993) studied the nature of dialogue in groups. Substantial differences were found in the ways in which effective and ineffective groups engaged with the content of the discussion, with highly interactive patterns of discourse being associated with effective learning. Children in effective groups talked more with more mathematical content, explicitly discussed the central idea, worked together co-operatively, proposed ideas, gave explanations with evidence, re-focused discussion more often, and responded to questions more often than children in the less effective groups. The researchers noted that teachers who are alerted to the characteristics of effective group discussion might be able to help children make it a better learning tool. Findings in small projects such as this are supported by evidence collected in other countries. In the United States, for example, Mulryan (1996) found that high achievers interacted more with their peers in co-operative small groups than did low achievers, and children in effective groups often used each other as resources, asking questions and checking information.

Gooding (1997) attempted to establish what children learned from group tasks, and examined the role of cognitive conflict in this process. Videotapes of Year 6 children working in groups were analysed, and learning outcomes were monitored with written tests. Gooding found that learning from the group task was effective and was retained better than learning from individual learning. The group with the most intense discussion retained this learning best. However, the children did not learn the ideas by explaining them to each other, but through their group appreciation of, and interaction with, the task. The learning from this group task was found to be gradual, retained long term, and linked a number of interconnected ideas about the basic concept (division). However, it is not a given that group work will help children to gain better numeracy concepts and skills. Gooding also found that the group dynamics in the girls’ and boys’ groups were different, with the boys’ style hindering the participation and learning of some boys.

Mulryan’s (1996) study (above), as well as Australian studies such as that of Zevenbergen (1995), showed that some low achievers were relatively un-involved, or passive, in co-operative small-group work. Research in England and Wales in the Leverhulme Numeracy Research Programme, and in Australia by Gervasoni (Early Numeracy Research Project), have shown that providing research-based advice on the ideal pedagogical structure is not a simple matter. In England and Wales, factors such as teachers’ expectations of the children were found to be more influential than whether the children worked in groups or not. Reporting on an intervention program in which both individual and group work were trialled, Gervasoni concluded that there was no clear answer as to which structure was most effective, as the effectiveness of the individual and small group program structures varied according to mathematical domain and year level. She suggested that if teachers had to choose between focusing remedial attention on groups or individuals, they needed to take into consideration both the specific domains in which children experience difficulty and the children’s year level. Gervasoni also noted that because teachers have the opportunity to assist more children, group structures have an efficiency advantage.

The development of social skills is important in primary education, and it is clear that children can be taught to work co-operatively in groups. Higgins (1994) used detailed analyses of videotapes to identify ways in which a teacher introduced tasks to children. Establishing the nature of appropriate interactions seemed important. For example, teachers need to express a clear expectation of the children who engage in collaborative dialogue. However, a case study by Gooding (1994) revealed some of the difficulties that teachers face in recognising effective group discussion. In this study, children in co-
operative groups made greater average gains than did students in control groups. Through videotape analysis, however, the researcher found that learning outcomes vary for group members, despite the children's common active engagement with, and discussion of, a task. This suggests that there is no easy way of alerting teachers to the situation and of advising them of ways to cater for, and capitalise on, the complexities of children's participation in small-group learning.

Grouping can also contribute to a supportive learning environment, but its use is problematic. Zevenbergen (1995), for example, studied how children working in groups are treated in slightly different ways because of teacher expectations. Social class was a significant factor. Watson and Chick (2001a) studied collaborative group work in a Year 5/6 classroom, using an open-ended task from the chance and data part of the mathematics curriculum. They focused on children's beliefs about collaborative group work and compared observations of children's learning and outcomes during collaboration with understandings displayed in individual interviews after the work was completed. They found that children's levels of co-operation were limited, and that there were discrepancies between the children's accounts and their actual behaviours. They concluded that, if this were a fairly typical classroom, then:

- preliminary work was required to set clear expectations;
- group composition needed to be considered carefully; and
- teachers may not be fully aware of the shortcomings displayed by many groups.

A study by Chick and Watson (in press) suggests that there is, in effect, little association between types of collaboration and children's views of group work.

The use of group work is frequently linked with formative assessment. This is because teachers can listen to, and analyse, children's discussions to gauge levels of knowledge, whether there are misconceptions, and the children's ability to apply mathematical ideas in a variety of contexts. For example, in the project, Assessment of Higher Order Cognitive Functioning during Co-operative Learning in Probability and Statistics, Watson, and Collis developed ways of using co-operative groups for the assessment of higher order processing.

Not all studies of group work have involved same-age children. De Lemos (1996) provided an overview and research-based evaluation of the Victorian First Steps pilot project for the first three years of schooling. This was an extensive pilot project that implemented multi-age grouping at junior primary level on a trial basis. De Lemos' longitudinal study focused on two samples of children who started school at the beginning of 1994, one of children enrolled in multi-age Preparatory to Year 2 classes, and the other a control group of children in single-age classes in schools not involved in the pilot project. With respect to numeracy, the research did not provide conclusive evidence that multi-age classrooms result in improved numeracy outcomes.

A study on teachers' attitudes to group work was carried out by White (1999). He found that teachers who were generally supportive of the idea of using more group work would allow children to work together and be exposed to each other's views. These teachers did not think it would place too great a demand upon resources, space, and equipment. In fact, they felt it might increase their efficient use. The teachers felt that group work might cater for all individuals' needs by allowing the more capable children to help those who were less capable. However, not all of the teachers who completed the project questionnaire had what the researcher called 'high intent' with regard to using group-work
in their own classrooms. Many were quite ambivalent, fearing that group-work would place too great a demand upon resources, and would result in many children not working well together because they lacked the necessary social skills.

The research on grouping discussed above reveals that teachers are generally supportive of the idea that group work allows students to work together and to be exposed to colleagues’ views. However, differences were noted in the ways that effective and ineffective groups work. Many children’s learning is enhanced as a consequence of working together on tasks but there was also evidence that low achievers may be relatively uninvolved or passive in this learning setting. There is little research and no clear evidence about the most effective group structures.

**The use of "real world" contexts**

As reported in Chapter 2, *International and National Developments*, many teachers are putting mathematics into realistic contexts, and being encouraged to do so. The Australian States and Territories have been helping practitioners in this respect by funding school-based programs and the development of appropriate materials. For example, as part of South Australia’s *Contextualising Mathematics Focus Schools* project in 1998, units of work were produced as guides for teachers. These illustrated and developed basic approaches for using everyday contexts within the experience of most children. While the units were developed primarily for schools with high proportions of Indigenous students, they are a useful resource for all teachers, and demonstrate ways that contextualised mathematics can be used to benefit all children.

Mulligan and Thomas (1995) undertook a critical review of current assessment practices, looking at multiplication and division in particular. They found that when young children were given a chance to demonstrate their knowledge in a variety of ways and contexts, they demonstrated an understanding of the structure of numeration and number operations far beyond the traditional expectations of curriculum developers and teachers.

The project *Research to Establish the Nature and Extent of the Relationship between a Student’s Mathematical Knowledge and Skills, and the Capacity to Use Mathematical Ideas and Techniques in Other Contexts: Strategic Numeracy Research and Development Project*, WA examines links between children’s understandings of mathematics and contexts. It is a cross-sectoral project in Western Australia, focusing on 800 children in the middle years. The relationship between children’s mathematical skills and their capacity to use these skills in situations other than in a mathematics classroom, and specifically in other lessons are being examined. The following questions are being addressed:

- Is there a relationship, and if so what is the nature of the relationship, between children’s achievement in numeracy basic skills tests, school mathematics performance, and their capacity to use mathematics to do numeracy-rich tasks across the curriculum?
- Does dealing with the numeracy issues explicitly, within other learning areas, have an impact on children’s numeracy learning?
- How do children deal with the numeracy demands of learning areas other than mathematics?
- How can teachers assist these same children to cope with these numeracy demands and assist children to improve their numeracy?
Grier (1993) investigated the gulf between children’s in and out of school numerical understandings. Differences between socially constructed number ideas and school mathematics were highlighted. Watson and Moritz (2000a) conducted research with Years 3, 6, and 9 children on the topic of statistical analysis in which words like ‘sample’ and ‘bias’ have colloquial meanings. One of the conclusions was that using examples from children’s personal experiences and the media should motivate the questioning attitudes required of future citizens. As an outcome of a further study, Watson and Moritz (1999a) also recommended that teachers develop tasks and ways of working that encourage children to continue to use intuitions built from experience to relate the data in graphs to real contexts. The researchers argued that such tasks should be used to encourage children to combine the traditional statistical skills of graphing with sensible out-of-school knowledge.

Whilst the evidence to date is sparse, the research findings discussed above imply that setting numeracy tasks in contexts that resonate with children’s experiences is beneficial to their conceptual understanding. More work is needed in this field.

**Chapter overview**

In this chapter, Australian primary numeracy research about teachers, children, and classrooms was reviewed.

In summary, the research on teachers revealed that:
- a set of Australian standards has been developed to describe excellence in the teaching of mathematics;
- from a range of research studies and projects, several characteristics of effective teachers of numeracy have been identified. There is consistency with the findings from international projects (see, also, Figure 2);
- for effective numeracy teaching, teachers need sound mathematical and pedagogical content knowledge. Levels of confidence, and beliefs and attitudes towards mathematics, are also important. Professional development programs and pre-service teacher education courses need to focus on these dimensions; and
- teacher involvement in research projects has been shown to be an effective means of professional growth.

The research on children has indicated that:
- among children with high mathematical potential, higher level thinking is promoted through problem solving in small group settings;
- students’ mathematical understandings are enhanced when teachers recognise the value of — and establish links with — children’s informal, out-of-school mathematics experiences;
- more positive attitudes and beliefs are associated with greater enjoyment, motivation, and higher numeracy achievement; less traditional teaching approaches foster more positive attitudes towards mathematics;
- one-to-one interviews and screening tests are an effective means of identifying children at risk;
• using individual children’s current knowledge as a basis for teaching is effective; and
• there was some evidence of a positive relationship between reading ability and mathematics achievement; children need opportunities to make links between their everyday worlds and the formal language of mathematics.

Findings from research on classrooms and related factors show that:
• there have been many early years numeracy projects focusing on the identification of children at risk and in the provision of intervention based on identified developmental learning frameworks; there have been some middle years and transition projects;
• small group learning settings can be more effective for some children than for others;
• small groups enable: interactive discussions in which explanations are supported with evidence; engagement with challenging mathematical content; co-operation with students using each other as resources, asking questions and checking information; and close monitoring by teachers with on-going assessment of children’s higher order thinking and conceptual development;
• evidence that practical teaching aids do not always help children to develop mathematical understanding; there is the possibility of cognitive overload and for misconceptions to develop;
• children who used calculators freely developed more sophisticated and flexible understandings than children who did not; teaching changes need to accompany calculator use;
• there is a lack of professional development and of supporting information on ways to integrate computer-based activities with other learning activities;
• it is not yet clear whether computer use enhances children’s numeracy learning outcomes;
• textbooks continue to be widely used to shape much of the teaching of numeracy;
• children can construct important mathematical ideas through solving well-designed novel problems;
• discussions arising from open-ended tasks and ‘good questions’ are advantageous for students’ learning of a wide range of concepts; children’s talk provides opportunities for error diagnosis and formative assessment; and
• open-ended tasks with appropriate scoring rubrics that are aligned to state and national curriculum standards can be used in large-scale assessment.
Chapter 5

Research on Curriculum and Processes

The specifics of classroom technique and strategy depend on the particulars of the teacher, the student, their relationships, and the material to be taught. It is worth underscoring that no technique will in and of itself necessarily lead to successful learning. (Schifter & Fosnot, 1993, p. 11)

The focus of this chapter is children’s conceptual development and mathematical thinking, together with curriculum issues related to numeracy. Children’s conceptual development is discussed under the key content strands of the Australian primary mathematics curriculum, as well as in terms of children’s developmental frameworks — that is, research on the typical sequences of development that children go through when learning different aspects of mathematics.

Concept development

The development of conceptual understanding is a major aim of mathematics teaching. Many mathematics educators emphasise the need to move away from what Mulligan (1990) calls the manipulation of “meaningless symbols” and the traditional predominance of “naked numbers” (p. 5) in order to put more emphasis on the development of understanding of concepts. This theme appears frequently in the research reviewed below in the key content strands of the mathematics curriculum: Algebra, Chance and data, Number, Measurement, and Space.

Algebra

There is some inconsistency in curriculum documents, at both state and national levels, in the Algebra strand. For example, A National Statement on Mathematics for Australian Schools (Australian Education Council, 1991) shows Algebra as a strand that should be addressed from the beginning of schooling in Bands A and B. On the other hand, Mathematics — A Curriculum Profile for Australian Schools (Curriculum Corporation, 1994) begins the Algebra strand at secondary levels. This is echoed in some state documents such as the Victorian Curriculum & Standards Framework II: Mathematics (Board of Studies, 2000). Within states there has also been some variation over time.

The level at which curriculum developers suggest that the teaching of algebra should commence also depends on how they wish to portray it. If algebra is seen as a way of thinking and communicating or a study of pattern and relationships, then it is more likely that initial experiences belong in primary school. If, however, algebra is seen as processes of symbolic manipulation, then it is likely to be placed later in the curriculum. These two perspectives are evident in relevant curriculum documents.

For example, A National Statement on Mathematics for Australian Schools (Australian Education Council, 1991) acknowledged a need to make links between algebra and all other strands, asserting that
Basic patterns of algebraic thinking are developed during the primary years. For example the notions of “in general”, “variation” and “function” and “unknown quantity” are implicit in much of the work of every other strand. These ideas should be fostered by emphasising algebraic thinking throughout Bands A and B. (p. 190)

The Victorian Curriculum & Standards Framework: Mathematics (Board of Studies, 1995), since superseded, presented a different perspective, stating that

> Centuries of mathematical work in geometry and arithmetic preceded the acceptance of the symbolism required for algebra to develop. Not surprisingly then, algebra is offered to students only after they have met the basic practices within the space and number strands. (p. 89)

Whichever perspective is taken, such documents acknowledge that algebraic understandings have their beginnings in early number work. For example, the authors of the Curriculum & Standards Framework: Mathematics (Board of Studies, 1995) states that “Beginning at level 5, [algebra] continues the work described in the number patterns and relationships substrand within levels 1–4 of the number strand” (p. 89). According to Herbert and Mousley (1997)

> Algebra is not a new topic for primary teachers and students — but a new emphasis that needs to be placed on ways of thinking about and representing mathematical ideas, and especially number ideas, during Years 1 to 8. When we encourage students to abstract, generalise, and/or predict, we are demanding algebraic thinking. When children explore patterns and record what they have found, they are working algebraically. When students double quantities in cooking, draw a graph to represent number of pets, or summarise their findings about the weather and use these to make predictions, they are working algebraically. Activities such as these involve representation of significant results and patterns in mathematics and/or everyday experience. Thus it is most important to recognise that much of what is done already in primary school mathematics classes is algebra. (p. 130)

It is expected that teachers of the foundation years understand their curriculum in terms of its role as providing a solid footing for later mathematics and lifetime use. The question is not so much about the level at which algebra is recognised as a strand in the curriculum, but about providing every student with rich learning experiences that will build appropriate understandings, language and skills (pedagogical and social, as well as mathematical) that can be drawn on and developed further as mathematics becomes more abstract.

Stacey and her colleagues have done much of the recent Australian work in this area. In the project entitled The Cognitive and Linguistic Demands of Learning to Use Algebra, Stacey’s research team studied over one thousand middle school and high school students learning algebra. They developed supplementary material to address difficulties arising from students’ reliance on intuitive language processing. Stacey and MacGregor (1997) and MacGregor and Stacey (1999) report how number work in the middle years of schooling can be extended to better prepare students for formal algebra. They nominate five aspects of number knowledge as being essential for algebra learning:

- seeing the operation, not just the answer;
- understanding the equals sign;
- understanding the properties of numbers;
- being able to use all numbers, not just whole numbers; and
- working without a practical context.
The authors provide classroom activities suitable for the middle years that highlight important features for classroom teaching together with samples of student work.

Swafford and Langrall (2000) focused on the third of Stacey and MacGregor’s aspects of number knowledge, the notion of equality. They investigated Year 6 students’ use of equations to describe and represent problem situations, prior to any formal instruction in algebra. The students involved demonstrated a capacity to generalise the problem situations and to write equations using variables, albeit in non-standard form. This implies that the concepts of variable and constant could be developed informally in primary education, but — more importantly — that equality must be taught well. This same point is made, and illustrative examples are presented, by Herbert and Mousley (1997).

Boulton-Lewis (1999) focuses on how children make sense of primary mathematics. She suggests that examining the strategies children use, demonstration and explanation of processes by teachers, effective choice and use of materials, and consideration of the demands made on children’s capacities to abstract and generalise information, are all aspects of primary mathematics that can help children make a smooth transition from arithmetic to algebra. Boulton-Lewis further suggests that there is an urgent need to rethink what we are doing, remove complicating strategies and unnecessary materials and teach so that all aspects of mathematics in the early years are seen as part of a meaningful system. We should also keep in mind the demand that various operations and strategies will make on children’s capacity to process information but at the same time recognise that motivation, provided by ownership of real problems and acknowledgment of children’s existing mathematical knowledge, might allow children to explore and make better sense of mathematics by themselves with a little help. (Boulton-Lewis, 1999, pp. 12–13)

Warren and English (2000) also see primary school children’s knowledge of arithmetic structure as foundational knowledge for their understanding of algebra. They studied 94 students in their final years of primary school, examining students’ understanding of and capacity to use arithmetic structures such as associativity — for example (6 + 4) + 3 = 6 + (4 + 3) — and commutativity — for example 6 x 4 = 4 x 6 — as well as their abilities to generalise problems. Warren and English identified a well known shortcoming of primary mathematics teaching: namely that students fail to abstract from their experiences the mathematical structures that are necessary for them to make a later successful transition from arithmetic to algebra. The authors suggest that the reasons for this could be:

• a focus in early mathematics education on discovering relationships rather than also exploring non-relationships;
• little opportunity for students to explore their own conjectures and inductions;
• teaching mathematics in a non-calculator supported environment making it difficult to explore the non-relationships that exist in subtraction and division;
• children making decisions purely on computational grounds without recognising the mathematical structures represented by number sentences;
• limited understanding of the notion of equality; and
• children not being able to express patterns in everyday language.

Mousley (2003) identifies reasons for a similar phenomenon observed in Year 6 classrooms, including the following:
• typical primary lessons focusing on finding the answer to specific sets of problems and not on looking for, describing, and making use of generalisable processes and structural properties of arithmetic;
• teachers’ own education (including primary and secondary schooling, but also pre-service teacher education and professional development) that has generally not encouraged conceptual, relational understanding of such processes and properties;
• resources used by teachers, including textbooks, reproduced worksheets, computer software, and curriculum outlines that almost invariably present mathematics as discrete topics and examples; and
• traditional lesson formats that do not allow a substantial part of the lesson (such as the last third) for discussion of links between problems, processes used by the children, underpinning concepts, and related ideas.

It should be noted that it is not only arithmetic that can be used to develop children’s ability to think algebraically — other aspects of the primary curriculum can also be used to develop the necessary ideas (see, for example, Outhred, 1993).

Overall, research in this area suggests that students fail to abstract from their primary school experiences the mathematical structures that are necessary for them to make a later successful transition from arithmetic to algebra.

**Chance and data**

While the topic of *Chance* (the study of probability) is not included formally in all Australian primary curricula, children in all states and territories study *Data*, using activities that teach them how to collect and record data (through observations and simple surveys), represent data (using written descriptions, pictures, tables, and graphs) and interpret their own and others’ representations of data.

The largest body of Australian research in the area of primary probability and statistics over the last ten years is that of Jane Watson and her colleagues in Tasmania. A summary of the foci of Watson’s projects is presented in Table 5. This body of inquiry and literature is remarkable given that this is only a selection of the works available, and that Watson and Collis wrote in a 1992 Australian Research Council research proposal that

The 1991 mathematics curriculum for Australian schools devotes 20% of its content to Chance and Data but no research has been carried out in Australia to determine the suitability of the suggested topics for students.

### Table 5.
**Research into probability and statistics by Watson and colleagues**

<table>
<thead>
<tr>
<th>Projects</th>
<th>Researchers</th>
<th>Details</th>
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<tbody>
<tr>
<td>Cognitive functioning in probability and statistics and its relationship to the school curriculum</td>
<td>Watson &amp; Collis</td>
<td>• developed a cognitive model of student and teacher understanding of probability and statistics</td>
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<td></td>
<td></td>
<td>• suggested ordering of topics in the curriculum</td>
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<td></td>
<td></td>
<td>• provided assessment procedures to use in longitudinal evaluation of the curriculum</td>
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<tr>
<td>Projects</td>
<td>Researchers</td>
<td>Details</td>
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| The assessment of higher order cognitive functioning during co-operative learning in probability and statistics | Watson & Collis                   | • developed methods for the assessment of higher order processing in probability and statistics  
• undertook a longitudinal evaluation of the implementation of the curriculum |
| The development of school students’ understanding of statistical variation | Watson & Shaughnessy              | • developed a longitudinal and cross-sectional model of school students’ understanding of the concept of statistical variation  
• trialled and evaluated teaching packages on variability  
• documented associations between outcomes related to variation and others in the chance and data curriculum |
| The chance and data curriculum and the development of school students’ statistical understanding: 1993–2000 | Watson                           | • evaluated the development of school students’ statistical understanding and made suggestions for future reforms  
• analysed a database of student responses to questions in probability and statistics  
• examined longitudinal data to model development of student understanding  
• produced digitised video extracts for use in further research |
| Data representation and interpretation by primary school students working in groups | Chick & Watson (2001)            | • related levels of students’ interpretation of a data set to their representation skills  
• sought an association between success and the types of collaboration observed in groups |
| The relationship of the concept of fair to the construction of probabilistic understanding | Lidster, Watson, Collis & Pereira-Mendoza (1996) | • focused on Years 3 to 9 students’ concepts of fair  
• found that most students had very little idea of how to test whether the dice were fair |
| Graphical representations of statistical associations by upper primary students | Moritz (2000)                    | • explored Years 4, 5 and 6 students’ graphical representations of statistical associations  
• described three levels of representing bivariate associations and of representing multivariate associations: unsuccessful, partial, and complete |
| The conjunction fallacy and longitudinal development of chance expression | Moritz & Watson (1999)           | • analysed Years 5 to 11 students’ understanding of various conditions and their conjunctions (such as averages)  
• revealed that chance expression improved with grade, and that incidence of conjunction errors was not associated with grade nor with chance measurement developmental level |
| Graphs: Communication lines to students?                              | Moritz & Watson (1997)           | • described comprehension difficulties by Years 6, 8, 9 and 11 students of graphs in authentic contexts |
| Development of the concept of statistical variation: An exploratory study. | Torok & Watson (2000)            | • documented Years 4, 6, 8 and 10 students’ understandings of aspects of variation present in three settings: an isolated random sampling situation and two real-world sampling situations  
• described four levels of responses and implications for teaching and future research |
<p>| Data cards: An introduction to higher order processes in data handling  | Watson &amp; Callingham (1997)       | • analysed Year 6 students discussion of data collection as a method of answering questions |</p>
<table>
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<tr>
<th>Projects</th>
<th>Researchers</th>
<th>Details</th>
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<tbody>
<tr>
<td>Factors influencing the outcomes of collaborative mathematics problem solving: An introduction</td>
<td>Watson &amp; Chick (2001)</td>
<td>• noted factors that influence the effectiveness of collaboration on open-ended probability and statistics tasks</td>
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<td></td>
<td></td>
<td>• identified phenomena that influence cognitive functioning, including cognitive, social, and external factors</td>
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<tr>
<td>Multimodal functioning in understanding chance and data concepts</td>
<td>Watson &amp; Collis (1994)</td>
<td>• compared two groups’ understandings of graphical presentations and ideas about the fairness of dice</td>
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<tr>
<td>Longitudinal development of chance measurement</td>
<td>Watson &amp; Moritz (1998)</td>
<td>• described the development of understanding of chance measurement</td>
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<tr>
<td>The development of concepts of average</td>
<td>Watson &amp; Moritz (1999d)</td>
<td>• examined Years 3 to 9 students’ ideas of average in differing contexts and noted the structural complexity of concepts</td>
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<td></td>
<td></td>
<td>• documented the prevalence of ideas associated with mean, median and mode</td>
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<td>• proposed that the concepts of mode and median should be introduced before mean in the school curriculum</td>
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<tr>
<td>The beginning of statistical inference: Comparing two data sets</td>
<td>Watson &amp; Moritz (1999c)</td>
<td>• described Years 3 to 9 students use of statistical inference</td>
</tr>
<tr>
<td>Development of understanding of sampling for statistical literacy</td>
<td>Watson &amp; Moritz (2000b)</td>
<td>• described the development of understanding of sampling using a longitudinal survey of students from Years 3 to 11</td>
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<td>• articulated a three-tiered framework for statistical literacy: defining terminology, applying concepts in context, and questioning claims made without proper justification</td>
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<tr>
<td>The longitudinal development of understanding of average</td>
<td>Watson &amp; Moritz (2000c)</td>
<td>• explored Years 3 to 9 development of understanding of the concept of average and identified six levels of response</td>
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<tr>
<td>Interpreting and predicting from bar graphs</td>
<td>Watson &amp; Moritz (1999a)</td>
<td>• analysed Year 3 students’ interpretations of a bar graph</td>
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<td></td>
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<td>• identified three levels of graph comprehension: reading the data, reading between the data, and reading beyond the data</td>
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<tr>
<td>Developing concepts of sampling</td>
<td>Watson &amp; Moritz (2000a)</td>
<td>• recorded Years 3, 6, and 9 students’ constructions of the concept of sample</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• identified and described six categories of sophistication</td>
</tr>
<tr>
<td>The role of cognitive conflict in developing students’ understanding of chance measurement</td>
<td>Watson &amp; Moritz (2001)</td>
<td>• explored the use of cognitive conflict in improving students’ understanding of chance</td>
</tr>
<tr>
<td>The development of chance measurement</td>
<td>Watson, Collis &amp; Moritz (1997)</td>
<td>• explored students’ understanding of chance measurement</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• developed a model for understanding chance measurement</td>
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Curriculum and Processes

<table>
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<tr>
<th>Projects</th>
<th>Researchers</th>
<th>Details</th>
</tr>
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</table>
| Assessing statistical understanding in Grades 3, 6 and 9 using a short answer questionnaire | Watson, Collis & Moritz (1994)               | • assessed students’ understanding of statistics and probability in Years 3, 6 and 9  
• discussed response differences and associated levels and types of cognitive functioning |
| Children’s understanding of luck                                        | Watson, Collis & Moritz (1995)               | • reported Years 3, 6 and 9 students’ understanding of the concept of luck  
• proposed a structure and implications for curriculum and teaching practice |
| Longitudinal understanding of conditional probability by school students | Watson & Moritz (1999b)                      | • examined Years 5 to 11 estimates of probability or frequency of conditional events  
• noted improvement in expressing probability numerically and in distinguishing conditional events, that events were better distinguished by the frequency than probability, and that understanding of conditional probability was related to development of basic chance measurement |

An over-riding point made by Watson and her colleagues is that children need to focus on the study of probability in the context of their everyday experiences, as they develop only a narrow understanding of chance events from “marbles in urns”. The ways that statistical words such as *sample*, *fair*, and *average* are used loosely in the media impacts on children’s understanding, and teachers need to have the knowledge and strategies to address such issues. They also need to know how to capitalise on (and confront where necessary) children’s intuitions — a point supported by the work of Way (1998). As Watson and Callingham (1997) noted, “Support and professional development will be required to assist teachers to achieve confidence in this area” (p. 16).

As a result of their research into data representation — graphing — Moritz and Watson (1997) point out that students need to be challenged in the classroom through the use of non-standard graphs, even those with errors, to question why the author has represented a message in certain ways, in order to be better prepared to notice misleading representations. Watson’s research team believes strongly that there is a need to include data handling activities such as graphing at all levels, and to connect them to the goals of the curriculum (Watson & Chick, 2001; Watson & Pereira-Mendoza, 1996).

Much of this work identifies and describes common patterns of understanding and misunderstanding about data representations. Pereira-Mendoza, Watson and Moritz (1995), for example, examined the interpretations of different graphs by students from Canada, Australia and Singapore and found three common misconceptions: the belief that graphs must have a pattern; difficulty in representing or interpreting intermediate values on line graphs or in data sets; and misuse or eccentric use of information. Chick and Watson (1998) express their concern that despite having previously encountered and produced various representational forms, virtually all students in a Year 5/6 class were unable to transfer that experience without being cued by the researchers. The authors stress the need for teachers to make these connections explicit in the upper primary years.

Asp, Dowsey and Hollingsworth (1994) researched students’ ability to read and interpret bar graphs, and use them for making predictions. Interview tasks and questions were developed to explore upper primary and lower secondary children’s prior knowledge, to
observe how they handled cases of missing data, and also their knowledge of scale and pattern. Results show that students had fairly well developed skills in reading, interpreting and predicting from pictograms and bar graphs, with increasing facility as ability level and year level increase. Results point to the need for future work on the effects of prior knowledge, scale, and pattern on students’ comprehension of pictograms and bar graphs.

In other work, Gerber and Boulton-Lewis, in an Australian Research Council-funded project entitled *Graphically Represented Quantitative Information in a Cognitive and a Didactive Perspective*, studied how graphical representations of quantitative information are conceived by teachers and students. In a New Zealand study, Rodrigues (1994) found that the majority of primary school children involved in the *Data Handling in Primary Science* project saw graphs in science as an end product to be displayed rather than as a record of what had been found that could be interrogated further. The children appeared to have a very limited understanding that different types of graph can be employed for specific purposes, and that this choice should be determined by the type of variable involved.

Given the importance of graphical and statistical literacy in both everyday numeracy and numeracy across the curriculum, children’s limited understandings in this area suggest that this should be a priority for future research and teacher professional development.

A further body of research related to Chance has been the work of Kath Truran and the late John Truran in South Australia. Based on her research into the thinking processes of children, aged from seven to twelve years, in relation to *random generators* such as coins, dice and spinners, Kath Truran suggested a range of activities for developing these concepts. According to Truran, teachers in selecting appropriate learning tasks should not assume particular levels of children’s understanding because of grade or age level. As well as having a good knowledge of their own students’ beliefs and misconceptions, teachers need to be aware of how much experience in similar activities the children have had as well as where the activity fits in the curriculum (see Truran, 1997).

In his Masters thesis, John Truran (1992) documented the development of children’s understanding of probability and applied his findings to classroom practice. His thesis concludes with a proposed teaching sequence. This work demonstrates the deep-seated nature of children’s beliefs about probability, and the fact that good understandings are not age related. Truran’s doctoral thesis (2001) investigated the teaching and learning of probability in South Australian schools from 1959 to 1994. He developed a model for examining the forces operating in school systems and used this to investigate three separate aspects of the teaching of probability:

- a general survey of the history of the teaching of the topic from 1959 to 1994 in South Australia, other states of Australia, and other countries;
- attempts that have been made throughout the world to assess the understanding of probabilistic ideas; and
- the influence on classroom practice of research into the teaching and learning of probabilistic ideas.

As with other aspects of mathematics, teachers need to focus not only on processes but when it is appropriate to use them. Wiest (1998) drew attention to the fact that having students decide when probabilities can be determined theoretically and when they must be found experimentally is an important thinking skill grounded in real-world sense making. Wiest demonstrated how upper-primary children can get involved in designing,
implementing and interpreting experiments, claiming that this allows for children’s desire for autonomy and self-initiated investigation, as well as enabling the construction of mathematical knowledge. Such work needs to attend to common misconceptions and to typical strategies and beliefs (based on prior experiences) that tend to inhibit the children’s development of probability ideas (Taylor, 1995).
Primary Numeracy

Measurement

Each of the four-yearly reviews of research in mathematics education published by the Mathematics Education Group of Australasia (MERGA) has had a section on research into the teaching and learning of Measurement (see, for example, Lowrie & Owens, 2000). While there is less research in this area than in areas such as the learning of number, it is still a rich area for Australian research.

The measurement curriculum is divided into discrete content areas. Evidence of the fact that teachers do not always recognise the developmental nature of the different content areas is provided by Outhred and McPhail (2000) who interviewed a sample of teachers about their understanding of the measurement strand, with most indicating that they place little importance on structuring their teaching of measurement.

Research into the learning and teaching of measurement has again been influenced by the Count Me In program — this time Count Me Into Measurement, a program that has been implemented in two hundred schools in New South Wales using a teacher-facilitator model of professional development. Central features of the project are: a conceptual framework of six levels of increasing complexity; practical activities; and sample lesson plans for teaching length, area, volume, and mass, from Kindergarten to Year 6 (Outhred & McPhail, 2000). The framework and associated learning program

- aim to develop students’ understanding of measurement concepts and language before more sophisticated strategies and processes are introduced;
- are based on research findings that indicate the importance of students’ knowledge of the unit iteration structure; and
- focus on the use of informal measurement units to develop understanding of measurement basics.

According to Outhred and McPhail, the authors of the program, the role of the conceptual framework is to highlight general measurement processes and principles, rather than focus solely on the specific quantities. Several related activities, involving practical applications of estimation and measurement, are provided at each level. Sample lesson plans illustrate mathematical language and teacher questioning as well as key aspects of the proposed measurement, recording and reporting back processes. Evaluation of the materials by 38 schools across the state (154 teachers) was positive (Outhred, 2001). The teachers approved of the structured, sequenced material, and many commented that the program had changed their approach to teaching measurement. Further, teachers’ assessments showed improved learning outcomes for the children involved in the program.

The remainder of the research into measurement is discussed below under three specific areas — length, area, and time. There is insufficient research into volume, capacity, or mass to warrant separate treatment here.

Length

Bragg and Outhred (2000a) point out that linear measurement knowledge is essential for its application to perimeter, area and volume, and in topics that rely on the understanding of scales such as directed number gauges and graphs. Bragg and Outhred (2000a; 2000b; 2001) investigated students’ understanding of linear measurement, focusing on
informal and formal units and the processes used to measure length. By Year 5 the majority of the students were able to use informal paper clip units to measure length and to identify linear units. However, few students in Years 1 to 4 showed an understanding of the linear nature of units when they were asked to show a centimetre unit length in a variety of contexts. While most high-ability students had a conceptual understanding of length, the majority of the lower-ability students did not acquire important concepts relating to the linear nature of units and took longer to acquire basic measuring skills. These results indicate that teachers need to identify units explicitly when they are teaching measurement because many students do not seem to have abstracted this concept in earlier grades as assumed. Further, the researchers state that teachers should not rely on basic “read-and-draw” measuring tasks to assess students’ understandings of linear measurement.

In her doctoral thesis, Willis (1990) aimed to clarify and extend what is known of the conceptual framework that underlies the teaching of measurement. In a study involving 126 subjects in Years 1, 3 and 5, no relationship was found to exist between the children’s ability to measure length, their knowledge of dimensional adjectives, and their ability to conserve length. A second part of this study involved oral questionnaires administered to teachers. Willis found that teachers differ in their personal definitions of measurement and that these personal definitions determine their perceptions regarding issues such as the ways in which measurement should be taught, the age when children are able to measure length, the requisite skills and understandings children should demonstrate prior to measuring and the activities which best foster measurement.

The research of Boulton-Lewis, Wilss and Mutch (1996) calls into question the normal curriculum sequence and timing. Seventy young children took part in a study to determine the strategies and devices they would choose to measure length in a range of measurement tasks. The children used a variety of strategies, which did not correlate well with the order of skills listed in curriculum documents.

**Area**

Owens and Outhred (1998) investigated children’s area concepts with tasks that asked them to visualise the tiling of given figures — an important spatial idea used in area measurement. Half the children in Years 2 and 4 had difficulty visualising the tiling of shapes, but children who had participated in spatial activities were generally more successful. Children who drew the tilings were more successful on some items, but children’s drawings indicated a varying awareness of structural features such as alignment and tile size. The researchers concluded that activities aimed at teaching the “length by width” formula and similar area measurements may not be effective because children do not see the need to cover the area accurately. They argue that the row and column structure of rectangular arrays needs to be taught because the children do not necessarily see this in tiled areas (see also Outhred, 1993). In further work, Owens and Outhred (1997) asked older children (aged 7–10) to visualise the tiling of given figures with different shaped tiles. An analysis of children’s drawings suggested that there was development from beginning tiling from the sides and corners, to an awareness of the need for having no gaps, regular patterns, alignment of tiles, and consistency of tile size. It is well known that there is considerable confusion between area and perimeter. This phenomenon was investigated by Kidman (1999). Children in Years 4, 6 and 8 were given
area judgement tasks involving rectangles. Their responses were analysed in terms of operations used and strategies exhibited. The children used both additive (perimeter) and multiplicative (area) rules, and seven different strategies. Children using an additive rule tended to rely on rulers or fingers for measuring length and align the rectangles vertically, while those using a multiplicative rule tended to use overlay and partitioning strategies. The misconception of area of rectangles being dependent on the sum of the rectangle’s dimensions was fairly constant across the grades, with about fifty percent of the children from each level using the perimeter rule to determine area. The findings of this research suggest that students may not have sufficient opportunity to explore, with practical activities, the spatial foundations of area and perimeter, and the relationships between them. Kidman suggests that structured classroom activities of a practical nature (for example the use of geoboards, tiles and dot paper) be more widely used to develop the notions of area and perimeter, with a delay in the presentation of area and perimeter formulae.

**Time**

In one of the few studies of time identified, Boulton-Lewis, Wilss, and Mutch (1997) undertook an analysis of primary school children’s abilities and strategies for reading and recording time from analogue and digital clocks. A large number of children in Years 1 to 6 were tested, with older students also being asked to describe their strategies. A sequence of time acquisition was proposed, based on a recent theory of cognitive development, namely hour, half hour, quarter hour, five minute, and minute times. Times after the hour would be more difficult and digital times would be learned sooner. The sequence was confirmed for Years 1 to 3, but irregularities occurred in Years 4 to 6.

In some research particularly relevant to teachers, O’Toole (1997) documents children’s thinking about time in one low SES, high ESL school, and analyses planning structures used to support and challenge their thinking. Teaching strategies, which enable students to experience success are identified, with specific focus on ESL students. Planning frameworks feature: the mathematics involved in time; stages of student thinking; description of materials and strategies for their effective use starting points; and questions to challenge students thinking.

Overall, Australian research into measurement over the past decade highlights the importance of devoting sufficient time to the development of underlying concepts before moving to paper and pencil activities and formulae.

**Number**

Those who set out to make their pupils “numerate” should pay attention to the wider aspects of numeracy and not be content merely to develop the skills of computation. (Cockcroft, 1982, paragraph 39)

While it is common for people to think of numeracy only in relation to number, in Australia the term numeracy usually incorporates other areas of mathematics, as well as people’s ability and disposition to make effective use of their mathematical skills (see, for example, Australian Association of Mathematics Teachers Inc., 1997). Nevertheless, the number component of primary mathematics is generally acknowledged to be a major part of the curriculum, and this is reflected not only in the time that teachers allocate to number
activities but also in the number of research projects and papers that were identified in this area.

**Counting**

The early years of schooling are crucial in providing a positive start to students’ numeracy learning, with counting and numeration being important foci for research into young children’s learning of number.

The work emanating from the United States from the team of researchers led by Les Steffe at the University of Georgia (see, for example, Steffe, von Glasersfeld, Richards & Cobb, 1983; Steffe, Cobb & von Glasersfeld, 1988) and from Karen Fuson at the Northwest University, Illinois (for example, Fuson, 1982; 1990) has been major influences on Australian research into children’s early number learning. In particular, research carried out in the United States into children’s construction and elaboration of the number sequence and the application of different counting strategies to addition and subtraction problems has been adopted as a basis for much of the Australian research in this area.

In their development of a learning framework of key aspects of early numeracy learning, the research team for the Early Numeracy Research Project (ENRP) studied a wide range of research into stages and levels of young children’s mathematics learning, and acknowledged Steffe’s foundation work. **Counting** is one of the domains described within the *Number* strand in the ENRP. Clarke, Sullivan, Cheeseman, and Clarke (2000) identify six growth points for counting. These were used by the research team in structured interviews that were developed both for the purposes of assessment of learning and as a strategy for teachers to engage with children’s learning. The growth points identified were:

- rote counting;
- counting collections;
- counting by 1s (forward/backward, including variable starting points; before/after);
- counting from 0 by 2s, 5s and 10s;
- counting from x (x > 0) by 2s, 5s and 10s; and
- extending and applying counting skills. (p. 192)

Wright (1991b) discusses Steffe’s five-stage model of children’s construction of the number sequence, namely: perceptual, figurative, initial number sequence, tacitly-nested number sequence, and explicitly-nested number sequence. Wright points out the relationship between these and Steffe’s five counting *types* — perceptual, figural, motor, verbal, and abstract. He provides characterisations of children at each of the five stages, together with activities that are intended as indicators of appropriate tasks for use with children at each stage. According to Wright, his theoretical model could be applied to number topics in the school curriculum, with the aim of improving students’ understanding and hence learning outcomes. Wright suggests that less emphasis should be put on current topics and activities such as the study of pattern, matching and ordering; with more emphasis (and classroom time) on the types of activities that would lead to stronger counting facility. Importantly, Wright questions the common view that problem solving and abstract mathematics are inappropriate for children who are just beginning school and suggests that there is a need to give problem solving a more central role in the early years.
In a later article, Wright (1996) suggests that current curricula and practice have not taken sufficient account of the research into early number learning and that teaching needs to be more informed by a knowledge of young children’s arithmetical thinking and how it is likely to advance (p. 36). In this paper, Wright reports on a study designed to investigate the effectiveness of an instructional approach that used activities focusing on advancing young children’s arithmetical knowledge in a collaborative, problem-centred setting in one first-year class in 1992. Using a methodology based on that adopted in second and third grade classes in the United States by Cobb, Yackel, and Wood (1992), children worked on a carefully sequenced set of activities over a period of seventeen weeks. They worked initially in a whole class situation only, but then used a pattern of whole class teaching, followed by children working in similar-ability pairs to solve arithmetical problems, concluding with a substantial period of teacher-led class discussion of solution strategies. Interviews based on the five-stage model developed by Steffe et al. (1983) were used to assess the children’s arithmetical knowledge before, during (mid-way), and after the program. Important gains were made by virtually all of the children over the course of the year, with relatively large gains made by the children who were initially most advanced. Comparison with children undertaking regular instruction in an earlier study suggests that children in this study made advances significantly beyond those usually made by first year children and that this could be attributed to the instructional approach. Significant features of the approach in this study were the incorporation of learning activities derived from recent research, a more challenging and extending approach to the teaching of number, and the proportion of time given to the drawing out of underpinning mathematical concepts in whole-class discussions.

These same features, including the use of a problem-centred or inquiry-based approach, were continued in the Mathematics Recovery project, a one-to-one withdrawal program developed for at risk first grade children. The aim was to advance their arithmetical knowledge to a level at which they were likely to learn successfully in a regular class. Almost all children taking part in the program were reported to have made major progress in terms of the arithmetical stages, with overall progress of participants notably exceeding that of their counterparts (see, for example, Wright, Stanger, Cowper & Dyson, 1996).

The Count Me In Too early numeracy program supplements the theory and methods of Mathematics Recovery. It synthesises research in early number learning with research into professional development, drawing on both Australian and international research. Stewart, Wright, and Gould (1998) report on a 1997 study of 866 Kindergarten (school entry) students, including 47 Aboriginal or Torres Strait Islanders, from schools identified as having low socio-economic indicators. These schools were selected to participate in the Count Me In Too project. Based on a comparison of initial and final assessments of children’s progress in early arithmetical strategies, forward number word sequences and numerical identification, the majority of the students were found to have met or exceeded expected syllabus outcomes. It is important to note that, consistent with earlier and smaller-scale studies, many students were found to begin the Kindergarten year already able to complete the tasks that were expected of them at the end of the year, although there was much diversity in levels. This supports the claim often made by researchers that many children are seriously under-challenged in their first year of school.

In Victoria, the Mathematics Intervention program, developed in 1993, featured elements of both Mathematics Recovery and Reading Recovery (see, for example, Clay, 1987). When first implemented in 1993, the Mathematics Intervention program tested children in
Year 1 who were considered at risk. This was later extended to Years 2, 3 and 4 (see Pearn, 1998; Pearn, Hunting, Merrifield & Mihalic, 1997). The initial ten-minute clinical interview included tasks that ascertained the facility of the children’s verbal counting skills, their knowledge of the number word sequence, and tasks that would help ascertain their counting stage level. Results from 1993 to 1998 indicate that most Year 1 children were successful in counting forwards by ones to twenty, and backwards by ones from ten, counting patterns of dots, and counting out exactly fourteen beads. They were less successful identifying the numbers between the numbers six and twelve or determining numbers before or after a given number.

There have been some Australian studies of counting capabilities and the development of subsequent mathematical skills of children with disabilities. Some of this work has tackled the assumptions that are frequently associated with particular disabilities. Warrell (1994), for example, questioned the belief that children with Down syndrome are not capable of abstract work in mathematics. Working in a teaching program with three children with Down syndrome, Warrell used counting principles to assist in the development of their number concepts. The study explored the possibility that teachers can guide these children to a conceptual level of counting competence rather than rely on drill and practice methods to produce rote learning. Results indicated that the children benefited from the program and that this aim was achieved. Warrell suggested that similar teaching tasks could be transferred to the classroom. This confirms the importance of basing classroom practice on research findings, as well as the need for teachers to be able to assess young children’s stages of development in order to create challenging activities that will take all children well beyond their current levels of understanding.

Pepper (1993) examined the relationship between preschool children’s counting competence and their ability to partition discrete quantities. According to Pepper, as well as counting skills, another skill that has the potential to develop markedly during the early childhood years is the use of a “dealing” strategy to distribute groups of discrete items (one basis of division). In her study, 76 preschool children participated in two interviews, one designed to investigate children’s sharing strategies and the other to examine their counting abilities. No significant relationship was found between children’s counting competence and their ability to distribute discrete items equally. In another study (Pepper & Hunting, 1998) 25 preschool children participated in an interview in order to examine what strategies other than counting they may have used to establish equal shares. Sharing tasks were designed to eliminate the need for counting, allowing for the use of visual cues such as subitising (for example, automatically recognising a group of six dots as “six”, as we do with dice or dominoes), or the use of estimation or measurement to establish equality. The research suggests that dealing competence does not relate directly to counting skills. Pepper and Hunting conclude that not only should teachers be able to involve young children in sharing problems regardless of their counting abilities, but that dealing tasks may assist children’s development of counting through the opportunities they present for children to check the size of shares.

Boulton-Lewis (1993) investigated the relationship between sequence counting and knowledge of place value in the early years of school. Basing her work on Fuson’s characterisation of sequence counting, and on the three levels of abstract counting described by Siegler and Robinson (1982), Boulton-Lewis interviewed 55 children in Years 1, 2 and 3, both early and late in the school year, in order to test their knowledge of the counting sequence and place value. Teachers were also interviewed to establish their
learning objectives for counting skills and knowledge, and for their teaching of place value. The results of work with children in this project showed that ability to count was closely related to place value knowledge, but that the level of sequence counting was more closely related to ability to explain place value than to knowledge of the counting sequence itself. Boulton-Lewis suggests that the difference in ability to explain counting and place value may be due to curriculum guidelines and teachers’ consequent practice of encouraging discussion when children are using materials to represent place value, but not discussing the structure of the counting sequence. While it appears that by Year 3 many children had worked out for themselves the structure of the number system, Boulton-Lewis stresses the importance of talking about it, as well as the need to make explicit for children the overall relationships between various mathematical concepts.

An area of concern arising from this study and others is the fact that most children counted well beyond both the levels outlined in curriculum documents and teachers’ expectations. As part of the Calculators in Primary Mathematics project, an extensive questionnaire was used to explore the extent to which teachers changed their expectations of children’s mathematical performance during their involvement in the project. Questionnaire results from seven teachers of Prep to Year 2 during their first two years in the project showed an increase in expectations for most of the items dealing with counting and large numbers, with a greater increase for the items dealing with negative numbers (Groves & Cheeseman, 1992). In general, teachers’ increased expectations reflected their observations of children’s performance. Nevertheless, teachers’ predictions remained conservative compared with actual levels of performance, with children performing much better than predicted by their teachers on items such as:

- continue 5, 10, 15, 20,...;
- count from 389 to 407;
- count in 10s from 960 to 1050; and
- count backwards in 10s from 50 to –70.

The items relating to counting that showed the greatest increase in expectations both involved counting backwards. Counting backwards on calculators was often cited by project teachers as leading many children to “discover” negative numbers — the area in which the greatest positive change in teachers’ expectations was observed.

One of the major ways in which the calculator was used in the project, especially with younger children, was as a tool for counting. This was reported as one of the unexpected findings. Children used the built-in constant function, which allows counting by any chosen number, to count from any desired starting point. So, for example, keying in $1 + = = = = =$, results in the numerals 1, 2, 3, ... being displayed successively on the calculator. Similarly, keying in $1 - 2 = = =$ results in the numerals –1, –3, –5 being displayed.

Groves (in press) draws on Pea’s (1985) distinction between the potential for technology to act in two different ways:

- as a cognitive amplifier — i.e., to “change how effectively we do traditional tasks, amplifying or extending our capabilities”; and
- as a cognitive re-organiser — i.e., as a tool whose use can “fundamentally restructure the functional system for thinking” (Pea, 1985, pp. 168; 170)

Groves focuses on some of the ways in which the calculator appeared to act as a cognitive re-organiser. A particularly striking example is recorded on the videotape Young
**Children Using Calculators** (Groves & Cheeseman, 1993), where a five-year-old boy is using his calculator to count by ones from one million during a “free exploration” time. When challenged by another child to reach “one million one hundred”, he initially said that there was no such number, but as he got to 1,000,079 he began to think that perhaps there was. When he finally reached 1,000,102 he was thrilled to see that he had “gone right past it”. While this boy clearly had an excellent grasp of counting and number recognition for a child in Prep, and could possibly have been able to carry out a similar activity without a calculator, it is extremely unlikely that he would have done so. According to Groves (in press), the calculator amplified and extended the boy’s capabilities by providing a dynamic display without the need to painfully record each number, thus allowing him to focus on thinking about the results and also to restructure his knowledge of large numbers well beyond one million. For more details of the many different ways in which calculators were used as a counting tool see Stacey (1994b) and Groves and Cheeseman (1995).

Overall, while much of the Australian research on counting has grown out of, and continues to be closely linked to, research in the United States, it now makes a significant contribution to knowledge in the area. Research on counting has also formed the basis for much of the research discussed elsewhere in this report, particularly in the sections on **Intervention**, **Developmental frameworks**, and **School-entry assessment**, as well as for significant state government projects and professional development programs.

**Place value and the number system**

Understanding place value is not a matter of simply “cracking” an arbitrary written code following adult explanation or some degree of exposure to computation. It is indissolubly linked to understanding the number system itself. Grasping it implies understanding a multiplicative recursive structure. (Sinclair, Garin & Tieche-Christinat, 1992, p. 93, cited in Thomas & Mulligan, 1999, p. 478)

Place value is one of the critical keys to an understanding of our number system. Traditionally place value has been considered important because of its use in standard written algorithms, but more recently an understanding of place value has increasingly been seen as a key component of number sense. This is especially relevant in an age where technology has all but removed the need to be proficient with many of the standard written algorithms. Place value plays an important role in estimation and the competent use of computational technology, as well as being the basis of our measurement and money systems.

Understanding of our numeration system is a complex process that begins at an early age with counting and develops across the whole spectrum of the primary school years to include an understanding of numbers into the thousands and millions as well as an understanding of our decimal system of notation. Despite this importance, the review of recent Australian research literature indicates that there has been far less research on children’s understanding of place value and the structure of the number system than there has been on early counting.

In a study designed to investigate whether the Montessori method of education assists the child to learn mathematics in general, and whole number place value in particular, Berry (1995) defined and analysed place value in terms of eleven skill groups. Interviews were conducted with seventeen nine-year-old children enrolled in Montessori schools in
Melbourne and Perth, using questions that involved the use of concrete materials as well as an ability to work in the abstract. According to Berry, the results support the proposal that the Montessori method of education assists the child to learn place value, with children in the study exhibiting strong understanding at a concrete level. There was, however, some variability in their ability to work in the abstract.

Using Ross’ (1986; 1989) conceptualisation of the five stages in the interpretation of two-digit numbers and a series of place value tasks as a basis for their research, Sierink and Watson (1991) interviewed 60 children from Prep to Year 4 in one Tasmanian primary school. Based on their performance on the seven tasks, children were evaluated as being in one of Ross’ (1986) five stages. A comparison of the 33 children from Years 2 to 4 with those in Ross’ study showed that fewer from the Tasmanian groups than the USA groups operated at lowest Level 1 — although combining Levels 1 and 2 resulted in similar percentages. Moreover, nearly half of the Tasmanian children were judged to be at Level 5, compared with only 20% of the American children. The authors argue that teachers need to be more aware of the complexities associated with the learning of place value and that the tasks used in the research could be useful as classroom activities to stimulate discussion about place value.

Sierink and Watson make a vital point: that while there is considerable emphasis on the use of concrete materials such as Dienes blocks, there is no guarantee that their use will result in children understanding place value unless the connections to place value are made explicit. Teachers need a repertoire of activities to develop and reinforce place value concepts at a more abstract level. This supports the findings of many overseas studies and particularly the work of Hart in the United Kingdom — as summarised in Chapter 4.

In a study of sixteen Year 3 children in one Brisbane school, Price (2001) investigated the development of place value understandings using two different treatments over a period of ten sessions. The children were placed in four groups, with two categorised as high achievers and two as low achievers. Two groups used physical base ten blocks and two used place value software with base ten representations. It was found that there was little difference in the learning that took place. Electronic feedback was found to be more positive and consistent, as well as reducing the need for attention from the teacher, although it was not as responsive to individual nuances. Based on his study, Price (2001, pp. 252–253) recommends that teachers should: challenge students’ ideas about numbers by asking them a variety of non-routine questions; use place value charts or other materials to help structure block arrangements; use materials that include groups of single materials instead of base ten blocks with young children; and be aware of and alert for signs of common misconceptions held by children about multidigit numbers.

Thomas (1996; 1998) conducted an extensive investigation into aspects of developing number knowledge that contribute to the apparent failure of children to make sense of our numeration system. A broad cross-sectional sample of over one hundred children from Kindergarten to Year 6 were given a structured, task-based interview, in order to assess their acquisition of key elements of counting, grouping and partitioning, regrouping, place value, number sense, and structure. Results show that by the end of Year 2, most children could represent two digit numbers with understanding. Older children demonstrated competence up to thousands, but 42% of the Year 6 children were not familiar with ten thousands. Children used many different strategies for calculations, drawing on place
value knowledge, but only some children used place value knowledge effectively (from 11% in Year 3 to 32% in Year 6). This suggests that children understand the place value system, but not deeply enough to use it to invent reliable methods when dealing with numbers outside their common experience. A disturbing finding was that there was little improvement between Year 3 and Year 6 in the tasks requiring an understanding of the recursive nature of repeated multiplication in the number system. Thomas and Mulligan (1999) argue that there is too much emphasis on the additive properties of numbers, citing the fact that while children in their study could count and group in tens, they did not relate these processes to a base ten structure. According to Thomas (1998), his research highlights the fact that the teaching of numeration as compartmentalised knowledge and processes restricts the construction of relationships. He urges a more holistic approach to teaching number so that children can be helped to build connections between their intuitive knowledge, various models that might be used and formal rules of numeration.

Difficulties with understanding decimal notation are well documented, with a lack of understanding of place value inherent in many of the misconceptions noted in students’ work (see, for example, Wearne & Hiebert, 1988).

In a New Zealand study, Irwin (1995) based her research on the fact that an earlier study of students aged 11 and 13 (Britt, Irwin, Ellis & Ritchie, 1993) showed a marked difference in students’ levels of understanding of place value of decimal fractions between schools in lower and higher economic areas. Irwin interviewed 48 students, aged 8 to 14, from schools in disadvantaged areas. She aimed to explore what relevant understandings students with a poor understanding of decimals brought with them to school. Her interviews with the six boys and six girls at each of the ages of 8, 10, 12, and 14 included four sections: an introduction, including questions about where they had seen decimals out of school; a pictorial representation task in which students were required to cut a “cake” into ten equal parts and then further subdivide one part into ten equal shares; questions to elicit whether or not students knew the names of the parts they had just constructed; and a calculator prediction and interpretation task. The eight-year-old students came up with a wide variety of places in which they had seen “numbers with a dot in them” although they had not seen decimals at school. The ten- and twelve-year-olds reported fewer out-of-school occurrences of decimals, giving much more emphasis to having seen them at school; and few connected the in-school and out-of-school uses. The fourteen-year-olds reported a range of out-of-school uses. Irwin found that when students were able to relate decimals to out-of-school contexts they displayed more understanding than they did with numbers alone. Although the results from the pictorial representation task indicated that students often had the necessary underlying concept for understanding decimals — that one object would be divided by 10, that tenths could be divided by ten, and so forth, each time giving a smaller portion — few students had appropriate words or symbols to express these ideas.

This lack of appropriate language made it difficult for them to abstract their understanding of concrete representation, or see the link between this concrete division model and the numerical representation of tenths, hundredths, and further subdivisions (Irwin, 1995, p. 342).

In contrast to Wearne and Hiebert’s (1988) findings, students in Irwin’s study, when attempting to make predictions for the results of calculator operations, appeared to try to make sense of their early attempts in terms of their existing pool of concepts. These
concepts included useful concepts such as “there can be lots of decimal Numbers between zero and one”, as well as ones that were likely to lead to confusion, such as “decimal numbers are smaller than zero, like negative numbers” (p. 343). Irwin confirms the need to coordinate new symbolic knowledge with existing informal knowledge, moving between “symbolic and concrete representations so that the learner is faced with any contradictions” (p. 343).

Stacey and Steinle (1999) report on preliminary findings from a longitudinal study of children’s understanding of place value in relation to decimal notation, based on earlier research using a “Decimal Comparison Test” that asks students to select the larger decimal in each of thirty carefully chosen decimal pairs. This test allows ten patterns of thinking to be diagnosed. The researchers report on the understandings of 3211 students in terms of four major categories of these ten patterns: apparent experts, longer-is-larger misconceptions, shorter-is-larger misconceptions, and unclassified. The progress of 64 students over about three years is also reported. In their earlier study using cross-sectional data (Stacey & Steinle, 1998), the longer-is-larger category was found to decrease from 32% in Year 5 to 5% in Year 10, suggesting that it is unlikely to be a commonly held misconception by adults. However the shorter-is-larger category was found to be consistently between 10% and 15%, suggesting that this misconception may still be held by adults, while the apparent experts plateaued at about 60% in Year 10, again suggesting that many adults have difficulty in understanding decimal notation. Data reported in Stacey and Steinle (1999) shows that almost all “apparent experts” in the longitudinal study remained so from one test to the next, while about one third of students in the other categories became apparent experts. In contrast, almost one half of students in the longer-is-larger category and one third of students in the shorter-is-larger category stayed in the same category on their next test. Results show a general, if somewhat slow, trend towards expertise. Future analysis of the data will take into account the ages of students, as age is likely to be an important factor in movement from one category to another.

Comparing decimal Numbers that have the same whole number part (for example, 3.032 and 3.04) requires both an understanding of place value and the fraction concept. In a study of 130 Year 5 Brisbane students, Baturo and Cooper (1995) identified nine different strategies, some indicating sophisticated understanding and others restricted understanding. Most students had a predominant strategy, which determined success or failure on particular items. Apart from a few careless errors, students who used certain strategies were always successful. These strategies were:

- the expert rule: correct comparisons in all situations by comparing the digits in like places from left to right;
- renaming: equalising the fractions by renaming tenths as hundredths; and
- benchmarking: based on estimation.

The whole number and fraction strategies were successful only if the item supported their use, and this never occurred for both together. Zero-ignored and expert-backwards (comparing like places from right to left, not left to right) were common strategies that led to incorrect results.

As part of her doctoral studies, Baturo (1998) developed a test to assess students’ understandings of place value and their ability to identify, regroup, count, order and estimate decimal Numbers including tenths and hundredths. Baturo (2000) summarises
some key cognitions that she found to be embedded in decimal number numeration processes. These were categorised into a model with three levels:

- Level 1 is associated with position, base, and order. This is considered baseline knowledge because all decimal Number knowledge is derived from this level. Zero plays an important role in the syntactic knowledge associated with reunitising.
- Level 2 is associated with unitising and equivalence. This is seen as linking knowledge because it is necessary for progression.
- Level 3 is associated with reunitising, additive structure, and multiplicative structure. These provide a superstructure for further knowledge.

Additive structure tended to dominate multiplicative structure, which Baturo thought may have been the result of over-extended representation of numbers with base ten blocks. Overall, the students “had great difficulty processing decimal Numbers and their teachers were unsure as to how decimal number numeration processes could be taught effectively” (p. 102).

In summary, Australian research into children’s development of place value concepts, including the underlying concepts involved in decimal notation, has often questioned the role of concrete materials. Moreover, teachers are urged to make explicit the connections between the materials and place value, and to have a repertoire of activities to develop and reinforce place value concepts at a more abstract level.

**Computation**

The 1990 report, *Reshaping School Mathematics*, produced in the United States by the National Research Council, recommended a “zero-based” approach to curriculum development — one that makes no *a priori* assumptions about the content of the curriculum, but instead starts from scratch. In such a curriculum, the inclusion of any topic needs to be justified on its own merits with no area being immune from scrutiny (National Research Council, 1990, p. 38).

A recurring theme in the Australian research reported in this section (and elsewhere in this chapter) is the role of and emphasis placed on standard written algorithms relative to mental and calculator computations. Groves and Stacey (1998) argue that calculators have a tremendous potential in developing children’s conceptual understanding and mental computation strategies before any formal teaching of algorithms, and that the amount of time given to paper-and-pencil algorithms in the school curriculum should be the subject of intense debate and experimentation. While there is probably agreement that some standard, written computational algorithms need to be included — even in a zero-based curriculum — their inclusion needs justification in terms of either their utility or some other clearly articulated grounds. Furthermore, as can be seen from the research reported here, it is not only a question of which algorithms to include, but also the extent to which they need to be “standard”, automated and able to be carried out with speed, efficiency and accuracy, and, even more importantly, when they should be taught.

**Addition and subtraction**

Extensive research into children’s addition strategies has been carried out since the early 1900s, particularly in the United States. Most of the models proposed for children’s solutions of single digit addition problems have been based on various levels of counting
and number fact retrieval. Counting is often characterised as “count-all” and “count-on”, with the latter being broken up into further levels depending on whether or not the child recognises that it is more efficient to count on from the larger number. Number fact retrieval strategies include direct retrieval and the use of known facts to derive new ones.

In an Australian study investigating the strategies used by 32 Year 2 children to solve single-digit addition problems, Christensen and Cooper (1991) found the range of strategies identified corresponded with those reported in the United States, although some major differences were observed. Children relied heavily on counting strategies, with children finding it easier to support their counting with blocks than fingers, while unassisted counting was the most difficult. More use was made of count-all strategies using concrete materials than was expected.

The Early Numeracy Research Project identified six growth points for the domain of Addition and subtraction strategies. These growth points, which extend beyond the addition of single digit numbers, are listed below.

1. Count-all (two collections);
2. Count-on;
3. Count-back/count-down-to/count-up-from;
4. Basic strategies (doubles, commutativity, adding 10, ten facts, other known facts);
5. Derived strategies (near doubles, adding 9, build to next ten, fact families, intuitive strategies); and
6. Extending and applying addition and subtraction using basic, derived, and intuitive strategies. (Clarke, 2001, p. 11)

According to Clarke, these growth points are intended to represent “big ideas” in mathematics and teachers need to realise that much learning takes place between growth points.

An Australian Research Council-funded two-year longitudinal study of 104 children in their second and third years at school, The Effect of Traditional Instruction on Children’s Spontaneous Cognitive Strategies for Computation sought to identify children’s spontaneous strategies for one-, two- and three-digit mental additions and subtractions, and to describe the effect of teaching standard written algorithms on these. Children were interviewed six times using a sequence of tasks moving through different numbers of digits, addition and subtraction, pictorial and symbolic presentations, different types of subtraction, and strategy-friendly or non-strategy examples. The researchers, Cooper, Heirdsfield and Irons (1993; 1996), found that while many children were lacking even the most basic understanding of concepts and procedures, even after years of instruction, many children were also very creative with numbers, manipulating them to suit the purpose of the task. Children’s use of strategies was complex, with many children being able to select the most appropriate strategy for a particular task. A small but significant number of children were found to be able to compute effectively and efficiently before instruction. There also appeared to be evidence that instruction in the use of standard written algorithms had a strong influence on students’ strategy choice for mental computation.

Heirdsfield and Cooper (1996) report on the children’s performance and strategy use for the subtraction tasks in the same longitudinal study. The study showed that while Year 2 children predominantly used a subtractive removing strategy for separation problems and
an additive building-on strategy for missing-addend problems, Year 4 children were more mixed in their strategy use. When presented with algorithmic exercises, in both vertical and horizontal form, children predominantly used subtractive strategies. Discussing the teaching implications of this study, Cooper, Heirdsfield and Irons (1996) suggest that primary students should be taught mental computation in a way that promotes flexibility and, in particular, that low performing students might benefit from direct teaching of mental strategies other than those mirroring standard written algorithms. Overall, they argue, less emphasis should be placed on the teaching of standard written algorithms and more on “identifying and developing children’s legitimate spontaneous strategies” (p. 160). Similar results and implications are reported by Heirdsfield (1999) in a discussion of two children’s mental addition and subtraction strategies after tracking over a five-year period from Year 2 to Year 6.

In a similar vein, Brinkworth (1998) also compares methods of subtraction used by children and teachers and suggests ways to encourage children to invent their own subtraction algorithms.

The use of various models for the development of both mental and written addition and subtraction has been the subject of considerable research, particularly in the Netherlands as part of Realistic Mathematics Education (see, for example, Klein, Beishuizen & Treffers, 1998). In Australia, Carney (1995) carried out a small-scale study into the effectiveness of the use of a range of structured aids, while findings from the project Thinking in Tens: The Use of Ten Frames to Develop Early Number Ideas and Link to the Development of Addition and Subtraction Basic Facts suggest that the tens frame is useful for developing children’s thinking in tens, assisting in “using doubles” and “make to ten” strategies for addition basic facts, with children being able to build up basic facts without resorting to primitive counting strategies.

A major new model for addition and subtraction to 100 is the empty number line, developed in the Netherlands as part of Realistic Mathematics Education (see, for example, Gravemeijer, 1994). This model has been adopted not only in the research carried out in the United States by Paul Cobb, Erna Yackel and others (see, for example, Cobb, Gravemeijer, Yackel, McClain & Whitenack, 1995), but is increasingly being recommended for classroom use in the United Kingdom and, in Australia, in Count Me In Too.

According to Klein, Beishuizen, and Treffers (1998), Realistic Mathematics Education (RME) came into prominence in the Netherlands after a national evaluation of primary mathematics showed an unacceptable level of procedural competency in areas such as subtraction. This led to a call for a new curriculum in which mental computation would play a central role in first and second grades. Criticism of the use of multibase arithmetic blocks (MAB or Dienes blocks) and Unifix as providing “a strong conceptual but weak procedural representation of operations on numbers” (p. 444) led to the introduction of the hundred square model in the 1980s. This was seen as preferable as it “embodied not only relations between numbers but also allowed the visualisation of addition and subtraction operations by having children draw arrows or jumps” (p. 444). Research showed that these different models differed in their effects on mental computation strategies, with blocks evoking a “split method” of decomposition or place value strategies (referred to as 1010) and the hundreds chart a “jump method” of sequential counting by tens from the first unsplit number (referred to as N10). While it was found that the hundred square provided a better
model for N10 than arithmetic blocks, the prestructured nature of the hundred square left little room for children’s informal strategies. In 1990 as part of the revision of the Dutch primary mathematics curriculum, the empty number line was proposed as a new didactic model for addition and subtraction. A structured bead string with 100 beads, arranged in alternating blocks of ten black and ten white, is used as an introductory model for the empty number line so that, for example, children could add 47 and 26 by finding 47 as 4 tens and 7 more and then use strategies such as adding on two more groups of ten to reach 67, then 3 more to reach 70 and then 3 more to reach 73. The empty number line — so called because it has no pre-recorded markings on it — can then be introduced to model the bead string and record children’s procedures by marking numbers and using arrows to indicate the additions (see Figure 3).

Figure 3. Two ways children might represent 47 + 26 on an empty number line.

Klein, Beishuizen, and Treffers (1998, p. 446) illustrate the use of the empty number line and how it can be used to model a bead string, showing three different strategies children might use for 38 + 25. Reasons given for using the empty number line as opposed to arithmetic blocks or the hundred square include:

- the suitability of the empty number line for modelling informal strategies;
- the opportunities it provides for raising the level of students’ activity by scaffolding their solutions through showing the operations, which are being carried out, and thus moving from being a model of the bead string to a model for representing mathematical solutions;
- its natural and transparent character; and
- students’ cognitive involvement in their actions with students concurrently solving computation problems and drawing jumps, rather than “reading off” answers.

Reporting on their study of 275 Year 2 students in ten classes using experimental materials based on the empty number line, Klein, Beishuizen, and Treffers (1998) conclude that the success of children on the difficult subtraction problems in the National Arithmetic Test confirmed the empty number line as a powerful model for instruction.

In a study of 107 third through sixth graders’ basic addition facts, Cumming (1994), found that 34% of errors involved the addition of zero. Dole (1991) in a study of eighteen students in their final year of primary schooling, compared the effectiveness of using manipulatives designed to meld concrete and abstract representations with the use of conceptual subtraction knowledge for the remediation of systematic error patterns in subtraction algorithms.

Boulton-Lewis and Tait (1993) investigated the representations and strategies for addition used by 55 Years 1, 2 and 3 children when presented with operations in symbolic form and asked to explain their procedures as they worked, using fingers, materials or paper and pencil, as preferred. Children were found to prefer verbal and mental strategies rather than algorithms and did not want to use fingers or materials unless they could not perform
the task in any other way. The authors suggest that children can have difficulties because teachers introduce procedures that are recommended in curriculum documents without being aware of the cognitive load they impose.

In a study comparing computer assisted learning with traditional methods in the teaching of two-digit addition to thirty students with intellectual disability, Sivakumar (1998) found that students exposed to computer assisted instruction performed better in two-digit addition than students taught by traditional methods, with males outperforming females and being more confident and motivated in the handling of computers. Hamilton (1991) in a study of six upper primary students who displayed considerable learning problems found that they increased their correct responses on addition number fact probes following the introduction of the two teacher developed arithmetic games.

### Multiplication and division

Much of the Australian research on multiplication and division in the past decade has originated from Joanne Mulligan’s doctoral thesis (Mulligan, 1991) and her subsequent research. Basing her two year longitudinal study on Carpenter and Moser’s (1984) investigation of children’s solutions to addition and subtraction word problems, Mulligan followed seventy children from Year 2 into Year 3, from the time when they had received no formal instruction in multiplication and division to the stage where they were being taught basic multiplication facts. Findings from the four interviews showed that 75% of the children were able to solve the five types of multiplication and five types of division problems using a wide variety of strategies, even though they had not received formal instruction in multiplication or division for most of the two-year period. A classification scheme for problem structures and solution strategies was developed, with three basic levels of solution strategies identified for multiplication and division:

- direct modelling with counting;
- no direct modelling, with counting, additive or subtractive strategies; and
- use of known or derived facts (addition, multiplication). (Mulligan, 1992a, p. 24)

According to Mulligan (1992a, p. 33) the levels of modelling, counting and the use of known facts, while more complex, were found to be analogous to those in Carpenter and Moser’s (1994) study of addition and subtraction. An analysis of children’s intuitive models revealed four models for multiplication, with the use of repeated addition predominant. Three intuitive models for division were identified: sharing one-by-one, “building up” (additive) and “building down” (subtractive). In view of her findings, Mulligan (1992a; 1992b) questions the reliance on sharing and repeated subtraction models for the teaching of division and suggests that the efficient use of multiple and group counting might be a more effective way of teaching division, while the development of problem-solving strategies for multiplication and division should be encouraged from the pre-school years.

Mulligan (1992c) also reports on a subsequent teaching experiment, with ten girls over a period of eight weeks, that aimed to assist them to see connections between and within a range of representations (spoken, concrete objects, pictures, real life situations and written symbols) and a range of problem contexts. Based on her analysis of the children’s representations, how these were related across problem situations and their explanations of the solution process, Mulligan recommends more emphasis on children’s intuitive
strategies and the use of teaching strategies that encourage mathematical representation and reflection (see also *Sixth Graders’ Understanding of Multiplicative Structures*).

A follow-up study (Mulligan, 1993) was conducted with the same sample of 45 primary school students in Year 6 to investigate their understanding of multiplication and division through word problems, including problems involving decimals, and to map the development of multiplication and division processes. Surprisingly, performance across many problem structures was generally lower than in Year 3, and much lower than expected for children entering secondary school, with about a third of the sample unable to solve half of the problems. Many of the children had stopped analysing the problems and focused instead on numerical manipulations. Among a number of classroom implications, Mulligan suggests that the use of additive and estimation strategies, especially efficient use of multiple and group counting, might be more effective than mastery of number facts. She further recommends that rather than depend on computational skills too early, teachers should incorporate the development of informal strategies and place emphasis on representations of decimals, attend to misconceptions about “multiplication makes bigger” and “division makes smaller”, and focus more on estimation strategies.

Mulligan (1998) describes the development of a six level framework of multiplication and division knowledge based on previous longitudinal research into children’s development of strategies and models for multiplication and division (Mulligan & Mitchelmore, 1995; 1996; 1997). This framework has been integrated into an interrelated learning framework in number (see Wright, 1998) which is used in *Count Me In Too*. Mulligan (1998) describes links between levels of development and key assessment tasks in order to assist teachers to promote the development of increasingly sophisticated multiplicative strategies by young children.

In another study that investigated the models young children associate with multiplication, and the strategies they use to solve multiplication problems, 115 children from Years 1 to 4 were interviewed while they solved an “area of a rectangle” problem and three word problems (Outhred, 1995; 1996). Children were encouraged to draw the problems. Their drawings demonstrated difficulties inherent in multiplication problems compared with addition and subtraction problems. Cartesian product problems proved difficult to understand, represent, and solve; with the jump from repeated addition to product rarely being made. According to Outhred (1996) there are major implications for teaching if children associate the equivalent set but not the area models (arrays of adjoining squares) with multiplication, as area models:

- can be used to demonstrate commutativity;
- generalise more readily to multiplication of large numbers; and
- are an integral part of learning about fraction, area, and volume concepts.

In contrast, any equivalent set model “becomes increasingly cumbersome for multiplication of large numbers, does not illustrate commutativity, and does not play an important part in the development of higher order concepts” (p. 189).

In a somewhat different study, Davis and Pitkethly (1990) report on interviews with seventeen second grade children who were shown video tape of three preschoolers involved in sharing activities. Almost all of the Year 2 children saw counting as an essential part of the process of “fair sharing” and believed that in order to determine whether dealing produced equal shares they should resort to an “action-based checking
procedure" such as counting rather than reflect on the underlying logical aspects of dealing.

**Basic number facts**

A knowledge of basic number facts is as important for informal computational strategies as for the use of standard written algorithms. While automatic recall of number facts is often seen by parents and the wider community as a major objective in primary school mathematics, much of recent research into children's acquisition of number facts has focused on children's thinking strategies, with the development of such strategies being seen as an important step between an understanding of the operations and automatic recall.

Bana and Korbosky (1995) report on a study of 390 Year 3 to 7 students, designed to provide baseline information on:

- students’ automatic recall of basic number facts for the four operations;
- their ability to transfer this knowledge to real-life situations;
- their understanding of and thinking strategies for subtraction and division; and
- the effects of a wide range of variables on these.

While results suggest good overall automatic recall, the gap between addition and subtraction suggests that students are not making the connection between these two operations, unlike multiplication and division where the link is much more obvious because of the similar language used. As expected, items involving doubling or squaring resulted in significantly better performance, as did, surprisingly, those involving multiplication by zero. Year level was a highly significant factor, although there was a plateau effect at Year 5.

The extent of understanding of the operations was consistent with performance on automatic response, with a wide range of student developed mental and semi-concrete strategies being used to explain subtraction and division. The authors recommend that the development of understanding of the operation must precede efforts to attain automatic recall, while teachers should encourage children to explore relationships and develop their own strategies as part of the overall process.

**Informal strategies for computation**

The debate about the role of standard written algorithms in the teaching of mathematics, in part due to the advent of the calculator, now has a history of almost a quarter of a century (see, for example, Plunkett, 1979), with much earlier writing also focusing on the importance of number sense and mental computation (see Brownell, 1945). The place of students' informal strategies in the development of computational skills has received considerable attention in Australia and New Zealand, as well as overseas. Some of this research is discussed below, while much of it is discussed elsewhere in this chapter.

Heirdsfield, Cooper, Mulligan and Irons (1999) report on a study of changes in 95 Queensland children's mental computation solution strategies for multiplication and division applied word problems over a three-year period from Year 4 to Year 6. Findings from the twice-yearly interviews suggest that some of the children's efficient procedures were replaced by less efficient written procedures performed mentally, while some children, after instruction, could not attempt the more difficult tasks that they had successfully completed in previous interviews. The authors argue for more flexible, child-
centred approaches to teaching operations in order to place more emphasis on alternative computational strategies. Heirdsfield, Cooper, and Irons (1999) report on the strategies used by one of the children in this study. Although "Adrien" was considered to be a higher-ability student, he was not regarded as a remarkable calculator. Nevertheless, he used his own efficient strategies to successfully multiply and divide two- and three-digit numbers before such calculations were taught. The efficiency and power of his strategies for multiplication declined over the teaching period as they became more like the traditional, right-to-left algorithms. Heirdsfield, Cooper and Irons argue that Adrien’s performance highlights the possibilities for teaching computation in such a way that children are allowed to develop and are supported to maintain their own spontaneous strategies rather than focus exclusively on the standard written algorithms.

While researchers in Australia and elsewhere frequently make similar calls for reforms to the teaching of computation, the challenges faced by teachers who attempt such approaches are highlighted by Buzeika’s (1999) study in two New Zealand classrooms at Years 3 and 4. Two teachers in the same school were provided with professional development over a period of six months in the use of invented strategies as a way of encouraging number sense, with the author joining classes three or four times a week over a five week period. Children working on multi-digit computations were encouraged to use mental methods as well as record the ways in which they worked and share their ideas in class. Children used a range of predominantly left-to-right strategies. While they completed fewer examples, they spent more time reflecting on their solutions and verbalising the processes used. One teacher commented that they were “thinking deeper”. While the teachers acknowledged the children’s success and were committed to the program, one teacher responded to the perceived pressure from parents and teachers in the higher grades by introducing, half way through the program, a standard written algorithm as a possible way of recording. Although the school had sanctioned the program, both teachers felt that invented strategies were seen as an interim step to the development of standard algorithms and recommended that any such approach should be entered into on a school wide basis. According to Buzeika (1999), for change to be implemented “the focus must come from the wider educational community and be backed up with support for teachers and schools” (p. 134).

In just such an attempt to involve the wider education community, Alistair McIntosh, Shelley Dole and Joy Edmunds are leading the Developing Computation: Strategic Numeracy Research and Development Project, Tas. This project is based on Enhancing Numeracy Outcomes, which commenced in 2000 in Years 3 to 6 in two ACT and four Tasmanian schools. The purpose of the current project, which involves 35 Years 2, 3 and 4 classes in nine government, Catholic and Independent schools in Tasmania in 2001 and 2002, is to support the development of informal written methods in Years 2 to 4, while investigating the effects of such a program on students’ number sense and computational ability. In 2002, the project has approximately fifty teachers involved. In particular, the project is exploring the interface between mental and written computation through developing informal written computation processes and documenting how the use of such processes affects student performance and student and teacher attitudes to computation. The project is also seeking to determine which classroom approaches to encouraging informal written computation and are most effective in developing students’ number sense and computational ability.
**Mental computation**

In most classrooms mental arithmetic is either neglected or appears confined largely to the basic facts — that is those calculations which we need to do mentally in order to do written computation. It is not geared to helping children to compute mentally in everyday life. (McIntosh, 1990, p. 1)

There is a substantial body of Australian research into mental computation, with much of the research having its origins in Alistair McIntosh’s interest in the area. As well as an initial focus on investigating mental computation strategies and performance, McIntosh’s work with Swan and other colleagues in Perth, and later with Dole and others in Tasmania and the Australian Capital Territory, has also sought to identify teaching and assessment practices to support the improvement of mental computation. Another group working in this area has been Cooper, Irons, and Heirdsfield in Queensland. Because of the inherent overlap between mental computation, the various aspects of computation and areas such as informal strategies for computation, number sense and children’s thinking, much of the research on mental computation is reported elsewhere in this report. Nevertheless, the sheer quantity of research reported in this section indicates the extent of Australian research in this area.

McIntosh (1996; 1998) reports on two Australian mental computation studies, the *Mental Arithmetic Project* and the *Mental Computation Test/Western Australia* — the Western Australian component of a cross-cultural study of mental computation among Years 3, 5, 7 and 9 students in Australia, Japan and the United States (see also McIntosh, Nodha, Reys & Reys, 1995). A major goal of the former project was to investigate the range of mental computation strategies used by Years 2 to 7 students, in order to underpin subsequent curriculum development in mental computation activities for primary teachers. Based on interviews with children, nine clusters of strategies (for example, using place value instrumentally or relationally, using doubling and halving, using fingers) were identified. More competent students were found to make less use of counting, used their fingers less, used effective strategies earlier, used place value strategies (such as removing a zero) mechanically, and were able to manipulate numbers and exploit their structural qualities more dynamically. A second goal of the project was to devise and trial classroom activities for primary teachers. Based on this research a further curriculum development project was undertaken, which resulted initially in the *Mental Arithmetic Project Schools Inservice Package*, a revised version of which was published as *Think Mathematically: How to Teach Mental Maths in the Primary Classroom* (McIntosh, de Nardi, & Swan, 1994).

The *Mental Computation Test/Western Australia* study, was designed to investigate three different aspects of mental computation among Years 3, 5, 7 and 9 students. A preference survey was used to identify the types of computation that students preferred to do mentally, while an attitude survey was used to measure their attitude towards mental and written computation, and a mental computation test was used to assess their performance on mental computation items. The preference survey demonstrated some weaknesses in conceptual understanding — for example, 40% of Year 5 students would not choose to calculate $100 \div 35$ mentally. The attitude survey showed children thought they did more written computation than mental computation in school, while they would use mental strategies more often outside of school. Children generally coped as well with items presented visually as with those presented orally, but only hearing the task tended to
encourage more flexible strategies. Boys were more sure of their ability to calculate mentally than girls, with girls more likely to be wrongly diffident of their ability. Whole number understanding was strong, but knowledge of fractions, decimals, percentages (and operations involving them) was generally inadequate for future needs. A detailed analysis of students’ error patterns for a selection of the 45 items is provided in Bana, Farrell, and McIntosh (1995).

McIntosh (1996; 1998) identifies a number of implications for teaching. Firstly, while most students demonstrated a range of strategies, few seemed to have acquired these through classroom instruction.

It would appear that competent mental calculators become so in spite of what happens in the classroom; or, more likely, they possess an early affinity with numbers which allows them to “play” with them, and they abstract from classroom practices of any kind skills and understandings which they adapt and use in mental computation and any other mathematical situations (McIntosh, 1998, p. 221)

Research is needed into how best to make teachers more aware of efficient strategies — especially as less competent students often do not receive help and do not appear to be able to abstract the required understandings for themselves. However “teaching” efficient strategies is not the answer as learnt rules (such as removing a zero) were misused badly through a lack of understanding. McIntosh concludes that children need encouragement to experiment and to verbalise their thinking and that a deep understanding of the essential features of place value is a critical means for achieving enhanced facility with mental computation.

The three instruments referred to above — the preference survey, attitude survey, and visual and oral forms of the mental computation test — were used in a cross-cultural study of mental computation among 2000 Years 3, 5, 7 and 9 students in Australia, Japan and the United States. McIntosh, Nodha, Reys, and Reys (1995) discuss three possible instructional approaches to mental computation, and compare the emphases placed on mental computation in the three countries. The approaches are:

• viewing it as a topic to be taught in much the same way as traditional written algorithms;
• a constructivist approach where students are encouraged to generate thinking strategies based on prior experience and knowledge; and
• the default approach of teaching only standard written algorithms and expecting students to extrapolate mental strategies from these.

In Australia, while the rhetoric suggests that mental computation is of prime importance, and should therefore be explicitly taught at all levels and involve creative problem solving, the reality is that in the majority of schools mental computation is completely overshadowed by the teaching of standard written algorithms. Similarly, in the United States, while the “reform agenda” calls for students to be engaged in finding and sharing invented strategies for solving computational problems, in most classrooms students are only engaged in learning the standard written algorithms and, where mental computation is addressed at all, it typically takes the form of direct instruction of a range of “mental computation” strategies. In general, mental computation precedes written computation throughout the primary grades in Japan, with written computation being used for more tedious computations and the soroban (Japanese abacus) being introduced as a computational alternative at the end of Year 4. Far from being merely a mechanical
computational device, the use of the soroban promotes mental images, which often lead to highly efficient mental procedures. While it is difficult to make comparisons of performance on the mental computation test because of the nature of the samples, results show that the performance of the Japanese students far exceeded that of the Australian and United States students at Years 2 and 3, but that the gap narrowed with age, and that by Years 8 and 9 the performance of the Australian students had surpassed that of the Japanese students. According to McIntosh, Nodha, Reys, and Reys (1995), an implication of their findings is that an early emphasis on formal algorithms (whether written or mental) is not necessarily beneficial and may represent a waste of time and effort "at the expense of conceptual learning and a wider curriculum" (p. 250). For further details of the Western Australian study see also McIntosh, Bana and Farrell (1995) and McIntosh (1996; 1998).

This work has been extended in Tasmania as part of the Baseline Standards in Mental Computation: A Preliminary Study (2000). This project aims to provide a reliable picture of the general range of competency and the range of ability at Years 3 to 10, in order to give a picture of the range of mental calculations, which students at these levels can process and hence provide a basis on which appropriate decisions can be made at successive grade levels. A further project Assessing and Improving the Mental Computation of School-aged Students has built on this research and extended it to Years K to 10 to provide a developmental sequence of mental computation competency and a coherent approach to the development of flexible mental computation strategies linked to practical classroom assessment processes.

According to Morgan (1999), the traditional teaching sequence of written followed by mental computation needs to be re-evaluated in light of research showing that the development of flexible mental strategies is influenced by the order in which mental and written techniques are introduced. While Morgan presented a mental-written sequence for introducing each of the four operations, findings from his survey of Queensland school personnel revealed that a majority disagreed with the proposition that an emphasis on written algorithms should be delayed to allow increased attention to be paid to mental computation. Noting that the constructivist view of mental computation as a process of higher order thinking is at odds with the direct teaching of mental computation skills, Morgan nevertheless concludes that there is a place for direct teaching of mental strategies and delineates a sequence for each of the four operations.

Heirdsfield (1998; 2000; 2001a; 2001b) reports on different aspects of a pilot study and a slightly larger in depth study of Year 3 children’s addition and subtraction mental computation strategies with the aim of going beyond merely reporting these to developing a comprehensive model of mental computation. Reporting on the pilot study of two children, Heirdsfield (1998) found that the child who was characterised as being flexible at mental computation had a well connected network of knowledge of number and the ability to apply this knowledge across different tasks. Number facts and an understanding of the numeration system were applied to mental computation, with a general “feel” for number evident in both mental computation and estimation. None of this was evident for the child characterised as being inflexible at mental computation. The results from this pilot study were confirmed in the larger study of 13 students, with students deemed proficient at mental computation (i.e., accurate and flexible) found to be “supported by a rich network of cognitive, metacognitive and affective components” (Heirdsfield, 2001b, p. 276). Students who were deemed accurate but inflexible were found to have a more limited and less connected knowledge base and were unable to “choose” a strategy as they relied on
teacher-taught strategies. These features of mental computation for children deemed proficient and those deemed accurate and inflexible are used to develop concept maps of a framework for each type. According to Heirdsfield (2001a), the importance of connected knowledge for proficient mental computation demonstrates the need for teaching practices to focus on the development of an extensive and integrated knowledge base, and for students to be encouraged to formulate their own strategies.

Computational choices

One indicator of number sense (and hence also numeracy) is the extent to which a person makes an appropriate choice of computational method for a given task, where appropriateness depends on a range of factors including the task, the tools at hand, the context, and the speed and degree of accuracy required. In a United States study of Years 2, 5, and 7 students, Reys, Reys and Hope (1993) found that written methods dominated students’ thinking — even when they were inappropriate. Some tasks were seen by students as being amenable to mental methods, with far fewer being seen as amenable to the use of calculators.

As part of a study of the longer term effects of calculator use in the Calculators in Primary Mathematics project, Year 3 and 4 children from six schools were observed while tackling a range of real-world problems and other computation tasks. Calculators and concrete materials were provided, as well as pencil and paper. Children with long term experience of calculators performed significantly better overall. These children also made more appropriate choices of calculating device and were better able to interpret their answers when using calculators, particularly where decimal answers were involved (Groves, 1993; 1994)

Price (1995) reports on a study investigating choices of computational method made by children in Years 5 to 7 to solve multiplication questions. Results indicated that the children favoured paper-and-pencil computation, even though a calculator was available. Significant relationships were found between computational choices and year level, number type and teacher presence. Reporting on the effect of teacher presence on students’ computational choice, Price (1997) states that results showed a significant difference in the balance between written and calculator methods when the teacher was present or absent, with students more likely to use written computation when the teacher was present. While children were not questioned about their choices, anecdotal evidence of children attempting to conceal the use of their calculator and their comments suggested that they believed that mental and written methods were preferred by the teacher.

Swan and Bana (1998) propose a model for computational processes that replaces the three traditional categories of mental computation, calculator and written (in the sense of standard written algorithms) with mental computation, calculator and recording — which is taken to include “informal jottings during the calculation, as well as the recording of more formal algorithmic steps” (p. 583). In a study of 75 students in Years 5 to 7, Swan and Bana (2000) explored the reasons given by students for their computational choices. Reasons given by students were placed in four categories: number magnitude, efficiency, knowledge of multiplication facts, and teacher influence. Despite schools and teachers being chosen to participate in the study because of their positive attitudes to calculator use and their stated beliefs about the need for students to develop a repertoire of
computational strategies, students believed that the teacher had an impact on the choice of computational method.

It is perhaps interesting to compare these findings with Berger’s (1999) suggestion that a reason for first year university students in a study in South Africa rarely using graphic calculators as a “thinking tool” was that the socio-cultural setting may have privileged one form of reasoning (algebraic) over another (graphical).

As in the area of counting, there has been extensive Australian research into various aspects of computation. Studies of children’s mental computation show that many competent children have acquired a range of efficient strategies almost “in spite of” what happens in the classroom. These and other studies highlight the need for children to develop a deep understanding of the essential features of place value and opportunities to experiment and to verbalise their thinking in order to achieve enhanced facility with computation.

**Fractions**

While there has been a significant decrease over the last decade in the emphasis placed on students’ ability to carry out operations with fractions, developing an understanding of the meaning of and notation for common fractions and the links with our decimal numeration system continues to be an important factor in numeracy.

The link between common fractions and the concept of a “fair share” is frequently regarded as being central to the development of an understanding of fraction concepts. In the Netherlands, *Realistic Mathematics Education* adopts a constructivist perspective in which constructs are bound to contexts, often using confrontation in order to achieve sense making. The extensive work of Leen Streefland as part of *Realistic Mathematics Education* (see, for example, Streefland, 1991) acknowledges not only the role of fair sharing in the development of fraction concepts but also the importance of starting from children’s social experience which may or may not support “fairness” as equating with what adults would recognise as “equal shares”.

Hunting (1996), in his report on several Australian studies of young children’s conceptions of division, sharing and fractions, notes the importance of teachers being aware of young children’s social interactions and the nature of “fair sharing” used in practical situations. In particular, the nature and extent of the dealing strategy used by pre-schoolers to share items into equal portions is described, and home and social factors that might be related to the skill of dealing are discussed.

In a study investigating the link between young children’s social understanding of fairness and the need for equal portions for a mathematically fair partition, 24 kindergarten to Year 4 children were interviewed and asked to share a pancake equally between three dolls. Watson (1997) identified four levels of development:

- sharing not associated with mathematical fairness;
- fair sharing related to number of pieces only;
- fair sharing employing *ad hoc* measurement; and
- fair sharing based on geometric principles. (p. 36)
The same data were analysed using the SOLO model\(^1\) with multimodal functioning (Watson, Campbell & Collis, 1996). This analysis showed that while sophistication increased with grade level, responses varied greatly, particularly in Years 1 and 2. Watson (1997) suggests that not only do early childhood teachers need to be aware that for children fairness in sharing does not depend solely on equal distribution, but also that it is possible that some of the same confusion may be present in division. According to Watson, Campbell and Collis (1993), results from the larger study, involving four different problems and students from Kindergarten to Year 10 provide evidence for a developmental progression within the ikonic mode during the school years, with a mutual interaction between ikonic and concrete symbolic mode development (see also Watson, Collis & Campbell, 1995).

By the time children begin to learn about fractions in school they already have considerable knowledge about whole numbers. This knowledge has sometimes been seen as interfering with children’s development of concepts relating to fractions. The project *A Longitudinal Study of Children’s Fraction Learning* was a two-year teaching experiment with ten eight- and nine-year-old children investigating their fraction learning and the role of whole number knowledge. A computer tool, *Copycat*, embodying fractions as operators, was used to allow children to explore relationships between inputs and outputs of discrete items. Based on results of the study, Hunting (1996) questions the traditional emphasis on continuous quantity and measurement experiences in the teaching of fractions as inhibiting children’s transition to conceptions of fractions as quantitative units. Hunting, Davis and Pearn (1996, p. 376), reporting results from teaching sessions in the same study related to pairwise comparison tasks, conclude that the operator interpretation of rational numbers is a viable approach to teaching basic fraction concepts, that could be used to complement other interpretations such as part-whole and measures (see also Davis, 1993; Davis, Hunting & Pearn, 1993).

Pearn (1996) also reports on a related study investigating the extent to which children’s thinking processes when solving fraction tasks might be associated with qualitative differences in their whole number knowledge. Twenty-eight Year 3 children were interviewed using various partitioning tasks, fraction tasks, and a ratio task. The most successful students demonstrated “proceptual” thought for both rational and whole number tasks, were flexible in their mathematical thinking and applied whole number knowledge appropriately to fraction tasks. Less successful students used procedural thought and were unable to relate previous knowledge to new contexts. Most children had difficulty in understanding the language of fractions, suggesting that more emphasis needs to be put on developing appropriate language before introducing formal work with fractions.

The research of Anthony and others emphasises *contextual knowledge*, where knowledge is held to be an integral part of the specific activity, context, and culture in which it is located. For example, Anthony and Walshaw (2003) viewed videotapes of 60 Year 4 and 50 Year 8 students, randomly selected from the New Zealand’s National Education Monitoring Project bank of student responses. From the children’s responses to rational number questions, the researchers aimed to identify the role that context plays in the

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\(^1\) The *Structure of the Observed Learning Outcome* (SOLO) taxonomy provides a systematic way of describing how a learner’s performance grows in complexity when mastering tasks.
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development of fraction understanding. An inventory of all solutions was produced. It was found that informal experience in varied contexts played a major part in many Year 4 students’ deliberations, resulting in answers like “2” or “2 pieces” rather than the expected fractions. Even when prompted, the children often gave more contextualised details, with the family dinner context and its principle of sharing dominating to the extent that the primary organisation of the number of remaining pieces assumed importance. The Year 4 students were much more likely to provide an answer to the request of “how much” in terms of pieces than Year 8 students who appreciated that a mathematical response was required and seemed more easily able to divorce themselves from contextual influences.

Studies of upper primary children’s understandings of fraction concepts in Papua New Guinea, Malaysia and Australia (see, for example, Clements & Lean, 1994; Ellerton & Clements, 1994) suggest that even when the curriculum allots a great deal of time to computation with fractions, children rarely link the formal language and symbols of fractions to their real-life experiences, particularly of sharing. Clements and Lean (1994) conclude that

Unless curriculum developers and teachers deliberately attempt to establish learning environments which are likely to link learners’ concepts with the children’s personal worlds... this “learned” mathematics will have no meaning outside the classroom. (p. 77)

Decimals

Given the extent to which our money and measurement systems are based on the decimal system, an understanding of decimals is a critical component of numeracy. It is well documented, however, that not only many students in schools but also many adults have difficulty in understanding the decimal system.

As part of an ongoing program of research into the learning and teaching of decimals by the team led by Kaye Stacey at The University of Melbourne, Steinle and Stacey (1998) report on a study aimed at identifying ways of thinking about decimal notation. A test of decimal understanding, based on students selecting the larger number from thirty pairs of decimals, was given to 2517 students at six primary and seven secondary schools across Melbourne. Ten incorrect ways of thinking about decimal notation are described, with eight of these grouped under two main categories of misconceptions: “Longer-is-larger” and “Shorter-is-larger”. While the variability of the results by school could be explained in part by socio-economic factors, it is disturbing to note Steinle and Stacey’s claim that there “seems to be clear evidence that certain misconceptions are learned from school instruction” (p. 555). In another study of students’ misconceptions regarding decimals, Condon and Hilton (1999) discuss primary and secondary school students' misconceptions, the reasons behind them, and some activities that can be used to address these.

Hunter and Anthony (2003) undertook a 6-month classroom teaching experiment on the development of a hypothetical learning trajectory for decimal understanding. Following individual interviews, four students were selected as case studies to represent the range of decimal misconceptions common to students within the middle school age group. The recursive and non-linear paths taken by students in their construction and reconstruction of decimal concepts subsequently influenced the choice of further activity in the teaching and learning cycle as the teacher and researcher revised and modified the instructional
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sequence. In order to extend students’ thinking contextual metric problems were introduced, requiring precise calculations while simultaneously connecting representations of number, ratio, and measurement. When translating across percentages students were seen to benchmark intuitively to decimal and fraction equivalents. As the teaching experiment progressed, students’ flexibility in and between modes of rational number representations increased as they made connections between the problem contexts, their informal and formal rational number concepts, and the notation system. In summary, it was found that the use of percentages as an introductory unidimensional representation embedded within contextual problems involving ratio/measure of water supported the students to construct a robust and effective overview of rational numbers. The students during discussion and activity spontaneously applied prior knowledge of proportional thinking in activity and this provided the teacher with a bridge to scaffold understanding of not only the continuous nature of decimals but also the notion of the unit whole and the referent unit.

Bana, Farrell and McIntosh (1997) use data from an international study of number sense in Australia, the United States, Sweden and Taiwan to investigate students’ misconceptions and error patterns in fraction as well as decimal concepts. The study involved students at up to four age levels: ages 8, 10, 12, and 14. Using a multiple choice format for questions, which nevertheless focused on students’ understanding rather than factual knowledge or computation, the items examined such things as the children’s ability to locate decimals and fractions on a number line, and their ability to identify the largest of a set of fractions, their awareness of there being an infinite set of decimals or fractions between two given numbers, and their ability to estimate products and quotients when decimals are involved. The study identified significant misconceptions about basic notions relating to decimals and fractions that persisted through the age groups. Based on their findings, the authors note that students appear to be required to compute with fractions and decimals before they have any real understanding of the concepts involved. They recommend much more emphasis on meaningful treatment of these concepts, with computation with fractions and decimals being delayed until the fundamental concepts are well established. This plea for a focus on conceptual development is echoed by many researchers in Australia and elsewhere.

According to Hunting, Oppenheimer, Pearn, and Nugent (1998), upper primary and junior secondary mathematics is characterised by an emphasis on rules and symbols, rather than on conceptual understanding. They believe that teachers generally assume that the conceptual basis has been introduced previously or is self-evident and that “students will reference that knowledge as they work through problems. But this does not seem to happen” (p. 271). In another of their studies, aimed at better understanding the way in which students make connections between common and decimal fractions, 49 Year 6 students’ responses to a task that involved matching common fractions and decimal fractions were examined. Students’ explanations revealed a range of different relational connections, often closely tied to procedural strategies. By far the most common mental object used to explain the similarities between common and decimal forms was a 100 scheme. However this may have been influenced by an earlier task where a 10x10 grid was presented. The authors acknowledge the role of reflection and explanation in clarifying mathematical thinking. However, they suggest that the inability of some students to recognise the flaws in their explanations of incorrect choices suggests that upper primary and junior secondary teachers need to spend more time teaching and reviewing
the basic meanings of fractions and decimals, as well as the interrelationships between them.

The importance of such teaching was emphasised in a New Zealand study that arose from a request from teachers for help in evaluating their current methods of teaching decimals. Teachers in fourteen Year 5/6 and Year 7/8 composite classes in four schools negotiated a procedure for evaluating teaching effectiveness and class progress (Irwin, 2000; Irwin, Lauaki, Jacobs & Marino, 2000). A pre-test and post-test of place value and decimal knowledge, suitable for assessing a wide range of understandings, were used to assess class progress. Teachers taught a topic for three weeks, with some planning individually and others together, but all keeping detailed records of their planning and samples of student work. At the end of the unit, teachers were interviewed. Interview questions included those relating to ways in which they taught the unit, the use they made of the pre-test information, as well as what they regarded as difficult aspects of decimals for students. Findings showed that every teacher taught differently — even those who planned together. Every class and the vast majority of students made progress. The main factors leading to students’ improvement appeared to be careful planning to meet their needs based on teachers’ knowledge of the underlying concepts, the use of a clear model that students could use to visualise decimal division, and careful bridging from visualisation to numerical forms. While several teachers mentioned the use of Dienes’ Multibase Arithmetic Blocks (MAB) as a possible model, none of those who used these found them particularly helpful, with students being confused by the different values assigned to the blocks from those used when working with whole numbers.

In an earlier study, Irwin (1995) also found that children aged 8 to 14 attending disadvantaged schools in New Zealand had a better understanding of decimals when the topic was related to out-of-school contexts. However, they lacked the appropriate language to describe the place value of decimals.

Helme and Stacey (2000) describe a small-scale study, with minimal intervention, in which four teachers made use of a different concrete model for decimals, Linear Arithmetic Blocks (LAB). LAB consists of hollow tubes of four different lengths, representing ones, tenths, hundredths, and thousandths. These aids can be used to represent decimals by being placed end-to-end or on an organiser somewhat similar to a spike abacus. However, in order to avoid inadequate conceptions which can arise from seeing decimals as simply indicating whole numbers of sub-units, LAB uses the quantity of length (not the measurement of length) to represent decimals — i.e. it is not intended to represent decimals by a whole number of millimetres, metres etc. The authors argue that LAB is a simpler model than MAB since the representation is based on length rather than volume, is unlikely to cause confusion with whole number representations since it is new for the students, and has the added advantage of being structurally similar to the number line. Even though these resources were freely available to the teachers in the research project, their use was unexpectedly low — although this seemed to have been the result of outside factors. However, teachers who did use them achieved an encouraging improvement in decimal understanding, measured against previous performance of the school over some years. This indicates that a small amount of deliberate attention to decimal concepts can make a difference. In a study comparing the use of LAB and MAB in two teaching experiments involving 30 matched students, Stacey, Helme, Archer, and Condon (2001) found LAB to be considerably more accessible for students, with more active engagement by students and deeper discussion.
Stacey and Sonenberg’s Australian Research Council-funded project *Improving Learning Outcomes in Numeracy: Building Rich Descriptions of Children’s Thinking into a Computer Based Curriculum Delivery System* set out to establish detailed knowledge of children’s understanding of decimals and the way that this develops from Years 4 to 10, as well as to describe the conditions under which children make transitions in their thinking. Teaching modules to advance children’s understanding were developed, incorporating both traditional and computer-based techniques. Much of the materials on the *Teaching and Learning About Decimals* website, http://www.edfac.unimelb.edu.au/DSME/decimals/ — now also available on CD-ROM — were developed in this project.

In summary, in terms of the teaching of decimals, student improvement has been found to depend on teachers’ knowledge of the underlying concepts, the use of a clear model and careful bridging from visualisation to numerical forms. A disturbing finding is that, for some children at least, certain misconceptions relating to decimals appear to be learned from school instruction.

**Percentages**

Given the amount of Australian research outlined in fractions, decimals and place value above, there appears to be relatively little research on the development, teaching or learning of concepts related to percentage. In the two publications that reported on children’s proficiency with percentages, this was merely one of several factors that were discussed in broader arithmetic projects. Bana, Farrell, & McIntosh (1995) concluded that the relationships between fraction, decimal and percentage forms need greater emphasis to ensure that the equivalences are well understood. In the other publication, McIntosh (1996), reporting the results of three major projects in Western Australia, stated that while whole number understanding was strong, knowledge of fractions, decimals, percentages (and operations involving them) was inadequate for the children’s future needs. These suggest the need for further Australian research in the area.

**Estimation**

Good estimation and approximation skills enhance our ability to deal with everyday quantitative situations.... [E]stimation needs to be an ongoing part of children’s study of numbers, and teaching should emphasise the development of a propensity to estimate. Children should... be helped to develop specific strategies to aid them in approximate computations.... By the late primary years, most children should... recognise that estimation [of measurement] is not simply guessing but rather informed judgement, and efforts to improve estimates should be made explicit. (Australian Education Council, 1991, pp. 108; 117; 144).

Given the importance of estimation in the context of numeracy and the emphasis placed by the *National Statement on Mathematics for Australian Schools* (Australian Education Council, 1991) on both computational estimation and estimation in measurement, there is surprisingly little Australian research in this area over the past decade.

Heirdsfield (1995; 1996) reports on an investigation of the relationship between mental computation, computational estimation, and number fact knowledge for addition and subtraction in Year 4 children. Based on three videotaped interviews relating to mental computation, computational estimation and number fact knowledge, and a written number fact test, Heirdsfield (1995) found that while students proficient at mental computation (i.e.,
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those who were both accurate and flexible) were proficient at estimation, the converse was not necessarily true. The predominant strategies used for computational estimation were “truncation, rounding, compensation, and truncation and compensation after solution” (Heirdsfield, 1995, p. 336). The apparent indiscriminate and inappropriate use of truncation and rounding resulted in the greatest percentage of errors — this was particularly true for rounding which might reflect the classroom emphasis on this as the predominant estimation strategy regardless of the appropriateness of the context. Students appeared to be capable of devising their own estimation strategies, with many of the strategies used not being taught. However many children were not confident estimators and preferred to either guess or carry out the exact calculation. Heirdsfield urges that a constructivist viewpoint be adopted to allow students to develop, verbalise, and share estimation (and mental computation) strategies.

In a study which sought to understand more fully the thought processes of Year 6 students when they estimate measurements and to determine possible gender differences in such thinking, Leeson (1995b) reports on findings from interviews with 50 female and 50 male Year 6 students, who were asked to explain how they had obtained their responses to fourteen multiple choice items on a measurement quiz. Items on the quiz asked students to choose from four alternatives the best estimate for the measurement of things such as the height of a door, the size of an angle, and the area of a slice of bread. Responses were later categorised into random guesses, estimation, elimination, and real-life experience. No significant gender differences were found with the boys and girls reporting the use of similar strategies. Elimination of unlikely choices was reported as the most used strategy, with the three items involving length and the one on angle (for boys only) and the one on the capacity of a saucepan (for girls only) being the only ones where over 10% of the students correctly used estimation. Students who made estimates either used referents such as their own or another’s height or else they “unitised”, typically using a metre ruler as their unit. Items such as those relating to the capacity of a bucket, something with the capacity of 60 litres, and the temperature of a healthy child elicited the most responses based on real-life experience. Students achieved highest on length and lowest on area, and sometimes confused the two.

In one of the few studies identified that dealt with estimation of measurement, Happs and Mansfield (1992) discuss how students construct mental images that aid estimation skills in the measurement of angles. Four strategies that students use to estimate sizes of angles were identified as being the use of mental images of a protractor, a right angle, a half-turn, and angles of a polygon.

Additional research on computational estimation is also reported in the next section.

**Number sense**

Number sense refers to a person’s general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgements and to develop useful and efficient strategies for managing numerical situations. (McIntosh, Reys, Reys, Bana & Farrell, 1997, p. 3)

While number sense is often regarded as an elusive quality and difficult to define, it is generally agreed to refer to a well organised conceptual network that enables a person to relate number and operation properties and use flexible, creative ways to solve number problems (see, for example, Greeno, 1991; Sowder, 1988). Clearly number sense is a
crucial component of numeracy and an essential requirement for the transfer of mathematical knowledge to real situations. In Australia, England and Wales, and the United States, curriculum frameworks and reports dating from the 1980s and early 1990s have called for an increased emphasis on number sense (see, for example, Australian Education Council, 1991; Cockcroft, 1982; National Council of Teachers of Mathematics, 1989).

The term “number sense” has only become prominent in the literature since the late 1980s, however the concept itself has a long history. For example, Dewey (1910) distinguished between children’s understandings of particular problems and the broader underpinning sense of meaning that allows some children to work flexibly with them, while Brownell’s extensive writings placed heavy emphasis on what we now call number sense (see, for example, Brownell, 1945). More recently, Skemp (1976) discussed how children who are competent learners of mathematics develop a well-organised “web” of understanding and use this flexibly.

Kaminski (1996a) provides a comprehensive account of various descriptions of number sense, focusing mainly on the work of United States researchers such as Judith Sowder, Robert Reys and Paul Trafton, largely based on work arising from the United States conference Establishing Foundations for Research on Number Sense and Related Topics (Sowder & Schappelle, 1989) and the Australian conference Challenging Children to Think When They Compute (Irons, 1992). This latter conference saw Australian, United States and Japanese researchers present a collection of papers arising from an invited conference immediately preceding the open conference held at the Queensland University of Technology in 1991. Most of the papers focused on mental computation, computational alternatives, number sense and estimation, and the links between these.

As one of the Australian contributors to this conference, McIntosh (1992) argued that number sense is not the same as the ability to calculate. He provides two anecdotes to illustrate that there is a difference between numeracy and the ability to carry out written computations with neither guaranteeing the other. He further argued that there is a case for more emphasis to be placed on the use of calculators and mental computation rather than written computation; with the mental computation needing to be of a type that promotes number sense, rather than the traditional “20-a-day” practice activities.

As part of a continuing collaboration since 1988 between Australian, United States and Japanese contributors to this conference, McIntosh, Reys and Reys (1992), developed a framework for the analysis of number sense using increasingly detailed components at three levels with the most general level dealing with the following three components:

- knowledge of and facility with Numbers;
- knowledge of and facility with Operations; and
- applying knowledge of and facility with numbers and operations to Computational settings.

This framework was further developed in the Number Sense in School Mathematics: Student Performance in Four Countries project with the three components being used to provide a six-strand classification that was then used to develop an extensive number sense item bank for use in assessing the number sense of students aged 8 to 14 in Australia, the United States, Sweden and Taiwan. Four group tests of number sense were compiled and administered to 1100 students aged 8, 10, 12, and 14 in Australia and the United States. Variations of the test were used in two smaller studies with ten- and
fourteen-year-olds in Sweden and twelve- and fourteen-year-olds in Taiwan. Among overall conclusions from these studies are the following:

- number sense (particularly of older students) can be assessed by written tests, but individual interviews are needed to reveal students’ thinking;
- conceptual understanding of decimals is generally weak, and conceptual understanding of fractions is very weak; and
- written questions testing number sense as opposed to skill acquisition or instrumental understanding are difficult to devise. (McIntosh & Dole, 2000b, p. 35)

Based on an analysis of items that were given to more than one age group, McIntosh, Reys, Reys, Bana, and Farrell (1997) conclude that number sense does appear to develop with age, although whether or not this improvement happens because of schooling remains an open question meriting further study. They further suggest that if number sense does develop through schooling then it would be worthwhile investigating which aspects of schooling and which pedagogical approaches are most effective in this regard.

All three studies, but particularly the Taiwanese one, suggest that number sense does not necessarily develop through the learning of standard written algorithms. These do not appear to have enabled students to develop a practical understanding of place value, estimation skills nor a “true feeling for the nature of fractions and decimals” (McIntosh et al., 1997, p. 53). The authors conclude that more curriculum development and action research are needed to develop effective teaching practices.

In the joint Australian and United States study, the same cohorts of students were tested on mental computation (see McIntosh, Bana & Farrell, 1995b). Results suggest that number sense and mental computation are linked, particularly after the age of 12 (McIntosh et al., 1997, p. 54). The authors conclude that an important implication of this is that one way to develop number sense is to develop mental computation ability, and they suggest that this should be given greater prominence in school curricula at the expense of time spent on the teaching of standard written algorithms. They also suggest that, with the increased emphasis on state-wide testing of numeracy, more emphasis should be placed on the assessment of mental computation, and argue that this can be assessed by group pencil-and-paper tests, where the questions are given orally, and students have about twenty seconds per item to write answers.

Findings from the Australian study suggest that

The average Australian 10-year old has a reasonable understanding of notation and place value of whole numbers and numbers with one, but not two, places of decimals. Understanding of fractions is limited to representations of simple fractions as parts of a whole and subsets of a set, but not as points on a line. (McIntosh, Reys, Reys, Bana & Farrell, 1997, p. 34)

For further details of this body of research and the earlier joint work in Japan, see also Reys, Reys, McIntosh, Emanuelsson, Johansson and Yang (1999), McIntosh, Bana and Farrell (1997) and McIntosh, Nodha, Reys and Reys (1995).

In a later study of 58 Years 3 and 5 students in Tasmania, McIntosh and Dole (2000a) administered separate pencil-and-paper tests for mental computation, number sense, and general mathematics over three days. Students were chosen for interview if their results on the three tests were either very similar or very different. Findings from the study suggest that:
the three tests were testing different things;

• students who score highly on mental computation tests and general mathematics tests may not be developing a “sense” of numbers;

• students who do not score highly on written tests of mental computation, number sense and general mathematics may still have good strategies for mental computation and a lot of “sense” about numbers; and

• mental computation and number sense need to become integral components of curriculum and assessment procedures at class, school, and system levels.

According to the authors, fostering the development of number sense and conceptual understanding of numbers and operations, and probing further to ascertain whether a student’s accuracy in mental computation is the result only of successful mental application of written strategies or is based on a more flexible range of mental strategies, may strengthen the relationship between mental computation, number sense and general mathematics ability.

The role of calculators in developing number sense has also been investigated over the past decade in Australia. The Calculators in Primary Mathematics project was a long-term investigation into the effects of the introduction of calculators on the learning and teaching of primary mathematics. The purpose of introducing calculators was not to make children dependent on calculators, but rather to enhance that elusive quality “number sense” by providing children with a rich mathematical environment to explore. As part of an extensive study of the long-term learning outcomes for children, four different tools — a written test, a test of calculator use and two different interviews — were used over the three-year period 1991 to 1993 at Years 3 and 4 levels, to determine the long-term effect of calculator use on children’s learning of number. The second interview, which focused specifically on number sense, was conducted with a sample of 85 children. The interview was based on a draft framework for number sense (from McIntosh, Reys, & Reys, 1992), including items on mental computation, knowledge of numbers (including ordering of numbers within and among number types, relationships between number types and place value) and estimation. Groves (1993; 1994a), and Stacey and Groves (1994) show that project children with long-term experience of calculators performed better than children without such experience on a range of computation and estimation tasks and some “real world” problems. The children who had had free use of calculators:

• exhibited better knowledge of number, particularly place value, decimals, and negative numbers;

• made more appropriate choices of calculating device; and

• were better able to interpret their answers when using calculators, especially where knowledge of decimal notation or large numbers was required.

According to Groves (1994) these results support the assertion that the presence of calculators provides a learning environment to promote number sense.

A current calculator project, entitled Calculator Support for Making Sense of Numbers in Primary School Classrooms, is being carried out by Sparrow and Swan at Year 6 level in four primary schools in Western Australia. This research aims to investigate if number sense can be developed in children using the calculator as a catalyst, after the introduction of standard algorithms. The project is using pre-tests and post-tests that are based on the number sense test developed by McIntosh, Reys, and Reys (1992).
Overall, it can be seen that Australian researchers investigating children’s conceptual development across a wide range of aspects of number frequently highlight the need for teachers to identify, value and develop children’s spontaneous, informal computational strategies. The place of standard written algorithms in the mathematics curriculum and their role in children’s development of number sense continues to be the subject of debate. A wide range of studies into children’s arithmetic strongly suggest the need to place much more emphasis on children’s understandings of fundamental concepts before the teaching of rules and procedures, as well as the need to coordinate new symbolic knowledge with children’s existing informal knowledge and their real-life experiences.

Space

A substantial body of research reporting on the development of spatial concepts has resulted from a collaboration between researchers at New South Wales universities and the NSW Department of Education and Training. The Count Me Into Space project has developed a comprehensive framework for the teaching of space based on extensive research work on visual imagery and spatial thinking. Owens (2000a), suggests a sequence of strategies as the usual development for spatial thinking for each of the key ideas of Part-Whole Relationships and Orientation and Motion. This sequence is:

- emerging strategies — in which students begin to notice parts and wholes and try to change shapes;
- perceptual strategies — in which students notice parts, compare, and make changes when they had the shapes present;
- pictorial imagery strategies — in which students have one or two images of shapes and can refer to parts and wholes and simple changes of the shape by relying on their imagery;
- pattern and dynamic imagery strategies — suggesting a development of imagery related to conceptual development and manipulation of images in the mind; and
- efficient strategies — in which students select and use a variety of imagery and conceptual understandings when working with shape investigations.

For each kind of strategy, descriptors both for investigating and visualising and for describing and classifying are given — and these can be used to inform planning for teaching as well as assessment.

This framework is being used in professional development for early years teachers in several districts of New South Wales. It should, however, be noted that it focuses only on two sub-strands for Space. The project has been evaluated by comparing the pre- and post-tests of children participating in the project with those from comparable schools. Overall, the project children performed significantly better (see Owens, 2000a). As part of the process used to improve tasks and lessons and to guide materials to be used in future developments — including videotapes for professional development — students and teachers completed questionnaires, and lessons were observed. Teachers in the project:

- realised that children were able to do more than they had expected;
- found the lessons provided an entry point for a more open-ended style of teaching;
- became more aware of children’s strategies;
- felt that they knew more after the project about how children learn relevant concepts;
• were more likely to devote more mathematics lessons to teaching the topic after the intervention than before;
• were more satisfied with their teaching of space;
• took time to feel comfortable with their assessment of their students’ performance of the tasks;
• extended their own mathematical knowledge (for example, became more familiar with the necessary language);
• extended their own pedagogical knowledge (for example, learned more about the purposes of the learning experiences, had more confidence to vary the lessons); and
• extended their own pedagogical content knowledge (for example, saw the value of drawing on children’s own language and experiences in developing more formal concepts, realised the value of specific types of teacher questioning, and learned how to achieve a sense of conversation).

As part of the same Count Me Into Space project, Mitchelmore and White (2001) worked with DET curriculum officers and consultants to design an *angles* unit on the basis of their research. They also developed a corresponding teaching package for Year 3, comprising background information, ten lessons and appropriate assessment tasks, together with a professional development two-day workshop package. The researchers recommended incorporation of the lessons into the program, after revisions informed by the trials. The authors reported that the participating teachers:

• showed a positive response to the ideas and materials;
• learned a great deal about angles;
• made various errors, suggesting that there is still room for improvement in their knowledge; and
• deepened their understanding of important pedagogical principles, including the value of hands-on materials, links to students’ environment, interactive lessons, use of correct terminology, careful sequencing of topics, and continually building on students’ previous knowledge.

The authors also reported that major difficulties experienced by students included:

• drawing a horizontal line to form an angle of slope
• representing the angle of turn (for example with the hands of a clock);
• seeing angles in three-dimensional situations (for example the angle of an open door); and
• focusing on the angle and ignoring irrelevant physical attributes of models.

Related to his work in this project, Mitchelmore has a series of reports on children’s ability to see similarities between realistic models of physical contexts and the corresponding real contexts. For example, Mitchelmore (1994) identifies contexts that appear to hinder the recognition of similarities, as well as specific abstractions that children need to make. He points out that concepts cannot be constructed from experience in a single context and outlines strategies to help children overcome difficulties and misconceptions with specific concepts of angle and related ideas (Mitchelmore, 2000).

Owens (1996; 1998) also reports on studies of difficulties that students have with angle concepts. She describes the ability to notice and analyse angles as being akin to problem solving, and uses case studies to illustrate how complex conceptualisations of angle
develop as a result of mental imagery, selective attention, manipulation of materials, and discussion.

A smaller project in which teachers worked closely with their own classes was the Early Assistance Action Research Project in South Australia. It attended to the question of “How do we extend children’s spatial ideas and their ability to express them?” The study focused on links between spatial numeracy and communication. The researchers found that young children have more developed spatial concepts than they can verbalise, so emphasis was put on supporting relevant language development. The children’s confidence in expressing spatial ideas and their ability to describe aspects of shapes and locations improved with this increased attention to language. This resulted in better learning and higher levels of confidence with primary school geometry.

Diezmann (1994) explored the role of diagrams in the learning of geometry. While diagrams are regarded as a useful form of communication for conveying geometric ideas, Diezmann found that children may experience serious difficulties in interpreting diagrams. She studied the behaviour of babies, Year 2 children, and Year 5 children when presented with a three dimensional shape and a corresponding diagram. Her results indicate that the interpretation of diagrams may be a constraint to effective communication in geometry. Diezmann proposes a five-stage model for the development of understanding of diagrams of three-dimensional objects:

- **induction** to representation;
- **syncretic representation** — where the child understands that the symbol “stands for” a referent, but confuses the referent characteristics with the pictorial characteristics;
- **naive conventional representation** — where the child has a novice understanding of the relationship between a symbol and a referent;
- **functional representation** — where the child is aware of the conventions, limitation and ambiguity of diagrams; and
- **multi-representation and multi-source** — where the child is aware that there are many possible drawings for a given object, and that there are many possible objects for a given drawing.

The levels of “diagram literacy” cover the transition from “re-presentation” of a referent to “representation” of a referent, with the model being useful in raising the awareness about the complexity of using diagrams.

In a further paper on developing spatial abilities and language, Diezmann (1997) focused on the USA’s National Council of Teachers of Mathematics (NCTM) recommendations for geometry. Diezmann identified a range of diverse everyday environments in which spatial encounters occur. This work suggests that teachers need to be made more aware of key concepts and experiences that can be drawn out of everyday environments in the early years of schooling.

Warren and English (1995) demonstrate the importance of developing this strong foundation. Using a series of spatial tasks involving conceptualisation and manipulation of unfamiliar plane shapes with children aged 4 to 12, Warren and English found that both children’s perceptions of shapes and their approaches to task solution were strongly influenced by language and experience, with these factors affecting children’s success at shape recognition. However, interview-based assessments by Everett and Mulligan (2000).
showed the influence of non-mathematical aspects of tasks. Although the quality of students’ visualisations improved over time, their representations of three-dimensional shapes continued to be affected somewhat by non-critical aspects of the shapes.

Harris’ (1991) research in a non-typical socio-cultural environment also shows the importance of building on children’s experience and spatial concepts. After spending some months in a remote rural Aboriginal community, with some observation and interviews of adults, Harris described her interpretation of the community’s perspectives on space, time, and money. Harris argues that the radically different views and socio-cultural experiences of these people should be recognised in both curriculum development and teaching.

Owens (2000b) asked whether there is a correlation between children’s competence with different types of spatial tasks — that is whether or not there is underlying spatial “trait”. Using a mapping task, a task involving tangram pieces, and one using shapes in a barrier game, she concluded that performance on the different types of tasks was not correlated.

Overall, as in the research on Measurement, the major work reported has originated from researchers from universities in New South Wales, working with the Count Me Into Space project. Other research into children’s spatial ideas and their ability to express them suggests that young children are more developed in spatial concepts than they can verbalise, and that their perceptions of shapes and their approaches to tasks are strongly influenced by language and experience. Children may also experience serious difficulties in interpreting diagrams and this may be a constraint to effective communication in geometry.

**Curriculum issues**

The question of what numeracy means to Australian curriculum developers and teachers has been raised by a number of researchers. For example, Willis (1998) sought answers to questions such as:

- What is numeracy, and what has mathematics got to do with it?
- Is numeracy one of the literacies, an aspect of literacy, or a complementary partner to literacy?
- Is numeracy at risk of being colonised by literacy thinking and practice?
- Is numeracy just “the basics” — and if so, basic to what? Are basics enduring or do they change — and if they are changing then what are the new basics?

While the terms numeracy, mathematics and mathematical competency often appear to be used interchangeably, Willis points out that different perspectives can suggest quite different curriculum content, pedagogical approaches, and assessment strategies. Moreover, given that mathematical ideas frequently underpin, and are used in, most school subject areas, the question of how to best prepare children for these demands is a matter of some importance. The term numeracy has been useful in highlighting the importance of developing students’ ability to use school mathematics outside of mathematics classrooms as well as the need for mathematics teachers to attend to cross-curriculum needs.
A popular practice in primary schools over the past decade has been the integration of various curriculum areas, including mathematics, into units of work of varying length. Reporting on the three-year Australian Research Council-funded project Integration of Mathematics, Science and Technology, Venville, Wallace, Rennie and Malone (2002) state that for the Years 6 to 9 children in their study, integration of mathematics with other curriculum areas enhanced numeracy. Modes of integration included thematic approaches and cross-curricular approaches incorporating special events such as fairs and local community projects. Successful integrated classroom environments held students’ interest and enhanced learning across the curriculum. Lessons were characterised by:

- high levels of teacher and pupil engagement and interaction;
- teachers and pupils having clear sense of direction;
- work that was cognitively challenging;
- emotional involvement of participants;
- high levels of trust and co-operation; and
- teachers regularly making links to the real world.

These features were present whether the class was taught by a team of teachers or a single teacher, and whether or not the content matter was chosen to fit a theme. The researchers concluded that, in the quest to engage pupils, the critical issue is one of good teaching rather than whether or not to integrate the curriculum.

Catering for numeracy needs in key learning areas other than mathematics is not merely a matter of teaching the necessary mathematics and assuming that children will be able to apply it in varied contexts. As Hogan (2000) points out, students need to:

- identify specific numeracy demands of a situation;
- cope with the mathematical demands;
- understand how the mathematics is shaped by the situation and vice versa;
- have the strategic skills required; and
- be able to identify further numeracy demands and possibilities present in the situation.

In a two-year project, Numeracy Across the Curriculum, carried out in nine Western Australian schools, Willis, Hogan, and Jeffery aimed to develop:

- a description of numeracy with examples from across the curriculum; and
- an approach to numeracy based on the practical experience of teachers and the needs of each learning area.

After documentation of numeracy practices within each school, teachers participated in workshops where their research data was discussed and reviewed. Staff then conducted their own research by observing and describing numeracy examples as they occurred in their classroom in all key learning areas. The research team reported that teachers generally improved their ability to develop strategies to act on numeracy issues that they identified. They started to note and engage with situations that had previously been missed by both teachers and students, or avoided by them, or covered by the teacher without demanding mathematical engagement from the students. The researchers proposed that curriculum documents need to establish more clearly the numeracy demands of the particular learning areas. As part of this project, they constructed a “Numeracy Framework”, in order to assist both primary and secondary teachers to:

- describe what is involved in being numerate in any particular situation;
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- identify the numeracy demands of a particular task;
- diagnose students’ numeracy problems;
- plan strategies to improve student numeracy; and
- ensure that there are sufficient numeracy demands included in their program plan.

(Hogan, Jeffery & Willis, 1998).

A further product of this project was a Numeracy Audit that outlines a process for schools to follow in developing teacher understandings of numeracy and at the same time review student numeracy.

In her thesis *Being numerate in Australian society: What is required?*, Hammond (1997) described methods used to determine what numeracy skills are needed in today’s society (see also Hammond, 1998; Clarke & Hammond, 1997). A list of numeracy skills was compiled from the relevant literature, and assessment tasks were developed around this framework. The resulting Basic Numeracy Framework details skills for the Number, Algebra, Chance and data, Measurement, Space, and Working mathematically strands at Years 6 and 9. In testing Year 6 children, Hammond found that Number and Measurement tasks resulted in the highest mean scores (although percentage tasks caused difficulties), while Space and Chance and data scored poorly.

In another study of numeracy demands in the wider context Cumming (1999) drew links between numeracy performance and life outcomes for employment, education and training. Cumming notes the continuing pervasive dependence on written tests in mathematics education that often use items with little obvious relevance or context. She stressed the need for:

- more conversations between students and teachers;
- more exploration of meaning and understanding through orality;
- less concern with curriculum content coverage; and
- recognition of the value of appropriate use of technology.

In terms of curriculum issues, the review of the literature suggests that, while individual authors have addressed the question of the meaning of the term “numeracy”, by and large it is used interchangeably with “mathematics” or “mathematical competency”. While the need to cater for numeracy needs in key learning areas other than mathematics has resulted in significant ongoing research and there has been some research examining numeracy demands in today’s society, there is still a need for further research in order to achieve the broader goal of fostering students’ capacities and disposition to make effective use of their mathematical learning.

**Developmental frameworks**

The starting point must be the child’s current understanding — our efforts must go into helping each child to make the connections which will promote the idiosyncratic personal understanding. (Pound, 1999, p. 51)

Any developmentally appropriate curriculum starts with children’s ideas. However, given the number of children in most classrooms and the difficulties of probing their thinking, as well as the inconsistencies in children’s thinking, there are considerable practical
obstacles to achieving this aim. A powerful compromise position is to base instruction on children’s typical understandings — and misunderstandings — as identified by research. A major feature of recent Australian research, including that taking place in some major state projects, has been the construction of developmental frameworks. As stated earlier in this chapter, much of this research was underpinned by the work of Steffe and his colleagues in the United States. This work, and later work by Cobb, Yackel, and Wood, included extensive teaching experiments based on:

- assessment through observation;
- a focus on settings, tasks, strategies and procedures;
- presentation of varied assessment tasks; and
- eliciting and supporting the use of children’s strategies.

As one outcome of this work, Steffe described levels of counting skills and competence that progressively need less sensory input. In a series of teaching experiments, shifts in cognition that allow children to move from concrete to abstract were used to map teaching trajectories to guide classroom activities and discussions.

This research was foundational to the work of Bob Wright in the *Mathematics Recovery* project (Wright, 1992; Wright, Stanger, Cowper & Dyson, 1996). Wright established an extensive empirical base for the analysis of arithmetic learning, including videotaped records of individual interviews with children as they attempted challenging mathematical tasks. Outcomes from this work were used as a basis for the development in 1996 of the *Learning Framework in Number* (Wright, 1998; Wright, Martland & Stafford, 2000). This framework of guiding principles, operational strategies and assessment items includes levels of competence in:

- addition and subtraction;
- forward counting;
- backward counting;
- identification of numerals; and
- understanding of place value.

An associated *Schedule for Early Number Assessment* (SENA) was also developed. This consisted of a series of 49 tasks for use in one-to-one interviews with children to elicit their levels of development. More than one hundred “at risk” first-grade students were withdrawn for one-to-one long-term teaching programs. Typically, the programs included a six-week orientation for teachers, incorporating the assessment of children and the planning of teaching sessions, as well as professional development days during the teaching period.

The learning framework was adopted, and further supplemented, by the Department of Education and Training in New South Wales in 1996. It was trialled in thirteen schools in NSW as *Count Me In*, and evaluated as successful, warranting further support (Bobis & Gould, 1998). Comparisons of learning outcomes also showed the potential of programs based on developmental frameworks to bring about significant improvement in children’s number knowledge. In 1997 the project was renamed *Count Me In Too* and used in 53 schools (Bobis & Gould, 1998; Stewart, Wright & Gould, 1998). The project, which now uses the *Count Me In Too Learning Framework in Number*, is a major state-wide initiative involving networked research (in schools, regions and state-wide in all sectors), teacher development, curriculum development, and materials development.
Based on *Count Me In Too*, a number of *Count Me In* projects have been developed in Tasmania and New Zealand. Schools are being supported to use the framework and materials to further develop teachers' knowledge of their students' mathematical needs and progress. There has also been significant interest in the project in Canada, England, and New Zealand, with facilitator training programs being established in these countries, together with nation-wide trials in New Zealand.

Another direction has been extension of the *Count Me In* principles to *Measurement* (see *Count Me Into Measurement*) and also *Space* (*Count Me Into Space*), together with the inclusion of sections on early multiplication and division. Six levels of multiplication and division knowledge described in order of increasing sophistication have been included in the basic framework, based on Mulligan’s (1998) work in Years K to 2.

The key ideas of Wright and his colleagues are also evident in other developmental frameworks and intervention programs (see, for example, Pearn, 1994; 1998).

According to Clarke, Sullivan, Cheeseman, and Clarke (2000), Wright's learning framework in number was one influence on the work of the *Early Numeracy Research Project* (ENRP) team in Victoria (see also Clarke, 2001). The researchers reported that an outcome of their initial data analysis was an *Emergent Numeracy Profile*. This was used as a basis for the design of structured, numeracy-specific teaching and learning materials to scaffold a hierarchy of skills, strategies, and dispositions concerned with mathematical thinking and problem solving. This was further developed into a comprehensive research-based learning and assessment framework — a framework of *growth points* of early numeracy learning. Clarke and his colleagues report that this framework took into account major research in numeracy learning as well as previous attempts to develop such frameworks. Data collected from individual interviews with over 5000 children was used to refine the framework. A summary of the framework’s finer details can be found in Clarke et al. (2000). Each of the three major strands are subdivided into learning domains, for each of which a set of four to six growth points have been identified and described. The strands and domains are:

- **Number** — including domains of Counting, Place value, Addition and subtraction strategies, and Multiplication and division strategies;
- **Measurement** — including domains of Length, Mass and Time; and
- **Space** — including domains of Properties and shapes; and Visualisation and orientation.

According to Clarke et al. (2000), the framework aimed to:

- reflect the findings of relevant research in mathematics education from Australia and overseas;
- emphasise important ideas in early mathematics in a form and language readily understood and retained by teachers;
- reflect, where possible, the structure of mathematics;
- enable the description of the mathematical knowledge and understanding of individuals and groups;
- form the basis of planning and teaching;
- provide a basis for task construction for interviews, and the recording and coding process that would follow;
- allow the identification and description of improvement where it exists;
• enable a consideration of those students who may benefit from additional assistance;
• have sufficient “ceiling” to describe the knowledge and understanding of all children in the first three years of school; and
• build on the work of successful, similar projects such as Count Me in Too (Clarke et al., 2000, p. 11).

Based on this framework, a multi-level professional development program was developed, together with a task-based one-to-one interview schedule. For detailed information about the three major components of the project — the framework, the interview, and the professional development project — see Clarke (2001) and Clarke and Cheeseman (2000). Data on growth in children’s understanding across the mathematical domains for Years K to 4 demonstrate significant progress by children in the 35 trial schools (Clarke, 2001; Gervasoni, 2000). Furthermore analysis of the data using a more sophisticated, rescaled model of growth points, and showed “a modest but consistent advantage in achievement from participation in teachers’ Professional Development” (Rowley & Horne, 2000, p. 22).

Based on this work of Clarke and his colleagues, a more complex framework of growth points in number, measurement and space was developed by Victoria’s Early Years Branch of the Department of Education, Employment & Training — the Early Years Numeracy Program, P-4 (EYNP). This new framework is organised into the domains of counting, place value, strategies for addition and subtraction, strategies for multiplication and division, time, length, mass, properties of shape, and visualisation and orientation. The EYNP framework is now being used as a basis for:

• materials production;
• research on teaching and learning;
• monitoring and assessment of students’ development;
• structured classroom programs;
• initial and ongoing professional development; and
• information for parents.

In Western Australia, the First Steps in Mathematics project sought to improve mathematics outcomes — particularly for students at risk of not achieving their potential — by using a developmental framework to increase teachers’ understandings of teaching and learning mathematics. Teachers were provided with means of linking the number, measurement, and space strands in the student outcome statements with curriculum content in each phase of schooling, with specific outcome statements being reflected on four levels of development. During the development process, detailed diagnostic maps and associated curriculum materials were produced, together with a structured professional development program. The materials include general outcomes for each strand, as well as specific outcomes with pointers that teachers can use to assess children’s progress.

Developmental frameworks can be used not only to inform curriculum, planning, and assessment of individual children, but also for describing the progress of entire cohorts of children. For example, Stephanou, Meiers and Forster (2000) report details about the Australian Council for Educational Research project, Longitudinal Literacy and Numeracy Study, which is set within a conceptual framework of developmental assessment.
Progress maps — continua describing increasing levels of achievement — provide frames of reference for monitoring students’ achievements over time, with numeracy scales used to map conceptual growth.

What’s ‘Making the Difference’ in Achieving Outstanding Primary School Learning Outcomes in Numeracy?: Strategic Numeracy Research and Development Project, NSW developed criterion-referenced achievement scale by which the numeracy development of students in the project can be assessed is being developed. The Rasch Simple Logistic Model is being used to attempt to construct a single unidimensional numeracy scale on which item difficulty and student ability can both be represented (Mulligan, 2001).

The New South Wales Secondary Numeracy Assessment Project (SNAP) has developed numeracy assessment items across a range of curriculum areas. This secondary-school initiative has now been extended into primary schools, with the assessment tasks being linked to Years 5 and 6 primary skills and concepts, so that primary teachers can also use the tasks to assess their students in order to better prepare them for the transition to secondary school. Using Item Response Theory (IRT), SNAP plots results of written tests on a grid of concepts and students. Teachers and schools are provided with detailed information on achievement by concept, as well as by child, class, and school. Common errors can be identified. Information on subgroups of students is being collated (by gender, ATSI, LBOTE) and cross-group analysis is being undertaken. State-wide performance data for each test item is available to schools, as well as individual student reports for parents.

The construction of developmental frameworks, based on extensive research, particularly in the early years, has been a major feature of recent Australian research. These developmental frameworks have been found to be not only useful for mapping students’ progress, but also for developing appropriate curricula, and, perhaps most importantly, as a means of linking teacher professional development to key mathematical concepts and their development.

Mathematical thinking

A major goal of mathematics instruction is to help children develop the belief that they have the power to do mathematics and that they have control over their own success or failure. This autonomy develops as children gain confidence in their ability to reason and justify their thinking. It grows as children learn that mathematics is not simply memorizing rules and procedures but that mathematics makes sense, is logical, and is enjoyable. (National Council of Teachers of Mathematics, 1989, p. 29)

Mathematical thinking plays a critical role in numeracy. When dealing with real-life situations, the ability to reason logically, to select appropriate strategies to tackle problems, to monitor one’s progress and apply reality checks, and to interpret, analyse and communicate mathematics, is as important in determining success as is a knowledge of mathematical skills.

The review of research into Mathematical thinking has been divided into four sections: Children’s problem solving; Children’s thinking strategies; The language of mathematics; and Visualisation. However the distinctions between these sections was not always clear-
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cut, resulting at times in a somewhat arbitrary allocation of the research and some overlap between the different sections.

Children’s problem solving

An earlier section of this report examines the role of problem solving and investigations as a teaching strategy to assist in the development of numeracy. This section reports on research into children’s problem solving as an aspect of their mathematical thinking.

Problem-solving strategies

One Australian researcher with a long history of research into various aspects of children’s problem solving, and mathematical thinking in general, is Lyn English. In the Australian Research Council-funded project *The Cognitive Competence of Low, Average and High Achieving Children in Solving Novel Mathematical Problems Involving Combinatorial and Deductive Reasoning*, English investigated nine- to twelve-year-old children’s strategies and reasoning processes in solving novel combinatorial and deductive problems. Children’s strategies in solving the combinatorial problems ranged from inefficient, trial-and-error procedures, to sophisticated strategies for generating all possible combinations. According to English (1993a), a particularly interesting finding related to the deductive reasoning of children classified as low achievers in mathematics. These children often appeared to quickly see relationships and connections between items of information, using these to streamline the solution process, and exhibited comparable self-monitoring processes to those of the other children. The study highlights children’s ability to reason logically in solving novel problems and, in the process, to independently develop more sophisticated procedures, suggesting that the mathematics curriculum needs to be broadened to include a range of novel problems in order to provide all children with the opportunity to have more control of their learning (see also English, 1996a; 1998a; 1999a).

In another related paper, English (1998b) reports on primary students’ views on the engagement potential of problem-solving activities, finding that computational problems had the least appeal, while non-routine problems focusing on reasoning rather than computation had the most appeal.

Even within the realm of computation, there are important decisions to be made about the nature of the tasks in which children are engaged. Watson and Mulligan (1990) used the SOLO taxonomy to analyse 34 K to Year 2 children’s solutions to a variety of multiplication and division word problems. Children in the early grades of school were found to be operating at as many as six levels. Furthermore, children may or may not need ikonic support materials in mathematical problem solving. So, while not discounting the importance of the use of physical materials in the infant school, Watson and Mulligan suggest that some children will be ready to do problem solving of a more symbolic nature, with the challenge for the teacher being to meet the need of individual children.

In a case study of three Year 6 children over the period of a school year, Lowrie (1998) investigated the importance of visual processing in problem solving. Based on an analysis of the methods used by the children, Lowrie concluded that while students tend to change their approaches to solving mathematics problems from visual to non-visual methods as task complexity is reduced, their beliefs and values about the nature of mathematics also
contribute to the methods they select, highlighting the impact the affective component of decision making has on the approaches used in problem solving. According to Lowrie, successful non-visual processing of a task may also suggest that the student has a “more complete” understanding of the particular problem. Furthermore, cooperative learning situations, which place visual and nonvisual “thinkers” together, may generate conflicts that promote higher-order reasoning within the group’s decision making environment (Lowrie, 1998, pp. 207–208).

**Problem posing**

Problem posing is recognised as an essential feature of mathematical activity and it has been suggested, particularly in the United States, that it should also form part of the school mathematics curriculum. While the seminal work of Brown and Walter (1983) promotes this view, other justifications for its inclusion range from the possible insight into children’s understanding that might be gained from observing children’s problem posing (English, 1996b) to a possible link between abilities in problem solving and problem posing (Silver, 1994). In the Australian Research Council-funded project *Developing Children’s Problem Posing, Grades 5–7*, English investigated the extent to which children’s number sense and novel problem-solving skills govern their problem-posing abilities in routine and non-routine situations. Using assessments of number sense and novel problem solving, children were characterised as strong in number sense but weak in novel problem solving, weak in number sense but strong in novel problem solving, or strong in both domains. A ten-week problem-posing program was developed based on a framework encompassing children’s recognition and utilisation of problem structures; their perceptions of, and preferences for, different problem types; and their development of diverse mathematical thinking. English (1997a) reports that, in contrast to the control group, children who participated in the program showed an improvement in their ability to construct their own problem contexts, with children’s responses suggesting that both number sense and the ability to solve novel problems play an important role in the creation of new problems based on existing structures, with their impact varying with the type of problem. Reporting on the part of this study that focused on children’s computational problem posing, English (1996b) notes that while most children were able to create problems for the addition and subtraction examples, little diversity was displayed, and that they had more difficulties creating appropriate multiplication and division problems. Moreover, children’s responses indicated a lack of connection between their informal, intuitive knowledge and “school maths”, suggesting that explicit attention needs to be placed on encouraging children to make these connections through everyday problem posing and “getting children into the habit of recognising mathematical situations wherever they might be” (p. 238). For more details of this project see also English (1998c), and English, Cudmore and Tilley (1999).

In a study of the way in which two Years 3 and 4 children constructed and designed mathematics problems for friends to solve, Lowrie (1999b) concluded that problem-posing experiences help children to think about problem solving in more sophisticated ways, while also helping teachers gain insights into children’s mathematical abilities.
Improving problem solving

In a review of recent research on cognitive, metacognitive and affective aspects of mathematical problem solving, Barkatsas (1995) describes mathematical problem solving as one of the most complex human activities and claims that this necessitates very thoughtful consideration about the best ways to teach it. Polya (1962) states that

Solving problems is a practical art, like swimming, or skiing, or playing the piano: you can learn it only by imitation and practice... if you wish to learn swimming you have to go into the water, and if you wish to become a problem solver you have to solve problems. (p. v)

However, this should not be taken too literally as Barkatsas then goes on, almost immediately, to state that

No book... [can offer] a universal perfect method for solving problems, but even a few small steps towards that unattainable ideal may clarify your mind and improve your problem solving ability... I wish to call heuristic... the study of means and methods of problem solving... [and offer] a down-to-earth practical aspect of heuristic: I am trying, by all the means at my disposal, to entice the reader to do problems and to think about the means and methods he uses in doing them. (p. vi)

Bentley (1996) examined the effectiveness of teaching heuristic strategies and metacognitive practices to Year 6 and 7 students engaging in problem solving in mathematics. Students who took part in an eight-week program based on the teaching of Polya’s four stage model of problem solving, heuristic strategies, and metacognitive skills showed an increase in problem-solving effectiveness compared with a control group. Knowledge of heuristics, however, does not of itself necessarily lead to success in problem solving, with an important feature of successful problem solvers being their ability to monitor and manage their progress in selecting and implementing appropriate strategies (Schoenfeld, 1995; Mason, Burton & Stacey, 1982). In one Australian Research Council-funded project, Organisation and Management of Knowledge in Geometry Problem Solving, Lawson investigated how the problem of having the required mathematical knowledge but not using it at appropriate times is influenced by the organisation of the knowledge base and students’ management of their problem-solving activity. In another study of ten Year 6 children’s problem solving, Taplin (1994a) explored how perseverance contributes to successful problem solving. Through an analysis of the strategy sequences used by successful problem solvers, Taplin developed a four stage model which was tried by 54 primary teachers with 105 children from Year 2 to Year 6 in order to explore the feasibility of training problem solvers to use this model, the extent to which teachers found it useful, and their recommendations for implementing the model (Taplin, 1994b, p. 593). Findings suggest that it is possible to “train” people to use the model, with both teachers and students believing it to be useful for enhancing problem-solving performance. Teachers’ recommendations for implementing the model included introducing it to young children and reinforcing it throughout schooling.

In a study of the effect of instruction on Year 5 children’s use of diagrams in problem solving, Diezmann (1998) investigated how instruction in using the “draw a diagram” strategy affected children’s problem-solving performance and explored the relationship between instruction and changes in children’s use of diagrams. While use of the “draw a diagram” strategy is frequently advocated, successful use of diagrams may not occur spontaneously and findings from this study suggest that instruction can improve children’s
use of diagrams in problem solving by developing the appropriate knowledge of diagrams as a problem-solving tool. According to Diezmann, effective use of diagrams in problem solving involves "visual literacy or graphicy", and therefore “the scope of literacy in the classroom needs to extend beyond numeracy, oracy, and written literacy to include literacy with various forms of visual representation, which includes diagrams” (Diezmann, 1998, Abstract).

**Metacognitive and affective aspects of problem solving**

In a year-long teaching experiment involving twelve Year 4 students, Siemon (1993) explored the possibility of changing their approach to problem solving. A metacognitive question and answer technique was used to reflect on the problem-solving process. While changes in children's approaches to problem solving differed, the program appeared to have an impact. According to Siemon, having some insights into the different goals, beliefs and values operating in the classroom provides teachers with a basis for challenging and changing those which actively operate against the negotiation of shared mathematical meanings and the use of more powerful generalisable strategies.

In a study of the effects of combined metacognitive and attribution training on achievement in reading comprehension and mathematics word problems, Walker (1995) found no commonality between metacognition in the reading comprehension and mathematics word problems. Findings provided only limited support for the role of metacognitive processes in reading comprehension, while self efficacy expectations and causal attributions appeared to be linked to success in both reading comprehension and mathematics word problems.

Wilson (1998; 2000) developed a new multi-method approach for the assessment of three key metacognitive functions for student learning: awareness (what they knew and had done before), evaluation (judgements regarding their thinking or strategy choices), and regulation (changes in the way they were working or plans to work the problem out). Year 6 students used a set of specially designed metacognitive and cognitive action cards to stimulate responses about their thinking during problem solving. Their attempts to solve the problems were videotaped, with the video tapes used in stimulated recall interviews. Students reported diverse metacognitive transitions and sequences when they tackled different types of problems, although most reported starting with the awareness function and ending with the evaluation function.

**Assessing problem solving**

In addition to giving information about how successful children are in getting the right answers to problems (the product), an assessment of problem solving should also give information about the problem-solving processes children use to arrive at their solutions. A number of Australian assessments of problem solving, which not only assess the processes children use but also provide teachers with information on which to base their teaching, have been developed.

In Profiles of Problem Solving (POPS), Stacey, Groves, Bourke and Doig (1993) use essentially unfamiliar tasks, set in familiar situations, to assess upper primary children’s problem-solving performance on five broad categories of processes: correctness of answer, method used, accuracy, extracting information, and quality of explanation. A significant feature of POPS is that it requires students to write an explanation of their
answers. The development of the problems and the marking scheme were underpinned by a research and development phase, which included initial interviews with 60 children and an analysis of data from 200 children using the final version of the test to develop the profile level descriptions. POPS also contains a substantial section on suggestions for further learning.

Another assessment of mathematical problem solving designed for upper primary and secondary students, is the Collis-Romberg Mathematical Problem Solving Profiles (Collis & Romberg, 1992). Tasks address cognitive development in five of the mathematics curriculum strands: number, space, measurement, algebra, and chance and data. Performance within each strand is measured in terms of the levels of responses in the SOLO taxonomy of learned outcomes, which are then related back to stages of cognitive development. For each strand, and for each of the levels, follow-up teaching suggestions are included.

Booker Profiles in Mathematics: Thinking Mathematically (Booker & Bond, 2001) contains a series of assessment items designed to gauge an individual’s capacity to think, reason and problem solve in mathematics. The kit focuses on the underlying concepts and processes involved in solving word problems, rather than content aimed at school year levels. Problems are categorised into five levels of difficulty: the operation to be used is relatively obvious; the operation required is not immediately obvious; the problem contains more information than is needed; further information needs to be gathered; and strategic thinking is required in order to determine a solution strategy (p. 31). Again, teaching suggestions to develop mathematical thinking are included.

In preparation for the research monograph Learning from Children: Mathematics From a Classroom Perspective, Doig and Lokan (1997) invited experts in a range of areas in mathematics education to explore, in the context of wider research findings, what the New South Wales Basic Skills Testing Program (BSTP) results tell us about children’s learning and understanding of mathematics. While many of the BSTP questions are routine, many could be categorised as “problems”, and these are typically set in “real-world” contexts. In her chapter of this monograph, Stacey (1997) discussed aspects of problem solving which can and cannot be assessed using the BSTP machine readable format, together with the effects of the use of everyday contexts on item difficulty. While it is impossible for assessments such as BSTP to truly assess performance in solving problems in everyday life, BSTP was found to provide useful information about some aspects of children’s ability to solve real problems. To provide a complete picture, teachers would need to use a range of assessment techniques.

In summary, studies of children’s deductive reasoning have shown that even children classified as low achievers in mathematics can reason logically in solving novel problems and develop sophisticated procedures, suggesting that the mathematics curriculum needs to be broadened to include opportunities for all children to attempt novel problems and have more control of their learning. In terms of improving children’s problem solving, although teaching heuristic strategies and metacognitive practices has been shown to increase problem-solving effectiveness, an important feature of successful problem solvers is their ability to monitor and manage their progress in selecting and implementing appropriate strategies.
Children’s thinking strategies

When children are consciously making decisions about the type of mathematical procedures ... to use, or how to modify these procedures based on available resources, or how to apply understandings previously learned to new situations, then they are thinking and reasoning mathematically. (Lowrie, 1999a, p. 9)

Children’s thinking strategies have typically been researched through observation and analysis of their thinking when attempting problem-solving tasks — despite the fact that “observing thinking” can present considerable difficulties in practice. Lowrie (1999a) describes the range of processes used by Year 3 children engaged in solving non-routine mathematics problems as an example of how teachers can ascertain whether children are thinking mathematically.

The Australian Research Council-funded project The Development of Children’s Competence in the Mathematical Domain of Combinatorics investigated the independent strategy development of seven- to twelve-year-old children. English (1993b; 1993c) found that changes in children’s strategies as they progressed on the problems suggested modifications to their knowledge of combinatorics, but that this alone was insufficient in explaining enhanced problem solution. Children’s ability to monitor their actions, detect and correct errors, and recognise problem completion also played a crucial role (see also English, 1991).

In a collaborative Australian Research Council-funded project A Longitudinal and Cross-cultural Study of the Analogical and Mathematical Reasoning Patterns of Young Children, English, together with Alexander (University of Maryland), tracked the development of young children’s mathematical and analogical reasoning processes as they progressed from Preschool to Year 2. The project also investigated whether different learning environments in Australia and the United States give rise to significant differences in children’s reasoning patterns. According to English (1999b), reasoning by analogy requires children to focus on relational properties rather than surface features and hence it is important to foster reasoning by analogy in children’s mathematical learning. In a study with older children, English (1998d) investigated ten-year-old children’s abilities to reason by analogy in solving addition and subtraction comparison problems. Children’s responses highlighted the importance of relational and conditional knowledge in children’s reasoning. English suggests that rather than providing children with instruction in rearranging relational statements in subtraction problems involving comparisons, children should be given an array of experiences that allow them to discuss, model and justify their problem interpretations and different approaches. She further concludes that although children reason by analogy in everyday life, they appear to require guidance to apply this to more formal problem solving.

In a study examining the effectiveness of three intervention measures designed to facilitate ten- to twelve-year-old children’s recognition of indeterminacy in reasoning with illogical syllogisms, English (1997b) found pre-adolescent children capable of the hypothetical reasoning involved in deduction. However, none of the eighty ten-year-olds was able to recognise the indeterminate nature of the illogical syllogisms, in contrast to the twelve-year-olds, many of whom nevertheless, even after intervention, had difficulty in achieving this recognition. According to English, the study demonstrates the need to attend to children’s knowledge of relational terms.
Byrt (1994) studied 171 Years 3 to 6 children’s use of formal methods to solve word problems over a two-year period. An analysis of results of a test covering five aspects of the Year 3 curriculum indicated that students increasingly use formal methods as they move to higher year levels. A possibly surprising finding was that a significant factor in the proportion of correct formal responses from students was the influence of individual teachers.

In an attempt to bridge the research-practice gap, Smith (2000) reported on a three-phase research project that led to the development of a pedagogical framework to promote mathematical thinking and understanding in mathematics classrooms. Adopting a social constructivist perspective on teaching and learning, the study used a school and university partnership based on the notion of teachers as reflective practitioners. The first two phases of the study sought to identify pedagogical practices that promote mathematical thinking. The third phase of the project resulted in the development of a framework underpinned by the following six requirements:

1. guided thinking in a supportive classroom environment;
2. verbalising thinking;
3. clarifying thinking;
4. linking thinking (labelled this to highlight the importance of students’ written representations of thinking);
5. learner centred experiences; and
6. learner centred assessment. (Smith, 2000, pp. 10; 12)

According to (Smith, 2000) an important message for educators is the need for a thinking curriculum to be accompanied by assessment practices that support thinking and sense making. She further argues that the involvement of practitioners as active participants in the research process adds authenticity to the framework, providing “descriptive images” of practices to promote mathematical thinking, which can lead to further investigation and elaboration.

Overall, research into children’s thinking strategies has been closely linked to research into problem solving, with children’s ability to monitor their actions, detect and correct errors, and recognise problem completion being found to play a crucial role in successful problem solving.

**The language of mathematics**

Becoming mathematically literate requires students to learn the “language of mathematics”. Skinner (1990) characterises mathematics as:

- a language of abstraction;
- a language that compresses experiences;
- a language that focuses on conventions and written symbols, most with no relationship to what they represent; and
- a language that uses familiar words in unfamiliar ways. (pp. 12–13)

It is typically these features of mathematics that are considered to present barriers to student learning of mathematics. Skinner describes a program focusing on the development of children’s use of mathematical language and recommends that, in order to enable children to link their own mathematical ideas with formal mathematical language, teachers should allow children to use their own words to explore and express their mathematical ideas and thinking.
Padula and Stacey (1990) also discuss the language-related difficulties young children have in understanding early mathematics instruction, with children needing to be able to

- learn words for new or previously established concepts;
- extend and restrict the meanings of familiar words; and
- learn new ways of combining words to generate meaning.

The authors maintain that there is a need for systematic planning of classroom activities that develop understanding of concepts.

In a study of linguistic and pedagogical factors affecting Australian and Papua New Guinean primary school children’s understanding of arithmetic word problems, Lean (1990) found that children in both countries used similar strategies and made similar errors, with the main variable determining difficulty being the semantic structure of the questions.

According to Dawe and Mulligan (1997), classrooms use a mixture of natural and “mathematical” English, with the latter presenting students with difficulties because of “its discontinuity with the modes of expression … children bring with them into the classroom” (p. 12). Dawe and Mulligan describe, analyse, and interpret the results of the New South Wales Basic Skills Testing Program (BSTP) in the light of research on language factors such as semantic difficulty and linguistic cues.

**Visualisation**

The importance of visualisation of mathematical concepts has long been recognised. Among a number of studies exploring the role of visualisation in the development of number concepts, Bobis (1996) investigated links between mental images and number sense of very young children. In a study of two kindergarten classes, teachers used “ten frames” and sets of cards with arrangements of dots representing the numbers from 1 to 10. Children were encouraged to manipulate their mental images and explain their thinking to classmates. Children’s descriptions of their visual images were found to vary widely. As they listened to one another, children began to think about arrangements in more than one way and independently elaborate and extend their own ideas. Bobis concluded that equipment and activities such as these help develop children’s knowledge of part-whole relationships and help children gain a richer knowledge of number than with counting alone, and that encouraging them to be flexible and inventive with their thinking strategies is important in promoting number sense.

In a study of links between understanding of the structure of numeration and representations of the counting sequence from 1 to 100, Thomas and Mulligan (1994) analysed 92 high-ability Years 3 to 6 students’ explanations and drawings of the numbers from 1 to 100, in order to infer their internal representations. Findings indicate that children who show evidence of dynamic internal representations and access to a variety of internal images have more developed relational understanding. This suggests that the active processing of images plays an important part in children’s development of number concepts (see also Mulligan & Thomas, 1998).

The ability to draw and use diagrams plays an important role in problem solving. While students are often urged to “draw a diagram”, they can experience a range of difficulties in generating effective diagrams. Diezmann (2000) compares the results from two studies of structurally dissimilar problems. She concludes “student difficulties on these tasks
appeared to be due to a lack of sense-making in mathematics rather than a difficulty with the problem structure of the generation of a particular type of diagram” (p. 228). The recently funded Australian Research Council project *A Longitudinal Study of the Development of Primary Students’ Knowledge About the Properties of Spatially-oriented Diagrams in Mathematics* will investigate children’s knowledge of the properties of diagrams and the formation of this knowledge.

Observations of primary students solving spatial problems were used by Owens and Clements (1998) to explore their thinking processes. Children were found to use concrete, dynamic and action imagery, as well as other visual processes such as disembedding part of the shape from the rest of the configuration. According to the authors, visualising facilitated, and often steered, the problem-solving processes.

In his doctoral thesis, Lowrie (1996) examined the problem-solving methods and strategies used by Year 6 primary school students in a variety of mathematical contexts. The study found that both visual and non-visual reasoning play an important role in problem solving, with students often using visual processing in the initial stages of problem solving, and then moving to more analytic, non-visual strategies when relevant patterns have been identified, or when a more informed understanding of the problem has been established.

Overall there has been a substantial amount of Australian research over the past decade into the children’s mathematical thinking, with most of this being in the overlapping areas of children’s problem solving and thinking strategies.

**Using mathematics**

Many of the research projects and studies described above involve children using mathematics — for example, to solve open-ended problems or in other school curriculum areas — but there has not been much research that has using mathematics as a central theme.

One exception is English and Lesh’s Australian Research Council-funded project *A Longitudinal Study of Primary School Children’s Mathematical Modelling Within Networked Learning Environments*. The research project is exploring children’s use of mathematical modelling in on-line networked learning environments. As English and Lesh point out, the task of preparing students for success in a technology-based society is especially difficult for primary school teachers. They note that while model-eliciting experiences incorporating computer-networked learning can meet this challenge, this area has received very little attention in primary education.

In another Australian Research Council project, *Practical Mechanics in Primary Mathematics: Fostering Links Between Children’s Spontaneous Concepts and Newtonian Mechanics*, Groves and Doig investigated ways in which practical activities can be used to foster links between Year 5 children’s spontaneous concepts and Newtonian mechanics. They studied how children interact with equipment-based practical mechanics activities and the extent to which ad hoc mathematical modelling could form a bridge between intuitive modes of explanation and scientific models of real problem-solving situations. Among other results, children’s use of graphs constructed from paper streamers to interpret and explain the motion of a ball along a track in a variety of situations was found
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to support the development of children’s understanding of speed as distance travelled in unit time. However the significant errors in the data obtained by the children highlighted the fact that constructing a mathematical model to explain observed phenomena relies not only on the data obtained but also on the observer’s conceptual framework (Doig, Groves & Williams, 1997).

Dunn (1994) reports on the results of a Masters project that used stories to engage young children who were judged by their teachers to be poor performers in both language and mathematics. Application of mathematical ideas to discussions about the stories in class, and extended discussions with the researcher about these aspects after teaching sessions, led to increased competence of the children in both mathematics and language.

In this mapping and review of research into numeracy it has been disappointing to find almost no research into children’s use of mathematics in everyday situations and other non-mathematical contexts. While there may be accounts of children applying mathematical concepts and skills in a variety of situations in professional journals, only two ARC-funded projects relating to mathematical modelling and a project on the application of mathematical ideas to discussions of stories in class were found.

Chapter overview

This chapter has provided a review of Australian primary numeracy research on children’s conceptual development and mathematical thinking, together with curriculum issues related to numeracy.

In summary, the review of research on children’s conceptual development revealed that:

- students fail to abstract from their primary school experiences the mathematical structures that are necessary for them to make a later successful transition from arithmetic to algebra;
- children need to focus on the study of probability in the context of their everyday experiences, with teachers attending to their misconceptions and strategies and beliefs based on experience;
- children have a very limited understanding that different types of graphs can be employed for different purposes;
- it is important to devote sufficient time to the development of underlying measurement concepts before moving to paper and pencil activities and formulae;
- there is a substantial body of Australian research on counting, closely linked to research in the United States, which has formed the basis for intervention programs, the construction of developmental frameworks, and professional development programs;
- the role of concrete materials in children’s development of place value concepts, including the underlying concepts involved in decimal notation, needs to be questioned, with teachers needing to make explicit the connections between the materials and place value and provide activities to develop and reinforce place value concepts at a more abstract level;
Curriculum and Processes

- there is a need for a more holistic approach to teaching number so that children can be helped to build connections between their intuitive knowledge, various models that might be used and formal rules of numeration;
- children need to develop a deeper understanding of the essential features of place value and be provided with opportunities to experiment and to verbalise their thinking in order to achieve enhanced facility with mental computation;
- student improvement in the learning of decimals depends on teachers’ knowledge of the underlying concepts, the use of a clear model and careful bridging from visualisation to numerical forms;
- teachers need to identify, value, and develop children’s spontaneous, informal computational strategies;
- more emphasis needs to be placed on children’s understandings of fundamental concepts before the teaching of rules and procedures with new symbolic knowledge needing to be integrated with children’s existing informal knowledge and their real-life experiences;
- young children are more developed in spatial concepts than they can verbalise, with their perceptions and approaches being strongly influenced by language and experience; and
- children may experience serious difficulties in interpreting diagrams and that this may be a constraint to effective communication in geometry.

The review of research on curriculum issues and developmental frameworks has revealed that:
- by and large the term “numeracy” is used interchangeably with “mathematics” or “mathematical competency”;
- there is significant ongoing research into ways of catering for numeracy needs in key learning areas other than mathematics and some research examining numeracy demands in today’s society;
- further research is needed to achieve the broader goal of fostering students’ capacities and disposition to make effective use of their mathematical learning;
- the construction of developmental frameworks has been a major feature of recent Australian research; and
- developmental frameworks have been useful for mapping students’ progress, developing appropriate curricula, and as a means of linking teacher professional development to key mathematical concepts and their development.

The review of research on mathematical thinking has revealed that:
- research into children’s thinking strategies has been closely linked to research into problem solving;
- children’s ability to monitor their actions, detect and correct errors, and recognise problem completion play a crucial role in successful problem solving;
- even children classified as low achievers in mathematics can reason logically in solving novel problems and develop sophisticated procedures; and
- the mathematics curriculum needs to be broadened to include opportunities for all children to attempt novel problems and have more control of their learning.

The review of research revealed very little research into children’s use of mathematics in everyday situations and other non-mathematical contexts.
Chapter 6

Research on Assessment

Assessment is an integral part of the learning process. Indeed, the major purpose of assessment is the improvement of learning. Assessment provides feedback about students' mathematical development to students and their teachers. This feedback should inform the future action of both learners and teachers. Assessment can be used to report students' progress to parents, prospective employers, and other educational agencies. (Australian Education Council, 1991, p. 21)

This chapter contains a summary of the range of large-scale assessment programs (international, national, and state-wide), standards, assessment techniques, and related research studies, in which Australian primary school children have participated.

While the large-scale studies have gathered data that Australian researchers can access and use as a basis for research, in the main this has not happened. However, studies of classroom and school practices that may lead to improved numeracy outcomes have been undertaken, and achievement results used as indicators of the success of the innovative practice.

Together, these aspects of assessment research provide a picture of assessment as a major facet of numeracy teaching and learning.

Large-scale assessment programs

In Australia, the last ten years have seen the development of large-scale testing programs that have allowed data to be collected by State and Territory authorities and by non-government systems. These have been implemented within the broader context of international assessment programs.

International programs

The Third International Mathematics and Science Study (TIMSS) has been the most significant international assessment program in the past ten years, in which large samples of Australia primary and secondary students participated. TIMSS data were gathered during 1994 and 1995 for three levels of schooling (9 year-olds, 13 year-olds, and Year 12 students). In relation to the primary level, children from 48 countries in the two adjacent grade levels with the largest proportions of nine-year-olds at the time of testing comprised one of the TIMSS cohorts (Population 1). In Australia, the children were in Years 3 and 4 in the Australian Capital Territory, New South Wales, Victoria and Tasmania, and in Years 4 and 5 in Queensland, South Australia, Western Australia and the Northern Territory (Lokan, Ford & Greenwood, 1997).

Lokan, Ford, and Greenwood (1997) reported the TIMSS findings for Australian children. Among the results that they reported were international comparisons of achievement, analyses of achievement by content area, results by gender and language spoken at
home, some State and Territory differences, and results for Indigenous children. Some of the pertinent Australian findings included:

- children in only six countries outperformed Australian children overall;
- Australian children, in the upper grade level, were equal top achievers in the content area of “space”;
- there were no statistically significant differences in mathematics achievement between boys and girls in either the upper or the lower grade;
- Indigenous children scored significantly lower than non-Indigenous children; and
- children who spoke English at home achieved higher than those whose home language was not English.

In discussing comparisons between the States and Territories, the authors were at pains to point out the need to take cognisance of State and Territory policy differences with respect to school entry age and in the different roles of the first year of schooling. The five highest achieving States and Territories were Queensland, Western Australia, South Australia, the Australian Capital Territory, and the Northern Territory where children’s performance levels were well above the international average. The performance levels of children in the other States were close to the international average.

Doig (2001) identified some of the strengths and weaknesses of the Australian children’s TIMSS performance in the context of international comparisons that “can guide future research and action” (p. 5). With respect to primary children, Doig (2001) and Lokan, Doig and Underwood (2000) noted the relatively poor performance of Australian primary children with whole numbers and, compared with lower secondary students, their poorer performance in data representation and analysis, as well as in geometry. Stacey (1997) also analysed the relative performances of Australian children of the various age groups tested in the TIMSS.

National benchmark standards

The State, Territory, and Australian Government Ministers for Education agreed to numeracy benchmarks for children in Years 3, 5, and 7 in April 2000. The benchmarks can be viewed at http://online.curriculum.edu.au/litbench/default.htm. The national benchmarks were developed for use in reporting minimum acceptable standards of literacy and numeracy achievement, in support of the national literacy and numeracy goal (cited earlier in this report).

The development process involved extensive consultation with government and non-government education authorities, independent curriculum and assessment experts (including academics), teacher professional associations and parent organisations. In establishing the benchmarks, guidance was also obtained from the TIMSS data, statewide assessment programs, State and Territory curriculum documents and professional judgments about appropriate and necessary standards. Similar work from overseas was consulted. O’Connor et al. (1999) and Lindsey, Pearn, Lokan, Doig and O’Connor (1999) reported on comparisons between the draft Year 3, 5 and 7 benchmarks, international curriculum documents and achievement data from TIMSS. In general, they found that statements of expectation were set at higher levels in most other countries; however, they noted that the Australian benchmarks are agreed minimum standards, but this was not
always the case elsewhere. Performance data from TIMSS was also examined to assist the benchmark development.

The national benchmarks enable the annual reporting by education authorities of aggregated children’s achievement against the agreed minimum standards. The data provide nationally comparable information on children’s achievement. This will facilitate the monitoring of trends over time, and in relation to the national goal. Importantly, the data produced include information on the performance of sub-groups of children. The data on children’s performance against the benchmarks can be viewed at http://www.curriculum.edu.au/mceetya/nationalgoals/index.htm.

State programs

All States and Territories have large-scale assessment programs in place. These tests monitor detailed children’s performance within education systems in terms of the relevant curriculum. National benchmark data are also obtained from these tests. All children in Years 3 and 5 are tested annually and all systems have agreed to move towards full cohort testing of Year 7 students. In almost all cases, the annual state-wide test results are made public, and are reported to parents, as well as to schools and teachers. The Catholic and Independent education sectors in each State participate in the annual assessment of children in literacy and numeracy through the statewide assessment programs or through an assessment program operated by the Australian Council for Educational Research (ACER). The appropriate educational authority in each State/Territory can be contacted for information on current testing programs.

Trend data are presented in several state-wide assessment program reports (data from reports are available from, for example, Performance Indicators in Primary School (PIPS), http://139.230.167.13/isp/pips/index.html; Doig 2001). The annual Council of Australian Governments (COAG) report on the provision of Government services in the States and Territories includes the results of these assessment programs. Doig (2001) pointed out that there are confounding factors that are involved in the interpretation of the findings of all large-scale assessments, for example, reading levels, and verbal reasoning abilities.

Aspects of state-wide assessment have been examined in research undertaken as part of masters and doctoral programs. Messenger (1998) explored the extent to which schools in Western Australia responded to the Education Department’s call for primary school accountability with respect to mathematics. Hungi (1998) explored data from the 1995 Basic Skills Tests in South Australia using Rasch modelling. The tests were found to be appropriately designed for low achievers and an increase in performance was found between Years 3 and 5.

The Victorian Early Years Numeracy program includes the New Zealand developed School Entry Assessment (SEA) — a series of New Zealand standardised performance tasks — as an option. Children entering school are assessed with the SEA kit in their first two months at school, within the context of the regular classroom. The numeracy task, “Check Out”, is in the form of a shopping game and is administered individually by classroom teachers, who then interpret the results in terms of their local curriculum frameworks.

In South Australia, a common framework is used to describe children’s knowledge and skills, using a consistent set of assessment criteria that can be added to, depending on
the child and the setting. This provides a baseline for children’s learning in the school setting as well as information to support planning and implementation of programs that build on children’s existing knowledge and skills (http://www.schoolentry.sa.edu.au/intro.html).

**Assessment techniques**

Among the current Strategic Numeracy Research and Development Projects, there are several projects that are developing one-to-one interviews and open-ended assessment tasks. In remote non-urban schools in the Northern Territory, for example, the project *Supporting Indigenous Students in Numeracy; Strategic Numeracy Research and Development Project, NT* is investigating the impact that the development and implementation of authentic (rich) assessment tasks have on the outcomes of middle years Indigenous students in a targeted group of schools. This project is developing a relatively small but representative item bank of authentic (rich) assessment tasks suitable for Indigenous students in middle years in remote non-urban schools in the Northern Territory. The tasks are diagnostic in purpose and are intended to assist teachers and schools in identifying the strengths and weaknesses of these students and to plan instruction accordingly. This research project is critically concerned with how teachers in participating schools can be trained to use the tasks with their own children and how, as a result of these assessments, teachers and schools are able to plan to improve their teaching and to modify their school’s mathematics program and report students’ achievement in numeracy. All students in the cohort (Year 3 to Year 6) should be able to make a start on each task. Tasks are graduated internally so that students can work through each task in steps of increasing complexity. Entry level will be aligned to pre Year 3 benchmark — Key Growth Point 3 of the Northern Territory Department of Employment, Education and Training’s (2002) *Northern Territory Curriculum Framework* (NTCF) — and Year 3 benchmark (Band 1 NTCF), with several ensuing parts aligned to the Year 5 benchmark (Band 2 NTCF), and a concluding section aligned to the Year 7 benchmark (Band 3 NTCF).

*What’s Making the Difference in Achieving Outstanding Primary School Outcomes in Numeracy?: Strategic Numeracy Research and Development Project, NSW* required measures of students’ ability beyond descriptive analyses of raw scores and percentages correct. Accordingly, the project has developed a *Numeracy Achievement Scale* that has appropriately graded items along a continuum for students aged 4.5 to 13 years representing key aspects of numeracy across all strands of the mathematics curriculum. The purpose of constructing the *Numeracy Achievement Scale* has been to assess individual student’s numeracy growth through interviews at two stages in the 2002 phase of the project. This required the construction and integration of a large number of items drawn from a number of sources. In order to establish the integrity of these items as a measure of numeracy it was essential to translate these items into a linear measure. The numeracy achievement scale can locate and map an individual student on a scale calibrated by tasks showing levels of attainment of a pre-defined standard of numeracy using a Rasch modelling approach. The scale being used can be easily aligned with standards and benchmarks used at State and national levels. This scale of achievement is independent of age and grade. Item difficulty is calculated...
through a process of calibration independent of the abilities of the individuals assessed in the data collection. Consequently, students can be located on a scale according to the total number of items they answer correctly. The degree to which this score summarises the individual’s profile of responses is found by identifying the ‘fit’ of the student’s response pattern to the model. For the purpose of trialling items, student measures of mean ability were derived from one numeracy assessment interview.

The Victorian Middle Years Numeracy Research Project used open-ended tasks and accompanying scoring rubrics to measure student performance in a large scale study (Siemon & Stephens, 2001). The use of scoring rubrics allowed the tasks and students’ responses to be aligned to different levels of the Victorian Curriculum and Standards Framework.

Stephanou, Meiers, and Forster (2000) reported progress on a numeracy scale, devised within the conceptual framework of developmental assessment, which enables the numeracy development of individuals or groups to be monitored. A sample of 10 children from each of 100 schools across the nation participated in the trials in the first and final terms of their first year at school, and in the first term of their second year. There were two data sources: assessment tasks produced by the ACER and work samples selected from the children’s normal classroom work. In administering the tasks, the children’s teachers conducted one-to-one interviews with the children. The use of common items and Rasch measurement allowed the Longitudinal Literacy and Numeracy Scale to be developed and calibrated. This means that, for example, if an item is answered correctly by 60% of the children at Survey 1 and another item by 60% of more advanced children at Survey 3, the second item must be more difficult because the same percentage of better performing children have answered it correctly. Thus, it is possible to use the scale to show the rate at which children develop various numeracy skills.

Ellerton and Clements (1997) interviewed 182 children to gather data about children’s responses to a multiple-choice test of basic numeracy, the Mathematics Competency Test, and found that many responses could be classified as ‘mismatches’. That is, responses were correct but the children did not have full understanding of the mathematical concept, or responses were incorrect when children had only a partial understanding. Ellerton and Clements were critical of research studies as well as classroom practices and tests in which aspects of mathematics were assessed by pencil-and-paper means and then taken as measures of numeracy achievement.

Given that tests are being used in all States and Territories, research into the effects of these – for example, on teachers’ planning, curriculum development, and forms of assessment used in classrooms – would seem warranted. However, there has been little Australian research into the advantages, limitations, and effects of various aspects of assessment.

**Diagnostic assessment**

Several diagnostic assessment techniques were associated with the intervention strategies discussed in the Intervention section of this report (see Chapter 4). Findings from other research studies and projects in which diagnostic assessment practice featured are presented and discussed here, although over the past decade it would appear that diagnostic assessment has not received much systematic research attention.
The *Queensland Year 2 Diagnostic Net* (Grieshaber, 1997) is a mechanism to diagnose children with inadequate literacy and numeracy levels. A four step process is involved: observation and mapping children’s progress; validating observations against devised assessment tasks; identifying children requiring intervention; and providing support to children needing this assistance, and reporting to parents. The *Net* also serves as a form of teacher accountability with respect to literacy and numeracy levels in society. Kable (1996) described a trial of the *Net*.

In order that New South Wales children reach an appropriate level of numeracy achievement (Stage 3), before moving into secondary school, the *Counting On: Transition 6–7* project focuses on assisting teachers to help children overcome common numeracy difficulties. A range of teaching strategies aimed at overcoming specific kinds of difficulties that children have with *Number* are included in a book by Palmer (1994).

**School entry assessment**

Projects and research studies specifically aimed at identifying children’s numeracy achievements before they enter school, as they enter school, and/or during their first year of schooling are described next. In these types of assessment programs, children ‘at risk’ and those considered to be in need of numeracy intervention are identified. Findings from other research studies, focusing on specific aspects of children’s early numeracy capabilities, are found in other relevant sections of this report.

*Project Good Start: Effective Numeracy Practices in the Year Before and the First Year of School* is a three-year, Australia-wide project aimed at improving the numeracy outcomes of young children. The project involves a quantitative study with a large, representative, national sample of children to gauge their numeracy development before school and during their first year of schooling; and a longitudinal, qualitative study of a sample group, examining before school and first year of schooling experiences that affect numeracy development over the two years.

Various State and Territory educational systems have implemented school entry assessment projects and programs with some extending beyond the first year of schooling. These include: *Early School Assessment Project* (New South Wales), *Early Years Assessment Project* and *School Entry Assessment* (SEA) program (South Australia) (see DETE, 1999), and *Early Numeracy Research Project* (Victoria).

As pointed out by Montgomery and Cheeseman (2000), much of what has been learned about the assessment of numeracy at school entry and in the early years of schooling also applies in later years. Montgomery and Cheeseman noted that it is physically and logistically impossible to evaluate and address the learning needs of each individual child. They suggest that attention to the following three broad considerations will cater for individual differences in attainment and a wide range of learning needs:

• establishment of a conducive learning environment;
• use of quality mathematics activities; and
• use of strategic teaching techniques.

The three points listed above include:

• providing opportunities for children to explain their mathematical understandings;
• encouraging various ways of solving mathematics problems;
• expecting children to reflect on their answers and strategies;
• using broad mathematics activities;
• Including open questions that encourage a range of mathematical answers;
• incorporating investigations;
• using concrete materials;
• engaging children with games;
• having children invent problems of their own;
• organising small group instruction;
• using partner work; and
• teaching techniques that provoke self assessment.

Practices affecting achievement

By far the greatest number of research studies categorised as lying within the general theme of assessment involves those in which measures of children’s numeracy achievements have been only one of the outcomes measured within the research design. Many of the projects in which assessment was a primary focus are discussed in other sections of this report (for example, findings based on the TIMSS, and performance of Indigenous children).

Test results have usually been used in research and reports of developments:
• to assess the effectiveness of teaching or curriculum developments, for example, Developing Computation: Strategic Numeracy Research and Development Project (Tasmania); Assessing Numeracy in Primary Schools: Strategic Numeracy Research and Development Project (Australian Capital Territory); Profiling High Numeracy Achievement: Strategic Numeracy Research and Development Project (South Australia), and, or,
• to gauge changes in children’s achievement, for example, Effective Teaching Strategies in Years 3, 4 and 5 Mathematics Classrooms: Strategic Numeracy Research and Development Project (South Australia).

The relationship of specific teaching practices to improvement in numeracy assessment results has been investigated in numerous studies. For example, changed teaching practices (such as, Indigenous Students Achieving Numeracy) and a range of teaching and learning strategies have been examined for effects on children’s numeracy achievements (for example, Numeracy Research in Primary Schools; What’s ‘Making the Difference’ in Achieving Outstanding Primary School Learning Outcomes in Numeracy?: Strategic Numeracy Research and Development Project, NSW; and Bleckly, Papps, & Hugo, 2000). Many of these projects are continuing and final findings have not yet been reported.

Smaller studies on other aspects of the relationship between particular teaching practices and achievement have been reported. Mir (1996), for example, found no significant differences in the achievement measures of three groups of children taught equivalent fractions using different teaching methods by the same teacher. It was not teaching methods alone that impact on children’s achievement. Significant factors included the teachers’ understanding of particular teaching approaches, and their knowledge of pupils'
thinking and strategies, and the underpinning mathematics. When these aspects of curriculum change were emphasised, the results were beneficial.

Bobis and Gould (1999) reported greater achievement gains by an experimental group, who were taught using the New South Wales *Count Me In Too* program, than a control group who were not. Similar findings were reported in the early stages of the *Early Numeracy Research Project* in Victoria.

There has been some useful cross-linking of research in this area. For instance, Doig and Lindsey (1992) arranged children’s responses to items on the New South Wales *Basic Skills Tests* (BST) in order of difficulty. They found that the order was consistent with the Collis and Biggs’ SOLO taxonomy (see Collis & Biggs, 1980) — a model of cognitive development that has been applied to mathematics learning — and had implications for curriculum development. Children’s responses to the BST questions have also been examined in relation to language factors, gender differences, and specific content areas - *number, measurement, space, and problem solving*. The findings were reported by Doig and Lokan (1997). For each perspective, the researchers defined the concept that was the focus of their analyses, highlighted items from the *Basic Skills Tests* that represented the concept, and presented detailed findings with respect to the children’s responses to the items. Implications of the findings for the classroom teacher were discussed, and comments from teachers, in response to the findings and implications, followed each chapter.

Affective factors and assessment are frequently linked, although there has not been much Australian research in this area in the past decade. Wither (1998) found evidence that it was change in achievement that formed the direction of causation of the well-established negative correlation between mathematical achievement and anxiety. That is, an increase in achievement would result in decreased levels of anxiety and vice versa.

Findings from several projects have indicated that improved teacher knowledge about appropriate mathematical pedagogy, and about mathematics, are associated with improved children’s learning outcomes. For example, findings from *Counting On* in New South Wales, the *Early Numeracy Research Project* in Victoria, and *Count Me In Too* (*Tasmanian implementation*) demonstrated this.

**Chapter overview**

In this chapter, research findings associated with large-scale assessment programs in which Australian children participated were reported. Also discussed were the findings from research studies and projects in which assessment issues were a major focus.

With respect to the large scale assessment programs, it was found that:

- in the international context, Australian primary children have performed well in comparison with children from many other countries and there were no gender differences noted in the overall performance, but some content areas of strength (space and measurement) and weakness (whole number operations, data representation and analysis, geometry) were identified; and

- States and Territories have large scale monitoring programs in which the numeracy achievements of children in grades 3, 5, and 7 are measured.
The findings from studies and projects with a focus on assessment issues have revealed that:

- there has been some research into improving the reliability of the techniques for measuring numeracy achievement;
- diagnostic assessment on a wide scale for primary-aged children is a feature of Queenslands’ Year 2 Diagnostic Net. Other state-based intervention programs in which children’s numeracy difficulties are assessed were discussed elsewhere in this report;
- in some States and Territories, school entry assessment programs have been implemented. Establishing what children know and the identification of children ‘at risk’ are among the aims of these programs; and
- evidence about the factors and teaching practices that can affect achievement is not strong but includes: teachers’ content knowledge, affective factors such as anxiety levels, and the materials and associated pedagogy used with children.
Chapter 7

Key Findings

This chapter, following the same thematic organisation as Chapters 3 to 6, provides an overview of the findings of the Australian research into primary numeracy. The most significant outcomes of research projects and publications that were reviewed in the preceding four chapters are summarised, and perceived gaps in the research are identified.

Equity and the school community

A major policy objective of the Australian Government is to provide all young people in Australia with strong foundations in numeracy and English literacy skills, with disadvantaged groups with lower numeracy and English literacy achievement — including Indigenous Australians — being a target for special attention.

In terms of the Australian research into primary numeracy reviewed here, considerably more research was located on individual and social-group differences between children, particularly their mathematical skills and knowledge, than on socio-cultural factors in the broader community that may affect children’s learning of mathematics, such as home environments, parental attitudes and community programs.

Equity

The dimensions of equity encompassed by the Adelaide Declaration (MCEETYA, 1999a) formed the basis of the primary numeracy research reviewed in Chapter 3. In keeping with Willis’ (2000) contention that whether or not children are “at risk” relates to whether their long term progress or mathematical growth is at risk, rather than the social grouping to which they belong, dimensions of equity encountered in the research were frequently incorporated into the research designs of studies or projects as some of the variables among a larger range. It may be desirable in many cases not to focus on specific target groups so long as comparative data can be drawn out of the findings if required.

Among the various themes grouped under Equity, research into the learning needs and mathematical achievements of Indigenous students was the most prolific. Research into gender issues was also well represented. There was noticeably less research, however, relating to disability, ethnicity, and rural issues, despite the fact that large proportions of the Australian population would fall under these umbrellas.

Disability and learning difficulties

The review of the research clearly supports van Kraayenoord, Elkins, Palmer and Rickards’ (2000) comments in their executive summary of the findings of the Students with Disabilities Project, that:
Based on the literature we found it difficult to get a clear picture of the numeracy achievement of students with disabilities. There was a lack of published literature in the area of numeracy development for students with all types of disabilities and what research we found was often dated. (Kraayenoord, Elkins, Palmer & Rickards, 2000, p. 1)

Not only was there a dearth of information for all groups of students with disabilities, and a shortage of literature on students’ numeracy achievement, but there has been greater emphasis on literacy than on numeracy when dealing with students with learning difficulties resulting from physical, social and psychological factors. It was noted that there was an abundance of teacher aides in classrooms, many with little numeracy training, who took on much of this numeracy instruction.

In many Australian states, there are state-wide, as well as localised, programs aimed at identifying children “at risk” of experiencing numeracy difficulties — particularly those with learning difficulties. Appropriate teacher professional development was clearly identified as a critical element contributing to the success of these programs, with teachers achieving success by using a more problem-solving, activity-based approach together with a wider range of assessment procedures. Inclusive programming that provided detailed and visible accountability was found to provide a powerful planning tool for teachers.

Based on the research reviewed, it is evident that there has been little specific research aimed at:

• identifying specific groups of children encountering learning difficulties in numeracy;
• the provision of quality numeracy teaching for target groups in the early years, including intervention programs where needed; and
• extending the current focus on the prevention or remediation of numeracy difficulties in the early years to target groups in the later years of schooling.

Very little research attention has been directed at the numeracy difficulties and needs of children with physical disabilities. Despite the range of physical impairments among Australian children, only one project on the general needs of children with disabilities, and two studies on specific impairments, were identified in the literature. It appears that knowledge specific to Australian children with physical difficulties has remained relatively static in relation to primary mathematics teaching and learning.

Ethnicity and language backgrounds other than English

Research findings on the numeracy achievements of children from language backgrounds other than English (LBOTE) appear inconsistent. Some research attention has been directed to the numeracy learning of bilingual children. Little is known, however, about the effects of bilingualism on either the learning outcomes for these children, or on how teachers in mainstream, multicultural classes deal with cultural diversity.

The Australian TIMSS data revealed that students born in non-English speaking countries from homes in which English is spoken at home were the top achievers; while students from homes in which a non-English language was spoken at home were the lowest achievers. In other research it has been shown that a lack of English language proficiency appears to be a major barrier to mathematical success.
Although socio-economic background is a confounding variable in research with LBOTE children, no Australian numeracy research was identified in which both ethnicity and socio-economic background were examined as separate or interacting variables, in relation to numeracy development at the primary level.

**Socio-economic status and rural students**

Research on children attending schools in disadvantaged areas has revealed a greater range in their numeracy achievement than for children from schools in other areas, together with low teacher expectations for these children.

The research indicates that successful numeracy learning for children in disadvantaged areas occurs when:

- families and community members are involved in numeracy programs;
- teachers’ expectations of children are high;
- high expectations are conveyed clearly to parents and other significant community members;
- teaching is shaped by the results of teacher-child interviews; and
- open-ended learning tasks are used.

Only one study of rural children who were not Indigenous was identified—a study of online learning of numeracy in rural and urban settings. A significant number of children live in rural settings, so this is a clear gap in the numeracy research field. Further, for many children, rural settings and socio-economic disadvantage go hand in hand, implying a need to expand knowledge about the impact that specific aspects of living, and being educated, in different rural settings has on numeracy learning.

**Gender**

Issues associated with the attainment of gender equity were found to be the most researched of all equity dimensions, apart from Indigenous issues. There has been, however, a marked decrease in the extent of this kind of research since the 1980s. There were no major projects from the last decade with gender as the main focus, although a number of researchers have included gender as a variable in their research designs.

Earlier research had revealed the strong nexus between societal attitudes and expectations, mathematics learning outcomes, and career decisions. Despite recognition of the power of longitudinal data in relation to this nexus, research of this kind has been uncommon in Australia.

The review of pertinent Australian research found:

- gender differences in overall numeracy achievement to be virtually non-existent, although there were small (non-significant) differences by content area that are consistent with those of the past;
- boys to be usually more confident than girls about their mathematical prowess; and
- gender-stereotyped attitudes and perceptions of girls’ and boys’ capabilities persisting, although to a lesser extent than in the past.

It is also worth noting that there is little research evidence at the primary level—in Australia or overseas—to support the perceptions and claims that boys, rather than girls, are now the disadvantaged group with respect to numeracy learning.
Indigenous students

Much of the reviewed research relating to Indigenous students fell into two categories: the reporting of large scale achievement data in which the results of Indigenous students were reported separately from those of other students, and action research involving small numbers of Indigenous students.

Data on the numeracy achievements of Australia’s Indigenous students consistently reveal that the numeracy levels of Indigenous students are generally lower than those of other Australians. Findings from several small, action-research based studies have revealed a range of factors that may be implicated in this persistent pattern of achievement. These include:

• different learning needs of Indigenous children;
• the language of instruction being different from home community languages;
• incompatibility between classroom practices and children’s background experience;
• low teacher expectations; and
• strained teacher-student relationships.

Success in addressing Indigenous students’ needs has been achieved through:

• appropriate professional development activities;
• drawing on and integrating community knowledge and Indigenous culture into the numeracy curriculum;
• promoting positive self-identity; and
• exploiting children’s preferred learning styles.

The school community

The research evidence generally supports the view that children’s learning will be enriched if there are strong educational partnerships between schools, their communities, and students’ homes, although there was no direct evidence that parental involvement in their children’s numeracy learning improved outcomes—perhaps because this has not been a focus of relevant research.

In Victoria, the Early Literacy Research Project (Hill & Crévola, 1998) developed nine interrelated design elements to model aspects of the school culture to be taken into account in order to improve learning outcomes in literacy, numeracy, and other curriculum areas. These design elements included affective, home, and school-related factors (e.g., leadership, teaching programs, school and class organisation, and special needs). The Hill-Crévola model has been used as the basis of several numeracy projects in Victoria, including the Early Numeracy Research Project.

In New South Wales, the current Australian Government-funded What’s “Making the Difference” in Achieving Outstanding Primary School Learning Outcomes in Numeracy? Strategic Numeracy Research and Development Project is attempting to identify educational policies, strategies, practices, and programs that contribute to outstanding numeracy outcomes for students.
In summary, while there has been a significant amount of research in the areas of *Equity* and the *School community*, particularly in the learning needs and mathematical achievements of Indigenous students and issues relating to gender, there has been considerably less research into issues relating to disability, ethnicity, socio-economic and rural factors, and into school organisation. Some suggestions for further research in these areas are outlined in Chapter 8.

### Teachers, students and classroom practice

Over a half of the total research reviewed focused on a range of issues associated with *Teachers, Students and Classroom practice*, with most of this research being carried out in classroom settings. Often the research related to more than one of these themes — for example, studies related to effective teaching do not fall easily into one or other of the themes of *Teachers or Classroom practice* as is evidenced by the fact that the UK study *Effective Teachers of Numeracy* (Askew, Brown, Rhodes, Johnson, & Wiliam, 1997), which examined the effect of grouping and other classroom practices on students’ gains on a test of numeracy, found no association between such aspects of pedagogy and student achievement. Instead, Askew et al. found an association between student achievement and teachers’ beliefs about how best to teach mathematics. High gains were associated with teachers who:

- focused on children’s mathematical learning, rather than on provision of pleasant classroom experiences;
- provided a challenging curriculum, rather than a comforting experience;
- recognised and emphasised deep connections between mathematical ideas; and
- had high expectations of initially lower attaining pupils.

Findings from similar studies are reported at the end of this section, under the heading of *Effective teaching of numeracy*.

### Teachers

A significant amount of Australian numeracy research in the past decade has focused on teachers, their training and professional development, teacher change, teacher’s beliefs and to a lesser extent their content and pedagogical content knowledge.

**Pre-service teacher education**

Findings from a number of studies relating to pre-service teachers indicate that:

- there is a strong correlation between student teachers’ levels of mathematics performance and their levels of self-confidence;
- student teachers often hold beliefs about mathematics and learning that constrain their access to rich and powerful ways of learning and teaching;
- many students in teacher education programs believe that calculator use should be avoided in primary mathematics;
- student teachers appear to have had little past experience with activities that might promote number sense or reflection on mathematical processes; and
many pre-service teachers believe they are insufficiently prepared in terms of mathematics content, pedagogy and pedagogical content knowledge, but believe they are sufficiently prepared in terms of their knowledge of mathematics curriculum.

According to Schuck (1996), there is often a dissonance between the beliefs of student teachers and teacher educators which needs to be challenged by teacher education programs, with students believing that good teachers are supportive and enthusiastic, and incorporate relevant, practical, “fun” activities into their teaching, but also believing that teachers are less likely to be empathetic to struggling pupils if they have a high level of knowledge of mathematics. Similarly, Smith and Lowrie (2001) found that contextual constraints can influence a beginning teacher’s classroom practice, suggesting that school-based structures and constraints need to be considered more at an undergraduate level.

Innovative practices found to be effective in mathematics teacher education included:

- a school-based program in which students took responsibility for teaching a small group of children throughout the year, with findings supporting the long standing psycho-dynamic theory that powerful emotional experiences involving practice and reflection are required if significant and effective change is to occur (Hill, 1994; 2000); and
- the use of an interactive multimedia resource (Mousley, Sullivan, & Mousley, 1996) based on a close analysis of one lesson used to support student teachers in their study of teaching, resulting in pre-service teachers demonstrating increased observation skills as well as improved ability to discuss teachers’ work in post-practicum discussions.

**Professional development and teacher change**

Findings from research on professional development indicate that effective programs:

- provide teachers with the time and appropriate resources to enable them to reflect on their teaching and make changes as and when they see fit, with a major impediment to change identified by teachers was a perception of a lack of time to adopt new practices;
- provide continuing support and encouragement while teachers are exploring possibilities and trialling new strategies in their classrooms;
- involve teachers in school-based and wider networks;
- are of sufficient duration (time span and contact hours) to allow significant changes to habitual beliefs and practices; and
- create opportunities for the exploration of, and reflection on theory-practice relationships.

Moreover, research in both Australia and overseas has emphasised the importance of professional development being:

- content-focused;
- situated in or near classrooms where teachers work; and
- rooted in the curriculum they teach.

Successful s such as *Count Me In Too* in New South Wales and Victoria’s *Early Numeracy Research Project*, and an evaluation of the use by teachers in Queensland of
the Year 2 Diagnostic Net, have shown the use by teachers of individual interviews with children based on a developmental framework to be a powerful tool for professional growth. By teachers becoming involved in researching pupils' mathematical understandings, teachers' own understandings of how children think mathematically and learn mathematics are enhanced, enabling them to develop teaching approaches and strategies to effectively help children to develop numeracy skills and understandings.

There have been few longitudinal studies of teacher change, with there being only limited knowledge about the longer-term effects of teacher change on students' numeracy outcomes.

**Teacher beliefs and knowledge**

Australian research in the last ten years on teachers’ beliefs is not extensive and appears somewhat fragmented. Little seems to be known about how teachers’ beliefs are related to effective numeracy teaching, although the findings related appear consistent with the more general findings, that is, that beliefs do affect behaviours. Much of the research on teachers’ beliefs relates to the use of calculators and, to a lesser extent, computers, with research showing that teachers' restricted views on the ways in which calculators could be used was one reason why their use lagged behind support for such use.

International research, such as that of Ma (1999) and Askew et al. (1997), has emphasised the importance of teachers' knowledge of deep connections between mathematical ideas for high student achievement — what Ma has termed profound understanding of fundamental mathematics. As stated earlier, Australian findings on pre-service teachers revealed that many felt that their mathematical backgrounds, as well as their knowledge of how to teach mathematics — their pedagogical content knowledge — were inadequate. While reports of growth in general pedagogical knowledge were numerous, no Australian project reviewed looked specifically at the mathematical content knowledge of practising teachers. The recently commenced ARC project Knowledge for Teaching Primary Mathematics: How Teachers’ Pedagogical Content Knowledge Develops and Affects Classroom Practices and Students’ Mathematics Achievement, which will investigate the influence of upper primary teachers’ mathematics-specific pedagogical knowledge on teaching practice and students’ learning outcomes, will begin to fill this gap in the Australian research.

**Students**

In projects and publications that focused on students as numeracy learners, Students at risk and Student attitudes received the most research interest, with less research into students’ Learning styles and Informal learning, and Gifted students receiving minimal attention.

**Gifted students**

There has been a lot of research with a focus on extending all students’ abilities to think and work mathematically. However, over the last decade there has been very little Australian research exploring issues associated with mathematically talented students. A project undertaken by the Australian Mathematics Trust was the only project located that had a focus on mathematically gifted and talented primary aged children. Otherwise, the
few small studies that included mathematically gifted students were limited in scope. Their findings indicated that among children with high mathematical potential, higher-level thinking was promoted through problem solving in small group settings.

As a society we have much to gain from nurturing those with talent who have the potential to benefit and enrich the lives of their communities, so the paucity of research on gifted and talented youngsters needs to be addressed. Gaps in the research here include:

- the identification of mathematically gifted and talented students;
- their specific needs;
- strategies that teachers can adopt to foster these children’s talents; and
- appropriate professional development.

**Students’ informal learning**

The number of studies on student’s informal learning was relatively small. However, the research indicates that:

- many children begin school with relatively high levels of knowledge, often not acknowledged by teachers;
- many Indigenous children face difficulties due to mathematically impoverished home environments;
- children’s mathematical understandings are enhanced when teachers recognise the value of — and establish links with — children’s informal, out-of-school mathematics experiences;
- students’ informal learning generally appears to be undervalued in the formal curriculum; and
- there is a need to bring the more social methods of learning that children experience out-of-school into the classroom.

Several current Australian Government-funded projects such as *Project Good Start: Effective Numeracy Practices in the Year Before and the First Year of School* will provide valuable information on effective pre-school learning environments, children’s numeracy learning at home, and strategies for successful transition from before-school settings to school.

**Learning styles**

In the research reviewed, learning styles arose mainly as one of the factors emerging in studies related to Indigenous students and students with disabilities. Focusing on children’s preferred learning styles and providing learning environments that do not conflict with traditional Aboriginal learning styles were found to enhance Indigenous students’ learning.

Only one small study was found in which children’s learning styles was a main focus of the research (Lewis, 1996). Findings did not support the contention that the use of computer programs in which teaching style is matched to the preferred learning style of students leads to increased attainment by students. Instead, it was proposed that in order to build mathematics ideas, students need to have access to several alternative representational codes — for example, verbal/linguistic, logical/mathematical, visual/spatial and motoric/kinaesthetic.
In view of the emphasis currently placed on learning styles and the notion of *multiple intelligences* in some teacher education and professional development, the extent to which these notions can be used to enhance learning outcomes for the wider student population is yet to be explored.

**Student attitudes**

There has been considerable research into students’ attitudes towards and beliefs about mathematics. The findings generally support the notion that more positive attitudes and beliefs about a range of issues related to mathematics teaching and learning are associated with greater enjoyment and motivation, as well as higher achievement.

Specific findings from the research reviewed include:

- a student perception that rules and terminology taught in school have no meaning outside the classroom;
- many students believe that it is important to be good at mental computation as mental strategies will be used more outside of school than in school;
- students respond positively towards the use of technology for mathematics learning, but have mixed beliefs about its effects on their learning;
- a positive correlation between students’ performance and their beliefs about themselves as learners of mathematics, beliefs about mathematics, and beliefs about the ways mathematics can be learnt;
- children’s self-concept being enhanced by teachers’ positive feedback on ability and performance;
- negative self-concept having detrimental consequences on attitudes towards mathematics; and
- positive self-identity being identified as a factor contributing to Indigenous students’ attachment to school and to their positive school outcomes.

The *Positive Self-identity for Indigenous Students and Its Relationship to School Outcomes* study recommended that a subsequent, long-term study be carried out to explore the self-identity of Indigenous students. Developing greater self-esteem is one of the key strategies being adopted by the Australian Government-funded *National Indigenous English Literacy and Numeracy Strategy, 2000-2004* to meet the objective of achieving English literacy, numeracy and school attendance outcomes for Indigenous students at levels comparable to those achieved by other young Australians.

**Students at risk**

There has been considerable research and many studies to identify children who are at risk of not learning mathematics successfully at an early stage of schooling, with the aim of assisting these children before they become further disadvantaged in their study of mathematics. Much of the research has been linked with intervention studies, which are discussed later in this chapter, with other research focusing on Indigenous students, as discussed earlier. The identification of students at risk has usually been done through the use of developmental frameworks, such as the assessment framework based on *growth points* developed by the *Early Numeracy Research Project* (ENRP, Pro003) or Queensland’s *Year 2 Diagnostic Net*.

Findings relating to low achieving children suggest that:
• many of their problems are related to poor language and reading skills
• low achieving students do not relate contextual word problems to their everyday life;
• such students rely on rules and procedures, with little thought as to whether answer are correct or make sense; and
• low achievers do not possess rich mathematical ideas or have powerful strategies that will enable them to use their mathematical knowledge to improve and enhance their mathematical thinking.

Much of the research has focused on young children, although very recently there has been some research into the identification of children in middle and upper primary years who are at risk. Research with this focus has shown that many children who have not been identified by their primary teachers as potentially having problems at secondary school suffer declines in engagement, self-regulation, and self-perception.

**Language factors**

Research on language factors affecting mathematics learning over the last decade has been fairly sparse. The research on children from non-English speaking backgrounds has been discussed earlier in this chapter. However it is important to note here a focus on the development of students’ understanding and use of the language of mathematics in English has been one of the factors shown to be effective in achieving numeracy gains for Indigenous students.

Findings from a number of small studies suggest that young children have language-related difficulties in understanding mathematics instruction, and that children’s concepts are sometimes more developed than they can verbalise. It has also been suggested that school mathematics needs to be “dejargonised” and that opportunities need to be provided for children to make links between their everyday worlds and the formal language of mathematics.

**Classroom practice**

Much of the research reviewed on teaching and learning in the area of numeracy is classroom-based and is discussed in different sections of this chapter. This section will focus on Intervention, Grouping, Use of resources, Use of specific teaching strategies, and findings related specifically to Effective teaching of numeracy.

**Intervention**

There have been many State and Territory projects and s focusing on the identification of children with numeracy difficulties in the early years of schooling. Teachers have been directly involved in a number of research projects and other s which have included the use of one-on-one interviews with children based on developmental frameworks. There have been reports of significant professional growth as a consequence. There have also been a number of intervention programs, which have aimed to identify students who have not achieved expected outcomes by a certain stage in order to give them extra attention. Such intervention programs differ from broad-based early years programs (“first-wave” programs) that aim to help all children achieve a solid foundation in numeracy.

Specific findings related to intervention programs include:
Key findings

• intervention can have positive effects not only on academic performance but also on general self-concept;
• both individual and group programs can be effective;
• while early-years intervention programs can result in short-term improvements in children’s competence and confidence, there may be a need for follow-up attention in later years; and
• student gains are difficult to sustain unless the interventions are articulated back into mainstream classroom pedagogy.

There was little reporting of the longer-term learning outcomes achieved by participants in intervention programs, especially as they proceed from primary to secondary school.

Grouping

Both in Australia and some countries overseas, there has been a strong trend in primary schools to make more frequent use of small groups. There has been a parallel increase in research on the effects of this practice. Australian teachers generally seem to support the idea that using group work allows students to work together and be exposed to each other’s views. However, substantial differences have been found in ways that effective and ineffective groups work, and there is research evidence that low achievers may be relatively uninvolved or passive in co-operative small-group work.

Effective small-group learning was generally found to be associated with:
• highly interactive group discussions that involved supporting explanations with evidence;
• genuine engagement with challenging mathematical content and a focus on central ideas;
• co-operative activity in which students use each other as resources, ask questions and check information; and
• close monitoring by teachers of group work, with on-going assessment of children’s higher order thinking and conceptual development.

There has been little research and no clear evidence about the most effective group structures. It is possible that, as the researchers in the British Leverhulme project found, the ways that teachers monitor participation and stimulate effective interaction are more important than the types of groups used.

Another obvious gap is research is into the effects of ability grouping, as well as the use of multi-age grouping at junior primary level — although the latter is currently being trialled in one longitudinal study.

Use of resources

Over the past decade, there has been as much research on calculators, and on computers, as on all other teaching aids. This is surprising because teaching aids are used frequently in so many primary classrooms. The relative newness and social appeal of technological aids has probably stimulated research in this area. State and Territory governments and professional associations have also encouraged developments in technology use. Textbooks, on the other hand — used daily in many classrooms — have received very little research attention.
Teaching aids. Findings from research over the past decade on concrete teaching materials appear to challenge earlier faith in their beneficial effects on children’s mathematical learning. The assumption that practical teaching aids always help children to develop mathematical understanding has been questioned, both by Australian researchers and internationally. There is now evidence that they may interfere with, rather than facilitate learning, particularly in the early years, and may add to the cognitive demands of numeracy tasks. The possibility for misconceptions and the extra layer of cognitive processing are two of the concerns.

There does not seem to have been much research about:

- which aids are most useful in promoting children’s numeracy learning (including children’s drawings and other representations, as well as teachers’ models) and, in particular, when such aids are most useful (what levels, which concepts);
- modes connecting the use of teaching aids with algorithmic thinking and recording, and ways of drawing out abstractions and generalisations.

Calculators. While there has been a considerable amount of Australian research in the past decade on the use of calculators, there has been little recent research into the effective use of this technology.

The research into calculator use has had very promising results. Overall, both Australian and international research has shown that children having free use of calculators have developed more sophisticated and flexible number understandings than children without long-term experience of calculator use.

It is clear that calculators can be used as effective tools for learning and teaching, with teachers participating in large-scale projects reporting increased expectations for children’s numeracy. Nevertheless, there is little evidence of widespread use of calculators as teaching aids, especially in the early years of schooling, and some parents, teachers, and even children have reservations about their use.

Research suggests that the use of calculators also results in changes in teaching, especially with regard to:

- the open-endedness of the tasks;
- the extents to which children are able to lead the learning process;
- the need for appropriate problems and investigations in everyday mathematics classes; and
- the provision of unhindered opportunities for children to make sensible shifts between mental, written, and calculator-assisted work.

Computers. Research into the use of computers has focused on three major areas:

- the use of particular software (both commercially produced and researcher designed) designed to enhance student learning;
- the use of computer assisted learning, integrated learning systems and instructional management systems; and, more recently,
- teaching and learning with information and communication technologies.

While some commercially produced software tools have been found to motivate children, enhanced learning outcomes were not necessarily found. However, research on the effects of the development of specific software packages in particular content areas reveals that they can have positive effects on children’s conceptual understandings. For
example, researcher designed computer games that focused on aspects of decimal understanding, were found to be effective in challenging children’s misconceptions about decimals.

Research into the use of computer-aided learning suggests that:

- some teachers and principals were very positive about its use;
- teachers who were experienced users of the software generally felt that skill development and complex learning could be supported by such a system;
- it should only be seen as another resource rather than a total approach;
- there were mixed findings regarding enhanced learning outcomes; and
- there is a lack of professional development and supporting information on ways to integrate computer-based activities without sacrificing the advantages of teacher-student engagement and interaction.

Recent developments in information and communication technologies have provided opportunities for some focus on on-line teaching, either to supplement regular classroom teaching or to substitute for (or complement) it in special cases such as remote students, students with disabilities, or adult learners studying basic numeracy and literacy content. In line with the recommendations of Yelland (2001), a number of recently funded ARC projects aim to inform the research and educational community about mathematical learning, social processes of learning with technology and the impact of computers on learning.

Given the recent growth in development of software for the teaching and learning of mathematics, there will always be gaps in our knowledge about effective pedagogical strategies that supplement and enrich the use of particular technological tools, as well as characteristics of software that can make a difference in terms of children’s abilities to reason and work mathematically.

**Textbooks.** While textbooks are widely used and shape much of the teaching (and hence the learning) of numeracy across the nation, little research into either the effects of textbook use or effective ways of using textbooks was located.

Topics appropriate for research in this area include:

- the effects of using different styles of textbooks (for example, those using formalist and constructivist approaches, closed and open questions, problem solving approaches);
- what teachers and administrators (as well as students and parents) think about the use of texts, and of different types of textbooks;
- whether textbooks are typically used with some groups of students more than others (particularly in relation to socio-economic divisions);
- why specific textbooks are chosen, and who makes the decision;
- whether the use of textbooks is related to teachers’ competence and confidence;
- relationships between textbook content and assessment processes; and
- how teachers adapt (or could better adapt) textbook content to meet the needs of particular students.

Again, there will always be a need to continue monitoring textbooks and other teaching resources for equity — that is, fair representation of Australia’s demographic profile.
The use of specific teaching strategies

This section discusses findings from research into Problem-solving and investigations — as a teaching strategy, as opposed to children’s problem solving which is discussed under the theme of Mathematical thinking. Both of these aspects of problem solving have received considerable research attention. This section also discusses research into Questioning and discussion, which has received somewhat less research attention, and Real world contexts — that is applications that are familiar to children’s experiences — which has had even less attention.

While Motivating students appears in the database as a sub-theme of Teaching strategies, and research has identified the ability to motivate children as a key component of effective teaching, this aspect is not reported here as it is discussed under a number of different themes, particularly under Mathematics self-concept and motivation.

Problem solving and investigations. Problem solving and the use of open-ended tasks are two areas of research that have attracted vast international attention. It is therefore not surprising to find considerable Australian research in the area.

Problem solving is regarded as an important aspect of mathematics for a number of reasons, including:

- its perceived potential to promote higher-order thinking;
- the opportunities it provides children to “talk mathematically”, and demonstrate and explain their thinking;
- the need for children to see mathematics as useful in real contexts;
- the opportunities it provides children to practise their numeracy skills by using their mathematical knowledge in everyday situations.

There have been many foci in the research on problem solving and investigations, including:

- the development of tasks designed to develop students’ understandings of particular concepts;
- problem contexts;
- the use of open-ended tasks and questions;
- classroom organisation during problem-solving and inquiry-based lessons; and
- the use of models to represent and to help abstraction, and generalised thinking.

It is clear from the research reviewed that:

- children can construct important mathematical ideas through solving novel problems;
- children of different abilities are able to become involved in the exploration of open-ended questions and to be challenged by them;
- working in groups to solve non-routine problems can provide opportunities for primary children to talk, think, and write mathematically;
- cognitive, social or interpersonal, and external factors can influence the effectiveness of collaboration during small-group problem solving;
- the use of open-ended tasks with appropriate scoring rubrics can provide important information for teachers regarding different levels of student understanding; and
• the traditions of school mathematics teaching, learning, and assessment regimes continue to make it a challenge for teachers to adopt open-ended problem-solving approaches in their teaching.

The review of the research literature related to the use of open-ended tasks indicates that there has been little empirical, large-scale research on whether the frequent use of open-ended tasks improves outcomes as measured by test performance, whether all children benefit, and which contexts for problems interest children or lead to greater problem-solving success.

Moreover, there is a need to acknowledge the growing international attention, especially since the two recent TIMSS video studies, focusing on what is regarded as the common Japanese lesson pattern of basing lessons on a highly structured, in-depth analysis of multiple student solutions to a single problem (see, for example, Stigler, 1996; Hiebert et al., 2003).

**Questioning and discussion.** Much of the Australian research into questioning and discussion has been focused on the use of open-ended tasks and “good questions”, discussed above.

Other research in this area covered a wide range of issues, with findings suggesting that:

- listening to children talk about their own mathematical experiences provides a rich source of information for teachers about children’s learning;
- children’s questions in small groups are generally at a lower level than those initiated by the teacher and hence children’s soliciting of help through questioning is not as effective as unsolicited help offered through questioning by teachers;
- children in effective groups talk more about mathematical content, propose more ideas, give more explanations with evidence, re-focus discussion more often, and responded to questions more often than children in less effective groups;
- in a study of two classes (Frid & Malone, 1995), conjecturing, criticising, explaining, testing and refining of ideas and procedures were seen primarily as the responsibility of the teacher, as students did not regard themselves or peers as a source of mathematical knowledge;
- teachers often see the enhancement of children’s self-esteem as the most important goal of whole-class discussion at the end of lessons, and consequently place less emphasis on mathematical aspects and what constitutes explanation and justification; and
- while there is high level of support among principals, teachers, and mathematics educators for the notion of communities of inquiry and purposeful mathematical dialogue, current practice is seen as falling far short of this goal, with a fragmented, outcomes-based curriculum and the use of tasks with very low cognitive demands being seen as obstacles.

A clearly identifiable gap is research on the types of robust tasks that can be used to support the development of sophisticated mathematical thinking or all students.

**The use of “real world” contexts.** The inclusion of realistic problem solving, including the application of mathematics to “real-life” contexts has been advocated in Australia and internationally for over 20 years. While materials for use by teachers with a focus on
relating mathematical ideas to real life have been developed in several Australian projects, relatively little other research was located from the past decade.

Findings from a small number of studies (some of which were also quite small in scale) suggest that:

- out-of-school learning of mathematics is more open-ended than school methods;
- students prefer open-ended tasks related to a real life experience or context;
- deciding when probabilities can be determined theoretically and when they must be found experimentally is an important thinking skill grounded in real-world sense making; and
- there is a need to develop tasks that encourage children to use intuitions built from experience to relate data in graphs to real contexts.
Effective teaching of numeracy

As discussed in Chapter 2 of this report, there is a lack of agreement in the international research on the characteristics of classroom practice that result in effective teaching of numeracy. This is due to a number of factors including: a lack of agreement on how to measure effective practice; claims that focusing on various aspects of organisational strategies diverts attention from other more relevant factors such as questioning at a high cognitive level; and difficulties in identifying pedagogical variables that result in statistically significant gains in attainment even in large-scale, longitudinal studies such as the Leverhulme Numeracy Research Programme in the United Kingdom (Brown et al., 2001). This view is supported by the recent report on the TIMMS 1999 Video Study (Hiebert et al., 2003), which found in its study of similarities and differences in the teaching of Grade 8 mathematics in seven countries that “if the learning goal for students is high performance on assessments in mathematics … there is no single method that mathematics teachers in relatively high-achieving countries use to achieve that goal” (p. 150).

Nevertheless, there is considerable convergence in both the international and the Australian research reviewed which suggests that, while effective teachers are not easily characterised and differences in terms of learning outcomes are often small, effective teachers:

- have high expectations that all children, at all levels of primary school will engage seriously with mathematical ideas;
- emphasise the understanding of mathematical concepts and the connections between these;
- structure purposeful tasks that enable different possibilities, strategies and products to emerge;
- choose tasks that are linked to real situations, engage children and maintain involvement;
- probe and challenge children’s thinking and reasoning;
- build on children’s mathematical ideas and strategies;
- are confident in their own knowledge of mathematics at the level they are teaching; and
- use assessment as a basis for development of methods and content, and the identification of problems before they affect further progress.

It is to be hoped that the extensive program of research funded through the Australian Government’s Numeracy Research and Development Initiative will make a major contribution in this important area of research.

Curriculum and processes

Over a third of the Australian research reviewed related to a range of issues associated with Curriculum and processes. Research on the development of particular mathematics concepts clearly dominated other aspects of research in this area, with research into
Mathematical thinking also being well represented. There was considerably less research into Curriculum issues and Developmental frameworks, and almost none into Using mathematics, a key aspect of numeracy.

Concept development

Of the content areas represented in the research into Concept development, by far the most research had Number as its focus. Of the other areas, much of the research into Chance and data was due to one highly prolific research team, while Measurement, an aspect of numeracy that is dominant in everyday encounters, received surprisingly little attention.

Algebra

While there is some debate as to whether Algebra has its beginnings in secondary school as a formal part of the curriculum, it is well recognised that primary school children’s knowledge of arithmetic structure provides the foundational knowledge for their understanding of algebra. Research however suggests that students fail to abstract from their primary school experiences the mathematical structures that are necessary for them to make a later successful transition from arithmetic to algebra. Suggestions for ways to smooth this transition include:

- describing and making use of generalisable processes and the structural properties of arithmetic, and of quasi-variable expressions in particular (Warren & Cooper, 2002; Fujii & Stephens, 2001; Fujii, 2003);
- providing classroom activities to address difficulties arising from students’ reliance on intuitive language processing, focusing on essential aspects of number knowledge, particularly the notion of equality;
- building on students’ capacity to generalise problem situations and to write equations using variables, to informally develop the concept of a variable;
- devoting substantial class time for discussion of links between problems, processes used by the children, underpinning concepts, and related ideas; and
- using problem situations in measurement and other non-number areas to develop children’s ability to think algebraically.

Given the importance of primary mathematics experience in later success with formal algebraic thinking, there has been very little research on ways of developing children’s quasi-variable thinking.

Chance and data

The need to make informed decisions in everyday life based on probabilistic and statistical data makes Chance and data a critical component of numeracy. In particular, the increasing use of graphical presentation of data in the media and elsewhere underscores the critical importance of graphical literacy.

During the past decade, Jane Watson and her colleagues in Tasmania have produced a remarkable body of research in the area of primary probability and statistics. Working originally with Kevin Collis, Watson explored students’ understanding of chance measurement in relation to the development of ideas of formal probability and produced a developmental model for understanding chance measurement. Based on the model,
Watson has suggested the ordering of topics in the curriculum and provided assessment procedures to use in longitudinal evaluation of the implementation of the curriculum. She has also implemented innovative pedagogies such as students viewing video recordings of other students’ conflicting responses to chance problems and deciding which response they preferred, with evidence that the resulting cognitive conflict improved their understanding.

Based on the analysis of large databases of student responses to questions in probability and statistics, Watson and her colleagues have examined longitudinal data to model development of student understanding in areas such as:

- the concept of “fair”;
- graphical representations of statistical associations;
- various conditions and their conjunctions (such as averages); and
- sampling.

Outcomes and findings from these longitudinal studies include:

- the development of a three-tiered framework for statistical literacy comprising
  - defining terminology,
  - applying concepts in context, and
  - questioning claims made without proper justification;
- a recommendation, based on the prevalence of ideas associated with mean, median and mode, that the concepts of mode and median should be introduced before mean in the school curriculum; and
- improvement in expressing probability numerically and in distinguishing conditional events with grade for Years 5 to 11 students, with understanding of conditional probability being related to development of basic chance measurement.

Other Australian research findings suggest that:

- levels of students’ interpretation of data sets are related to their representation skills;
- students, when dealing with graphs in authentic contexts, commonly do not
  - appreciate scaling difficulties,
  - relate a graph as relevant in the context of a standard interpretation task, or
  - apply numeracy skills for calculations based on data in graphical representations; and
- students need to be challenged in the classroom through the use of non-standard graphs to be better prepared for misleading representations.

Watson and colleagues, as do other Australian researchers, emphasise the need to focus on the study of probability in the context of children’s everyday experiences, and the need for teachers to attend to common misconceptions and intuitive beliefs that tend to inhibit the children’s development of probability ideas.

Given the importance of graphical literacy in both everyday numeracy and numeracy across the curriculum, children’s limited understandings in this area suggest that this is likely to be a rich area for future research and teacher professional development.
Measurement

The research reviewed suggests that teachers are unaware of the importance of structuring their teaching of measurement in terms of students’ conceptual development in the various sub-strands of Measurement. As part of Count Me Into Measurement, a conceptual framework of six levels of increasing complexity, and sample lesson plans including practical activities for teaching length, area, volume, and mass have been developed for Kindergarten to Year 6. These are based on research findings that indicate the importance of students’ knowledge of the unit iteration structure and are aimed at developing students’ understanding of measurement concepts and language before more sophisticated strategies and processes are introduced, with a focus on the use of informal measurement units to develop these. Evaluation of the implementation of this program in 38 schools across New South Wales showed highly positive outcomes.

Almost all of the research relating to specific sub-strands was in length or area, with a small number relating to time. Linear measurement knowledge is essential for perimeter, area and volume, and in topics that rely on the understanding of scales, such as directed number gauges, and graphs, yet research suggests that while most high-ability students have a conceptual understanding of length, the majority of the lower-ability students do not acquire important concepts relating to the linear nature of units. Teachers are urged to identify units explicitly when they are teaching measurement and also not to rely on paper-and-pencil measuring tasks to assess students’ understandings of linear measurement.

Among findings from studies investigating children’s understanding of area, young children were found to have difficulty visualising the tiling of shapes, suggesting that activities aimed at teaching the “length by width” formula and similar area measurements may not be effective because children do not see the need to cover the area accurately and may need explicit teaching of the row and column structure of rectangular arrays.

In some research particularly relevant to teachers, O’Toole (1997) documents children’s thinking about time in one low SES, high ESL school, and analyses planning structures used to support and challenge their thinking.

Overall, Australian research into measurement over the past decade highlights the importance of devoting sufficient time to the development of underlying concepts before moving to paper and pencil activities and formulae.

Number

The two major areas for research into Number in the past decade have been Computation and Number sense. The areas of Fractions, Counting, Decimals, and Place value, while producing less research, were still well represented in our review. However Estimation and Percentages, two aspects of number that are often found in everyday contexts and applications, attracted very little research attention.

The need to base curricula and practice in early number learning on research into young children’s arithmetical thinking and how it can be developed is increasingly being recognised. The substantial amount of Australian research into children’s early number learning — often originating from the work of Steffe and von Glasersfeld — has informed the development of frameworks for teachers to use in assessing young children’s stages of conceptual development. These frameworks have underpinned research and professional development programs, such as Count Me In Too and Victoria’s Early
Numeracy Research Project, that aim to enhance children’s numeracy outcomes by enabling teachers to engage with their students’ mathematical learning in order to create challenging activities for all children. Teachers working with “at risk” students in intervention programs are urged to share their knowledge with classroom teachers in order that they too can use this knowledge to develop appropriate classroom activities for these students.

Studies looking at children’s learning of number in the early years of school suggest that many children are seriously “under-challenged” and emphasise the need to give problem solving, as well as abstract mathematics, a more central role. The call for a more holistic approach to the teaching of number, emphasising connections rather than compartmentalised knowledge, is made by many researchers.

The role of concrete materials in the development of place value concepts, including the underlying concepts involved in decimal notation, is often questioned. Teachers are urged to make explicit the connections between the materials and place value, as well as to have a repertoire of activities to develop and reinforce place value concepts at a more abstract level.

In terms of the teaching of decimals, student improvement has been found to depend on teachers’ knowledge of the underlying concepts, the use of a clear model and careful bridging from visualisation to numerical forms. Teachers have not necessarily found place value blocks to be particularly helpful and an alternative model, Linear Arithmetic Blocks, has been suggested (Helme & Stacey, 2000). A disturbing finding is that, for some children at least, certain misconceptions relating to decimals appear to be learned from school instruction.

The use of various models for the development of both mental and written addition and subtraction has been the subject of considerable research. Tens frames have been found useful for developing children’s thinking in tens, with children being able to build up basic facts without resorting to primitive counting strategies. A major new model for addition and subtraction to 100, the empty number line, has been developed in the Netherlands as part of Realistic Mathematics Education. This model, which has been adopted in many places, including Australia in Count Me In Too, is seen to provide a natural and transparent means to model children’s informal strategies and has been found in the Netherlands to be a powerful model for instruction.

Researchers investigating children’s conceptual development across a wide range of aspects of number frequently highlight the need for teachers to identify, value, and develop children’s spontaneous, informal computational strategies. Studies of children’s mental computation show that many competent children have acquired a range of efficient strategies almost “in spite of” what happens in the classroom. Such efficient strategies are flexible, taking into account the numbers involved, unlike those that merely mirror standard written algorithms. Research is needed on how best to make teachers more aware of efficient strategies, although “teaching” efficient strategies is not the answer as learnt rules are often misused through a lack of understanding. Rather, a deep understanding of the essential features of place value and opportunities to experiment and to verbalise thinking are seen as the means for achieving enhanced facility with mental computation.

The place of standard written algorithms in the mathematics curriculum and their role in children’s development of number sense continues to be the subject of debate. A wide range of studies into children’s arithmetic strongly suggest the need to place much more
emphasis on children’s understandings of fundamental concepts before the teaching of rules and procedures, as well as the need to coordinate new symbolic knowledge with children’s existing informal knowledge and their real-life experiences.

In terms of the connection between numeracy and the ability to calculate, McIntosh, one of Australia’s foremost researchers into number sense and mental computation, argues that there is a difference between numeracy and the ability to carry out written computations, with neither guaranteeing the other. If indeed number sense does not necessarily develop through the learning of standard written algorithms — as seems to be the case — yet does develop through schooling, then it would be worthwhile investigating which aspects of schooling and which pedagogical approaches are most effective. Furthermore, number sense and mental computation appear to be closely linked, with one way to develop number sense being to develop mental computation ability. This suggests that mental computation should be given greater prominence in school curricula at the expense of the teaching of standard written algorithms, and that more emphasis should be placed on the assessment of mental computation. Among the outcomes of a continuing international collaborative program of research into number sense, mental computation and computational estimation has been the development of a framework for the analysis of number sense and the development of paper-and-pencil tests for number sense and mental computation.

Research into the role of calculators in children’s early number learning suggests that they have a tremendous potential in developing children’s conceptual understanding and mental computation strategies before any formal teaching of algorithms, with their presence providing a learning environment to promote number sense. While appropriate choices of computational methods is an indicator of number sense, a significant factor in students’ computational choices appears to be their belief that mental and written methods are more highly valued than using the calculator. Rather than categorising computation as mental, calculator and written (in the sense of standard written algorithms), it has been suggested that written be replaced by recording, which then includes children’s informal jottings during the calculation as well. One recently commenced project, Developing Computation: Strategic Numeracy Research and Development Project, Tas is supporting the development of informal written methods in Years 2 to 4, while investigating the effects of such a program on students’ number sense and computational ability.

In summary, findings from the extensive body of research on Number suggest that:

- many children are seriously “under-challenged” in their learning of number in the early years of schooling;
- explicit connections need to be made between concrete materials and the concepts being developed, with further activities needed to develop and reinforce concepts at a more abstract level;
- teachers’ knowledge of the underlying concepts, the use of a clear model, and careful bridging from visualisation to numerical forms are important factors in the effective teaching of decimals;
- more emphasis is needed on children’s understandings of fundamental concepts before the teaching of rules and procedures;
- new symbolic knowledge needs to be coordinated with children’s existing informal knowledge and their real-life experiences; and
• the use of calculators as teaching aids can enhance children’s conceptual understanding and mental computation before the formal teaching of algorithms. As a result of their findings, researchers have called for:

• more emphasis on problem solving and abstract mathematics;
• a more holistic approach to the teaching of number, emphasising connections rather than compartmentalised knowledge;
• more focus on children’s spontaneous, informal computational strategies;
• greater prominence in school curricula for mental computation at the expense of standard written algorithms; and
• more emphasis on the assessment of mental computation.

Space
As in the research on Measurement, the major work reported has originated from researchers from universities in New South Wales, working with the Count Me Into Space project. A developmental framework for two of the sub-strands of Space, based on extensive research into visual imagery and spatial thinking, is accompanied by suggested lessons and sets of assessment tasks, which have been used for professional development for early years teachers. Detailed evaluations showed project students performing significantly better than those from comparable schools, while teachers reported many positive aspects, such as feeling that they had extended their mathematical and pedagogical content knowledge. Also as part of the Count Me Into Space project, an angles unit with a corresponding teaching package has been developed.

Children’s ability to see similarities between realistic models of physical contexts and the corresponding real contexts has also been the subject of research, with findings showing that concepts cannot be constructed from experience in a single context, with some concepts, such as turns, being most easily modelled abstractly, while others, such as slopes, are most easily related by comparison. Strategies to help children overcome difficulties and misconceptions with specific concepts of angle and related ideas have also been identified.

Research into children’s use of diagrams — which play an important role in the communication of geometric ideas — shows that children may experience serious difficulties in interpreting diagrams and that this may be a constraint to effective communication in geometry. Other research into children’s spatial ideas and their ability to express them suggests that young children are more developed in spatial concepts than they can verbalise, and that their perceptions of shapes and their approaches to tasks are strongly influenced by language and experience.

Spatial encounters occur in a range of diverse everyday environments, suggesting that teachers to be made more aware of key concepts and experiences that can be drawn out of everyday environments. While the importance of building on children’s experience needs to be emphasised, a study of adults’ concepts of space, time and money in a remote rural Aboriginal community highlights the radically different views and socio-cultural experiences of people in “non-typical” environments.
In summary, findings from the research on Space indicate that:

- positive results in terms of student learning outcomes and teacher satisfaction resulted from the use of a developmental framework and lessons in the Count Me Into Space project;
- children’s interpretations of diagrams can constrain their effective communication in geometry;
- language and experience strongly influence young children’s perceptions of shapes and approaches to tasks, with spatial concepts often being more developed than children can verbalise; and
- teachers need to be made more aware of key concepts and experiences that can be drawn out of everyday environments in the early years of schooling.

**Curriculum issues**

While specific issues relating to the curriculum – such as the sequencing of topics or calls to delay the teaching of standard written algorithms until children have a deep understanding of the operations – have been dealt with in other sections, the issue of what numeracy means for Australian curriculum developers and the relationship between numeracy, mathematics and literacy still need to be addressed. The review of the literature suggests that, while individual authors have addressed the question of the meaning of the term “numeracy”, by and large it is used interchangeably with “mathematics” or “mathematical competency”.

Research into enhancing numeracy through integration with other curriculum areas suggests that successful integrated classroom environments hold students’ interest and enhance learning across the curriculum as well as require teachers to regularly make links to the real world.

The need to cater for numeracy needs in key learning areas other than mathematics has resulted in significant ongoing research – for example Sue Willis, John Hogan and Mark Jeffery’s *Numeracy Across the Curriculum*, which resulted in the development of a Numeracy Framework emphasising the need to focus on contextual and strategic knowledge as well as mathematical knowledge in order to provide a rich description of numeracy (see, for example, Hogan, 2000). Other research has examined numeracy demands in today’s society, including links between numeracy performance and life outcomes for employment, education, and training, and the mathematical competencies and skills required.

**Developmental frameworks**

The construction of developmental frameworks, based on extensive research, particularly in the early years, has been a major feature of recent Australian research.

As mentioned earlier in this report, research by Steffe and others in the United States has underpinned much of the Australian research into children’s early learning of counting and arithmetic. This in turn led to the development of Wright’s *Learning Framework in Number*, which defined levels of competence in children’s identification of numerals, forward and backward counting sequences, understanding of place value, and addition and subtraction problems. This learning framework underpinned the New South Wales Department of
Education and Training’s intervention program in which “at risk” first-grade students were withdrawn for one-to-one long-term teaching programs — the beginnings of the Count Me In program. Comparisons of children’s learning outcomes showed the potential of Count Me In to bring about significant improvement in children’s number knowledge, leading to the adoption of the Count Me In Too Learning Framework in Number, as well as significant interest and the development of similar programs in the project in Canada, England and New Zealand. Similar, though less extensively research-based, frameworks have also been developed in New South Wales in the areas of Measurement and Space.

As part of a large-scale New South Wales project investigating the educational policies, strategies, practices and processes that contribute to outstanding numeracy outcomes for students, What’s ‘Making the Difference’ in Achieving Outstanding Primary School Student Learning Outcomes in Numeracy?, a criterion-referenced achievement scale by which the numeracy development of students in the project can be assessed is being developed. The Rasch Simple Logistic Model is being used to attempt to construct a single unidimensional numeracy scale on which item difficulty and student ability can both be represented.

Another example of a developmental framework has been the framework of “growth points” of early numeracy learning developed in Victoria by the Early Numeracy Research Project. Building on the findings of relevant research in mathematics education from Australia and overseas, this framework attempts to emphasise important ideas in early mathematics in the areas of Number, Measurement, and Space in order to allow the description of children’s mathematical knowledge and understanding to form the basis of planning and teaching. The framework was used to develop a task-based, one-to-one interview schedule that was used with children as part of a multi-level professional development program. Use of this interview by teachers has been seen as a powerful professional development tool to engage teachers in investigating the mathematical knowledge of their students and hence enabling them to better plan appropriate sequences of learning tasks. This research has also led to the development of a new framework including counting, place value, strategies for addition and subtraction, strategies for multiplication and division, time, length, mass, properties of shape, and visualisation and orientation, as part of the Victorian Early Years Numeracy Program, P-4, which is being used as a basis for materials production, monitoring and assessment of students’ development, and professional development.

The Western Australia, First Steps in Mathematics project developed detailed diagnostic maps and associated curriculum materials, together with a structured professional development program aimed at enabling teachers to link the number, measurement and space strands in the student outcome statements with curriculum content in each phase of schooling.

In summary, Australian research into stages of children’s development, particularly in number, has resulted in the construction in several States of research-based learning frameworks in number, as well as in measurement and space in New South Wales.

These developmental frameworks have been found to be useful for:

- mapping students’ progress;
- developing appropriate curricula; and
- linking teacher professional development to key mathematical concepts and their development.
Moreover, their use in projects such as *Count Me In Too* has been found to result in improved learning outcomes for children and increased satisfaction for teachers.

**Mathematical thinking**

The critical role of mathematical thinking in the application of mathematical knowledge and skills to real-life situations gives it a central role in numeracy.

Of the research reviewed under *Mathematical thinking*, the themes of *Children’s thinking strategies* and *Children’s problem solving* attracted considerable attention, although it should be noted that there was significant overlap between these two themes. Considerably less research was located dealing with *Visualisation* and *The language of mathematics*.

**Children’s problem solving**

Studies of children’s deductive reasoning have shown that even children classified as low achievers in mathematics can reason logically in solving novel problems and develop sophisticated procedures, suggesting that the mathematics curriculum needs to be broadened to include opportunities for all children to attempt novel problems and have more control of their learning. Research into children’s problem posing indicates a lack of connection between children’s informal, intuitive knowledge and “school maths”, suggesting that explicit attention needs to be placed on encouraging children to make these connections by encouraging children to recognise mathematical situations in everyday life.

In terms of improving children’s problem solving, while teaching heuristic strategies and metacognitive practices has been shown to increase problem solving effectiveness, an important feature of successful problem solvers is their ability to monitor and manage their progress in selecting and implementing appropriate strategies, with Australian research showing perseverance to be a contributing factor. Among metacognitive and other models developed by Australian researchers is a four-stage model, based on the strategy sequences used by problem solvers who persevered after reaching a stage where they recognised that they had not reached a satisfactory answer. Teachers and students evaluating the use of this model believed it to be useful for enhancing problem solving performance, with teachers recommending it be introduced to young children and reinforced throughout schooling.

Another area that affects successful problem solving is the effective use of diagrams, which suggests that numeracy and literacy needs to be expanded to include literacy with various forms of visual representation, including diagrams.

In an attempt to provide information to teachers not only about the correctness of children’s problem solutions but also about the problem solving processes they are using to arrive at their solutions, a number of Australian assessments of problem solving have been developed. These also provide teachers with information on which to base their subsequent teaching. While assessments such as the New South Wales *Basic Skills Testing Program* (BSTP) — which contains items that could be categorised as “problems set in “real-world” contexts — cannot truly assess people’s performance in solving problems in everyday life, they can provide useful information about some aspects of
children’s ability to solve real problems. However, in order to provide a complete picture, teachers need to use a range of assessment techniques.

In summary, findings from the research on *Children’s problem solving* indicate that:

- even children classified as low achievers in mathematics can reason logically in solving novel problems and develop sophisticated procedures;
- there is a lack of connection between children’s informal, intuitive knowledge and “school maths”;
- children need to be encouraged to recognise mathematical situations in everyday life;
- successful problem solvers monitor and manage their progress in selecting and implementing appropriate strategies;
- the use of metacognitive models can enhance problem solving performance;
- the effective use of diagrams affects successful problem solving; and
- teachers need to use a range of assessment techniques to obtain a complete picture of children’s problem solving.

*Children’s thinking strategies*

Research into children’s thinking strategies has been closely linked to research into problem solving.

Findings from the research on *Children’s thinking strategies* indicate that:

- children’s ability to monitor their actions, detect and correct errors, and recognise problem completion play a crucial role in successful problem solving;
- although children reason by analogy in everyday life, they appear to require guidance to apply this to more formal problem solving; and
- while students increasingly use formal methods as they move to higher year levels, teachers play a significant role in student success in formal responses.

In an attempt to bridge the research-practice gap, a research-based pedagogical framework to promote mathematical thinking and understanding in mathematics classrooms has been developed. The need for a thinking curriculum to be accompanied by assessment practices that support thinking and sense making has been stressed, as well as the value of providing teachers with “descriptive images” of practices to promote mathematical thinking.

*The language of mathematics*

The abstract, compressed nature of mathematical language, with its focus on conventions and written symbols, and its use of familiar words in unfamiliar ways, present barriers to student learning of mathematics.

Research suggests that in order to enable children to link their own mathematical ideas with formal mathematical language, teachers should:

- be aware of the discontinuities between children’s out of school use of language and mathematical language;
- allow children to use their own words to explore and express their mathematical ideas and thinking; and
• systematically plan classroom activities that develop young children’s understanding of concepts.

**Visualisation**

The visualisation of mathematical concepts has long been recognised as important for mathematical thinking. Australian research has looked at the role of visualisation in the development of number concepts, spatial understanding, and problem solving.

An important area of research in both Australia and internationally has been the links between mental images and number sense of young children. Investigation of the role of equipment, drawings and activities to enhance children’s thinking strategies and promote number sense suggest that the active processing of images plays an important part in children’s development of number concepts.

One of the few identified studies dealing with estimation of measurement has found that students estimate sizes of angles by using strategies such as mental images of a protractor, a right angle, a half-turn, and angles of a polygon.

Both visual and non-visual reasoning have been found to play an important role in problem solving, with students often using visual processing in the initial stages of problem solving, and then moving to more analytic, non-visual strategies. While drawing a diagram is recognised as a powerful problem solving strategy, students have been found to experience a range of difficulties in generating effective diagrams in problem solving.

In summary, the research into Mathematical thinking has been dominated by research into the two overlapping areas of Children’s thinking strategies and Children’s problem solving and has focused largely on cognitive aspects of problem solving. As noted earlier, there has been considerably less research on Visualisation and even less on The language of mathematics, which is closely linked to another area with very little research, namely Algebra.

Unlike other countries such as the United States and Japan (see, for example, Yackel & Cobb, 1996; Carpenter & Franke, 2001; Fujii & Stephens, 2001), there has been very little focus in Australia on systematically developing children’s algebraic reasoning, processes of generalisation and proof, or their ability to represent and interpret mathematical expressions signifying the relationships between quantities, which is a rich strand of the mathematics curriculum in Japan from Year 3 onwards.

No doubt further research in Australia will produce knowledge about the use of specific types of tasks and teaching approaches to develop these important areas of mathematical thinking.

**Using mathematics**

In a mapping and review of research into numeracy it has been disappointing to find almost no research into children’s use of mathematics in everyday situations and other non-mathematical contexts. While there may be accounts of children applying mathematical concepts and skills in a variety of situations in professional journals, only two ARC-funded projects relating to mathematical modelling and a project on the application of mathematical ideas to discussions of stories in class were found.
Assessment

Many of the research projects and publications on primary numeracy used assessment of achievement as a major criterion for effective teaching. While such research can be found under the theme of Achievement in the database, it is discussed elsewhere in this report. In this section, only those studies whose major focus was assessment are included. These studies are reported under the themes of Assessment programs, an area with a considerable amount of research, Assessment techniques and School entry, both of which attracted a reasonable amount of attention, the latter especially in recent years, and Diagnostic assessment, for which relatively little research was located.

Assessment programs

In the international sphere, the most significant international assessment program in the past ten years has been the Third International Mathematics and Science Study (TIMSS). Findings relating to Australian children’s performance in TIMSS showed that:

• children in only six countries outperformed Australian children overall;
• Australian children were the highest achieving English-speaking children;
• Australian children, in the upper grade level, were equal top achievers in the content area of space;
• Australian primary children performed relatively poorly in the area of whole numbers;
• there were no statistically significant differences in mathematics achievement between boys and girls;
• Indigenous children scored significantly lower than non-Indigenous children; and
• children who spoke English at home achieved higher scores than those whose home language was not English.

Nationally, numeracy benchmarks have been agreed upon and common assessment items are being included in state-wide testing programs. The national benchmarks were developed for use in reporting minimum acceptable standards of literacy and numeracy achievement, in support of the national literacy and numeracy goal.

While comparisons between the draft Years 3, 5 and 7 benchmarks, international curriculum documents and achievement data from TIMSS suggest that statements of expectation are set at higher levels in some other countries, the Australian benchmarks are agreed minimum standards, which is not necessarily the case elsewhere.

The data provides nationally comparable information on student achievement, which will facilitate the monitoring of trends over time and in relation to the national goal. Importantly, the data produced includes information on the performance of sub-groups of students.

All States and Territories now have large-scale assessment programs in place. These tests monitor detailed student performance within education systems in terms of the relevant curriculum. National benchmark data is also obtained from these tests.

The advantages, limitations and effects of adopting alternative forms of system-wide assessment on the mathematical achievements of children have not received much research attention, nor have what forms of assessment take into consideration differences
in children’s learning styles, and the difficulties and limitations that assessment places on those with language, physical and cognitive disabilities.

**Assessment techniques**

Over the past decade, research on *Assessment techniques* has been concentrated in three main areas: techniques for assessing particular aspects of mathematical learning — for example, problem solving or statistical understanding; developmental assessment — for example, the construction of developmental frameworks in early number learning or mental computation; and the use of innovative assessment styles — particularly so-called *Rich assessment tasks* (RATS).

Research into the first of these — techniques for assessing particular aspects of mathematics learning — has been discussed elsewhere in this report.

Much of the research into developmental assessment has been discussed in the section on *Developmental frameworks*. However there has also been a growing interest in and use of Item Response Theory (IRT) and Rasch measurement to devise numeracy scales to enable the numeracy development of individuals or groups to be monitored — for example, the *Numeracy Achievement Scale* (Stephanou, Meiers, & Forster, 2000).

There is also an increasing interest in the use of developmental assessment frameworks as a catalyst for driving changes in instructional decision making — for example, in the recent project *Developmental-based assessment and instruction*, but also in projects such as the *Early Numeracy Research Project (ENRP)* and *Count Me In Too*.

In keeping with criticisms of research studies and classroom practices that use traditional pencil-and-paper tests as the sole measures of numeracy achievement, there has been a growing interest and use of *Rich Assessment Tasks* (RATS). Projects such as Victoria’s *Middle Years Numeracy Research Project* have incorporated the use of rich assessment tasks into the design and development of their *Student Numeracy Profile*, with results suggesting that their use has the potential to provide useful insights into instructional strategies for middle years’ students. Rich assessment tasks have also been a key component of recent and current projects in several States and Territories, including the Northern Territory, where their use with middle years Indigenous students is being investigated.

Findings from many of the current Australian Government *Strategic Numeracy Research and Development Projects* will make a significant contribution to research in these areas.

**School entry assessment**

The importance of mathematical learning in the early years cannot be overestimated.

Much of the focus for *School entry assessment* has been on the identification of children “at risk” and those considered to have need of numeracy intervention. Various State and Territory educational systems have implemented school entry assessment projects and programs with some extending beyond the first year of schooling.

*Project Good Start: Effective Numeracy Practices in the Year Before and the First Year of School*, a current Australian Government *National Numeracy Research and Development Project*, is just completing a quantitative study of a large, representative, national sample of children to gauge their numeracy development before school and during their first year of schooling, together with a longitudinal, qualitative study of a sample group, examining
before school and first year of schooling experiences that affect numeracy development over the two years. However, ways of smoothing the transition between pre-school and school continues to be an area that has attracted little research attention either in Australia or overseas.

**Diagnostic assessment**

The research located related to *Diagnostic assessment* has been discussed under *School entry assessment, Developmental frameworks and Intervention*. In addition, material containing teaching strategies aimed at overcoming specific kinds of difficulties that children have with number was located.

While the transition from primary to secondary school has been identified as a particularly critical period of children's schooling, and several numeracy-based projects and research studies have included data gathered from year levels that straddle this transition, assessment issues were not the major focus of any of these projects. In fact, overall it appears that diagnostic assessment, other than in the early years of schooling, has not received much systematic research attention.

This chapter has given an overview of the findings of the Australian research into primary numeracy and identified some of the gaps in this research. In the next chapter we will suggest directions for future research that could assist in the filling of these gaps.
Chapter 8

Directions for Future Research

The potential of research to inform practice is unquestionable. This project has sought to produce an information base of key Australian research into numeracy at the primary school level for use by a range of numeracy education stakeholders, in order not only to assist in policy development and planning of numeracy teaching programs, but also to identify directions for future research targeted at effectively improving students' numeracy.

The project has identified a wide range of research into primary numeracy over the past decade, from all States and Territories of Australia, drawing on, and contributing to, international research. Perhaps the most striking feature of this Australian research has been its sheer quantity and diversity.

Significant strengths of Australian research into primary numeracy during the past decade include:

- a substantial, coherent and continuing body of research into children’s early number learning;
- a significant body of Australian and collaborative international research into number sense, estimation and mental computation;
- the extent of numeracy research in areas other than Number, particularly in Chance and Data, and, to a lesser extent, Measurement;
- a substantial body of research into the use of calculators as teaching aids, with consistent positive findings relating to children's development of number sense, but continuing evidence that they are not being used as often or as effectively as is possible, especially in the early years;
- large-scale research investigating effective teaching of numeracy in the early and middle years of schooling, using developmental frameworks and rich assessment tasks to provide teachers with the opportunity and tools to engage with their students’ understandings of primary mathematics; and
- a significant amount of exploratory research into areas such as the use of open questions, children’s reasoning, and inquiry based approaches to teaching.

Nevertheless, we pointed out in Chapter 7 that there are significant gaps in Australian primary numeracy research. Based on our review of this research, this final chapter focuses on identifying, for policy makers and the wider research community, possible directions for future Australian research.

The suggestions for further research made below have been identified either as major gaps of as areas where some fruitful research, and especially national and state-wide projects, have laid foundations for the development of leading-edge knowledge about the teaching and learning of mathematics in Australian primary schools.

Equity

It is important for research into numeracy teaching and learning to recognise the diversity of students, communities, and educational settings. In particular, students from
disadvantaged backgrounds, such as those from low socio-economic backgrounds, Indigenous students, and students with disabilities, including those with learning difficulties, present particular challenges and may require different responses from teachers and schools for them to gain appropriate numeracy skills. Some of the gaps in the research related to these issues and areas in need of further research are discussed below.

**Disability and learning difficulties.** In keeping with the findings of van Kraayenoord, Elkins, Palmer and Rickards (2000), this review suggests a need for research in the area of numeracy development for students with all types of disabilities, with more of a focus on numeracy development, rather than literacy.

This gap in the research suggests two major research questions:

- How can we improve the identification of children encountering learning difficulties in numeracy? 
- How can we improve numeracy teaching and learning for children with physical disabilities?

The Australian Government has committed funding of $4.5 million under the National Literacy and Numeracy Strategies and Projects Programme to assist in equipping teachers to better meet the needs of students with disabilities and learning difficulties. Projects at the national and state levels will focus on more effective teaching and learning practices for students with disabilities and learning difficulties in the early and middle years of schooling. Findings from these projects are likely to be of great value in this area.

**Indigenous students.**

In order to extend and validate the knowledge about the learning needs of Indigenous students gained from the smaller research projects, there is a need for larger-scale research studies that address Indigenous students' numeracy needs. Nation-wide, co-ordinated research, involving members of Indigenous communities, is needed to expand understandings of the needs of Indigenous learners and the development of appropriate numeracy curricula.

The Australian Government’s current National Indigenous English Literacy and Numeracy Strategy, which involves all Australian States and Territories in supporting the identification and dissemination of effective practice models and teaching methods drawn from earlier pilot projects, is likely to provide a strong basis for further research in this area.

**Students**

The paucity of research on gifted and talented youngsters needs to be addressed both on equity grounds and the potential benefit to the community. Research needs to focus on:

- the identification of mathematically gifted and talented students;
- their specific needs;
- strategies that teachers can adopt to foster these children's talents; and
- appropriate professional development.

At the other end of achievement scales, much of the research on children who are at risk of not learning mathematics successfully has focused on young children. There is a need
for more research into the identification of children in middle and upper primary years who are at risk.

There is also a need for research into the long-term effects of intervention programs for those children who have been identified as being at risk of falling behind in their study of mathematics. In particular, we need to know the effects of remedial programs where interventions take the form of withdrawal programs, as suggested by Luke et al. (2003).

**Classroom practice**

Luke et al. (2003) have also called for a focus on renewing mainstream pedagogy, including the need for a “much fuller, research-based understanding of what is going on every day in school classrooms … [and] more systematic emphasis on intellectual demand and student engagement” (p.8). We would see many of the possible directions for future research identified below as resonating with this call.

Further research is needed into the most effective group structures, comparing the effectiveness of group work not only with individual work, but also with models of whole class teaching — such as that used in Japanese schools — that focus on problem solving and student explanation rather than teacher exposition.

There is a need for research into which aids are most useful in promoting children’s numeracy learning, when such aids are most useful, and ways of drawing out abstractions and generalisations.

Regarding the use of technology, further research is needed to identify effective pedagogical strategies that supplement and enrich the use of particular technological tools, as well as characteristics of software that can make a difference in terms of children’s abilities to reason and work mathematically. Furthermore, given the lack of use of calculators, despite the positive research findings, there is a need to carry out further research into ways in which classroom teachers can be assisted in incorporating calculators into their everyday teaching practice.

In a climate where the use of tasks with low cognitive demand is seen as one reason for the frequent absence of purposeful mathematical dialogue in the classroom, further Australian research is needed into the development of conceptually focused, robust tasks that can be used to support the development of sophisticated mathematical thinking.

The two recent video studies coming out of the *Third International Mathematics and Science Study* (TIMSS) suggest that there is potential for fruitful research that is aimed at providing an understanding of every day classroom practice in countries with different cultures and patterns of teaching, as a means of achieving better understanding of one’s own practice and looking at ways of extending its boundaries. This could involve international collaborative research into primary mathematics practice, particularly into aspects of Japanese practice, which uses a common lesson pattern based on a highly structured, in-depth analysis of multiple student solutions to a single problem.

**Concept development**

Despite the quantity of research into the development of mathematical concepts, further research is needed in many of the areas that have a strong link to everyday numeracy. In particular further research is needed into:

- children’s understandings in the area of graphical literacy;
Primary Numeracy

- computational aspects of measurement and ways in which connections between measurement and our decimal system can be used to support students’ acquisition of numeracy skills;
- which aspects of schooling and which pedagogical approaches are most effective in developing children’s number sense;
- the development and use of new models and materials for instruction in mental and written computation; and
- the types of tasks and teacher actions that can enhance students’ learning in the area of space.

**Developmental frameworks**

Further research at both the pre-school and primary-secondary transition levels could include case studies in the development a common language that enables clear communication of children’s capabilities and needs between pre-school staff, primary and secondary teachers, and parents.

**Mathematical thinking**

Further research is needed in Australia on the development of tasks and teaching approaches to develop important areas of mathematical thinking, including

- algebraic reasoning;
- processes of abstraction;
- generalisation;
- proof; and
- representing and interpreting mathematical expressions signifying the relationships between quantities.

**Using mathematics**

It has been disappointing to find almost no research into children’s use of mathematics in everyday situations and other non-mathematical contexts. Given that using mathematics is where children’s numeracy can be put into practice, this is clearly an area where further research is needed.

**Assessment**

The advantages, limitations, and effects of adopting alternative forms of system-wide assessment have not received much research attention, and this must be a significant budget item for relevant government departments. Hence we suggest that further research is warranted in these areas.

Despite the fact that some State and Territory educational systems have implemented school entry assessment projects and programs, there still appears to be a need for further research into the development of appropriate tools for profiling pre-school children’s mathematical development in order to smooth the transition between pre-school and school.


**Teachers**

Our review of the research literature on teacher education suggests that further research is needed into:

- the ways in which technology can be used to provide student teachers with opportunities to engage in powerful experiences through the structured analysis of carefully selected video and other records of learning and teaching;
- teacher change and its longer-term effects on students’ numeracy outcomes (and especially longitudinal studies);
- the extent to which professional development results in sustained change is needed;
- ways in which teachers’ pedagogical content knowledge can not only be assessed but also improved.

Clearly, while there is an impressive array of Australian research into primary numeracy, there are still many areas where further research is needed. In order to achieve the goals agreed by State, Territory, and Australian Government Ministers for Education, that all students should attain the skills to be numerate, it is important that numeracy research remain a high priority for Australian research.


Primary Numeracy


Groves, S. (In press). Calculators, computation and number sense – Some examples from the Calculators in Primary Mathematics project.


Primary Numeracy


Primary Numeracy


Primary Numeracy


Primary Numeracy


Primary Numeracy


Appendix A

Project personnel

Associate Professor Susie Groves (Project Director)
Centre for Studies in Mathematics, Science and Environmental Education
Faculty of Education
Deakin University

Associate Professor Judith Mousley (Research Team)
Centre for Studies in Mathematics, Science and Environmental Education
Faculty of Education
Deakin University

Dr Helen Forgasz (Research Team)
Faculty of Education
Monash University
(previously Deakin University)

Ms Kathy Savige (Website Designer and Research Assistant)
Faculty of Education
Deakin University

Ms Angie Bloomer (Project Manager)
Faculty of Education
Deakin University

Ms Brenda O’Donnell (Research Librarian)
Deakin University Library
Appendix B

Advisory Committee

Ms Margaret McCulloch
Assistant Director
Benchmarking, Assessment and Numeracy Policy Section
Literacy and Special Programmes Branch
DEST

Mr John Barbour
Assistant Director
Benchmarking, Assessment & Numeracy Policy Section
DEST

Mr Peter Gould (Australian Education Systems Officials Committee nominee)
NSW Department of Education and Training

Ms Pauline Duffy (National Council of Independent Schools Associations nominee)
Association of Independent Schools, Victoria

Dr Thelma Perso (Australian Education Systems Officials Committee nominee)
Senior Curriculum Officer in Mathematics
Education Department of Western Australia

Ms Trish O’Toole (Catholic Education Commission nominee)
Catholic Education Office, SA

Associate Professor Alistair McIntosh (Numeracy expert)
Faculty of Education
University of Tasmania

Associate Professor Kaye Stacey (Numeracy expert)
Department of Science & Mathematics Education
The University of Melbourne

Mr Will Morony (Numeracy expert)
Australian Association of Mathematics Teachers

Ms Vicki Steinle (Deputy National Numeracy Coordinator)
National Co-ordinator of Numeracy Research Projects
DEST
Appendix C

Consultants

Australian consultants

Dr Jack Bana
Edith Cowan University
Mt Lawley, WA

Ms Rosemary Callingham
Faculty of Education
University of Tasmania
Hobart, TAS

Ms Anne Carrington
de Lissa Institute Of Early Childhood and Family Studies
University of South Australia, Magill, SA

Ms Debbie Efthymiades
Curriculum Services Branch
School Services Division, NTDE
Darwin, NT

Dr Peter Galbraith (MERGA President)
Graduate School of Education
University of Queensland, QLD

Ms Beth Lee
45 Curlewis Crescent
Garran, ACT

Dr Joanne Mulligan
School of Education
Macquarie University, NSW

International consultants

Dr Mike Askew
School of Education
Kings College
London, UK

Dr Andy Begg
University of Waikato
Hamilton, New Zealand

Professor Carolyn Kieran
Department of Curriculum and Instruction
Université du Québec à Montréal,
Québec, Canada

Professor Judith Sowder
Center for Research in Mathematics & Science Education
San Diego, USA
Appendix D

List of Consultations

Australian Capital Territory: Judith Mousley & Brenda O’Donnell

Thursday 10 May 2001
9:00 am  Rick Owens (ACT Department of Education) & Steven Thornton (Division of Communication and Education University of Canberra)
11:00 am  Robert Fitzgerald (School of Education, Signadou Campus ACU) & Joan Robson (School of Education, Signadou Campus, ACU)
1:00 pm  Library: Signadou, ACU
3:00 pm  John Hogan - Redgum Consulting (ACT3) - telephone interview
3:15 pm  Annabelle Cassels - telephone interview

Friday 11 May 2001
9:00 am  Library: O’Connor Centre
10:00 am  Beth Lee (Garran)
1:00 pm  Wayne Hawkins (Division of Communication and Education University of Canberra)
2:30 pm  University of Canberra Library

Victoria: Helen Forgasz, Susie Groves, Judith Mousley

Tuesday 8 May 2001
12:30 pm  Doug Clarke & Jill Cheeseman, Australian Catholic University

Tuesday 5 June 2001
11:30 pm  Pauline Duffy (AISV: Director: literacy and numeracy projects) – telephone interview

Thursday 14 June 2001
8:30 pm  Pam Hammond (Early Years of Schooling, Numeracy Team, DEET, Victoria)

Monday 27 August 2001
5:30 pm  Associate Professor Di Siemon, RMIT

Thursday 30 August 2001
3:00 pm  Professor Sue Willis, Monash University (about WA First Steps Project)

Tuesday 11 September 2001
9:00 am  Mr Gerard Lewis & Ms Cath Pearn, CEO
**Tasmania: Helen Forgasz**

**Wednesday 2 May 2001**
12:30 pm Pre-visit to Tasmania. Rosemary Callingham, consultant (Tasmania) visiting Deakin University, Victoria (Susie Groves & Helen Forgasz)

**Thursday 7 June 2001**
10:30 am Doug Bridge, Principal Project Officer, Office of Education, Department of Education
11:30 am Vicky Nicholson, Senior Curriculum Officer, Aboriginal Education, Equity Standards Branch, Department of Education
2:00 pm Joy Edmunds, Literacy and Numeracy Support, Department of Education
3:30 pm Miriam Solomon, Literacy and Numeracy Support, Department of Education
6:30 pm Denise Neal, primary teacher involved in numeracy projects over the past decade

**Friday 8 June 2001**
3:00 pm Professor Sue Willis, Monash University (about WA First Steps Project)
11:00 am Dr Jane Watson (Reader), University of Tasmania
10:15 am Dr Shelley Dole, University of Tasmania
10:45 am Tracey Muir and Louise Fisher, teachers involved in “Teaching and working mathematically” project
1:30 pm Associate Professor Alistair McIntosh, University of Tasmania

**Saturday 9 June 2001**
7:00 pm Pat Jeffery, retired primary teacher involved in numeracy projects over the past decade

**New South Wales: Susie Groves & Judith Mousley**

**Friday 29 June 2001**
10:30 am Professor Judith Sowder (Centre for Research in Mathematics & Science Education, San Diego - USA consultant)

**Saturday 30 June – Thursday 5 July 2001**
During the Mathematics Education Research Group of Australasia [MERGA] conference and the DET Numeracy Day, meetings were held with the following people:
Mr Peter Gould, Chief Education Officer, Mathematics, NSW DET
Ms Diane McPhail, Curriculum Support Directorate
Dr Paul White, Australian Catholic University (Mount St Mary Campus)
Mr Peter Howard, Australian Catholic University (Mount St Mary Campus)
Associate Professor Mike Mitchelmore, Macquarie University
Ms Dawn Bartlett, Student Assessment and Reporting
Professor Bob Perry, University of Western Sydney
Dr Kay Owens, University of Western Sydney
Dr Tom Lowrie, Charles Sturt University
Associate Professor Joanne Mulligan, Macquarie University
Dr Noel Thomas, Charles Sturt University
Associate Professor John Pegg, University of New England
Associate Professor Bob Wright, Southern Cross University
Ms Jan Stone, Independent Schools Association (telephone interview)

Friday 6 July 2001
10:00 am Ms Susan Busatto, DET
1:30 pm Ms Sue Moffat, CEO Sydney

Queensland: Susie Groves & Judith Mousley

Sunday 8 July 2001
7:30 am Dr Robyn Zevenbergen, Griffith University, Gold Coast

Monday 9 July 2001
9:30 am Professor Lyn English, Queensland University of Technology
2:00 pm Dr Cal Irons, Queensland University of Technology
6:00 pm Mr Graham Meiklejohn, Queensland School Curriculum Council

Tuesday 10 July 2001
9:00 am Dr George Booker, Griffith University, Mt Gravatt
1:00 pm Ms Rhonda Eggerling & Ms Nola Simpson, Education Queensland

Northern Territory: Susie Groves & Helen Forgasz

Thursday 26 July 2001
1:00 pm Ms Debbie Efthymiades, Ms Josie Roberts, Mr Geoff Gillman & Ms Connie Emslie, Northern Territory Department of Education

Friday 27 July 2001
10:00 am Ms Debbie Efthymiades

Western Australia: Susie Groves & Helen Forgasz

Wednesday 23 May 2001
1:00 pm Pre-visit to Western Australia. Beth Powell (Murdoch University) visiting Deakin University, Victoria (Susie Groves)

Monday 30 July 2001
8:45 am Dr Elena Stoyanova & Ms Jocelyn Cook, EDWA
9:45 am Dr Thelma Perso, EDWA
11:30 am Dr Jack Bana
1:30 pm Ms Glenys Reid & Ms Wendy Devlin, EDWA
Friday 27 July 2001
8:45 am      Dr Len Sparrow, Curtin University
10:00 am     Mr John Hogan, Murdoch University
1:30 pm      Dr Anne Chapman, UWA

South Australia: Helen Forgasz

Wednesday 1 August 2001
2:00 pm      Sue Emmett (School Entry Assessment)
             Margot Rose (Early Years Project Officer)
             John Bleckly (Curriculum policy officer: Literacy and numeracy across the curriculum)
             Jackie Walter (Mathematics Curriculum Officer)

Thursday 2 August 2001
9:45 am      Trish O'Toole (Catholic Education Office)
11:30 am     Will Morony (AAMT)
Appendix E

Web-based data collection instrument

Primary Numeracy Research

[A project funded by the Australian Government Department of Education, Science and Training]

Research team: Susie Groves, Judith Mousley, and Helen Forgasz

We are seeking your help to collect the details of relevant publications and projects for entry into the project’s database, for use in the project report, and later for the bibliography on CD-ROM disc. We are gathering information about publications from completed (and also on-going) Australian research projects (externally funded or not). Their focus must be on numeracy in primary education.

Please review:

- the purpose, background, and outcomes of the project;
- the publications that are already listed on the data base.

If you know of any other publications and or the details of any projects that you believe could be relevant, please complete as many of the following fields as possible. Please use a separate form for each publication and project.

Completed forms should be emailed as attachments (in .rtf format) to Angie Bloomer (Project Manager): angieb@deakin.edu.au

FOR ALL PUBLICATIONS AND PROJECTS

Your name:

E-mail address:

**PUBLICATION DETAILS**

Full citation details, in APA format:

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Abstract:

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Electronic copy of the paper is attached: YES / NO

If not attached, copy available from: (over)
## PROJECT DETAILS

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<td>Name, address and email of mail contact person:</td>
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THANK YOU
Appendix F

Focus Group Meetings

Melbourne
Monday 12 November 2001, 4:30 – 7:30 pm
Convenor: Associate Professor Susie Groves
Project team: Helen Forgasz & Susie Groves
Participants: 12 teachers, numeracy co-ordinators, and teacher educators

Darwin
Thursday 15 November 2001, 4:30 – 7:30 pm
Convenor: Ms Debbie Efthymiades
Project team: Judith Mousley & Susie Groves
Participants: 9 teachers, numeracy co-ordinators, and teacher educators

Gold Coast
Thursday 22 November 2001, 4:30 – 7:30 pm
Convenor: Associate Professor Robyn Zevenbergen
Project team: Helen Forgasz & Judith Mousley
Participants: 10 teachers, numeracy co-ordinators, and teacher educators
A mapping, review and analysis of Australian research in numeracy learning at the primary school level

FOCUS GROUP FEEDBACK

FOCUS GROUP: Melbourne  Darwin  Gold Coast

NAME:

SCHOOL / INSTITUTION / AFFILIATION:

YOUR VIEWS OF THE CD

1. What was your first impression of the CD?

2. Do you like the appearance of the CD?
   Please explain your response:

   How could the appearance of the CD be improved?

3. Did you find the CD easy to navigate?
   Please explain your response:

   How could navigation of the CD be improved?
4. Please rate each of the search functions for ease of use 
(1 = very easy to 5 = very difficult)
  Publications by author
  Project by title
  Themes
  Keywords
  Site map
Other comments on the search functions:

How could the search functions be improved?

5. Please comment on each of the following aspects of the contents of the data base:
  Scope of data included

Organisation of the data

Key words used

Information provided for each publication and project

6. Please comment on each of the following aspects of the packaging of the final version of the CD:
  What additional instructions for use are needed? What (if anything) would you want in hard copy to go with the CD?
7. How would you be likely to use the report of the project? Would you read only the executive summary? Would you be likely to read all or part of the full report? Would you print those parts of the report you wished to read or would you read them from the CD?

8. What were the three best aspects of the CD for you?

9. What were the three worst aspects of the CD for you?

10. What is missing from the CD that you think should be there?

11. Is there anything that should be deleted from the CD?

12. If you got stuck at any stage, what were you trying to do at the time? How did you get “unstuck”? 
USE AND DISSEMINATION

1. Would you use this CD if it were available to you?
   If yes, how would you use it? When would you use it?

   Would you be more likely to use it if it were on a CD or on a website?

2. Who else do you think would use it? (Please tick)
   Teachers
   Curriculum developers
   Teacher educators
   Other (please specify)

   Would it be better to disseminate this information on a CD or on a website?

3. In your opinion, what would be the best overall dissemination strategy for the findings of this project?

PLEASE USE THE SPACE BELOW FOR ANY OTHER COMMENTS
Appendix G

Numbers of entries in the Primary Numeracy database by theme and sub-theme

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