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A recent article in *New Scientist* by Bryant Furlow argues that children’s intelligence is boosted by playing games. In this article, the third in a series, the mathematics behind common card games will be examined.

**Happy Families**

The traditional pack of cards for *Happy Families* is even fun to look at and read: Mr Bun the Baker, Mrs Bun the Baker’s wife, and little Master (or Miss Bun), happy, smiling, white-coated chef-hatted, roly-poly bun-shaped bakers – and many others – a happy community of tradespeople.

Here are the basic rules of *Happy Families*. The aim is to be the player who succeeds in making the most complete sets of families.

Other games have a related format. For example, in *Authors* (see Gough, 2000), the sets of matching cards consist of an author, and three of his or her works – a general knowledge work-out. The Shakespeare set, for example, might consist of a card showing the Bard of Avon musing, quill poised, and three other cards, one for ‘Hamlet’, another for ‘Macbeth’ and a third for ‘Midsummer Night’s Dream’.

Clearly the *Happy Families* and *Authors* format is equivalent to the standard 52-card pack, with four suites (♥, ♦, ♣, ♠), and four of each type of card, for example, a 6 of Hearts (♥), a 6 of Diamonds (♦), a 6 of Clubs (♣), and a 6 of Spades (♠). This is not as appealing as pictures of book-writers and characters from books, and is not suitable for developing a lot of general knowledge or trivia, but is readily available and very adaptable for other uses.

- The cards are dealt out to all players, as evenly as possible, although some players may have one card less than others. (Sharing, or division, with or without remainders, is one obvious mathematical idea to explore, at this point.)
- Each player looks at his or her cards, not letting others see them.
- Each player sorts his or her cards into families, as much as possible. At this stage, and later, as soon as a player has a complete family set, this is placed face-down in front of the player.
- Players take turns. In each turn a player asks any other player if he or she has a particular named card, e.g. Master Chuff, the son of the train driver. It is important that a player must already hold at least one member of the family that is being named! If the answer is ‘Yes’, the named card must be handed over. The player may continue asking for particular cards, in this way, until the answer is ‘No’. When this happens, the player’s turn ends.
- During each turn, as family sets are completed they are laid face-down in front of the player who has collected that family.
- Play continues until all the families have been collected.
- The winner of the round is the player who ends the game with the most completed families. Score one point for each complete family collected.
- Play continues for an agreed period of time, and the player with the highest score wins the overall game; or play until one player reaches an agreed total score, such as 50 points, thereby winning the overall game.
Old Maid

The children’s card game *Old Maid* (or *Old Bachelor*, in the gender-reversal version) has a format of matched pairs, such as two identical Cyclists, two identical Butchers, and so on, as well as one single card which has no matching pair, a grim (or cheerful) spinster. The aim is to collect as many matching pairs as possible, while not being the losing player who is left at the end with the unmatchable solitary old person. A French version, using an ordinary card pack, but with the Jacks of Hearts, Diamonds and Clubs removed, is called ‘Le Vieux Garçon’ — *The Old Boy* — the Jack of Spades. A German version uses a special pack of pairs of cards, with one singleton called ‘Black Peter’ — the name of the game — often showing a fierce black cat wearing boots and a hat (*The Way to Play*, p. 309).

The rules for playing *Singleton* (a non-ageist name for the otherwise traditional but not so politically-correct game, *Old Maid*) are similar to those for *Happy Families*.

A further modification occurs with the game *Fish* (or *Go Fish*). Using a *Happy Families* pack, or a standard pack of cards, the usual rules for *Happy Families* apply, with two exceptions.

- When dealing the cards at the start, an extra hand is dealt. For example, if 4 people are playing, the pack is shuffled, and dealt out as fairly as possible, as though there were 5 players. The extra hand becomes the ‘pond’ where players ‘fish’.
- When a player asks for a particular card, and gets the answer ‘No’, that player is told to ‘Fish’. This means the player picks up the top card of the extra hand (unless all the cards in the fish-pond have already been taken up in this way).

The rules of *Fish* may vary considerably. For example, rather than forming an extra hand, players start with a deal of, say, seven cards each, and all the remaining cards are placed face-down in the middle as the ‘stock’ or ‘fish-pond’. Also, if a player asks, for example, ‘Do you have a three?’, if the player who is asked has any threes, these must all be handed over (Frey, 1970, p. 281).

Several important features arise with these familiar games. Notice the large amount of luck or chance in *Singleton*: luck in the original deal, and luck in choosing a card from the player on the left. This is not a game of skill, except the skill of keeping a straight face. Do not let other players know that you have just picked up the dreaded Singleton, or that the card a player is apparently about to choose is the one you want to get rid of.

By contrast, notice the lesser amount of luck, and greater involvement of both memory and logic in playing *Happy Families*. A luck-dominated game, such as *Singleton*, is useful for becoming familiar with handling cards, taking turns, and ordinary game-manners, and with the format of a pack of cards. But too much luck, and not enough thinking, condemn a game as fit for beginners only.

Once the mechanics of play have been established it is better to play a game with scope for thinking, or change the rules to decrease the amount of luck, and require some brain work. For example, using a standard card pack, and the format of making pairs while avoiding the singleton, you can require that a ‘pair’ must not only match in number but also in colour — e.g. $7\spadesuit$ is a pair for $7\clubsuit$, but not for $7\heartsuit$ or $7\diamondsuit$ (*The Way to Play*, p. 309).

A very different kind of make-pairs game is *Concentration* (also known as *Memory*, *Picture Lotto*, or *Pelmanism*, after a Mr Pelman who had a method of practising and boosting the power of memory). All the cards of a standard pack (or a special pack, with pictures) are dealt out face-down across a table. Players take turns. In each turn a player turns over any two cards. If they match (e.g. two 8s), the player picks up the pair, and scores 1 point for losing, with overall winner being the lowest scoring player. Or score -5 for losing, and +1 for every pair made.
point, and may continue, turning over another pair. If the pair revealed do not match, they are turned down again, and the player’s turn ends. The game finishes when all the cards have been paired. The winner is the player with the most pairs.

Rummy

However, all of these traditional family games, entertaining though they may be for children of the right age, are really only preparation for better games that offer adult-level thinking and life-long playability. **Rummy** is one of these.

**Rummy** exists in many variants, such as **Gin Rummy**, **Canasta** and **Rummicub**. The Chinese game **Mahjong** uses a ‘pack’ of 144 domino-like tiles, but is essentially an elaborate version of **Rummy**. Similarly, anything that can be played with a pack of cards can be played, possibly with some modification, using a set of dominoes, possibly using suits of numbers (for example, all the dominoes with a 6 at one end), as well as a suit of doubles.

The card games discussed so far can be considered to be examples that illustrate general distinguishing features. For example, the kind of pack used, the setting-up or dealing, the method of playing, the overall aim, the kind of ‘families’ that players attempt to make while playing, and so on.

**Rummy**

- Shuffle a standard pack, and deal 10 cards each to 2 players; or 7 cards each to 3 or 4 players; or 6 cards each to 5 or 6 players. Cards are dealt face-down, of course.
- The remaining pack, called the ‘stock’ is placed face-down in the middle of the table.
- The top card of the ‘stock’ is placed face-up alongside, to start a separate ‘discard’ pile.
- Players take turns. Each player aims to be the first to make a complete set of card combinations, thus ‘emptying’ or using up all the cards in his or her hand.
- In each turn a player ‘draws’ the top card from the ‘stock’, or picks up the top card on the ‘discard’ pile, and then tries to use his or her cards in the following ways.
- The player is aiming to make sets of three or four of a kind (e.g. three Aces) or three or more in a sequence (e.g. 2, 3, 4, of any suit). Usually, when a player has formed 3 or 4 of a kind, or 3 or more in a row, these are placed face-up on the table in front of the player. (Why? The scoring makes it preferable to minimise the number of cards a player is left holding at the end of the game.)
- The player may also add a suitably matching or sequential card to any other player’s visible set of cards of a kind, or cards in a sequence. For example if another player has already placed down three 8s, the player may lay another 8 from his or her hand, making this four 8s. Similarly, if another player has already laid down 4, 5, 6, and 7 the player may lay either a 3, or a 8, turning the four-in-a-row into five-in-a-row.
- The player is also attempting not to be caught with high-scoring cards in his or her hand, not yet placed face-up on the table in a suitable set-of-a-kind or set-in-a-row.
- The player ends his or her turn by placing one card face-up on the discard pile. If a player is able to use all of his or her cards in this turn, putting down sets-of-a-kind or sets-in-a-row, or playing onto the sets of other players, and has only one card left, before discarding, this may be played on the discard pile, thereby emptying the player’s hand, and ending the game. However, if the player has started his or her turn by picking up the top card on the pile of discards, the player is not allowed to end the turn by discarding that same card.
- Play continues until one player has successfully emptied his or her hand, making sets-of-a-kind, sets-in-a-row, and possibly laying other cards on the sets of other players, and possibly discarding the final card. This ends the game.
- Scoring is simple. The winner scores the total of cards still held by other players (not counting the sets they have placed face-up in front of them). Aces score as 1. If the winner has not previously laid down any sets, but does so in this final turn, this is called ‘going rummy’, and the winner’s score is doubled.
Consider how to extend such a table to include other card games, and other game-features. For example, consider number of players as a distinguishing feature. As soon as we start considering families of card games, we realise that one-player activities sometimes correspond in important details with multi-player games. The many versions of card Patience (or card Solitaire) show different ways of dealing, drawing and using cards to form winning sets or cards or family patterns. The game Spite and Malice (sometimes called Spit) is essentially a two-player two-pack version of Patience, each player competing, and interacting, to be first to complete a played-out Patience sequence. That is, the ‘family’ being made is the complete pack, sorted or built in an ordinary Patience-based process.

Other card games fit more loosely into this make-a-family format. Poker, for instance, involves make-a-family with the extra feature that different families are worth different amounts, and the most valuable family wins. This idea is also used, slightly differently, in Mah jong, where certain family combinations are higher scoring, but more difficult to make. The Japanese season-of-the-year game Hanafuda (The Way to Play, pp 118–121, and Game, Set and Match — Maths!, p. 32) also scores different families in different ways. We can imagine a modified version of Rummy, in which players compete to form a poker hand, and at the end of play, the highest hand wins, measured in

<table>
<thead>
<tr>
<th>Card pack type</th>
<th>Singleton (Old Maid)</th>
<th>Happy Families &amp; Authors</th>
<th>Fish</th>
<th>Rummy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of players</td>
<td>2 or more</td>
<td>2 or more</td>
<td>2 or more</td>
<td>2 or more; 4 is preferable</td>
</tr>
<tr>
<td>Dealing</td>
<td>All dealt fairly</td>
<td>All dealt fairly</td>
<td>All dealt fairly</td>
<td>A hand of 5 to 10 cards</td>
</tr>
<tr>
<td>Getting more cards</td>
<td>Help yourself</td>
<td>Ask for a specified card</td>
<td>Ask for a card or fish from extra stock</td>
<td>Draw from stock; or pick up a discard</td>
</tr>
<tr>
<td>Aim of play</td>
<td>Make pairs &amp; avoid the singleton</td>
<td>Make families</td>
<td>Make families</td>
<td>Make 3 or 4 of a kind, or 3 or 4 in a row</td>
</tr>
<tr>
<td>Winner</td>
<td>Player with most pairs</td>
<td>Player with most families</td>
<td>Player with most families</td>
<td>First to empty his or her hand</td>
</tr>
<tr>
<td>Loser</td>
<td>Player left with the singleton</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
</tbody>
</table>

Hunt the Secret

- Start by setting up the cards to be used as follows. For the given number of players, choose that number, plus one, of ‘top’ cards, that number of ‘middle’ and that number of ‘bottom’ cards. The ‘top’ cards start with the Kings, highest, then Queens, then Jacks. The ‘middle cards’ start with 10s, then 9s then 8s. The ‘bottom’ cards start at the lowest with Aces, then 2s, then 3s. For example, with five players, we use a diminished pack of four Kings and two Queens, four 10s and two 9s, and four Aces and two 2s.

- Randomly choose one ‘top’, one ‘middle’ and one ‘bottom’ card, and place this Secret Target set face-down in the middle of the table.

- Collect all the other ‘top’, ‘middle’ and ‘bottom’ cards, shuffle them, and deal them evenly to each player.

- Players look at their hand of cards, and then take turns. In each turn a player either makes a ‘guess’ or a ‘declaration’.

- A player’s ‘guess’ might be something like this: ‘I think the Secret Target is King of Clubs, 9 of Hearts, and Ace of Diamonds’. This is a challenge to the other players to contradict this guess, if possible. Working clockwise (from left to right) from the guesser, the next player who can contradict at least one part of this guess, does so by secretly showing the guesser one of the guessed cards that is held by that contradicting player. After one player has made such a contradiction, no further contradicting occurs in this turn. A player who can contradict part of a guess must not refuse to do so if no other players have been able to make a contradiction.

- A ‘declaration’ is made this way. The player secretly writes down what the player believes the Secret Target is. The player then secretly examines the Secret Target. If the player is correct, the player is the winner, and the game is finished. Otherwise the player is not allowed to make any more ‘guesses’, and continues in the game only to provide any possible contradictions to other players’ continuing guesses. If, during a player’s turn, a player has made a ‘guess’, the player may then make a ‘declaration’. Otherwise, after other players have responded, if possible, to the ‘guess’, the player’s turn ends.

- Players should keep notes of ‘guesses’, revealed contradictions, and any other helpful information.
poker-playing priority.

Trump-taking games, such as Whist, Euchre, Five Hundred, Solo and Bridge, use families of cards (based on the four standard suits, and no-trumps) as the basis for putting sets of cards together to see whose card scores highest, and therefore wins the point.

The familiar game of Cluedo can also be seen as a card game in this aim-to-make-a-family pattern. The special feature is that the family of cards that is to be made, or discovered is the secret combination of cards (or Cluedo information) that identify who committed the murder, what weapon was used, and the room where the murder occurred. The playing essentially consists of asking other players about the information, or cards, they hold. This resembles the asking-for-a-named-card process in Happy Families.

Hunt the Secret is a modified version of Cluedo, omitting any moving around a board, using cards from a standard pack (also see Code-Buster in Game, Set and Match — Maths!, p. 63). Players use ‘top’, ‘middle’, and ‘bottom’ cards from a standard pack.

Stealing Bundles is a simple children’s game which resembles Rummy, and these other make-a-family games. Importantly it most closely resembles Casino, an adult card game which, despite the name, is more a game of skill than a game for gambling. In this case, the make-a-family format applies to simple accumulation, which is triggered at each step by simple matching of a pair.

Stealing Bundles
- Two or four people play, using a standard pack of cards. (Why not have three players?)
- Each player is dealt four cards, face-down, and four cards are placed face-up in the middle of the table.
- Players take turns. In each turn a player may ‘claim’ one of the cards in the middle. The player does this if he or she also holds a card of the same numerical value or rank. The player picks up this card, and places it, along with the matching card from his or hand, as a face-up stack in front of the player. This is the start of the player’s ‘bundle’. A player may successively ‘claim’, in one turn, as many cards in the middle as match cards held by the player, and these are placed successively, face-up on top of the player’s ‘bundle’.
- A player may also ‘steal’ another player’s ‘bundle’, if the ‘bundle’ is matched by one of the cards held by the player. That player plays the matching card on top of the ‘bundle’, and places the ‘stolen’ cards face-up on the top of his or her own ‘bundle’.
- However, if none of the cards in the middle match any of the cards held by the player, that player must end his or her turn by placing one of his or her cards face-up in the middle. This is known as ‘trailing’.
- That is, a player may ‘claim’, and may ‘steal’, depending on what cards the player currently has in his or her hand. But if the player is unable to ‘claim’ or ‘steal’ the player must ‘trail’, and the player’s turn ends.
- When all four cards have been used, four new cards are dealt from the remaining stock of cards, and play resumes.
- The game ends when all cards have been dealt and played. The winner is the player with the biggest ‘bundle’ at the end of the game.

References

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