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Engaged to Learn Pedagogy: Theoretically Identified Optimism Building Situations

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Associations between resilience or optimism (Seligman, 1995) and the inclination to explore unfamiliar challenging problems in mathematics have been identified (Williams, 2005, 2008). This raised questions about how to build optimism to enhance mathematical performance. In this study, a theoretical framework was formulated to study optimism-building situations (Seligman, 1995; Csikszentmihalyi, 1992). By interrogating ‘Engaged to Learn’ pedagogy (Williams, 2000) through a video-stimulated interview study, situations theoretically expected to be optimism building were identified.

Introduction

Performances on ‘mathematical literacy’ for Australian students are generally not improving and some indications that the performances of girls are dropping was evidenced in the Organisation for Economic Co-operation and Development’s (OECD), Programme for International Student Assessment (PISA). Australia’s performance relative to other countries has decreased slightly. Ability to work with unfamiliar mathematical problems is a crucial part of ‘mathematical literacy’ (Thompson, & De Bortoli, 2006): the ability to use mathematics in out of school situations to make sense of the world. Importantly, PISA 2006 found:

[S]tudents who are confident in their own abilities and well motivated tend to do better at school. Positive approaches not only help to explain student performance but are also themselves important outcomes of education.” (Thompson, & De Bortoli, p. 15)

Martin and Marsh’s (2006) study of ‘academic resilience’ and its role in student motivation at school, and Williams’ (2005) findings linking ‘optimism’ with successful mathematical problem solving are consistent with the need to develop a positive approach to mathematics in our students. This study examines situations expected to develop a positive approach to learning or an optimistic orientation to successes and failures.

Background

Martin (2003b), Martin and Marsh (2006) and Williams (2005) identified similar constructs associated with student capacity to overcome adversities associated with learning. Martin’s ‘academic resilience’ is linked to student participation in school learning in general and my construct of optimism (derived from Seligman, 1995) is linked to student inclination to explore unfamiliar mathematical ideas. This construct is a subset of Seligman’s ‘explanatory style’ called ‘exploratory style’. I was interested in why some students were not inclined to explore new ideas, but rather remained within the confines of what they already knew. Seligman’s optimistic orientation, which relates to student orientation to successes and failures, possesses some of the constructs within ‘Attribution Theory’ (Weiner, 1974). It also includes a ‘pervasive-specific’ dimension, and conditions under which optimistic orientation can be increased have been identified (Seligman, 1995). An optimistic child perceives successes as ‘permanent’, ‘pervasive’, and ‘personal’ and failures as ‘temporary’, ‘specific’, and ‘external’. Indicators of these characteristics were
displayed in interviews of students who creatively solved problems to develop new mathematical ideas (Williams, 2003) through statements like: “it always takes me a long time to understand when we first start a new topic” [failure as temporary]; “I go over and over it until it makes sense” [success as personal]; “and then I get it!” [success as permanent]; “[I could not find the total angle because] I was facing the [angle] points up and out when they needed to face in” [failure as specific]; “last year I did not do as well in maths; the teacher took too big a leaps” [failure as external]; and “I am good at working things out for myself” [success as pervasive]. These dimensions fit factors Martin identified within academic resilience: a) ‘self-efficacy’ (equated to confidence) fits with the perception of success as pervasive; and b) ‘persistence’, illustrated by “If I can’t understand my schoolwork at first, I keep going over it until I understand it” (Martin, 2003a, p. 46) fits with failure as temporary and success as personal. Martin’s findings (in a large-scale quantitative study) link these constructs and higher school performance. This adds strength to the need to study optimism-building situations.

Theoretical Framework

Seligman (1995) identified success through ‘flow’ experiences (Csikszentmihalyi, 1992) as optimism building. Flow is a state of high positive affect during creative activity. Conditions for flow during mathematical problem solving include a spontaneous student-set challenge that requires the development of new (to the students) mathematical ideas to overcome it (Williams, 2002a). The Engaged to Learn Model (Williams, 2000, 2005) underpins a teaching approach intended to create flow opportunities for students working in groups. Study of students learning through this approach has shown it does elicit frequent creative activity (Williams, 2000, 2002a, 2007). ‘Collaboration’ occurs during flow situations when students work together on mathematical ideas that are just outside the present understandings of all group members. This approach presupposes teacher faith in students to challenge themselves if they have opportunity to do so. The strength of the Engaged to Learn Model lies in the accessibility of the tasks through a variety of mathematical pathways that can use mathematics of varying degrees of sophistication and can include student-developed representations. As the students control the difficulty of the mathematics they use to explore within a teacher-set focus, and the size of the challenge they try to overcome, two of the inhibiting factors to motivation identified by Martin and Marsh (2006) are likely to be eliminated or reduced in magnitude. Students need no longer feel anxious about the size of the challenge they face because they have set it themselves. They can select the difficulty of the mathematics they use so ‘failure avoidance’ associated with not having control of the learning situation should be diminished. The research questions for this study are: Are there situations within the implementation of Engaged to Learn pedagogy that are optimism building? Or, are there situations where students spontaneously set their own challenges associated with mathematical complexities they discover, and decide to work outside their present conceptual understandings to explore these complexities?

Research Design

To identify flow situations in a Grade 5/6 classroom, the Learners’ Perspective Study methodology (see Clarke, Keitel, & Shimizu, 2006) was adapted to capture group interactions on three cameras, and interim reporting by groups on a fourth camera. Student reconstruction of their thinking in class, and student indicators of optimism were captured.
through video-stimulated post-lesson interviews. Mixed image video with the student’s group at centre screen, and groups reporting as an insert in the corner was used to stimulate interview discussion. Post-lesson video-stimulated interviews were undertaken individually with four students after each lesson. Students were selected from at least two of the groups. Decisions were based on their positions in relation to the cameras and interactions that occurred. In the interviews, students controlled the video remote and discussed parts of the lesson they considered important. Flow situations were identified using student reports of when new learning occurred, and positive affect and/or intensity in class or the interview.

The researcher formed the groups informed by the classroom teacher’s knowledge of the students, and video of this class working in groups. The aim was to group students who could think at the same pace so they had opportunities to create new ideas together and thus sustain group flow. This same pace of thinking differs to same student performance on skills tests containing only routine problems. The researcher (RT) and teacher (T) team-taught with the RT as the primary implementer of the tasks.

The sequence of lessons for each task followed the same underlying structure associated with Engaged to Learn pedagogy. The same cycle of activities was repeated several times. Where full cycles not completed in a session, the cycle continued in the next session after five minutes for groups work to refresh student memories. The activities in each cycle were:

- Introduction (at start of task) and Refocusing Group Work at start of new cycle
- Group Work (10-15 mins)
- How to ‘prime’ reporters (30 seconds)
- Priming Reporters (1-2 minutes)
- Group Reporting (1-2 minutes per group)
- Opportunity for students to add comments, raise questions, summarise (5 minutes)

Crucial to this pedagogy was the RT and T not hinting or affirming, but asking questions to elicit further thinking. The design of each task fits with the table of tasks features employed to elicit complex thinking in Williams (2002b), and more recent findings about the role of dynamic visual images in supporting complex thinking (Williams, 2005).

The Fours Task (see Figure 1) was undertaken in October, during one eighty-minute session. It is presented first here because data from this task is used first in this paper.

| Make each of the whole numbers from one to twenty inclusive using: |
| | Four of the digit four and no other digits |
| | Any or all of the operations and symbols |
| + | + | - | x | / | ÷ | √ | ( ) | ^ 2 |
| Think about how to make all the sums as fast as possible. |

*Figure 1. The Fours Task*

This task required students to generate sums to produce each of the natural numbers from 1-20 using four of the digit 4 and a restricted number of various operation and symbols. See Williams (2008) for more information about how this task was implemented and student responses to it. The decimal point was included in the array of operations and
symbols because these Grade 5/6 students had not previously undertaken work on how to use whole numbers and multiplication with a number containing a decimal component to generate whole numbers. This was expected to be a complexity groups might identify.

The Volumes of Cuboids Task (see Figure 2) was undertaken over three eighty-minute sessions. Students were provided with small cubes and asked to find how many different boxes (rectangular prisms) could be formed that each contained 24 cubes.

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**Task**

**Part 1:** Make boxes with 24 of these cubes. How many can you make? How do you know that you have got them all? Can you make a mathematical argument for how you know you have got them all?

[Intention: elicit novel building-with and recognizing to support constructing]

**Part 2:** Late in Lesson 2, introduce a game for group competition. A ‘box’ with a volume of 36 little cubic blocks had been hidden in a big coloured container. Groups had 5 minutes to develop strategies. The aim is to be the first group to find the ‘box’ dimensions. Each group can ask a question that all class members could hear. RT and T will give Yes / No answers. Each group can state what they think the dimensions are when they are sufficiently sure. They cannot have a second turn at stating this until all groups have had a first turn. [Intention: to elicit consolidating and increased elegance to support constructing]

Note: Two terms were introduced at the start of the task.

- **Box:** was elaborated by the students identifying the features of a large cuboid prior to the task. The RT drew attention to both cubic and non-cubic examples during this discussion.
- **Volume:** was defined as the amount of 3D space taken up by the box and measured in cubic centimetre blocks for this task.

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Reduction in the number of cubes per group from one session to the next, and increase in the magnitudes of the volumes students considered was intended to shift students from counting to analysing the underlying structure of boxes (cuboids). The task provided many opportunities to discover complexities around the structural arrangement of the cubes making up the boxes, and why factors seemed to be important.

**Analysis and Results**

Many aspects of Engaged to Learn pedagogy were identified as contributing to what were theoretically optimism-building situations (or flow situations). Four of these aspects are described and illustrated in herein: a) complexities to discover within tasks; b) questions from ‘expert others’ (Vygotsky, 1978) to all groups during ‘priming’ time without a requirement that they be answered; c) limited concrete resources later in a task; and d) sharing of ideas generated by groups with the rest of the class at intervals.

**Complexities To Discover**

In the Fours Task, various groups responded in different ways when they encountered and thought about whether and how the decimal point could be used. One group immediately decided it was not possible to use it, and another group generated and thought about one sum before they ran out of time (see Williams, 2008). This demonstrates that building the opportunities for discovering mathematical complexities into a task does not
necessarily mean groups will recognise them as complex and decide to explore them. It also demonstrates that the initial exploration of an idea may occur before the nature of the complexity is discovered. Trying one example was not sufficient for the group to realise that different examples could be more productive. It is possible that a question from an expert other to that small group at the stage at which they had an answer to the sum they made might have assisted them to think about what was possible. Possibly a question like “It always give a decimal answer does it? Can you make an argument for that?” asked in an innocent way with the teacher walking away might have sustained exploration. This is a question for future exploration when this task is used in the future. An example of a discovered complexity that did result in a flow situation is described under c) limited concrete resources.

‘Questioning’ of Expert Other During Priming Time

Before each reporting session, groups had one to two minutes to ‘prime their reporter’. This involved the group deciding what they wanted to say, the reporter practicing their report in front of the group, and the group refining this report so it matched what they had intended. The questions the RT asked that were audible to the whole class before and during this priming stage led sometimes to students spontaneously ‘stepping beyond’ what they had known to generalise their findings. A group of three (Eliza, Gina, and Patrick) were priming Patrick to report. An illustration of a general comment made by the RT during priming time is included along with subsequent comments from the group under study:

RT [You might report on another number you have found or] you might be thinking big picture and trying to work out some neat ways to get there fast

Although this comment was not structured as a question, it had the function of posing a question, and the group began to think about ‘big ideas’ associated with their findings. Group members had already worked out that including -4 +4 in a sum made that part of the sum zero. The RT’s ‘question’ led to this group spontaneously asking themselves when this could be useful, and how they could explain what they knew. Following is part of the lesson transcript during the group activity of priming the reporter:

[Key to transcript: - pause for change of idea, … pause]

1. Patrick We could say that if you're going to use- I don't know- if you do minus four plus four cause they cancel each other out and you can get a low number or something I don't know

2. Eliza Sometimes part of the sum doesn't count because things cancel each other out

3. Patrick I'm not primed for it

4. Gina Yes you are

5. Patrick Sometimes some sums … um they cancel each other out because

6. Eliza Yes so a part of the sum doesn't weigh- sometimes some sums they cancel each other out because so a part of the sum doesn't weigh

Patrick focused the group on -4 +4 and how this could be useful [Line 1: ‘you can get a low number’]. The tentative nature of his comments indicated these were new ideas. Eliza focused on what this combination of numbers and operations did to the sum [Line 2]. The different tries at rewording by different group members indicated they were working
out what they knew and how to communicate it [Line 1: ‘cancel each other out’, Line 2: ‘doesn’t count’, Line 6: ‘doesn’t weigh’]. As they interacted, the group tried to articulate why they could make this claim [Line 2, 5, 6: because]. As a result of the RT suggesting groups might think ‘big picture’ this group spontaneously focused beyond their present findings about \(-4 + 4\), on how their finding could be useful for generating sums fast. In doing so, they maneuvered a theoretically optimism-building situation (flow situation) by spontaneously setting their own challenge (the usefulness of \(-4+4\)) and developing new mathematical understandings as they overcame it.

**Limited Concrete Resources**

Late in the Volumes of Cuboids Task, one group was investigating what boxes could be made that contained 32 cubes, with only 24 cubes on their table (see Williams, 2007). The group made a box with six layers of four cubes, and then, because they did not have any more cubes, they represented the last two layers of four cubes as flat shapes (two squares each subdivided into four squares) on a sheet of paper. They then counted in ones to make sure they had 32 cubes. It was during their development and counting of this new representation that this group became aware that the cubes were in layers and that they could count by fours. Eliza explained this novel representation in her interview:

We had [sketching] six [lots of 4] stacked up like that … then we had … a drawing on a piece of paper … we needed that to pretend there was another bit of eight”

The group participated intensely as they recognised they did not have enough cubes, and thought about the representation Gina had drawn to help overcome this. They were working outside their present understanding because they did not have sufficient cubes to construct their box of 32 cubes. New understandings developed as a result of the creative representations they developed. Flow conditions existed which theoretically signals an optimism-building situation.

**Sharing Ideas With Class**

Group reports to the class as a whole were found useful to some students as a way of identifying additional mathematical complexities that they wanted to explore. For example, Patrick frequently displayed a willingness to explore unfamiliar ideas from the reports of other groups and volunteered this as one of the ways he learnt during these research tasks. He focused on: a) what students had started but reported being unable to complete; b) what he identified as incorrect; or c) what was reported as incorrect by groups. For example, a group reporting during the Volumes of Cuboids Task had made a 24-cube box when they had intended to make a 12-cube box. The reporter for the group showed and described the box: “the length was two- the width was two and the height was six” and stated that the group were still working on how to make the 12-cube box. Patrick stated in his interview:

“You know how they got it wrong- it made me think about (pause) how they could get it right (pause) um (pause) thinking that- it was 2 2 (pause) 2 2 6 (pause) and (pause). If it was 24- they got 24 and they have to get 12 what if they changed the 6 to 3 and that would just halve it and instead of 24 they would have 12.”

As Patrick’s group had just realized that these boxes were made up of layers of blocks (see part c) above), and that the number of blocks the box contained could be found by counting the number of blocks in one layer and then multiplying by the number of layers, he used this new understanding to creatively consider the problem encountered by the
other group. He halved the number of stacks in the height by considering the physical representation rather than worked numerically. Patrick spontaneously formulated his own challenging question and extended the ideas he had just developed to answer it thus engineering flow conditions.

Discussion and Conclusions

Situations that are theoretically optimism building or flow conditions were created by students or student groups who spontaneously focused their own challenging questions then used mathematics in unfamiliar ways to answer these questions. Engaged to Learn pedagogy was found to contain situations that could build optimism. These included the nature of the tasks which provided opportunities for students to discover complexities, the types of questions asked by the RT to elicit generalisation, the gradual withdrawal of resources to assist students to transfer from using concrete aids to thinking without them, and the sharing of group ideas at intervals through the task to increase the number of ideas students may decide to use. Further study of Engaged to Learn pedagogy is needed to identify other aspects that support the development of situations that theory suggests as optimism building.

Further study is required to find whether the types of optimism building situations identified theoretically are supported by empirical evidence. Seligman (1995) states that engagement with flow activity over time, and the successes that accompany this activity should build optimism. My study funded by the Australian Research Council (2009-2012) is designed to generate empirical evidence to find whether such situations do build optimism. Students will be studied as they progress from Grade 4 to Grade 6 in 2009 to 2011 in classes where Engaged to Learn pedagogy is used for at least three tasks across each school year. Changes in student optimism, and factors contributing to these changes will be examined. Associations between changes in optimism and changes in problem solving capacity will be examined. The study reported in this paper does not address how to encourage students to take up the opportunities to engage in optimism-building situations. This is an important area for further study.

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