This is the published version:


Available from Deakin Research Online:

http://hdl.handle.net/10536/DRO/DU:30018351

Reproduced with the kind permission of the copyright owner.

Copyright : 2008, International Group for the Psychology of Mathematics Education
DESIGNING OPEN-ENDED PROBLEMS TO CHALLENGE PERSERVICE TEACHERS’ VIEWS ON MATHEMATICS AND PEDAGOGY

Leicha A. Bragg  
Deakin University

Cynthia Nicol  
University of British Columbia

This study examines preservice elementary teachers’ reported experiences of posing open-ended mathematics problems. Responses of 33 students in a mathematics teacher education course were analysed for the strategies participants used, what they learned and the challenges encountered from an opportunity to collect digital images and pose open-ended problems related to those images. Results indicate that preservice teachers reported a shift in the ways they viewed mathematics and how it might be taught. The school curriculum both constrained and provided possibilities for preservice teachers in noticing mathematics beyond the textbook and mathematics classroom. This study adds to our understanding of teaching as a learning practice and the art of posing mathematical problems as a significant aspect of that practice.

INTRODUCTION

Selecting, adapting and/or extending mathematics problems are a significant pedagogical practice for teachers. Mathematical “tasks convey messages about what mathematics is and what doing mathematics entails” (National Council of Mathematics, 1991, p. 24). They can provide a context for student learning about mathematical concepts and skills as well as mathematical inquiry. Tasks can also help students frame ideas about what it means to do mathematics. As Schoenfeld (1989) argues, students develop beliefs about the discipline of mathematics from their experiences with classroom mathematics activities. What counts as a good mathematics task has varied interpretations. Henningsen and Stein (1997) refer to worthwhile tasks as high-level tasks having the potential for high cognitive demand by students. Sullivan and Lilburn (2002) define good questions for mathematics teaching as having three features: 1) requiring more than recalling a fact or skill; 2) educative for both students and teachers; and 3) having possibly several acceptable answers. Whereas Gutstein (2006) argues that good tasks include those that are culturally relevant, that is, those that are related to students’ lives, offer the possibility of teaching for social justice, and “rely more on students’ own meaning making rather than with outside sources like the teacher or answer sheet” (p. 103). These definitions share an openness that offers students opportunities to explore mathematics in meaningful ways. They recognize that what makes a task ‘good’ does not necessarily reside in the task itself but rather in the relationship between the task and the student (or the teacher).

Learning to develop, adapt, select and pose good tasks is neither simple nor trivial. Teachers who have had few opportunities to experience posing their own
mathematics problems or even asking “what if?” questions when they themselves were students may find it challenging to now select and pose more open-ended mathematics problems as teachers. Moreover this challenge may be amplified for those elementary teachers who come to teaching without a strong background in mathematics. How do teachers learn the practice of selecting and posing good mathematics tasks?

The field of mathematics education does not currently have a well-developed knowledge base on particular ways in which teachers learn to pose non-routine or open-ended mathematics tasks. We do know that it is extremely difficult for teachers to maintain with students the high cognitive demand of potentially high-level tasks (Henningsen & Stein, 1997). We also know that changes in problem posing strategies are possible and that preservice teachers can move from posing traditional single step problems to more open-ended cognitively complex problems (Crespo, 2003; Sinclair & Crespo, 2006). One factor that seems to support this change is opportunities for teachers and preservice teachers to explore new kinds of problems in varied contexts.

Our study adds to this research and examines preservice teachers’ experiences with opportunities to pose new kinds of problems: those that are open-ended and grounded in images of real-life activities. In this study we explored elementary preservice teachers’ perspectives on posing open-ended tasks inspired by a set of digital images that preservice teachers collected for the specific purpose of investigating mathematics with students. We offer an example of the kinds of images preservice teachers collected and the kinds of related problems they posed (see Nicol & Bragg, forthcoming for a more detailed analysis of these posed problems). We focus in this paper more specifically on preservice teachers’ strategies for developing problems in the context of a mathematics teacher education course, what they report they learned, and the challenges encountered from an opportunity to collect images and pose open-ended problems related to those images.

**THEORETICAL CONSIDERATIONS**

For many preservice teachers’ their prior experiences with mathematics has situated their knowledge and beliefs of mathematics as procedural, rule-bound, and closed. Supporting beginning teachers as they examine their underlying knowledge, beliefs and practices about teaching and learning mathematics is challenging. Lampert (2001) and others propose that teaching and learning can be understood as learning practices. A focus on teaching as learning practices draws attention to the activities teachers attend to in the activity or practice of teaching mathematics. Crespo (2003) and Nicol (1999) suggest that learning to pose mathematical problems, listen to and interpret student responses, and respond to students are central learning practices for teaching. Crespo (2003) describes a context in which preservice teachers developed their problem-posing practices through penpal letter writing activities where preservice teachers and Grade 4 students exchanged mathematical problems. Preservice teachers in Crespo’s study were not provided with explicit direction on
defining or creating open-ended problems, leaving us to wonder how preservice teachers might respond with more explicit instruction.

Situated theories of learning and cognitive apprenticeships suggest that what is learned is intricately tied to the context in which it is learned (Lave, 1996; Lave & Wenger, 1991). From this perspective knowledge is inseparable from the activity, context, and culture in which it is developed and used. Heckman and Weissglass (1994) contend that “a key and vital factor in acquiring knowledge through cognitive apprenticeships is situating the learning experience in an environment that is real to the student” (p. 30). This requires learning to pose mathematically and pedagogically interesting problems that connect to students’ lives. Where do preservice teachers see mathematics? How do they see or notice mathematics in their own or their students’ lives? Inspired by Richard Philip’s Problem Pictures CD-ROM (http://www.problempictures.co.uk/index.htm) that offers hundreds of digital photos as a source of mathematics problems together with Sullivan and Lilburn’s (2002) description and examples of open-ended mathematics problems our study examines the experiences of preservice teachers who developed open-ended mathematics problems around their personal collection of digital photographs. How did preservice teachers approach this task, what did they learn and find challenging, and to what extent does this problem-posing task offer insight into learning practices of teaching?

**CONTEXT AND DATA COLLECTION METHODS**

The Problem Pictures task was posed to elementary preservice teachers as a course assignment in a 13-week mathematics teacher education course taught by Author A. The task involved preservice teachers in collecting their own photos with digital cameras, selecting four photos from their collection, analysing the photos, and then posing 3 to 4 open-ended mathematics problems associated with each photo. Preservice teachers were encouraged to collect photo images that they thought would be engaging to students and would offer opportunities to explore interesting mathematics related to the elementary school curriculum. They collected images over a 2-week period and collated and submitted their pictures and problems in a PowerPoint file. A range of contexts were chosen by preservice teachers as places to pose problems. Figure 1 is representative of the kinds of photos and problems developed by participating preservice teachers.

![](image)

Problem: You are a giant spider and this is your web. If you catch one or two flies in every “hole” in your web, how many flies might you catch for supper? BOO!

Figure 1. Problem picture photo and question designed by preservice teacher.
Participants for the study were enrolled in a 3 hour per week, 13 week mathematics education course as part of a two-year post-baccalaureate teacher education program in a large Canadian university. Students enrolled in this course were also members of the Diversity cohort—a programme option for students entering the teacher education program with interests on issues of diversity, social justice, and equity. Thirty-three of the 40 students volunteered to participate in the study. Participants’ backgrounds included Asian-Canadian (14), First Nations (4), and Caucasian (15). Participants were in their first year and first term of the teacher education program.

Data collected included researcher field notes, a written response survey completed by students upon completion of the course and copies of students’ work in the form of the Problem Pictures assignment (as described above). For this paper we draw upon researcher field notes and students’ written survey responses. The survey was administered through SurveyMonkey (an online survey program) and was developed to learn more about preservice teachers’ experiences with the Problem Pictures task. It involved 15 open response questions asking students to share their thoughts about how they approached the assignment, what they learned and did not from it and what they found useful and challenging. Four questions were selected for analysis in this paper. These questions specifically examined participants’ approach to creating open-ended problems based on original photos, the challenges the preservice teachers faced in this assignment and the impact of this task on their future as an educator.

A qualitative computer program, Nvivo, was employed to collate and analyse the data gathered from the online survey and field notes. A preliminary phase of analysis consisted of reviewing the students’ responses and implementing a coding scheme. The responses were coded by the researchers independently according to the common themes that emerged and then cross-checked for commonality and consistency. The themes were categorised and reviewed again for emerging sub-themes. The data from the interviews are presented in a narrative form, and the interpretation presented in the discussion. These data are seen as broadly representative of the general views of the participating preservice teachers. Field note excerpts supplemented these data from the researcher’s perspective.

RESULTS

Preservice teachers’ reported varied responses in their strategies for approaching the Problem Pictures assignment; however most (85%) stated that they began by looking around them (indoors and outdoors) for mathematical contexts. These preservice teachers indicated that they began by seeking out mathematically-centred photos and developing their questions based on these images. Heather (all names used are pseudonyms) detailed this process:

Firstly, I took my digital camera and snapped photographs of what I thought could turn into a mathematical question. Capturing photographs proved to be harder than I thought,
because I did not just randomly snapped pictures. I would stare at a potential scene for a few good seconds, thinking if I could come up with at least two "good" diverse questions, and if I could not, I would just move on and started to walk elsewhere.

The formulation of open-ended questions is challenging for experienced teachers who do not have the added restriction of matching the questions to an original photo as outlined in the assignment criteria. It appears that the preservice teachers’ limited knowledge of the mathematical curriculum was an added barrier and made the task more difficult than anticipated.

Some preservice teachers chose to stage their photos based on personal interests, as illustrated by Ava’s comment:

I first approached this assignment by taking photos around UBC that I felt contained possibilities for good questions. However, I felt uninspired. I then took some photos around my neighbourhood (playground, streetscape, etc.) but still did not feel great about what I was coming up with. After turning ideas over in my head, I decided to set up a Scrabble board with math words. After that the questions wouldn't stop coming! I decided to make a list of things I really enjoyed doing: baking, basketball, and playing with my nephew inspired me for the remaining pictures. The questions came easily once I felt a connection and excitement with the photos. I used the IRPs [curriculum documents] as a guide and tried to cover a variety of the Prescribed Learning Outcomes with the questions.

Preservice teachers sought images that depicted school mathematics (particularly the topics of space and shape) more than images that were connected to their own lives, interests and passions. That preservice teachers did not readily see their own passions and interests as a resource to collect photos and develop problems indicates the disconnect many felt with mathematics and their personal lives. An informal analysis of their photos and problem contexts confirms this claim.

As some preservice teachers (48%) became more familiar with the provincial curriculum documents and the nature of the Problem Pictures assignment their approach to collecting photos changed. Rather than selecting and taking photos that inspired possible open-ended questions, these students began searching for photos that would match questions they had already posed. Sophia’s comment is indicative of this change:

At first, I just took pictures of things that I thought I could formulate questions around. But once I referred [sic] to the IRPs [curriculum] it seemed like that wouldn't necessarily work. Instead I ended up looking at the IRPs [curriculum] and then generating questions and ideas for potential photos. So in the end, I really thought of the questions first, and then went out and took the pictures that I had in mind for those questions.

As these preservice teachers began to focus on the mathematical needs of their students they created Problem Pictures based on the requirements of the curriculum documents. Thus for some collecting photos became an act of finding images to illustrate the problem rather than finding images that could ground or situate the problem.
Mathematical and Pedagogical Possibilities

Participants were asked to comment on what they learned through the Problem Pictures task. Most students (95%) reported that collecting images and exploring open-ended problems with the photos helped extend or challenge their previous views about the nature of mathematics and how it might be taught. Connie’s observation was representative of this realisation, “I learned that math is really all around me, and that it is useful to me in everyday life, not just in school for homework from textbooks and tests from teachers.” Preservice teachers further stated that creating open-ended tasks challenged their previously-held views about the nature of mathematics. This can be seen in Yvette’s comment:

It taught me that there is such thing as "doing math without ONE correct answer". Prior to taking this course, I held the WORST fears and anxieties toward math and teaching math, most likely due to my poor math performances in 11th and 12th grade. Looking back, it certainly would have been nice to have these types of questions to do back then to build confidence.

Although many preservice teachers in the course expressed some mathematics anxiety, it is interesting to note that even those who stated they had enjoyed mathematics and was successful with it as a school student reported their experience with the Problem Pictures task changed their ideas about mathematics. Isabel stated:

I love math, but I think it was because I could get the right answers most of the time because I was good at the repetitiveness of close-ended questions. However, these open-ended questions… I actually enjoyed them more… they provoked more enthusiasm and excitement in math.

For preservice teachers, such as Isabel, multiple possible solutions to questions provided a less apprehensive lens and a more exciting context through which to view and experience mathematics.

Creating open-ended questions, reported some preservice teachers (25%), gave them a sense of empowerment through being able to create mathematical problems beyond the textbook and classroom walls. This process provided them with a sense of ownership of their questions. Heather stated, “Because these questions are thought of by me, I would feel much more comfortable explaining and teaching the concepts of these questions because I made them up as compared to teaching through the textbooks.” On the one hand, this expression of autonomy is impressive for a beginning teacher. However, discarding textbook problems developed by experienced teachers and researchers that could be a source of possible problems is worrisome. It is important for preservice teachers to not only develop but also consider and critically evaluate well-constructed open-ended questions designed by experts in the field (see Sullivan & Lilburn, 2002).

DISCUSSION AND CONCLUSION

Our results indicate that preservice teachers designing open-ended questions based on photographs prompted a shift in their understanding of pedagogical approaches and
ways in which they viewed mathematics. However, at least one-third of the 
participants stated that developing open-ended questions was complex. One 
participant stated, “I debated with my classmates for hours trying to figure out 
whether or not my question was open-ended or not.” This statement supported in-
class observations of the preservice teachers’ struggle with the idea of open-
endedness. As Ilana wrote, “…it was hard to distinguish between a very good close-
ended question and an open-ended one.” It was observed that many students initially 
viewed an open-ended question as having multiple, complicated steps to achieving 
one correct answer.

A second challenge for preservice teachers involved their limited experience with the 
mandated school mathematics curriculum. Unfamiliarity with the curriculum made it 
difficult for preservice teachers to pose mathematically appropriate and suitable 
problems for particular grade levels. This unfamiliarity may explain some students’ 
reported struggle to find the mathematics within a particular photo scene. However, 
students’ understanding of mathematics also framed what they were able to attend to 
or notice that was mathematical within a photo. That some preservice teachers turned 
to the elementary school curriculum for a list of mathematical topics and concepts 
(e.g., number, patterns, relations, space and shape) that could be used to analyze a 
photograph speaks to the structure the curriculum offered them for noticing school 
mathematics outside the textbook. Thus the curriculum both constrained and provided 
possibilities for preservice teachers to explore and create open-ended problems.

Further analysis that includes an examination of preservice teachers’ collected 
problems and photos can help determine the extent to which the preservice teachers 
were able to develop mathematically interesting problems. Sinclair and Crespo 
(2006) found that when preservice teachers were intent on posing problems for their 
students they created mathematically less interesting problems than if they were to 
create problems for themselves. However, Crespo (2003) previously found that 
having an authentic audience, such as preservice teachers posing problems to young 
children, was one supporting factor that helped preservice teachers move from posing 
procedural computation-type problems to more open-ended problems. Our study 
results indicate that preservice teachers’ overwhelmingly reported the mathematical 
and pedagogical benefits of creating open-ended problems related to photos. As Del 
best stated, “…now I carry my digital camera around and have noticed more math in 
real life.” Nonetheless, we wonder if the kinds of problems preservice teachers create 
in this context are mathematically interesting (Sinclair & Crespo, 2006) and if 
preservice teachers consider themselves as the authentic audience or their future 
students. These questions are important if we are to further our understanding of 
teaching as a learning practice and the art of creating and posing mathematical 
problems as a significant aspect of that practice.

References


PME 32 and PME-NA XXX 2008 2 - 207


