The Construction of Knowledge: Theoretical Approaches

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The construction of knowledge by students continues to be a central concern of mathematics education, and research has shown the importance of theoretical approaches for our understanding of construction of knowledge. Tommy Dreyfus has not only made substantial contributions to this field but has forged links between researchers in this area, and encouraged new researchers. This symposium provides an introduction to Tommy's research for some MERGA members, a chance to reflect further on it for others, and opportunities for everyone to develop new insights by connecting ideas in these papers.

Tommy Dreyfus *introduces "Abstraction in Context".*

Building on ideas of the Freudenthal School and of Activity Theory, abstraction is defined as an activity of vertical reorganisation, achieved by means of actions on mental or material objects, through which previous constructs are combined, connected, structured and developed into novel ones. According to this definition, abstraction is not objective and universal but depends on the learning context and the social context in which it takes place. This view of abstraction is consonant with Davydov's view that abstraction proceeds from an undeveloped and often vague initial form to a consistent and elaborate final form, and with van Oers' criticism of decontextualisation as a basis of abstraction. The three epistemic actions of Recognizing, Building-with and Constructing form the basis of a model for describing and analysing processes of abstraction. The model has proved useful to describe processes of abstraction for a wide variety of mathematical contents in both, classroom and laboratory settings. Recent studies on partially correct constructs and on justification have exhibited the analytic nature of the model; these studies will be used to exemplify the approach in separate presentations.

Mike Thomas *discusses "Constructing Versatile Mathematical Conceptions".*

In this talk we will describe the idea of versatile mathematical thinking as comprising three aspects: process/object versatility—the ability to switch at will in any given representational system between a perception of particular mathematical entities that may be seen as a process or an object; visuo/analytic versatility—the ability to exploit the power of visual schemas by linking them to relevant logico/analytic schemas; and representational versatility—the ability to work seamlessly within and between representations, and to engage in procedural and conceptual interactions with representations. Examples, taken from algebra, calculus and linear algebra, will be presented to illustrate the role of these three aspects in building conceptions, and some advantages of versatile mathematical thinking. In addition, ways in which such versatility may be mediated through an informed use of technology, and the implications for teachers' pedagogical technology knowledge (PTK) will be discussed.

Jill Brown *examines "Constructing Knowledge through the Perceiving of Affordances of a Technology-Rich Teaching and Learning Environment (TRTLE)".*

This theoretical approach is intended to find evidence of how mathematical knowledge is constructed in a TRTLE. Gibson's affordances and Valsiner's zone theory are used as lenses on interactions in classrooms and outcomes of those interactions. The affordances of a TRTLE are the offerings of that environment, of which the people are a part, for facilitating or impeding teaching and learning. For learning to occur some of the existing affordances of the TRTLE must be perceived and acted on. What makes this perception possible is analysed using Valsiner's Zone of Free Movement (ZFM) and the Zone of Promoted Action (ZPA). These are ever changing in nature. The ZFM describes what learning is possible and is characterised in a TRTLE by what is currently available (technologies, affordances, allowable actions). The ZPA describes what learning is promoted or encouraged and is characterised by the particular set of activities, artefacts, actions being promoted by the teacher or students. The ZPA is non-binding in nature and hence students may choose to act outside the ZPA.

Gaye Williams *examines "Cognitive artefacts and abstracting: What can we assume?"*

Synthesis of the cognitive elements of Dreyfus, Hershkowitz, and Schwarz's Recognizing, Building-with and Constructing Model or RBC Model, and Krutetskii's 'mental activities' frames this exploration of a surprising finding: insights about the usefulness of the Cartesian Axes System as a tool to interrogate linear functions does not presuppose a connected understanding of variable. Dreyfus' theoretical perspective that "abstraction is not objective and universal but depends on the learning context and the social context in which it takes place" is illustrated in the case described. Data from lesson video, and a video stimulated student interview illuminated the idiosyncratic nature of the constructing process that led to insight even though cognitive artefacts associated with understanding variable were not possessed. This study informs teaching and research by raising questions about what can and cannot be assumed when students construct new knowledge. It makes a theoretical contribution to links between cognitive artefacts assembled and mathematical structures developed.