Abstract: Problem solving is often seen as being the core of mathematics. While there are many examples of teaching for and about problem solving, there are relatively few examples of teaching mathematical content through problem solving. This paper uses data from three, apparently quite different, mathematics lessons from Australia and Japan to explore different ways in which mathematics can be taught successfully through problem solving and to analyse some of the characteristics of such lessons. It also attempts to identify some of the supports and constraints for adopting a problem solving approach to the teaching of mathematics that exist in the quite different contexts of Japan and Australia.

Introduction

Instead of starting with mathematical ideas, and then applying them, we should start with problems or tasks, and as a result of working on these problems the children would be left with a residue of mathematics … that mathematics is what you have left over after you have worked on problems. (Davis, 1992, p. 237)

Teachers, in the United States at least, have been urged to make problem solving the focus of school mathematics for at least 25 years (see, for example, National Council of Teachers of Mathematics, 1980). However, Smith (1996) and Simon (1997) argue that attempts to reform school mathematics have undermined teachers' sense of efficacy by condemning the traditional expository model of teaching without replacing it with a clear new alternative.

In terms of classroom practice, problem solving can mean many things including: teaching for problem solving; teaching about problem solving; and teaching through problem solving. The first usually refers to providing learners with the experience of solving non-routine problems. In a classroom entirely based on an expository approach, with few opportunities for students to engage in genuine problem solving in the sense of tackling problems for which they have not been supplied with an obvious way of finding a solution, just including opportunities for such experience would be a radical change in teaching.

The second aspect refers to attempts to teach problem solving strategies and skills explicitly. This is often referred to as concentrating on the process aspects of mathematics. These two aspects cannot be separated successfully as it is important to concentrate simultaneously on the experience of solving problems and the strategies and skills involved.

The third aspect refers to attempting to teach “standard mathematical content” through problem solving. This is by far the most difficult aspect, with relatively few examples to provide teachers with new models of practice (see Gravemeijer, McClain & Stephan, 1998, for one such example).

There is growing international interest — especially since the two recent video studies arising from the Third International Mathematics and Science Study (TIMSS) — in research aimed at providing an understanding of everyday classroom practice in countries with different cultures and patterns of teaching (see, for example, Stigler & Hiebert, 1999). While Australia has performed well overall in recent international studies of performance in mathematics, the Australian video-data from the TIMSS Video Study (Hollingsworth, Lokan, & McCrae, 2003) showed that, in marked contrast to the problems set for students in countries such as Japan, about three-quarters of Australian problems were low in procedural complexity, and repetitious. Moreover, while there has been considerable emphasis and improvement in classroom environment in Australian schools, there has been a lot less emphasis on approaches to teaching and learning that stress higher-order thinking and overall intellectual demand and expectations (Luke et al., 2003).

Nevertheless, there are examples of Australian lessons that do involve students in learning mathematics through genuine problem solving and lead to higher-order thinking. This paper uses data from three, apparently quite different, mathematics lessons from Australia and Japan to explore different ways in which mathematics can be taught through problem solving and to analyse some of the characteristics of such lessons.
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This lesson occurred during the last week of the school year. The teacher, who we will refer to as Helen, had, at the beginning of the year, inspired what became a small-scale study of innovative teaching practices in mathematics in primary schools (see Groves, Cheeseman & Beeby, 1990) in which approximately 20 teachers were interviewed about their practice and observed teaching one lesson. According to Helen, she was adopting Cambourne’s model of learning (see, for example, Cambourne, 1988) with its “seven conditions of learning”1, not only in her literacy teaching, where it was usually applied, but in all subject areas. As this was a rather unusual claim, I asked to interview her so that she could explain how it applied to mathematics — something she did very convincingly. While all of the other teachers in what then became our study were observed teaching, we had not visited Helen’s classroom. When we asked to do so at the end of the year, we were told that there would be no more mathematics lessons to observe, but we could come if we wished to watch her class packing up the classroom materials during the last week of school — a frantic time in Australian schools, being just before Christmas as well as the end of the school year.

The lesson was conducted with approximately 30 five- and six-year old children. Children were asked to work with their friends in small groups to pack up one of a number of different types of materials — for example, the Unifix blocks (interlocking plastic cubes with side length approximately 2 cm, normally used for counting, place value, and simple addition and subtraction tasks), or the pencils. Within the context of the “problem” of packing up the materials, children carried out many mathematical tasks that would often be posed in Australian schools without a real context — for example, counting collections of objects by grouping them in tens and units. They were also given instructions that lead to high level, relatively autonomous mathematical activity. For example, the children packing the pencils were told “we don’t want to keep the really small ones; you will have to decide how long they must be for us to keep and then find a way of deciding which ones to pack up and which ones to throw away”.

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While all of the teachers who took part in our study had been recommended as exemplary teachers whose mathematics teaching was innovative, and several teachers had identified problem solving as a focus of their teaching, Helen was one of only four of the twenty teachers who we regarded as engaging their students in genuine problem solving and higher-order thinking, placing high expectations on their students and providing conceptually complex tasks with high intellectual demand. A characteristic of these four teachers was the depth of their understanding of the mathematics curriculum at not only their own year level but across the primary years, as well as their ability to listen to and respond to children’s explanations in an affirming but challenging way.

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The lesson commenced with the teacher presenting children with the problem “How many desks are there in this room?” and asking them to solve the problem in as many ways as they could. Desks in the class were arranged as shown in Figure 1. To help them with their solutions, the teacher handed out A5 sized pieces of paper showing the arrangement of desks as in Figure 1. From the their question as to whether they should count the desk of the child who was absent, it was clear that the children knew that there were 40 desks in the room.

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- immersion — learners need to be immersed in texts of all kinds;
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- approximation — learners must be free to approximate (“mistakes” are essential for learning); and
- response — learners must receive relevant, timely feedback from more knowledgeable others.
Figure 1. Arrangement of desks in the class

Children spent about 10 minutes finding different solutions, with one boy finding eight or nine solutions. Keiko-sensei then called the class to attention and asked children to explain their solution methods to the class. Children were eager to share their solutions.

As each child explained their solution, Keiko-sensei wrote it on the blackboard, with the position on the board depending on the type of solution. So, for example, a solution where the child had circled each block of five desks and said “5 plus 5 plus 5 plus 5 plus 5 plus 5 plus 5 plus 5 plus 5 plus 5 plus 5 plus 5 plus 5 plus 5 plus 5 plus 5 plus 5 plus 5 plus 5 = 40” was written on the central part of the board as 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40.

She also counted the number of fives aloud and wrote next to it, “eight fives”. Each child’s solution was “labelled” with the child’s name, using, as is common in Japan, small magnetic cards on which the children’s names were “stored” on the board.

All solutions that involved a single type of grouping were written in this central “column” of the board, while solutions such as “15 plus 15 plus 10” were written on the left side as 15 + 15 + 10 = 40.

As well as “eight fives”, the following solutions were written in the central column: 20 + 20 = 40
10 + 10 + 10 + 10 = 40 and, amid much excitement,
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Children were asked to raise their hands if they had found each solution. The teacher counted the number of children aloud and wrote this number under the name card on the board. About half the class had found each of these solutions.

A child came to the board and used one of the two large copies of Figure 1 displayed on butcher’s paper on the board to show how he had found the solution of “twenty twos”. Several different solutions were recorded on the left of the board—for example, 15 + 20 + 5 = 40 25 + 15 = 40. The teacher then asked the children what the difference was between the solutions in the central column and those to the left of the board. After a long wait, she wrote above the solutions in the central column that they used “the same number”.

Different children then came to the front and drew how they had found the solutions to “eight fives”, “four tens” and “two twenties”. Amid shrieks of excitement from the children, Keiko-sensei then put another solution on the board:
1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 20.

Starting at the bottom of the board, she then wrote the symbolic form for each of the multiplications in the central column—namely, 1 x 40 = 40, 2 x 20 = 40, 10 x 4 = 40, 20 x 2 = 40, and 5 x 8 = 40.

The lesson ended with children pasting their solutions into their exercise books.

An Australian Year 7 Lesson on the Area of a Triangle

This double lesson, was taught by Gaye Williams, the author of a book containing a detailed theoretical and practical approach to learning through problem solving (Williams, 1996). The class comprised approximately 24 Year 7 girls in Australia. It was videotaped as part of a suite of five “exemplary problem solving lessons” for

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teaching purposes at Deakin University. The lesson was part of a sequence of lessons on the topic of the area of a triangle.

Girls worked in groups of four, trying to find a rule for determining the area of a triangle. One group already knew the rule and was trying to find a rule for the area of a trapezium. The teacher introduced the problem by saying: You can draw as many triangles as you like …. What you want to do is to try and find the amount of space inside them; see if you can find any patterns; think about whether those patterns always happen; and try some more if you think you need to try more, until you think you know how to tell someone how to find the amount of space inside a triangle. I mean there might not even be a rule — except these people [the group working on the area of a trapezium] think there is.

The girls were given 10 minutes to make as much progress as they could, before one person from each group was asked to report on what their group was thinking about. Initially, some groups struggled with the difference between area and perimeter and tried to use irrelevant information such as the angle sum of a triangle.

G1: I think we measure the angles. It will give us the inside if we add them all up.
G2: It's always meant to add up to 360 ….
G1: I'm not sure whether it would give us the outside or the inside.
T: What are you trying to find out? … What is area? … I'm not going to answer.

As the groups worked, the teacher moved around the room, asking questions and observing students working, very much in the manner of the Japanese kikan-shido “between desk walking” or “purposeful scanning” (see, for example, Kepner, p. 7), using this as an opportunity for selecting the order of reporting. Gaye also sometimes suggested specific aspects she wanted the group to report. While each group could choose who would report, there was an understanding that each member would report at some stage of the investigation. During the initial reporting, the teacher reminded the girls that they were not allowed to contradict but only to ask for further explanations.

After some considerable time, at least one group came up with the standard rule for the area of a triangle of “base times height divided by two”. Gaye commented on this group’s report: We have a couple of interesting things here. I had a question to ask, but I didn’t need to ask it. I was going to ask “Can they really say they have a pattern when they have only worked with one triangle?” And then Kathryn went on and said they’d worked with heaps of triangles! That’s OK. It looks like they really have a pattern. But I hope they looked at some really unusual triangles to make sure it seemed to be happening all the time. But then I loved Sarah’s question because when you have found a pattern that’s the beginning not the end — that’s when you have to think “well if it really is so, why is it so?”

Before discussing these lessons further, it should be made clear that neither of the Australian teachers is “typical”, and, while we are often lead to believe that all Japanese lessons are similar to the one discussed here, it is more likely to be the case that lessons that introduce a topic are more strongly problem focussed than later lessons in a sequence (Ito-Hino, 1995).

**Characteristics of the Three Problem Solving Lessons**

[Students’] understanding increases significantly with their discovery of concepts they have built out of their own prior mathematical knowledge. (Williams, 1996, p. 2)

In each of the three lessons described here, the teachers took what is usually regarded, at least in Australia, as standard mathematics to be taught by either exposition or what Simon (2003) refers to as empirical activity, and transformed it into challenging and problematic, yet accessible content.

In Helen’s case, her deep understanding of the mathematics curriculum and her commitment to providing children with conceptually challenging tasks enabled her to engage young children in learning mathematics through problem solving. According to Pehkonen (1999), instead of asking what is the role of the teacher, we should ask what is the teacher’s task, and answers that it is to reconcile children’s interests, spontaneous concepts, and engagement with everyday experiences with a powerful conceptually-driven mathematical agenda.

As stated earlier, the Japanese lesson was the first of a series of lessons on multiplication. Ito-Hino (1995) describes Japanese teaching as involving students in three stages: “using informal approaches and [building] on previously learned concepts and procedures when attempting to solve problems, … [acquiring] insight into
mathematical ideas based on their own interpretations, and … [seeking] algorithmic procedures” (p. 233). While it is not uncommon for a whole class period to be devoted to a single problem, the sequence of instructional tasks span more than one lesson and make different uses of the different stages. So, for example, “teachers always enjoy seeing how students attempt to solve the very first problem in a chapter … using only their previous knowledge” (as was the case in the Year 2 class) but it is difficult for “students to deal with an idea or concept after they were informed about algorithmic procedures” (Ito-Hino, 1995, p. 235; 244).

The importance of the development of conceptually focussed, robust tasks to support the development of sophisticated mathematical thinking should not be underestimated (Groves & Doig, 2002). In Japan, this is supported through the use of Lesson Study, which aims to research the feasibility or effectiveness of a lesson (see, for example, Kepner, 2002). A common framework for lesson planning in Japan adopts the following steps: Posing a problem, Students’ problem solving; Whole class discussion; and Summing up. Each of these is accompanied by entries under the column headings of Main learning activities; Anticipated student responses; and Remarks on teaching (Shimizu, 2002). This common lesson pattern, based on students’ actual and anticipated solutions of a single problem, together with an in-depth analysis of these solutions, promotes the problematising of the mathematics curriculum.

In the case of the lesson on the area of a triangle, possibly the most outstanding feature from the point of view of Australian lessons in general, was the way in which content that is almost always regarded as requiring only exposition or brief demonstration became the focus for an extended problem solving investigation. However, it was also evident that a great deal of effort had been made by the teacher to establish an environment where risk-taking was both supported and simultaneously minimized — for example, as stated earlier, the teacher reminded the girls during a report they were not allowed to contradict but only to ask for further explanations. This was one of many “rules” that formed part of explicit social norms operating in her classroom (see Williams, 1996, for further details). While in Japan, teachers frequently make a point of using students’ incorrect solutions as a stepping stone to the class developing their understanding, in Australia a great deal of successful effort has gone into establishing safe classroom environments, but there is much less emphasis on establishing a common intellectual purpose, promoting higher-order thinking and having high expectations of students’ mathematical explanations.

**Conclusion**

Stigler and Hiebert (1999) argue that because teaching is a cultural activity, change needs to be continual, gradual and incremental. While international comparative research is problematic, the purpose of studying international classroom practices is not to mimic them, but rather to support our reflection on our own practice. A critical step in articulating new models of mathematics teaching is to identify characteristics of successful practice. This necessarily brief paper attempts to identify some of these characteristics, as well as some of the supports and constraints that exist in the quite different contexts of Japan and Australia.

**References**


Problematising The Mathematics Curriculum — Some Examples From Australia And Japan

Susie Groves
Deakin University — Burwood

Abstract: Problem solving is often seen as being the core of mathematics. While there are many examples of teaching for and about problem solving, there are relatively few examples of teaching mathematical content through problem solving. This paper uses data from three, apparently quite different, mathematics lessons from Australia and Japan to explore different ways in which mathematics can be taught successfully through problem solving and to analyse some of the characteristics of such lessons. It also attempts to identify some of the supports and constraints for adopting a problem solving approach to the teaching of mathematics that exist in the quite different contexts of Japan and Australia.

Introduction

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The second aspect refers to attempts to teach problem solving strategies and skills explicitly. This is often referred to as concentrating on the process aspects of mathematics. These two aspects cannot be separated successfully as it is important to concentrate simultaneously on the experience of solving problems and the strategies and skills involved.

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There is growing international interest — especially since the two recent video studies arising from the Third International Mathematics and Science Study (TIMSS) — in research aimed at providing an understanding of everyday classroom practice in countries with different cultures and patterns of teaching (see, for example, Stigler & Hiebert, 1999). While Australia has performed well overall in recent international studies of performance in mathematics, the Australian video-data from the TIMSS Video Study (Hollingsworth, Lokan, & McCrae, 2003) showed that, in marked contrast to the problems set for students in countries such as Japan, about three-quarters of Australian problems were low in procedural complexity, and repetitious. Moreover, while there has been considerable emphasis and improvement in classroom environment in Australian schools, there has been a lot less emphasis on approaches to teaching and learning that stress higher-order thinking and overall intellectual demand and expectations (Luke et al., 2003).

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The lesson commenced with the teacher presenting children with the problem “How many desks are there in this room?” and asking them to solve the problem in as many ways as they could. Desks in the class were arranged as shown in Figure 1. To help them with their solutions, the teacher handed out A5 sized pieces of paper showing the arrangement of desks as in Figure 1. From the their question as to whether they should count the desk of the child who was absent, it was clear that the children knew that there were 40 desks in the room.

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She also counted the number of fives aloud and wrote next to it, “eight fives”\(^2\). Each child’s solution was “labelled” with the child’s name, using, as is common in Japan, small magnetic cards on which the children’s names were “stored” on the board.

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Starting at the bottom of the board, she then wrote the symbolic form for each of the multiplications in the central column— namely, \[ 1 \times 40 = 40, \]  \[ 2 \times 20 = 40, \]  \[ 10 \times 4 = 40, \]  \[ 20 \times 2 = 40, \]  and \[ 5 \times 8 = 40 \]

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Girls worked in groups of four, trying to find a rule for determining the area of a triangle. One group already knew the rule and was trying to find a rule for the area of a trapezium. The teacher introduced the problem by saying:

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mathematical ideas based on their own interpretations, and … [seeking] algorithmic procedures” (p. 233). While it
is not uncommon for a whole class period to be devoted to a single problem, the sequence of instructional tasks
span more than one lesson and make different uses of the different stages. So, for example, “teachers always enjoy
seeing how students attempt to solve the very first problem in a chapter … using only their previous knowledge”
as was the case in the Year 2 class but it is difficult for “students to deal with an idea or concept after they were
informed about algorithmic procedures” (Ito-Hino, 1995, p. 235; 244).

The importance of the development of conceptually focussed, robust tasks to support the development of
sophisticated mathematical thinking should not be underestimated (Groves & Doig, 2002). In Japan, this is
supported through the use of Lesson Study, which aims to research the feasibility or effectiveness of a lesson (see,
for example, Kepner, 2002). A common framework for lesson planning in Japan adopts the following steps:
Posing a problem, Students’ problem solving; Whole class discussion; and Summing up. Each of these is
accompanied by entries under the column headings of Main learning activities; Anticipated student responses; and
Remarks on teaching (Shimizu, 2002). This common lesson pattern, based on students’ actual and anticipated
solutions of a single problem, together with an in-depth analysis of these solutions, promotes the problematising
of the mathematics curriculum.

In the case of the lesson on the area of a triangle, possibly the most outstanding feature from the point of view of
Australian lessons in general, was the way in which content that is almost always regarded as requiring only
exposition or brief demonstration became the focus for an extended problem solving investigation. However, it
was also evident that a great deal of effort had been made by the teacher to establish an environment where risk-
taking was both supported and simultaneously minimized — for example, as stated earlier, the teacher reminded
the girls during a report they were not allowed to contradict but only to ask for further explanations. This was one
of many “rules” that formed part of explicit social norms operating in her classroom (see Williams, 1996, for
further details). While in Japan, teachers frequently make a point of using students’ incorrect solutions as a
stepping stone to the class developing their understanding, in Australia a great deal of successful effort has gone
into establishing safe classroom environments, but there is much less emphasis on establishing a common
intellectual purpose, promoting higher-order thinking and having high expectations of students’ mathematical
explanations.

**Conclusion**

Stigler and Hiebert (1999) argue that because teaching is a cultural activity, change needs to be continual, gradual
and incremental. While international comparative research is problematic, the purpose of studying international
classroom practices is not to mimic them, but rather to support our reflection on our own practice. A critical step
in articulating new models of mathematics teaching is to identify characteristics of successful practice. This
necessarily brief paper attempts to identify some of these characteristics, as well as some of the supports and
constraints that exist in the quite different contexts of Japan and Australia.

**References**


