# Comments on 'Information measure for performance of image fusion'

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The unsuitability of using classic mutual information measure as a performance measure for image fusion is discussed. Analytical proof that classic mutual information cannot be considered a measure for image fusion performance is provided.

*Introduction:* Image fusion aims to merge two or more images to produce a new image that is better than the original ones. An image fusion system takes as an input two or more source images and produces one fused image as an output. Image fusion performance measures depend mainly on estimating the amount of information transferred from both source images into the resulting fused image. In the above-named Letter Qu *et al.* proposed using mutual information measure as a performance measure for image fusion [1]. This Letter notes the problem with using mutual information measure for image fusion performance assessment.

*Mutual information:* Mutual information measures the distance between joint statistical distributions for two random variables $X$ and $Y$ from the case if they are totally independent. It uses cross entropy between the joint distribution $p_{XY}$ and the best case distribution of being totally independent random variables as follows:

$$I(X,Y) = \sum_{x,y} p_{XY}(x,y) \log_2 \frac{p_{XY}(x,y)}{p_X(x)p_Y(y)} \tag{1}$$

where $p_{XY}$ is the joint probability for $X$ and $Y$, $p_X$ is the probability distribution of $X$, $p_Y$ is the probability distribution of $Y$, and $x$, $y$ are sampling variables. This metric works fine for estimating dependability between random variables, since it is symmetric and reaches zero if $X$ and $Y$ are totally independent where $p_{XY}(x, y) = p_X(x)\, p_Y(y)$ which leads to:

$$I(X,Y) = \sum_{x,y} p_{XY}(x,y) \log_2 \frac{p_X(x)p_Y(y)}{p_X(x)p_Y(y)} = 0 \tag{2}$$

*Problem with image fusion:* In [1] Qu *et al.* used mutual information measure to estimate the joint information between source images $x$, $y$ and the fused image $f$ as follows:

$$M_F^{XY} = I(F,X) + I(F,Y) \tag{3}$$

where $X$, $Y$ and $F$ are the normalised histograms of source images $x$, $y$ and the fused image $f$, respectively. Applying mutual information measure to image fusion raises problems regarding the boundness of the metric since:

$$I(X,X) = H(X) \tag{4}$$

where $H(X)$ is the entropy of the random variable $X$ since $p_{XX} = p_X$. This means that $I(F,X)$ and $I(F,Y)$ are not measured at the same scale. Therefore, $M_F^{XY}$ mixes two unnormalised quantities and becomes biased towards the source image with the highest entropy which is not always the right decision with all fusion algorithms.

*Proposed solution:* This problem has motivated statisticians to develop other variations of the mutual information to be a proper metric as presented in [2–5]. In [5] Horibe proposed a normalised mutual information measure (NMI) as follows:

$$NMI(X,Y) = \frac{I(X,Y)}{\max\{H(X),H(Y)\}} \tag{5}$$

where $H(X)$, $H(Y)$ are the entropies of $X$ and $Y$, respectively. In [4] Kvalseth generalised Horibe's normalised information measure to:

$$NMI(X,Y) = \frac{I(X,Y)}{D} \tag{6}$$

and the candidates for $D$ are:

$$D_1 = \min\{H(X)H(Y)\} \tag{7}$$

$$D_2 = \frac{H(X)+H(Y)}{2} \tag{8}$$

$$D_3 = \max\{H(X),H(Y)\} \tag{9}$$

where the denominator $D$ is some appropriate norming quantity. He also criticised Horibe's selection of $D_1$ in [5] and proved that $D_2$ is the proper normalising factor to use. In [2], Strehl and Ghosh chose to normalise by geometric mean of $H(X)$ and $H(Y)$ to comply with the constraints of Hilbert space as follows:

$$D_4 = \sqrt{H(X)H(Y)} \tag{10}$$

Equation (3) actually mixes two joint entropies measured at different scales. This causes instability of the measure and makes it biased towards the source image with the highest entropy while the other image might also have added valuable information into the fused image. In order to maintain the same scale for mutual information between the fused image and both source images, both $I(F,X)$ and $I(F,Y)$ should be normalised as Horibe proposed in [5]. We suggest choosing $D_2$ to normalise with since $D_1$ and $D_3$ are proven to be unfair by Kvalseth in [4] and $D_4$ causes the measure to diverge as the contrast of one or more source images decreases near zero. Therefore, (3) should be changed to:

$$M_F^{XY} = 2\left[\frac{I(F,X)}{H(F)+H(X)} + \frac{I(F,Y)}{H(F)+H(Y)}\right] \tag{11}$$

where $H(X)$, $H(Y)$ and $H(F)$ are the entropies of $X$, $Y$ and $F$, respectively.

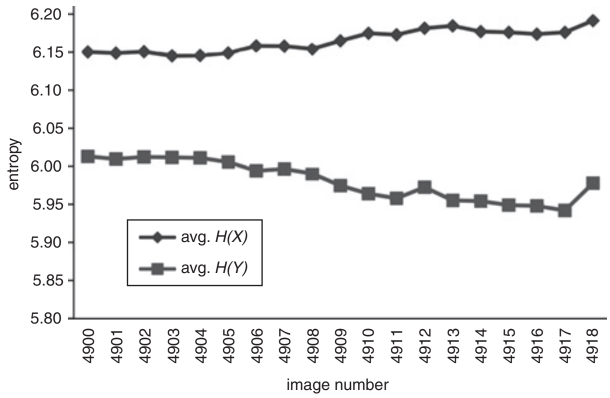


**Fig. 1** *Entropy of visual and IR images*

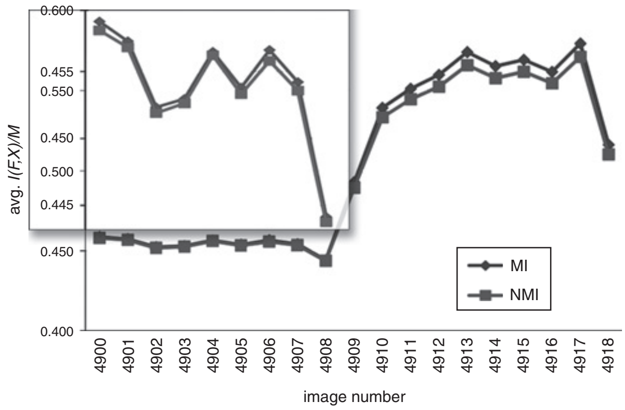


**Fig. 2** *Ratio between $I(F,X)$ of visual and fused images $M_F^{XY}$*
Graph is magnified and overlayed for images 4900–4908

*Comparison and discussion:* This Section describes an experiment that runs averaging, principle component analysis (PCA), max intensity, wavelet and Laplacian pyramid image fusion algorithms on the 'Trees' image sequence prepared by TNO Human Factors in [6] and [7]. The experiment shows that classic mutual information (MI) is biased towards the source image with the highest entropy. The experiment runs five image fusion algorithms on 38 registered source images featuring visual and infrared information (19 images each). Fig. 1 shows that in all cases a visual image has a higher entropy value than its corresponding infrared image. Fig. 2 compares the ratio of $I(F,X)/M_F^{XY}$ of the classic mutual information measure and its normalised version. The results show that MI records significantly higher differences as $H(X) - H(Y)$ increases.

*Conclusion:* This Letter comments on the work of Qu *et al.* in [1]. The discussion presented in this Letter concluded that mutual information measure used in (3) should be normalised to give the correct estimation of transferred information from source images into the fused image. This Letter also suggested using Kvalseth's normalisation in [4] and modified (3) to (11). Results show how the error between using classic mutual information and the normalised version increases as the difference between entropies of source images increase.


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