Trade Liberalization and Income Inequality:
A Theoretical Analysis

By
Su Wu, Bachelor of Economics

Submitted in fulfilment of the requirements for the degree of
Master of Economics
Deakin University (March, 1999)
I certify that the thesis entitled

Trade Liberalisation and Income Inequality: A Theoretical Analysis

submitted for the degree of

Master of Economics

is the result of my own research, except where otherwise acknowledged, and that this thesis in whole or in part has not been submitted for an award, including a higher degree, to any other university or institution.

Full Name: [Redacted by Library]
(Please Print)

Signed: [Redacted by Library]

Date: 30/7/199
ACKNOWLEDGMENTS

I'm deeply grateful to my supervisor, Prof. B. Hazari, who inspired my interest in this topic and guided my research from the beginning to the end. His perceptive suggestion and invaluable comments help me going through all the difficulties in writing this thesis. I'm also greatly indebted to Mr. D. Neath for his efforts in providing generous guidance and correcting grammatical errors.

I would like to thank all the staff in School of Economics, Deakin University for their encouragement and help. I also wish to express my gratitude to my family for their firm support and deep love.
TABLE OF CONTENTS

CHAPTER 1: INTRODUCTION ......................................................................................... 1
  1.1. INTRODUCTION ......................................................................................... 1
  1.2. INCOME INEQUALITY AND TRADE THEORY ................................................ 2
  1.3. RATIONALE FOR THE THESIS ................................................................ 3
  1.4. SCOPE OF THE ANALYSIS ........................................................................ 4
  1.5. STRUCTURE OF THE THESIS .................................................................. 5

CHAPTER 2  INCOME INEQUALITY IN THE HECKSCHER-OHLIN, THE SPECIFIC
  FACTOR AND OTHER MODELS OF TRADE: A SYNOPTIC SURVEY .................... 6
  2.1. INTRODUCTION ......................................................................................... 6
  2.2. HECKSCHER-OHLIN MODEL ..................................................................... 6
  2.3. SPECIFIC FACTOR MODEL OF TRADE ....................................................... 11
  2.4. INCOME DISTRIBUTION CONSEQUENCES IN THE H-O MODEL AND THE
       S-F MODEL ................................................................................................. 13
  2.5. THE ROLE OF NON-TRADED GOODS ......................................................... 16
  2.6. THE RELATIONSHIP BETWEEN INTERNATIONAL TRADE IN GOODS AND
       FACTORS ................................................................................................. 18
  2.7. A MODEL WITH ILLEGAL IMMIGRANTS .................................................... 20
  2.8. RECENT STUDIES ...................................................................................... 20
  2.9. CONCLUDING REMARKS .......................................................................... 22

CHAPTER 3: TRADE, TECHNICAL PROGRESS AND INCOME INEQUALITY: A THREE-
  FACTOR, TWO-GOOD MODEL ........................................................................ 24
  3.1. INTRODUCTION ......................................................................................... 24
  3.2. MODEL ..................................................................................................... 26
  3.3. CONSEQUENCES OF TRADE LIBERALIZATION: ........................................... 31
  3.4. CONSEQUENCES OF TECHNICAL PROGRESS ............................................ 34
       Appendix 3.1: The derivation of equations of change – section 3.3 .................. 40
       Appendix 3.2: Relationship between p and r ............................................... 42
       Appendix 3.3: The derivation of equations of change – section 3.4 ............... 43
       Appendix 3.4: Validity of the model ............................................................ 45

CHAPTER 4: TRADE, TECHNICAL PROGRESS AND INCOME INEQUALITY: A THREE-
  FACTOR, THREE-GOOD MODEL ..................................................................... 46
  4.1. INTRODUCTION ......................................................................................... 46
  4.2. MODEL ..................................................................................................... 47
  4.3. CONSEQUENCES OF TRADE LIBERALIZATION: ........................................... 52
  4.4. CONSEQUENCES OF TECHNICAL PROGRESS: ......................................... 55
       Appendix 4.1: The derivation of equations of change – section 4.3 ............... 60
       Appendix 4.2: Relationship between r and P in a model with non-traded good..... 61
       Appendix 4.3: The derivation of equations of change – section 4.4 ............... 63
       Appendix 4.4: The derivation of equations of change – section 4.4.b ............. 64

CHAPTER 5. TRADE, TECHNICAL PROGRESS AND INCOME INEQUALITY: A MODEL
  WITH ILLEGAL IMMIGRANTS ................................................................. 67
  5.1. INTRODUCTION ......................................................................................... 67
  5.2. MODEL ..................................................................................................... 69
  5.3. CONSEQUENCES OF TRADE LIBERALISATION: ........................................... 73
  5.4. CONSEQUENCES OF TECHNICAL PROGRESS: ......................................... 77
       Appendix 5.1: The Equation of Change – Section 5.3 .................................... 82
       Appendix 5.2: The Equation of Change – Section 5.4.a ............................. 83
       Appendix 5.3: The Equation of Change – Section 5.4.b ............................. 84

CHAPTER 6: SUMMARY ......................................................................................... 86
  6.1. CONCLUSION: .......................................................................................... 86

REFERENCES: ..................................................................................................... 88
Chapter 1: Introduction

1.1 Introduction
This thesis analyzes the effects of trade liberalization and technical progress on factor prices and national income, in the presence of international capital mobility. A trade theoretical framework is used for this purpose.

Ever since the time of Adam Smith and David Ricardo, there has been consensus among economists that given certain assumptions, free trade is better than no trade. Following this line of argument, in recent years trade liberalization has been a ubiquitous phenomenon in many countries. However, it is now argued that the income distribution effects of trade liberalization are not desirable. This argument is supported by the fact that trade liberalization is associated with the rising income inequality between skilled labour and unskilled labour in the developed countries. This inequality problem has been examined in the context of traditional trade theorems. The Stolper-Samuelson (S-S) theorem derived from the standard H-O model states that a fall in the prices of a product would lower the return to factor used intensively relative to the factor not used intensively in producing the product. It implies that, a decrease in the price of imported goods caused by trade liberalization, will result in the immiserization of the unskilled workers in developed countries, since the imported goods are unskilled-labour intensive.

Trade liberalization has been alleged to result in changes in income distribution within the liberalized economies. In particular, it is argued that the skilled workers have gained at the expense of unskilled workers. However, other factors, such as technology, education and so on, may also contribute to the growing dispersion between wage rates of skilled labour and those of unskilled labour. Numerous theoretical models have been used and intensive empirical tests have been conducted to address the income inequality problem. Among them, trade liberalization and technical progress have been identified to be the main factors contributing to the
growing income inequality in the developed countries. This thesis explores these issues on the basis of the H-O model and its extensions. We use the general equilibrium analysis to show whether trade liberalization and technical progress can result in increasing income inequality between skilled and unskilled labour in the developed countries.

1.2. Income inequality and trade theory
Trade liberalization is an important phenomenon that has now occurred in both developed and developing countries. However, this liberalization process has contributed to a widening gap between skilled and unskilled labour wage rates in the developed countries. Several data sources have documented that wage inequality between skilled and unskilled labor increased sharply in the United States from the mid-1970s though the 1980s and into the 1990s. Such a trend can also be observed in the newly industrialized economies over recent years. However, the coincidence of increasing wage inequality with trade liberalization does not establish that there is any causal relationship between them. Many other things have also changed in the developed countries, such as technology, exogenous resource growth, tastes and other factors.

Rising wage inequality has been a source of major concern for many economists. The topic has recently been discussed by Davis (1996), Kremer and Maskin (1996), Richardson (1995) and Wood (1994, 1995), among others. Current emphasis has been on sorting out the various causes of rising income inequality. The theoretical and empirical issues involved have received extensive attention in the international trade literature. Lately there has been a surge of research on the possible connections between trade liberalization or technology and increasing income inequality. Some economists point out that trade liberalization will increase the wage gap between skilled labour and unskilled labour. Others come to the conclusion that international

---

1 See Freeman (1995);
2 See Bhagwati and Dehejia (1994), Batra (1993), Wood (1994, 1995) and Leamer (1994);
trade is not responsible for increasing income inequality. Instead, they argue that technological change is the real culprit, which bids up the wages of skilled workers relative to the wages of unskilled workers.

While the previous studies have been very impressive, there are still many areas that need to be researched. Firstly, there are few theoretical attempts to analyze these problems in a general equilibrium setting, using either the standard or extended Heckscher-Ohlin model or the Specific Factor model. Secondly and surprisingly, free trade in the presence of internationally mobile capital has not been systematically treated in the existing literature on income inequality. Lastly, the important issue of illegal immigrants is seldom discussed in this literature.

1.3. Rationale for the thesis

The investigation into the relationship between trade, technical progress and income inequality is of great theoretical and empirical importance. First, it is important to know the consequences of trade liberalization. The recent argument on whether trade is a major contributing factor to income inequality in the developed countries has invoked rising concerns about the adverse effect of trade liberalization in many aspects. Uncertainty about its income distribution effects creates resistance to a continued trade liberalization process even if such reform benefits society as a whole. Second, if the move toward free trade promises to alter the distribution of income in the economy, generally the losers do not favour the move toward trade liberalization, unless the gainers are willing to subsidize the losers. In reality, compensation transfer occurs.

This paper takes up these issues by examining in detail the income consequences of trade liberalization and technical progress. The main theoretical results have some implications for government policy. In the event that trade liberalization does

---

3 See Sachs and Shatz (1994), Lawrence and Slaughter (1993), Krugman and Lawrence (1994), Bound and Johnson (1992);
Contribute to income inequality, the policy question arises: whether to pursue free trade or restricted trade. If national income worsens after trade liberalization, what kind of trade-impeding policies can be implemented to reduce income inequality? Or if national income is to improve, what actions can be taken to make sure that the losers are fully compensated? However, in this thesis, we simply apply the compensation principle by assuming that if national income improves at the expenses of some group being worse off, it is always possible for the government to redistribute income so that the society as a whole is better off.

1.4. Scope of the analysis
The purpose of this thesis is to develop a theoretical model to examine the impact of trade liberalization and technical progress on income distribution between skilled labor and unskilled labour in an economy.

This thesis employs the specific factor model with the following features. First, it should capture several aspects of the reality of booming international trade and investment in the world. The 1980s and 1990s have witnessed growing global integration where the mobility of capital, goods and services has increased steadily. Second, it is concerned with the inflow of illegal immigrants into the developed countries. The number of illegal immigrants in the United States and European countries has grown quickly since 1970s. These two factors also work toward the income dispersion between the wage rates of skilled workers and unskilled workers.

These features are of great importance in this thesis. For example, the problem associated with previous research efforts is that mostly they have focused on free trade in goods alone. Trade in goods no longer dominates world trade. Movements of capital, as well as migration between countries are equally significant and can have sizeable impacts on trade in goods. Traditional trade theories may not be valid when international factor movement is taken into consideration. The object of this thesis is to develop a model integrating commodity trade, capital mobility and illegal
immigration in order to analyze the effects of exogenous changes on income distribution in a particular country.

1.5. Structure of the thesis
This thesis is organized in the following form: Chapter two presents a literature review for the two main trade models – the Heckscher-Ohlin and the Specific factor model - which is used to explain the income inequality problem. Chapter three outlines the structure of a simple two-good, three-factor model and examines the effects of trade liberalization and technical progress on income distribution. In chapter four, the model is extended to include non-traded goods. The analysis in chapter three and four is conducted under the assumption that capital is internationally mobile while labour is not. Chapter five employs a more applied model with illegal immigrants to consider the case of income inequality in the developed countries in the presence of illegal workers from the developing counties. Finally, chapter six contains conclusions.
Chapter 2  Income Inequality in the Heckscher-Ohlin, the Specific Factor and Other Models of Trade: A Synoptic Survey

2.1. Introduction

In this chapter, we attempt to present an overview of the current state of the pure theory of international trade relating to income inequality. Only selected topics relevant to this thesis in the pure trade theory are covered. The following two sections consist of a review of pure theory of international trade in the two traditional models: the Heckscher-Ohlin (H-O) model and the Specific Factor (S-F) model. It seems appropriate to give the mathematical presentation of the model as utilized in Jones’s (1965) approach. In section 2.4, we narrow down to the income distribution aspects of these two models. An intuitive explanation of the results will be given with the aid of a diagram. Then we consider extensions of the models by relaxing some assumptions. A non-traded good is incorporated into the model in section 2.5, followed by analysis of the model under an assumption of free factor movement between countries in section 2.6. The model is extended to cover illegal immigrants in section 2.7. Recent theoretical and empirical studies of the income inequality problem will be presented in section 2.8. Finally, section 2.9 contains concluding remarks.

2.2. Heckscher-Ohlin model

Theoretical and empirical issues involved in examining problems of income inequality have received extensive attention in the international trade literature. Most of the theoretical studies have used either the standard H-O or the S-F model. The H-O model, first postulated by two Swedish economist Eli Heckscher and Bertil Ohlin and later on mainly due to the work of Samuelson, has dominated international trade theory for much of this century. Four main propositions – the H-O, the factor-price equalization (FPE), the Rybczynski, and the Stolper-Samuelson
(S-S) theorems are derived from this model and have been viewed as "the central body of international trade theory"\(^4\).

A standard H-O model consists of two countries, each producing two commodities using two intersectorally mobile factors of production. We specify this model for one country only, as the other country is not required for our present analysis.

The economy produces two goods, labelled \(X_1\) and \(X_2\) respectively, using factor endowment of labour \(\bar{L}\) and capital \(\bar{K}\), which are inelastically supplied to the economy. We assume that \(X_1\) is labour intensive and \(X_2\) is capital intensive. The production functions are given below:

\[
X_j = F_j(L_j, K_j) \quad (j = 1, 2) \tag{2.1}
\]

where \(L_j\) and \(K_j\) are respectively the amounts of labour and capital used in the sector \(X_j\). Each production function is constant return to scale and diminished return to factor.

Under perfect competition, firms producing positive outputs earn zero profit, which implies that the unit cost must equal output prices. Let \(P_j\) be the market prices of good \(X_j\), and \(w\) and \(r\) be the prices of the factors of production \(L\) and \(K\) respectively. We have:

\[
a_{L1}w + a_{K1}r = P_1 \tag{2.2}
\]
\[
a_{L2}w + a_{K2}r = P_2 \tag{2.3}
\]

where \(a_{ij}\)'s \((i = L, K)\) represent variable input-output coefficients.

Full employment condition require that:

\[
a_{L1}X_1 + a_{L2}X_2 = \bar{L} \tag{2.4}
\]
\[
a_{K1}X_1 + a_{K2}X_2 = \bar{K} \tag{2.5}
\]

\(^4\) See Chang, Ethier and Kemp (1980);
Let hat ($\hat{}$) denotes the relative change of any variable, for example, $\hat{y} = dy/y$.

Differentiate equations (2.2) to (2.5), and rearrange terms to give:

$$\theta_{L1} \hat{w} + \theta_{K1} \hat{r} = \hat{P}_1 - \left[\theta_{L1} \hat{a}_{L1} + \theta_{K1} \hat{a}_{K1}\right]$$  \hspace{1cm} (2.6)

$$\theta_{L2} \hat{w} + \theta_{K2} \hat{r} = \hat{P}_2 - \left[\theta_{L2} \hat{a}_{L2} + \theta_{K2} \hat{a}_{K2}\right]$$  \hspace{1cm} (2.7)

$$\lambda_{l1} \hat{x}_1 + \lambda_{l2} \hat{x}_2 = \hat{L} - \left[\lambda_{l1} \hat{a}_{l1} + \lambda_{l2} \hat{a}_{l2}\right]$$  \hspace{1cm} (2.8)

$$\lambda_{k1} \hat{x}_1 + \lambda_{k2} \hat{x}_2 = \hat{K} - \left[\lambda_{k1} \hat{a}_{k1} + \lambda_{k2} \hat{a}_{k2}\right]$$  \hspace{1cm} (2.9)

where $\theta_j$ is relative share of factor $i$ in the total cost of $j^{th}$ commodity, and $\lambda_j$ is the fraction of factor $i$ employed in the production of the $j^{th}$ commodity; for example, $\theta_{L1} = wL_1/P_1X_1$ and $\lambda_{l1} = L_1/L$.

By solving equations (2.6) to (2.7), we obtain the following:

$$\hat{w} = \frac{\theta_{K2} \hat{P}_1 - \theta_{K1} \hat{P}_2}{|\theta|}$$  \hspace{1cm} (2.10)

$$\hat{r} = \frac{\theta_{L1} \hat{P}_2 - \theta_{L2} \hat{P}_1}{|\theta|}$$  \hspace{1cm} (2.11)

where $|\theta| = \begin{pmatrix} \theta_{l1} & \theta_{k1} \\ \theta_{l2} & \theta_{k2} \end{pmatrix}$.

Since $X_1$ is labour intensive, $|\theta| > 0$. Suppose that $\hat{P}_1 > \hat{P}_2$, we have:

$$\hat{w} > \hat{P}_1 > \hat{P}_2 > \hat{r}$$  \hspace{1cm} (2.12)

Therefore, we find that the real wage rate increases relative to rental rate of capital as the price of $X_1$ rises relative to that of $X_2$. This is the essence of S-S theorem, which examines the impact of commodity price changes on factor prices with factor endowments fixed.

Stolper-Samulson Theorem states that:

A rise in the price of a commodity will increase the real reward of the factor used intensively in the sector and decrease the real reward of the other factor, provided that both goods are produced.
Specifically, Jones in 1965 obtained the magnification effect from the S-S theorem.

We now turn to the relationship between factor endowment and outputs when good prices are kept constant. Solving equations (2.8) and (2.9) for the changes in outputs gives rise to:

\[
\hat{x}_1 = \frac{\lambda_{x2} \hat{L} - \lambda_{x2} \hat{K}}{|\lambda|}
\]

(2.13)

\[
\hat{x}_2 = \frac{\lambda_{x1} \hat{K} - \lambda_{x1} \hat{L}}{|\lambda|}
\]

(2.14)

where \(|\lambda| = \begin{pmatrix} \lambda_{x1} & \lambda_{x2} \\ \lambda_{k1} & \lambda_{x2} \end{pmatrix}\). Since \(X_1\) is labour intensive, \(|\lambda| > 0\).

Following the magnification effect, if \(\hat{L} > \hat{K}\), then it follows that:

\[
\hat{x}_1 > \hat{L} > \hat{K} > \hat{x}_2
\]

(2.15)

This can be summarized by the Rybczynski theorem, which is concerned with the responses of commodity outputs to changes in factor endowments at constant prices\(^6\).

*Rybczynski Theorem states that:*

An increase in the endowment of a factor will increase by a greater proportion the output of the sector, which uses the factor intensively, and decrease the output of the other sector.

There are two other important results derived from this model. One of them is the H-O theorem of trade\(^7\). It states that, in a simple two-country, two-good and two-factor model, a country should export that commodity which uses its abundant factor intensively, and import that commodity which uses its scarce factor intensively, given identical technology across countries and other assumptions. In this way, welfare in both countries will be improved. The other is the FPE theorem, which is

---

\(^5\) See Stolper & Samuelson (1941);
\(^6\) See Rybczynski (1955);
\(^7\) See Heckscher (1919) and Ohlin (1933);
concerned with the relationship between commodity prices and factor prices⁸. It proves that, under certain conditions, international commodity trade will tend to equalize the prices of each factor of production between two trading countries, in spite of factor immobility.

Effects of other exogenous parameter changes on the endogenous variables can also be derived from H-O model. For example, the effect of technological changes on outputs and factor prices depends on the nature of technical change. As shown originally by Johnson (1955), at constant goods prices, a Hicks-neutral technical change in an industry can lead to an expansion in that industry and a contraction of the other industry. It seems that technical change is no different from a change in factor endowments. But, in fact, unlike endowment changes, technical changes also leads to an adjustment in factor prices. Technical progress in one industry will raise the rental rate of the relatively intensive factor and lower the price of the relatively scarce factor in that industry. Findlay and Grubert (1959) further analyzed the case of biased technological change. They concluded that there will be a fall in the output of a sector when there is technical progress in the other sector which saves on the other factor used intensively in that sector. But the result of technical progress saving the factor used scarcely in that sector is ambiguous.

However, the assumptions defined in H-O model are highly restrictive⁹. In addition to the assumption of free trade and no transport costs, it is also assumed that:

1. The home country and the foreign country are similar in many dimensions. They have the same tastes and therefore have identical demand for goods¹⁰. The technology is identical across countries.

2. There is no factor intensity reversal in the sense that one sector is always labor intensive relative to the other sector.

---

⁸ The original source for factor-price equalization theorem is Samuelson (1948, 1949) and Lerner (1952);
⁹ An detailed description of H-O model can be found in Jones (1965);
¹⁰ The sufficient conditions for H-O model differs in terms of the definition of "factor abundance"; the physical or the price definition. If the price definition of factor abundance applies, the condition of identical consumption pattern is not necessary. Details see Bhagwati (1965);
3. Both countries are incompletely specialized in production.
4. All markets are perfectly competitive.
5. Constant returns to scale and diminishing returns prevail in the production.
6. Factors are perfectly mobile across sectors but immobile across countries.

Attempts have also been made to extend the simple two-country, two-commodity, two-factor (2 x 2 x 2) model to the many-country, many-factor, many-commodity (n x m x r) framework\(^{11}\). In this case, most of the propositions derived from the 2 x 2 x 2 model carry over to the n x m x r model, but in a weaker form. As for the H-O theorem, Deardoff (1979) proved in his many-commodity, two-factor, many-country model that if the goods are ranked in order of factor intensities, all of the exports of a country must lie higher in the list than all of its imports. As for the factor-price equalization theorem, it may only be valid when the number of commodities is at least as great as the number of factors. When the number of commodities is smaller than the number of the factors, factor prices depend not only on the international prices of commodities, but on the factor endowments of the countries\(^{12}\). If the number of commodities r is greater than the number of the factors m, factor prices can be determined solely by the prices of m commodities, but we need to identify which are the m commodities. The S-S theorem and the Rybczynski theorem also apply to some cases but not necessarily to all.

2.3. Specific Factor model of trade
Since 1970, the H-O model has shared the center stage with another international trade model, the Specific Factor model, one that differs from the former in that one of the two factors is immobile between sectors. Beginning with Jones (1971) and Samuelson (1971), several economists gave attention to this alternative model,

\(^{11}\) For reference, see Jones (1956), Bhagwati (1972) and Ethier (1984);
\(^{12}\) Factor-price equalization may still achieve under certain strong and bizarre assumptions;
which provided a more interesting description of an open economy with imperfect factor mobility\(^{13}\).

In the simple 3 x 2 S-F model, one of the factors of production such as capital is assumed to be specific to a particular sector so that no capital movement between the sectors exists, while the other factor - labour, can move freely between the sectors. Consequently, the wage rate of the mobile factor, labour, will be the same in the two sectors. However, the returns to the specific factor capital will differ across sectors. In the S-F model, there are more factors than goods in this model, so commodity prices will generally not suffice to determine factor prices. Factor endowments and commodity prices jointly determine factor prices. Thus, the Rybczynski theorem and the F-P-E theorem do not hold in the S-F model.

The general outcome of a change in commodity price in this model is: a rise in the prices of one good benefits the factor specific to that sector but hurts the factor specific to the other sector, with a higher rental rate to mobile inputs. By comparison with the H-O model, there is no magnification effect on changes in the price of the mobile factor.

As far as changes in factor endowments are concerned, they play a role in influencing factor prices, as well as determining the level of output\(^ {14}\). An increase in the endowment of a factor specific to a sector leads to an increase in the output of that sector and a decline in the output of the other sector. The rental rate to the mobile factor rises, while those to sector-specific factors decline. On the other hand, an increase in the endowment of the mobile factor lowers the return to that factor and increases those to sector-specific factors. Output levels of both sectors rise. The sector with the higher elasticity of demand for the mobile factor weighted by its share in income expands relatively more.

\(^{13}\) This model is also called Ricardo-Viner model by Samuelson (1971), or Jones-Neary model. See Ricardo (1817), Viner (1931,1950), Jones (1971), Neary (1978a, 1978b);

\(^{14}\) Explanation has been given in Jones (1971);
Some results of the S-F model, which may appear to be more plausible, contradict directly the H-O model results. Compared with the H-O model, S-F model takes into account a further consideration on the factor side. Sector-specificity characteristics result from many reasons. One of the most likely reasons is that some factors have comparative advantage in producing one particular good. S-F model is more applicable in examining the short-run effects of exogenous disturbances. In the short run, the amount of factors in each sector of the economy is fixed. In the long run, factors will flow between sectors in response to the rental differentials until all factors of the same quality earn the same rental rate. Mussa (1974), Mayer (1974), Neary (1978a, 1978b), and Grossman (1983) regarded sector specificity of capital as a short-run phenomenon\textsuperscript{15}. In addition to its more accurate description of the short-run effects on the economy, the S-F model also has provided a framework for studying the dynamic adjustment process and stability\textsuperscript{16}. It is actually an important tool for analyzing a variety of trade policy issues. Therefore, many other issues have been analyzed in the context of the S-F model. Samuelson (1971), Mussa (1974), Jones (1975), Dixit and Norman (1980) also investigate the properties of the S-F model with many commodities.

2.4. Income distribution consequences in the H-O model and the S-F model

Both the H-O model and the S-F model can be used to analyze the income distribution effects of trade liberalization and technical progress.

Presumably developed countries are unskilled-labor scarce relative to developing countries. In a strict 2 x 2 x 2 H-O model, both countries use both skilled-labour and unskilled-labour in the production of the two traded goods. According to the H-O theorem, the developed countries have comparative advantage in the production of skilled labour intensive goods, and therefore, should export the skill-labour intensive products and import the unskilled labour intensive products. Consequently, the S-S

\textsuperscript{15} See Wong (1995), P58;
\textsuperscript{16} See Neary (1978b) & Mussa (1982);
theorem tells us that a decrease in the price of imported goods caused by trade liberalization will cause the immiserization of unskilled labour.

These results can be simply illustrated using the unit price curve diagram, shown in figure 2.1. The developed country produces at the intersection of the two unit-price curves, point E. The position of these two curves tells us good 1 is unskilled labour intensive and good 2 skilled labour intensive. Factor prices are indicated as $W_1$ and $W_s$ on the vertical and horizontal axes respectively. Suppose that trade liberalization leads to a decrease in the price of the imported goods. This shifts the unit price curve $P_1$ downward to $P_1'$. As a result, at the new intersection point $E^*$, factor prices change. The wage rate of the unskilled labour declines to $W_1^*$ while that of skilled labour rises to $W_s^*$.

Figure 2.1 Income effect of trade liberalization in H-O model

![Diagram](image)

An expansion of the S-F model also allows for an explicit analysis of the separate returns to skilled and unskilled labor. Suppose skilled labour and unskilled labour are used in the production of exported goods and imported goods respectively while

14
capital is mobile between sectors. Free trade with other countries will inevitably result in a fall of the wage rate of unskilled labour.

We can illustrate this result by using figure 2.2. In figure 2.2, VMPK_j is the value of marginal product of capital in the j^{th} sector and O_1O_2 is total endowment of capital in the economy. The initial equilibrium capital rental rate r_0 is determined at the intersection of the VMPK_j curves, point A, and the equilibrium wage rate to sector-specific labour W_1 and W_s, can be found by reading off the two unit price curves c_1 and c_2 in the right-hand panel respectively. At the initial equilibrium, we artificially set them equal at point C.

Consider the effects of a decrease in the relative price of the imported good 1. VMPK_1 will shift downwards to VMPK_1', resulting in a new equilibrium at point B. Capital moves from sector 1 to sector 2, while r_0 falls to r_0'. The returns to the specific factors also change. As shown in the right-hand panel, the unit price curve c_1 shifts leftward while c_2 remains constant, as a result of a decrease in the relative price of good 1. Therefore, W_1 declines and W_2 rises.

Figure 2.2 Income effect of trade liberalization in S-F model
Either of the two major theoretical approaches has, implicitly or explicitly, provided support for the argument that unskilled labor in the developed countries is destined to suffer from free trade. In reality, even the possibility of trading with poor countries could put downward pressure on the wages of unskilled labour in the developed countries. Some economists have applied the same approaches to the analysis of technical progress, as we have mentioned before.

However, the S-S theorem may not hold in the real world because one or more of the sufficiency conditions are unrealistic. And the results under the theoretical H-O and the S-F model may be modified when any of the assumptions are changed. As we know, the models of international trade basically differ from one another in the specification of assumptions, variables and their relationships. In some cases, a minor variation from the original model may lead to an absolutely different outcome.

In recent years, there has been a great deal of research conducted to analyze the widening of wage inequality in developed countries by using a model extended from the traditional H-O model and S-F models. An extensive body of literature on the extension of the trade model has been produced since the 1970s. In the following section, we examine the development of a model with non-traded goods. Then in section 2.6, we discuss the trade model in the presence of internationally mobile capital. Section 2.7 addresses the case of illegal immigrants.

2.5 The role of non-traded goods

The study of trade models with non-traded goods has expanded enormously in recent years. Each country produces goods that are neither for export nor import. They are just produced for domestic consumption. The existence of non-traded

---

17 Discussion of distortion theory can be found in Batra (1973), Bhagwati (1971), Hazari (1978), and Kemp (1969), Magee (1971);
18 The implications of non-traded goods for trade models have been considered by McDougall (1965), Komiya (1967), Ethier (1972) and Jones (1974);
goods results from various reasons, such as high costs of transport, different tastes, or the nature of the goods. Economists have long recognized that non-traded goods play an essential role in the analysis of many problems of international monetary theory (Ethier 1972). However, it was only in 1965 that McDougall introduced non-traded goods to his analysis of the transfer problem. Then, Komiya (1967) was first to study the role of non-traded goods in the pure theory of international trade. Since then, many studies on the non-trade good model have been undertaken.

The model, which is extended from the traditional $2 \times 2 \times 2$ H-O model, generally takes the form of three commodities (one exportable, one importable and one non-tradable good) and many factors of production. It has often been assumed that, while prices of traded goods are determined on the international market, the prices of non-traded goods are determined exclusively by domestic supply and demand conditions. However, the presence of non-traded goods is influenced by and is also able to influence the traded sector in some way. For instance, any exogenously given changes in the prices of the traded goods may lead to a change in the prices, as well as the output of the non-traded goods. The supply and demand responses required by the necessity that the markets for non-traded goods clear internally, indirectly change the market for traded goods.

Some of the basic theorems of the pure theory of international trade must be modified when non-traded goods are introduced. Among other, Komiya (1967) found that only the factor-price equalization theorem and the Rybczynski theorem hold true in his two-factor, three-good model. Ethier (1972) analyzed the validity of traditional trade theorems in his model with two commodities (one non-traded and one composite traded good) and two factors of production (capital and labor) by assuming incomplete specialization. Following Ethier's model, Rivera-Batiz (1982) examined the effects of technological change and changes in factor endowments within the context of a model with an equal number of goods and factors (two goods - one non-traded and one composite traded good and two factors). They all pointed
out that, while some of the results can be extended under specific condition, most are affected.

2.6 The relationship between international trade in goods and factors
So far, we have surveyed much of the trade theory that has only been conducted in terms of the classical paradigm that goods are internationally mobile but factors are not. International factor mobility has not received enough attention in the trade literature. Firstly, historically trade in goods has dominated international business; and secondly, it is generally believed that the theory of international trade in goods can be extended to cover international factor mobility. International factor movement is growing significantly over time and can have sizeable impacts on a country’s trade and welfare. It is important to examine whether trade theories still hold when international factor movement is taken into consideration.

Initially, the economic reality of international capital movement was incorporated into modern international trade theory. Mundell (1957), who examined the consequences of allowing capital to be internationally mobile in the H-O framework, firstly developed the relationship between international trade and capital mobility. In his 2-commodity, 2-factor and 2-country model, Mundell examines the implications of a small tariff levied by the country that imports the capital-intensive good. By applying the factor-price equalization theorem to his analysis, he shows that international trade in goods and factors are perfect substitutes in the sense that tariff barriers encourage international capital flow. However, this result seems incompatible with the explosion in both trade and factor flows in recent decades.

A fatal shortcoming of Mundell’s work is that his conclusion is derived from the H-O framework, which is constructed under a series of unrealistic assumptions. There are many cases in which Mundell’s finding can not hold true. Purvis (1972), arrives at a quite different conclusion by relaxing the assumptions of identical technology between the trading countries. Markusen (1983) argues that if the basis for trade is differences in factor endowments, as in the H-O model and Mundell’s, trade and
factor movements are likely to be substitutes. However, if the basis for trade resides in other factors, such as returns to scale or imperfect competition, commodity trade and factor movements could be complements as well.

Among the trade models allowing for international factor movements, Jones (1970, 1980) and Ferguson (1978) discuss the determinants of trade patterns. If capital is internationally mobile in the H-O model, trade patterns are determined by comparative labour costs. If technology differs between countries, patterns of trade will reflect comparative as well as absolute advantage. Kemp (1966), Markusen & Melvin (1979), Brecher & Choudhri (1982) and Rivera-Batiz (1982) use two-factor, two-good models to examine the effect of protection, in terms of tariffs, on the economy. Such a question is also addressed in a two-good, three-factor model by Batra and Casas (1976), Ruffin (1981), and Jones and Easton (1983). Jones and Scheinkman (1977) and Chang (1979) follow on to conduct higher-dimensional treatments. But the answer to the question has become quite complicated. Hazari (1982), within a model with two traded goods and one non-traded good using a sector-specific factor, demonstrates that foreign capital inflow raises welfare. Using a two-good, two-factor model, Jones (1984) finds that when capital is perfectly internationally mobile, an increase in the tariff is always welfare immiserizing regardless of factor intensities and of whether capital is inter-sectorally mobile or not. Finally, Hatzipanayotou and Michael (1997) develop a more general model of two traded goods and one non-traded good, with two or more factors of production that are either intersectorally mobile or not. He demonstrates that under free trade, an exogenous inflow of foreign capital is always welfare improving. This reconfirms Hazari’s (1982) result. He also finds that an increase in the import tariff is always welfare immiserizing when capital is perfectly internationally mobile. This reconfirms Jones’s result.
2.7. A model with illegal immigrants

The most common version of such trade models allows for free international capital movement, while labor is immobile internationally. Trade theorists have only occasionally examined international labour movement. With the surge of immigration in developed country since the 1980s, some new theoretical models on international trade in goods with human capital movement have been produced.

One approach has concentrated on the economic consequences of migration for the trading economies. Most commonly, these analyses focus on the brain drain problem confronted by the emigrant countries in the literature. For example, the Bhagwati and Hamada (1974) model examined the effect of “brain drain” in a model where emigration leads to a highly-paid class of high-skilled professionals in the emigrant country and hence to an escalation of the fixed wage in the Harris-Todaro model. Consequences for the host countries are also examined. For instance, Kuhn and Wooton (1991) developed a 3x3 model to examine the effect of an increase in the factor endowment, including immigration, on the rewards of factors and then subject U.S. data to empirical testing.

Another interesting topic relates to the illegal immigration problem in host countries. Most countries have complicated policies and quantitative restrictions on the inflow of migrants. However, as long as the economic, political and social incentives to move are significant, some will choose to move illegally. The existence of illegal migrants has important effects on the host countries. However, theory in this field needs to be further developed.

2.8 Recent studies

The income inequality problem in the developed countries since the 1980s has been a source of great concern to many economists. The coincidence of trade

---

19 See Johnson (1967), Bhagwati & Hamada (1974) and Hamada (1977);
20 See Ethier (1986), Bond and Chen (1987) and Djajic (1987);
liberalization in those countries with the increasing wage inequality does not prove any causal relationship between them. Various theoretical and empirical researches have been conducted aimed at sorting out the main forces driving wage inequality.

Most of the researches are empirical studies aimed at establishing whether there is a link between changes in some variables and income inequality. For the United States, the analyses have tended to support the conclusion that trade has not been the primary cause of increasing income inequality (Freeman 1995). For example, Bhagwati and Kisters (1994), Lawrence and Slaughter (1993), Sachs and Shatz (1994), Krugman and Lawrence (1994), Bound and Johnson (1992) all conclude that trade has had only a minor effect on the decline of the wage rate of unskilled labour. They argue that the rising income inequality is mainly attributed to technological change rather than trade. Some come to a different conclusion. Batra (1993), Bhagwati and Dehejia (1994), Wood (1994, 1995), Leamer (1994) find that trade liberalization has played an important role in widening the wage gap between skilled-labour and unskilled-labour. The different results are related to the different theoretical frameworks, research techniques and data used by the economists.

However, there is less progress in theoretical studies of the income inequality problem. Up until now, few theoretical models analyze widening income inequality within a general equilibrium framework. Borjas and Ramey (1995) developed a model in which the impact of trade on the relative wages of less skilled workers depends on the market structure of the import-competing industry. Oda (1996) used a 2-good, 3-factor S-F model to examine the effects of trade, technical progress and education on factor prices. By assuming capital is intersectorally but not internationally mobile, he found that both trade liberalization and technical progress would result in a widening income inequality between skilled labour and unskilled labour. Choi (1997) developed a model for examining the impact of free trade on wage inequality in the developed countries and the sensitivity of wage inequality indices to changes in other causal factors, such as technological progress, changes in relative factor endowments and consumer expenditure, within a general equilibrium
framework. His North-South trade model is presented where there are three factors of production: sector-specific high-skilled labour, immobile unskilled labour and mobile unskilled labour and therefore three types of income inequality indices are derived. He finds that overall wage inequality does not necessarily deteriorate with trade liberalization. In order to cure income inequality, the developed country should recast factor market structure.

While previous studies on income inequality have been very impressive, there are still some fairly open areas in which more research is needed. For example, free trade in the presence of international mobile capital has not been systematically treated in the existing literature on income inequality. Moreover, the issue of illegal immigrants is also seldom discussed in this literature.

2.9 Concluding remarks
In recent decades, a rich body of literature has grown which focuses on the validity of international trade theorems in various models developed from the H-O framework. However, most of the researches were conducted by simply assuming that factors are internationally immobile. It has become more and more clear that there are many issues relating to international trade that require further study.

In view of the restricted nature of existing work in the field of trade theory, this paper will provide a general equilibrium analysis of the impact of trade liberalization and technical progress on wages, employment and social welfare for a single economy in the presence of capital and labour mobility. Our analysis of the income inequality problem departs from earlier models in the following two ways: On the one hand, our analysis is conducted within the general equilibrium framework. The economy in question is treated as a set of interconnected markets where commodity prices and factor prices are jointly determined. On the other hand, our model does capture several aspects of the reality of international trade. We develop a model,
integrating trade and factor mobility, to examine the effects of changes in commodity prices, productivity and factor endowment on factor prices.
Chapter 3: Trade, Technical Progress and Income Inequality
- a Two-Good, Three-Factor Model

3.1. Introduction:
Wage inequality is a phenomenon that has now become of pressing concern in many countries. In Europe and the United States, a widening gap between wages for the skilled and non-skilled labour has been associated with the trade liberalization process. Although such coincidence does not prove any causal relationship between them, trade liberalization has been a source of major concern to many economists in discussing this issue. An alternative explanation of wage inequality problem may be discovered by examining the consequences of technical progress taking place in these countries.

This topic has been recently discussed by both labour and trade economists, including Davis (1996), Kremer and Maskin (1996), Richardson (1995) and Wood (1995), among others. Various theoretical and empirical researches have been conducted to sort out the main forces driving wage inequality. But very few attempts have been made to analyze this problem in a general equilibrium model. One plausible model that links trade and wage inequality is the traditional H-O general equilibrium model. According to this model, trade between the developed countries and the developing countries leads the prices of skilled-labour intensive goods to rise relative to that of unskilled-labour intensive goods, which will then cause wages of skilled-labour to increase relative to those of unskilled-labour. The H-O model can also been used to examine the effect of technical progress, which will increase wage inequality if it is concentrated in the skilled labour-intensive sector. However, several restrictive assumptions have been made in the standard H-O model and some of them are clearly unrealistic.

This thesis aims to clarify the reasons for the increase in income inequality by using the general equilibrium model. The model is extended from the S-F model with
more realistic assumptions. This chapter uses a simple general equilibrium model to analyze this issue. In later chapters, the model is extended in several directions.

In this chapter, we introduce a two-good, three-factor model\textsuperscript{21} to examine the income distribution consequences of trade liberalization and technical progress. Aggregate income change is also analyzed. The model expands on the specific-factor model by adding some features that are characteristic of many countries. The economy in question is assumed to produce two commodities with the help of two sector-specific factors - non-skilled labour and skilled labour, and one mobile factor - capital. It also integrates international free good and capital movement into one model by assuming that capital is internationally mobile.

Exploring the implications of all possible variations of the unknowns in the model would be tedious. Because of the nature of the problem we have studied in this case, it is sufficient to only consider the effects of parametrical changes on the relative factor prices and national income. In examining the income inequality problem, a change in income distribution can be shown by calculating \((\hat{w}_2 - \hat{w}_1)\), either using changes in the nominal wage or real wage\textsuperscript{22}. However, it is still necessary to show the changes in real rewards of the factors. Therefore, we simply choose to use proportional changes in nominal wages \(\hat{w}_1\) and \(\hat{w}_3\), and then examine whether the magnification effects (Jones 1965) still holds to show the changes in real wages. We also present the result for national income in the main body of this chapter because this variable can show whether trade liberalization or technical progress is in the national interest of the economy in question. We treat the results as desirable for the economy in question if national income is going to improve no matter what the consequences on income distribution to specific groups are.

\textsuperscript{21} This model is based on the specific-factor model without foreign capital presented by Masao Oda (1996);

\textsuperscript{22} It does not matter whether changes in nominal wage or real wage is used, since the term of relative price changes cancels out when using changes in real wage;
3.2. Model:
We consider a small open economy that produces two commodities $X_1$ and $X_2$. Commodity $X_1$ is produced with the help of non-skilled labor $L_1$ and capital $K_1$; while commodity $X_2$ is produced with skilled labour $S_2$ and capital $K_2$. We assume that the economy in question imports commodity $X_1$ and exports commodity $X_2$.

The production functions are assumed to exhibit constant returns to scale and diminishing returns to factors. All firms are assumed to be profit-maximisers. The production functions are given below:

$$X_1 = F_1(L_1, K_1, t_1)$$  \hspace{1cm} (3.1)

$$X_2 = F_2(S_2, K_2, t_2)$$  \hspace{1cm} (3.2)

where $t_j$'s represent the technology level in $j^{th}$ ($j = 1,2$) sector. Note that $L_1$ and $S_2$ are specific to the production of commodities $X_1$ and $X_2$ respectively, with capital perfectly mobile between the two sectors. It is also assumed that capital is internationally mobile across the countries. The country in question is assumed to be a net importer of foreign capital at the initial equilibrium point.

The factor utilisation equations are:

$$a_{L_1} X_1 = L_1$$  \hspace{1cm} (3.3)

$$a_{S_2} X_2 = S_2$$  \hspace{1cm} (3.4)

$$a_{K_1} X_1 + a_{K_2} X_2 = K^d + K^f = K$$  \hspace{1cm} (3.5)

Where $a_{q_j}$'s denote variable input-output coefficients; $L_1$, $S_2$ and $K^d$ the inelastic supply of the factors - skilled labour, non-skilled labour and domestic capital. $K^f$ represents the endogenously determined supply of foreign capital. As is standard in this literature, the $a_{q_j}$'s are functions of the factor prices and the state of technology, given as follows:

$$a_{q_j} = a_{q_j}(w_1, w_2, r^*, t_j)$$  \hspace{1cm} (3.6)

where $i = L, S, K$ and $j = 1,2$. So we have, e.g. $a_{L_1} = a_{L_1}(w_1, r^*, t_1)$, $a_{S_2} = a_{S_2}(w_2, r^*, t_2)$.
The economy in question can be illustrated by the Edgeworth box diagram in Figure 3.1. The total capital endowment, including domestic capital (measured from O to O′) and foreign capital (measured from O′ to O″), is given by the length of the box OO″. The vertical distance OO₁ represents unskilled labour endowment, which is only employed in the production of X₁. The vertical distance OO₂ shows skilled-labour endowment, which is only used in the production of X₂. Under the assumption of full employment, any point along the OO″ line represents an efficient allocation of the factors of production between the two sectors. As shown in Figure 3.1, the initial equilibrium for the economy is given by point A, where two isoquant curves X₁ and X₂ intersect with each other. The slopes of dotted curves BA and CA give the initial wage-rental ratio for sector X₁ and X₂ respectively.

![Figure 3.1](image)

We shall assume that the country is small in both the commodity market and the factor market for foreign capital, hence, the two commodity prices P₁, P₂ and the
return to foreign capital $r^*$ are given from outside. From the assumption of perfect competition, the pricing equations for the system are:

\begin{align}
    a_{x1}w_1 + a_{x1}r^* &= P \\
    a_{x2}w_s + a_{x2}r^* &= 1
\end{align}

(3.7) (3.8)

where $w_1$ denotes the wage rate of unskilled labour and $w_s$ the wage rate of skilled labour. And for analytical convenience, we take commodity $X_2$ as the numeraire. Then the prices are expressed in terms of $P_2$. We introduce a new term, $P = P_1 / P_2$, defined as the relative price of commodity $X_1$.

The price determination procedure is shown in Figure 3.2. In quadrant 1, the unit price curve for commodity 1 is drawn and in quadrant 2 for commodity 2. Rental rate on capital always moves opposite to the wage movement along the unit price curve. A small open economy takes the rental on capital as given, and hence, $r$ is specified exogenously. Given the values of $r$ and $P$, equilibrium values of $\bar{w}_l$ and $\bar{w}_s$ are attained as shown by $\bar{w}_l^*$ and $\bar{w}_s^*$ in the Figure. We assume that $\bar{w}_s^* > \bar{w}_l^*$.

Figure 3.2
The endogenously determined amount of foreign capital inflow is shown in Figure 3.2. The value of the marginal product curve for the economy as a whole is drawn as $VMP_k$ curve in Figure 2. The supply of domestic capital is shown by $\dot{K}_d$ and by the vertical line $\dot{K}_d A$. The foreign supply curve for capital is shown by the curve $r^*B$. Equilibrium is attained at point $e$ where the value of the marginal product of capital equals the foreign rental rate of capital. The amount of foreign capital imported is shown by $\overline{K}_d K'$. The inflow of foreign capital is determined by the following equation:

$$K' = K'(P,t_j)$$  \hspace{1cm} (3.9)

In this case, we shall assume that$^{23}$:

$$\frac{P}{K'} \frac{\partial K'}{\partial P} = E_{K', P} > 0$$  \hspace{1cm} (3.9.1)

where $E_{K', P}$ denotes the price elasticity of demand for foreign capital.

Therefore, if $P$ decreases after liberalising trade, there will be an outflow of foreign capital from this economy. As shown in Figure 3.3, after trade liberalization, the $VMP_k$ curve shift leftward. The new equilibrium point for the economy in question is $e'$. The amount of capital outflow is equal to $\overline{K}K'$.

$^{23}$ The proof of the relationship between $K'$ and $P_1$ is given in Appendix 3.2;
We do not specify the relationship between the level of technology $t_j$ and the flow of foreign capital $K'$ here for the following reasons: First, intuitively we can establish a positive relationship between $t_j$ and $K'$. For the given constant price level, technical progress in any sector will lead to a rise in the factor prices. Domestic rental rate of capital tends to rise after technical progress. There will be an inflow of foreign capital in the economy in question. Secondly, consider the case of technical progress in the production of $X_2$. The system of equations is sufficient to solve all the unknowns, without referring to the relationship between $t_j$ and $K'$.

Market-clearing conditions require that:

$$D_1(P, I) = X_1 + M_1$$  \hspace{1cm} (3.10)

$$D_2(P, I) = X_2 - E_2$$  \hspace{1cm} (3.11)

where $D_j$ indicates domestic demand for commodity $X_j$, $M_1$ the imports of commodity $X_1$ and $E_2$ the exports of commodity $X_2$. And $I$ is national income.
Finally we introduce the national income equation from both demand and supply sides of the market\(^{24}\):

\[
P D_1 + D_2 = P X_1 + X_2 - rK' \tag{3.12}
\]

This completes the specification of the model. Our model consists of twelve equations [(3.3) – (3.7), the four implied by (3.8), (3.9) and the two from (3.10) – (3.12)] in twelve unknowns \((a_{l1}, a_{l2}, a_{k1}, a_{k2}, X_1, X_2, W_1, W_2, K', D_1, D_2 \text{ and } I)\) with six exogenously-given variables \((P, t_1, t_2, K^d \text{ and } L \text{ and } S)\).

The next step is to analyze the system by using comparative static techniques. In section 3.3, we differentiate the system of the model with respect to changes in relative price of imported goods resulted from trade liberalization. In section 3.4, we modify the model and differentiate the equations with respect to changes in the level of technology \(t_2\). The procedure involved is to derive the equations of change for the system of the model, and then solve for the unknown variables. To focus on the income inequality problem, we only present the solutions to factor prices and national income in the main body of this chapter. The other results are reported in the appendix on equations of change.

### 3.3. Consequences of trade liberalization:

In this section of the thesis, we examine the consequences of trade liberalization for factor returns and national income as captured by the above twelve equations. Trade liberalization has occurred in many countries that have liberalized in recent years - both developed and less developed countries. This process consists of both lowering the prices of imported goods and increasing that of exported goods, which implies

\(^{24}\) According to the Walras's law, if two of equations (3.10) to (3.12) are satisfied, the third equation must hold true;

\(^{25}\) We assume that foreign capital owner repatriate back the entire capital income in the absence of any foreign income tax;
that the price of imported good will fall relative to that of exported good. Therefore, we can see that \( \hat{P} < 0 \).

From equation (3.7) we know that:

\[
\hat{w}_i = \frac{\hat{p}}{\hat{\theta}_{l_1}} < 0
\]  \hspace{1cm} (3.13)

where \( \theta_{l_1} \) is the relative share of unskilled-labour in the total cost of commodity \( X_1 \).

We also know from equation (3.8) that:

\[
\hat{w}_s = 0
\]  \hspace{1cm} (3.14)

Since \( 0 < \theta_{l_1} < 1 \), it is easy to show that \( \hat{w}_s = 0 > \hat{P} > \hat{w}_i \). The relative changes in factor prices are proportionally greater than the changes in commodity price induced by trade liberalization. The magnification effects of changes in goods prices on wages are still valid, implying that the real wage rate of unskilled-labour falls after trade liberalization.

The above results are presented graphically in Figure 3.4. Since \( P \) declines, the unit price curve for commodity \( X_1 \) in quadrant I shifts leftward. As a result, \( w_1 \) will decrease while the exogenous variable \( r^* \) remains unchanged. Unit price curve for commodity \( X_2 \) in quadrant II keeps constant, so there is no change in the \( w_s \).

Figure 3.4
Hence it follows that:

$$\hat{\omega}_s - \hat{\omega}_i = -\frac{\hat{P}}{\theta_{l1}} > 0$$  \hspace{1cm} (3.15)

In this model, wage inequality increases as a result of trade liberalization. Unskilled workers become worse off relative to skilled workers.

Before we proceed to discuss the national income consequences for the economy in question, we need to mention again that foreign capital will flow out from this economy after trade liberalization. An explanation for this capital outflow has been given before.

To find out the change in national income from the fall in the relative price of imported goods, we develop the following welfare criterion. Following Caves and Jones (1985), we define change in real national income as follows:

$$dI = PdD_1 + dD_2$$  \hspace{1cm} (3.16)

By assuming utility-maximisation, national income consequences can also be derived from equation (3.9):

$$\hat{I} = -\alpha_i\hat{P} > 0$$  \hspace{1cm} (3.17)

where $\alpha_i = \frac{M_P}{I}$ denotes the relative share of national income represented by the value of imported goods.

In the presence of perfectly mobile capital, national income necessarily improves after trade liberalization due to the direct effect of the improvement in the term of trade. Trade liberalization in the economy in question results in a decrease in relative price of the imported commodity. The whole society benefits from consuming relative cheaper imported goods, and if calculated in terms of percentage changes in national income, the benefits is up to $-\alpha_i\hat{P}$. However, although trade liberalization leads to an improvement in national income, it only works in favour of the skilled-labour group as against the unskilled labour group.
3.4. Consequences of technical progress

Another important factor contributing to income inequality is technical progress incurred in the process of economic development. In this section, we apply the same framework of analysis to an examination of the process of adjustment to technological progress in the economy.

Consider that Hicks-neutral technological progress takes place in the production of $X_2$. We assume that neutral technological progress only takes place in the production of $X_2$, because technology is produced by skilled labour and in this case skilled labour is specific to the export-industry. We still assume that the factor market is perfectly competitive and the owners of the factors are profit-maximisers. We also assume that, for the moment, the prices of traded goods remain unchanged.

Hicks-neutral technical progress taking place in the production of $X_2$ implies that improvement in the technology in this sector induces equal percentage reductions in inputs of skilled-labour and capital to gain same level of output. Equation (3.2) is now replaced by

$$X_2 = F_2[S_2, K_2, t_2] = t_2 F_2[S_2, K_2]$$

(3.18)

where $t_2$ is a parameter representing the level of technology in sector $X_2$ and is assumed to be equal to unity initially.

We also have an alternative expression of technological change. Consider the cost-minimizing input-output coefficient $a_{ij}$, which depends on both relative factor prices and the state of technology. In terms of the relative rates of change, $\hat{a}_{ij}$ can be decomposed as

$$\hat{a}_{ij} = \hat{c}_i - \hat{b}_{ij}$$

(3.19)

$^{26}$In a general equilibrium model, it is the sector-biased technological changes, not the factor-biased technological changes, that determine the effect on factor prices (see Leamer 1993, 1996). Therefore, only sectoral technical progress is assumed to take place;

$^{27}$The approach of disaggregating $\hat{a}_{ij}$ comes from Jones (1965);
where $c_{ij}$ denotes the relative change in the input-output coefficient that results from a change in factor prices for a given technology. And $\hat{b}_{ij}$, defined as $\frac{1}{a_{ij}} \times \frac{\partial a_{ij}}{\partial t}$, is a measure of the contribution of technological change to the input-output coefficient at constant factor prices\(^{28}\). Technological progress is assumed to be neutral in the Hicksian sense, implying that at unchanged relative factor prices, factor proportions used in the production of $X_2$ do not change. Specifically, we can say,

$$\hat{B}_2 = \hat{b}_{L_2} = \hat{b}_{K_2} = \hat{i}_2$$  \hspace{1cm} (3.20)

where $\hat{B}_2$ represents the uniformity of the relative changes in $a_{ij}$ due to Hicks-neutrality.

We also further develop the national income equation by adding the expression for total value of domestic factors of production:

$$P_1D_1 + D_2 = P_1X_1 + X_2 - r^*K' = \bar{w}_1\bar{L} + \bar{w}_2\bar{S} + r^*\bar{K}^d$$  \hspace{1cm} (3.21)

Other equations making up the model are unchanged.

We begin to examine the impact of technological changes in the $X_2$ sector on endogenous variables. Suppose now that technological changes occurs in the production of $X_2$, implying that:

$$\hat{i}_2 > 0$$  \hspace{1cm} (3.22)

Totally differentiating equation (3.7) and (3.8), we get:

$$\hat{\bar{w}}_1 = 0$$  \hspace{1cm} (3.23)

$$\hat{\bar{w}}_2 = \frac{\hat{i}_2}{\bar{\theta}_{L2}} > 0$$  \hspace{1cm} (3.24)

Since there is no change in the production of $X_1$, the wage rate of unskilled labour remains constant. Technical progress in the $X_2$ sector works toward a reduction in

---

\(^{28}\) Please be noted that $b_{ij}$ is expressed in positive terms;
the cost of factors of production. It is clear that for the same commodity price to prevail there must be an increase in factor prices. As a result, the wage rate of skilled labour rises. Therefore, unskilled workers become worse off relative to skilled workers. Income inequality between skilled labor and non-skilled labor increases.

The consequence of technical progress in the production of $X_2$ can be illustrated in Figure 3.5\textsuperscript{29}. In figure 3.5, the initial equilibrium is shown with the isoquant curve $X_2$ tangential to the factor-price-ratio curve $AB$. The tangencies at point $M$ indicate that in producing one unit of good $X_2$, $OC$ units of capital and $OD$ units of skilled-labour are employed. Suppose that Hicks-neutral technical progress takes place in the production of $X_2$. It implies merely the renumbering of the isoquant curve for $X_2$, so that the factor proportions chosen in response to any factor-price ratio remain unchanged for all factor-price ratios. Now the new isoquant curve $X_2'$, lies below the old one for the same level of output. However, with the fixed good price $P_2$ and rental return to capital $r^*$, the wage rate of skilled labour will eventually rise in order to remove the cost-reducing effect of technical progress. This can be seen from the shift leftward of the factor-price-ratio curve $A'B'//(AB)$ to $EF$, implying a lower relative price of capital to skilled labour. This new factor-price-ratio curve is tangential to the new isoquant curve $X_2'$ at point $N$ with capital and labour employed $OG$ and $OH$ respectively. As a result, the capital-labour ratio rises, since $\frac{OC}{OH} > \frac{OC}{OD}$. Therefore, due to Hicks-neutral technical progress, less skilled-labour and less capital is required to produce one unit of $X_2$. However, the reduction in the requirement of skilled-labour is larger than that of capital since relatively cheap capital is used to substitute for the relatively expensive labour.

\textsuperscript{29} This technique is originated by Lerner (1932), and revived by Findlay and Grubert (1959);
However, the process has not been completed. The above information then can be used in the Edgeworth box to determine the impact of technical progress in sector $X_2$ on the economy as a whole. Unlike the result from H-O model where technical progress in one sector will have an impact on both sectors, fixed unskilled-labour endowment in sector $X_1$ and international mobility of capital specified in the this model ensure that output of $X_1$ will not be influence by the technical progress in $X_2$. Therefore, the lower-left part of the Edgeworth box remains the same as shown in figure 3.1. Now, we just need to rebuild the upper-right part of figure 3.1 to show the effect of technical progress on the production of $X_2$.

As already explained in figure 3.5, after technical progress takes place, less capital and much less skill-labour is used to produce the same quantity of $X_2$. New production point is given by $G$ with a higher wage-rental ratio (given by the slope of curve $EG$) and a higher capital-labour ratio (given by the slope of curve $GO_2^*$). Under the assumption of full employment, more capital is needed to produce commodity $X_2$ at the new capital-labour ratio. Therefore, foreign capital flows in.
The amount of the inflow of foreign capital is equal to \( O_2^m O_2^M \) \((GO_2^m \parallel AO_2^M)\). Eventually, the economy in question will produce at point A (different from the initial production point A) along a higher level of isoquant curve \( X_2'' \) with the corresponding new sector wage-rental ratio (represented by the slope of dotted curve DA)\((DA \parallel EG)\). Output of \( X_2 \) will increase.

Differentiating the national income equation (3.20), we obtain that:

\[
\dot{I} = \alpha_2 \dot{i}_2 > 0 \tag{3.25}
\]

where \( \alpha_2 = \frac{X_2}{I} \) is the relative share of the value of export goods in national income.
Therefore, national income is always going to improve at the expense of some particular groups being worse off.

We come to the conclusion that technical progress in the skilled-labour intensive exportable sector in the economy in question works in favour of skilled labour and against non-skilled labour and therefore increase the income inequality between the two groups. However, national income will unambiguously improve even if the unskilled labour group loses.
Appendix 3.1: The derivation of equations of change – section 3.3

To discuss the effect of trade liberalization on the variables, we can convert the equations in the system into the equations of change. Let hat (^) denotes the relative change of variables.

Taking the \( P \) and \( r^* \) as exogenously given, equation (3.7) and (3.8) represent two equations in two endogenous variables \( w_1, w_5 \). Totally differentiating these two equations by using the cost minimising condition to obtain:

\[
\theta_{t_1} \hat{w}_1 = \hat{P} \quad \text{(A3.1.1)}
\]

\[
\theta_{s_2} \hat{w}_5 = 0 \quad \text{(A3.1.2)}
\]

where \( \theta_q \) is the relative share of the \( q \)-th factor in the total cost of the \( j \)-th commodity.

It is clear that, in this model factor prices are independent of factor endowment and depend exclusively on goods prices. Equations (A3.1.1) and (A3.1.2), then, can be converted into equation (3.13) and (3.14) respectively, which show that the wage rate of unskilled labour falls while that of skilled labour remains unchanged.

Equation (3.3) to (3.5) give us a system of 3 equations in 3 unknowns. Given \( L_j \), \( K^d (j = X_1, X_2) \), we can determine the unknowns - \( X_j \) and \( K' \).

\[
\hat{X}_1 + \hat{a}_{L_1} = \hat{L}_1 \quad \text{(A3.1.3)}
\]

\[
\hat{X}_2 + \hat{a}_{s_2} = \hat{S}_2 \quad \text{(A3.1.4)}
\]

\[
\lambda_{K_1} \hat{X}_1 + \lambda_{K_1} \hat{a}_{K_1} + \lambda_{K_2} \hat{X}_2 + \lambda_{K_2} \hat{a}_{K_2} = \lambda_{K'} \hat{K}' \quad \text{(A3.1.5)}
\]

Where \( \lambda_q (i = L_1, L_2, K_1, K_2) \) is the proportion of the \( i \)-th factor employed in the \( j \)-th commodity and \( \lambda_{K'} \) the proportion of foreign capital used in the economy.

---

\(^{30}\) The “hat” calculus is popularised by Jones (1965, 1971);
The above equations can be further simplified after we introduce the concept of input-output coefficients $a_{ij}$. And the relative changes of input-output coefficients $\hat{a}_{ij}$ can be expressed in terms of the factor prices as follows\(^{31}\):

\[
\hat{a}_{L1} = -\theta_{k1} \sigma_1 (\hat{\hat{w}}_1 - \hat{\hat{r}}) \tag{A3.1.6}
\]
\[
\hat{a}_{S2} = -\theta_{k2} \sigma_2 (\hat{\hat{w}}_s - \hat{\hat{r}}) \tag{A3.1.7}
\]
\[
\hat{a}_{Kj} = \theta_{Lj} \sigma_j (\hat{\hat{w}}_j - \hat{\hat{r}}) \tag{A3.1.8}
\]

where $\sigma_1 = (\hat{a}_{k1} - \hat{a}_{L1}) / (\hat{\hat{w}}_1 - \hat{\hat{r}})$ and $\sigma_2 = (\hat{a}_{k2} - \hat{a}_{S2}) / (\hat{\hat{w}}_s - \hat{\hat{r}})$ are the elasticity of substitution between factors in the production of commodity $X_1$ and $X_2$ respectively and both of them are positive.

Substituting equations (A3.1.6) - (A3.1.8) and (A3.1.1) - (A3.1.2) into (A3.1.3) and (A3.1.4), we obtain the expression for the relative changes in output with respect to changes in commodity prices:

\[
\hat{x}_1 = \frac{\theta_{k1} \sigma_1 \hat{\hat{p}}}{\theta_{L1}} < 0 \tag{A3.1.9}
\]
\[
\hat{x}_2 = 0 \tag{A3.1.10}
\]

This implies that the output of the importable sector will decrease while that of the exportable sector remains constant.

By totally differentiating equation (3.12) and then substituting equation (3.10) into it, we obtain the following result, as shown in equation (3.17) of the text:

\[
\hat{f} = -\alpha \hat{\hat{p}} > 0 \tag{A3.1.11}
\]

Since $\hat{\hat{p}} < 0$, national income for the economy in question will inevitably improve after trade liberalization.

---

\(^{31}\) $\hat{a}_{ij}$ is a function of the factor prices and of the state of technology $t_i$. In this case by assuming that technology is constant, $a_{ij}$ depends solely upon the relative factor prices;
Appendix 3.2: Relationship between $P$ and $r$

In equation (3.9.1) in chapter 3, $\varepsilon_{K/R}$ is assumed to be positive. It is derived from the relationship between $P$ and $r$ in a simple three-factor and two-good model without foreign capital.

From the assumption of perfect competition, price equations follow that:

$$a_{L}w_{L} + a_{K}r = P \quad \text{(A3.2.1)}$$
$$a_{S}w_{S} + a_{K}r = 1 \quad \text{(A3.2.2)}$$

Factor endowment equations are defined as:

$$a_{L}X_{L} = L \quad \text{(A3.2.3)}$$
$$a_{S}X_{S} = S \quad \text{(A3.2.4)}$$
$$a_{K}X_{K} + a_{K}X_{S} = K \quad \text{(A3.2.5)}$$

We now have five unknowns - $w_{L}, w_{S}, r, X_{L}$, and $X_{S}$ in five equations. By differentiating equations (A3.2.1) to (A3.2.5), and using the cost-minimising condition, we have:

$$\theta_{L} \hat{w}_{L} + \theta_{K} \hat{r} = \hat{P} \quad \text{(A3.2.6)}$$
$$\theta_{S} \hat{w}_{S} + \theta_{K} \hat{r} = 0 \quad \text{(A3.2.7)}$$
$$\lambda_{K} \sigma_{L} \hat{w}_{L} + \lambda_{K} \sigma_{S} \hat{w}_{S} = (\lambda_{K} \sigma_{K} + \lambda_{K} \sigma_{K}) \hat{r} \quad \text{(A3.2.8)}$$

Suppose that $P$ declines after trade liberalization. From equation (A3.2.6) to (A3.2.8), we can get the relationship between $P$ and $r$:

$$\hat{r} = \frac{D_{1}}{D} \hat{P} \quad \text{(A3.2.9)}$$

where $D_{1} = \lambda_{K} \theta_{L} \sigma_{K} (\lambda_{K} \sigma_{K} + \lambda_{K} \sigma_{K}) > 0$

and $D = \lambda_{K}^{2} \theta_{S} \sigma_{K}^{2} + \lambda_{K}^{2} \theta_{L} \sigma_{K}^{2} + \lambda_{K} \lambda_{K} \sigma_{K} \sigma_{K} (\theta_{L} + \theta_{S}) > 0$. 

---

42
Since $D_1 > 0$ and $D > 0$, we find that $\frac{\dot{r}}{\dot{P}} > 0$.

Therefore, the relationship between $r$ and $P$ is always positive, independent of the factor intensity in $X_1$ or $X_2$. The decline of $P$ after trade liberalization will lead to a decrease in the domestic rental rate of capital $r$. As shown in Figure 2, VMPK curve will shift leftward. The amount of foreign capital imported in this country will decrease.

So the relationship between $P$ and $K^f$ is positive. As expressed in equation (3.9.1), $\mathcal{E}_{K^fP} > 0$.

**Appendix 3.3: The derivation of equations of change – section 3.4**

Equations (3.3) to (3.5) and (3.7), (3.8) give us a system of 5 equations with 5 unknowns. Given $t_2$, $P_j$, $L$, $\bar{S}$, $K^d$ and $r^*$, we can easily determine the five unknowns - $w_j$, $X_j$ and $K^f$.

Totally differentiating equations (3.3) to (3.5) and (3.7),(3.8), we obtain:

\[
\begin{align*}
\theta_{\lambda t} \hat{\hat{w}}_1 &= 0 & \quad \text{(A3.3.1)} \\
\theta_{s z} \hat{\hat{w}}_s &= \hat{t}_2 & \quad \text{(A3.3.2)} \\
\hat{\hat{X}}_1 + \hat{\hat{a}}_{L1} &= 0 & \quad \text{(A3.3.3)} \\
\hat{\hat{X}}_2 + \hat{\hat{c}}_{S2} &= \hat{t}_2 & \quad \text{(A3.3.4)} \\
\lambda_{K1} \hat{\dot{X}}_1 + \lambda_{K1} \hat{\dot{a}}_{K1} + \lambda_{K2} \hat{\dot{X}}_2 + \lambda_{K2} \hat{\dot{c}}_{K2} - \lambda_{K^f} \hat{\dot{K}}^f = \lambda_{K^f} \hat{\dot{t}}_2 & \quad \text{(A3.3.5)}
\end{align*}
\]

Equations (A3.3.1) and (A3.3.2) can be easily converted into equation (3.23) and (3.24) to show the effect of technical progress on factor prices. Wage rate of skilled labour in the production of the $X_2$ where technical progress takes place will rise while that of unskilled labour in the production of $X_1$ keeps constant.
Then by introducing the term of the elasticity of substitution $\sigma_j$ and substituting the result of $w_1$ and $w_2$ into equations (A3.3.3) and (A3.3.4), we can derive the solution for the output of $X_1$ and $X_2$.

\[
\hat{X}_1 = \theta_{k1} \sigma_{k1} (\hat{w}_1 - \hat{r}^*) = 0
\]

\[
\hat{X}_2 = \left( \frac{\theta_{k2}}{\theta_{s2}} \sigma_{k2} + 1 \right) \hat{y}_2 > 0
\] (A3.3.6) (A3.3.7)

Then, substituting equations (A3.3.6) and (A3.3.7) into equation (A3.3.5) to yield:

\[
\hat{K}^f = \frac{\lambda_{k2} \sigma_{k2} \hat{y}_2}{\lambda_k \theta_{s2}} > 0
\] (A3.3.8)

Output of $X_2$ will always increase due to technical progress in this sector while that the output of $X_1$ remains the same. Foreign capital will further flow into the economy in question to assist the production of $X_2$ since the wage rate of skilled-labour is now higher and entrepreneurs would rather substitute capital inputs for labour inputs.

Totally differentiating equation (3.21) and with some manipulations, we can obtain the effect of technical progress on national income. The effect is shown in equation (3.25) of the text. It tells us the national income for the economy in question will inevitably improve when there is technological change in the export sector.

**Appendix 3.4: Validity of the model**

International capital mobility could lead the economy in question to specialise in production, though the result is not inevitable. Throughout the whole paper, we confine attention to non-specialised production in the economy in question. We simply assume that, with given stocks of labour and capital, there exists equal commodity prices and equal rate of returns to capital between the economy in question and the rest world and both are diversified in the production.
However, one question may still arise that whether there exists such equilibrium under which both the economy in question and the rest world are incompletely specialised in production. This question has long been discussed in the literature. As for the two-country model in which labour is immobile but capital is perfectly mobile internationally, Jones (1967) claims that countries tend to specialise in production under free goods trade and capital movement. However, Inada and Kemp (1969), Chipman (1971) argue that there are some quite general condition for the existence of an equilibrium situation with diversification in the presence of perfectly capital mobility. Uekawa Yasuo (1972) further derives more sufficient conditions for the existence of an efficient production point with diversification.
Chapter 4: Trade, Technical Progress and Income Inequality
- a Three-Good, Three-Factor Model

4.1. Introduction:
In the previous chapter, we have examined the income distribution effect of trade liberalization and technical progress in a simple 3-factor, 2-good general equilibrium model. In that model, we assume the economy in question only produces two commodities, one for export and another for import. Now in this chapter, a non-traded good is introduced and the model is extended to a 3-factor, 3-good model.

Each country produces not only traded goods, but also the goods for pure domestic consumption. The existence of non-traded goods results from various reasons, such as high costs of transport, different tastes, or the nature of the goods. For example, many services are nontradable. And it is interesting to note that the amount of non-traded goods actually accounts for a large portion of GDP in most countries. Therefore, it is necessary to integrate non-traded goods into models analyzing international trade.

The model with non-traded good, which is extended from the traditional 2 x 2 H-O model, generally takes the form of three commodities (one exportable, one importable and one non-tradable good) and two or more factors of production. It has often been assumed that, while prices of traded goods are given exogenously, the prices of non-traded goods are determined endogenously in the model by domestic supply and demand conditions. However, when a non-traded good is introduced, the responses to any changes in the economy become much more complicated because of the repercussions through the demand and supply of the non-traded good. The presence of non-traded goods is influenced by and is to influence the traded sector in some way.
Some of the basic theorems derived from the traditional H-O model must be modified when non-traded goods are introduced. In the context of income inequality problem, the conclusion drawn from previous chapter in a simple model without non-traded goods may not readily extend to the case with non-traded good. Therefore, in this chapter, we introduce a non-traded good in our model, while retaining its other basic properties. It is assumed that the non-traded good is produced with the help of non-skilled labour and capital. In the extended model with three factors of production (skilled labour, unskilled-labour and capital) and three commodities (exportable, importable and non-tradable), we still assume that capital is internationally and intersectorally mobile and labour sector-specific\(^{32}\). The object of this chapter is to analyze the issue of changes in the distribution of income associated with trade liberalization and technical progress in such a more general model with foreign capital. National income consequences are also analyzed.

4.2. Model:

We consider a small open economy that produces three goods: the importable \(X_1\), the exportable \(X_2\) and a third non-traded good, \(X_N\). Both commodity \(X_1\) and \(X_2\) are produced with the help of non-skilled labour \(L\), skilled labour \(S\) and capital \(K\); while commodity \(X_N\) is produced with non-skilled labour \(L_N\) and capital \(K\). The neoclassical production functions are given below:

\[
\begin{align*}
X_1 &= F_1(L_1, S_1, K_1) \tag{4.1} \\
X_2 &= F_2(L_2, S_2, K_2) \tag{4.2} \\
X_N &= F_N(L_N, K_N) \tag{4.3}
\end{align*}
\]

where \(L_j\), \(S_j\) and \(K_j\) \((j = 1, 2, N)\) are the unskilled labour, skilled labour and capital used in the \(j^{th}\) sector respectively. Note that skilled labour is specific to the production of commodities \(X_1\) and \(X_2\) while unskilled labour and capital are perfectly mobile across all the three sectors. It is also assumed that capital is

\(^{32}\) To be more realistic, in the present model, we assume that both imported good \(X_1\) and exported good \(X_2\) are produced with the help of non-skilled labour, skilled-labour and capital. However, factor intensities in the production of \(X_1\) and \(X_2\) are different;
internationally mobile and the country in question is a net capital importer. The amount of foreign capital imported is denoted by $K'$. 

The factor utilisation equations are:

$$a_{L1}X_1 + a_{L2}X_2 + a_{LN}X_N = \bar{L}$$  \hspace{1cm} (4.4)

$$a_{S1}X_1 + a_{S2}X_2 = \bar{S}$$  \hspace{1cm} (4.5)

$$a_{K1}X_1 + a_{K2}X_2 + a_{KN}X_N = \bar{K}^d + K' = K$$  \hspace{1cm} (4.6)

Where $a_{ij}$'s denote variable input coefficients; L, S and $\bar{K}^d$ the inelastic supply of the factors - unskilled labour, skilled labour and domestic capital. The term $K'$ shows the endogenously determined supply of foreign capital.

We shall assume that the country is small in both the commodity market and the factor market for foreign capital, hence, the two commodity prices $P_1$, $P_2$ and the return to foreign capital $r^*$ are given from outside. The pricing equations for the system are:

$$a_{L1}w_1 + a_{S1}w_S + a_{K1}r^* = P$$  \hspace{1cm} (4.7)

$$a_{L2}w_1 + a_{S2}w_S + a_{K2}r^* = 1$$  \hspace{1cm} (4.8)

$$a_{LN}w_1 + a_{KN}r^* = P_N$$  \hspace{1cm} (4.9)

Where the terms $w_1$ and $w_S$ denote the wages of unskilled and skilled workers in the economy. And for analytical conveniences, commodity $X_2$ is taken as the numeraire. $P = P_1/P_2$, represents the relative price of commodity $X_1$, while $P_N$ is the relative price of commodity $X_N$.

As is standard in this literature, the $a_{ij}$'s are functions of the factor prices and the state of technology $t_j$, so that:

$$a_{ij} = a_{ij}(w_1, w_S, r^*, t_j)$$  \hspace{1cm} (4.10)

where $i = L, S, K$ and $j = 1, 2, N$. 

48
In figure 4.1, the price determination procedure is shown. A small open economy takes the rental on capital as given, and hence, \( r \) is specified exogenously. Given the values of \( r^* \), \( P \), the net unit price curves for commodity \( X_1 \) and commodity \( X_2 \) are drawn in quadrant I. The intersection of the two net unit-price curves determines the equilibrium factor prices. The values of \( W_i^* \) and \( W_j^* \) are attained as shown by \( W_i^* \) and \( W_j^* \) in quadrant I. We shall assume that \( W_i^* > W_j^* \). Once \( W_i^* \) is attained, the position of the unit price curve for commodity \( X_N \) can be determined in quadrant II, with the exogenously-given \( r^* \).

The endogenously determined amount of foreign capital inflow is shown in Figure 4.2. The value of the marginal product curve for the economy as a whole is drawn as the \( VMP_x \) curve in Figure 4.2. The supply of domestic capital is shown by \( \dot{K}^d \) and by the vertical line \( \dot{K}^d A \). The foreign supply curve for capital is shown by the curve \( r^*B \). Equilibrium is attained at point e where the value of the marginal product of capital equals the foreign rental rate of capital. The amount of foreign capital

---

49
imported is shown by $\overline{K}^d K^f$. We define the following function to take the inflow of foreign capital into account:

$$K^f = K^f (P, t_1)$$  \hfill (4.11)

In this case, We shall assume that:

$$\frac{P}{K^f} \frac{\partial K^f}{\partial P} = \varepsilon_{K^f P} < 0$$  \hfill (4.11.1)

where $\varepsilon_{K^f P}$ denotes the price elasticity of demand for foreign capital. In previous chapter, the sign of $\varepsilon_{K^f P}$ has been justified to be positive. Now in the extended model, $\varepsilon_{K^f P}$ is negative, due to the differences in the production structure of the economy in question. Explanations in details are given in Appendix 4.2.

Therefore, if $P$ decreases after liberalizing trade, there will be an inflow of foreign capital into this economy. As shown in Figure 2, the inflow of foreign capital due to trade liberalization equals $K^f \tilde{K}$.

Figure 4.2

---

33 See Appendix 4.2;
Fore the same reasons explained in the previous chapter, we do not specify the relationship between the level of technology $t_j$ and the flow of foreign capital $K^f$.

We still assume that the economy in question imports commodity $X_1$ and exports commodity $X_2$. The market clearing condition requires that:

$$ D_1 = X_1 + M_1 $$  \hspace{1cm} (4.12)
$$ D_2 = X_2 - E_2 $$  \hspace{1cm} (4.13)
$$ D_N(P, P_N, I) = X_N $$  \hspace{1cm} (4.14)

We shall note that the non-traded good market is locally stable. The price of the non-traded good, $P_N$, is determined endogenously, through the influence of domestic demand and supply for the non-traded good. To achieve equilibrium in the domestic market, the demand for the non-traded goods $D_N$, which is a function of relative prices and income, must be equal to the supply of the non-traded good $X_N$.

Finally we introduce the national income equation from both sides of the market:

$$ P D_1 + D_2 + P_N D_N = P X_1 + X_2 + P_N X_N - \rho^* K^f $$  \hspace{1cm} (4.15)

This completes the specification of the model. The present model with non-traded good comprises eighteen equations [(4.4) – (4.9), eight equations implied by (4.10), (4.11) and three equations from (4.12) to (4.15)], in eighteen unknown variables (eight of $a_{ij}, X_1, X_2, X_N, W_1, W_s, P_N, K^f, D_1, D_2$ and $I$) with seven exogenous variables ($P, t_j, K^d, L$ and $S$).

We then follow the same approach as used in chapter three to examine the effects of trade liberalization and technical progress on income distribution and national income.
4.3. Consequences of trade liberalization:

In this section of the thesis, we examine the consequence of trade liberalization on factor returns and national income as captured by the above eighteen equations. We still assume that trade liberalization will result in a lower relative prices of imported goods. Thus, we have \( \hat{P} < 0 \).

By totally differentiating equations (4.7) - (4.9), we can obtain solutions for the following variables:

\[
\hat{w}_1 = \frac{\theta s_1}{|\theta|} \hat{p} \quad (4.16)
\]

\[
\hat{w}_s = - \frac{\theta s_2}{|\theta|} \hat{p} \quad (4.17)
\]

\[
\hat{p}_x = \frac{\theta_{s_2} s_2}{|\theta|} \hat{p} \quad (4.18)
\]

and \( |\theta| = \theta_{s_1} \theta_{s_2} \frac{w_s}{w_1} (s_2 - s_1) \), where \( s_j \) denotes skilled-labour to unskilled-labour intensity in the \( j^{th} \) sector. Therefore, the sign of \( |\theta| \) depends on relative factor intensity in the traded sectors. If \( s_2 > s_1 \) (\( X_2 \) is more skilled-labour intensive than \( X_1 \)), \( |\theta| > 0 \). If \( s_2 < s_1 \) (\( X_1 \) is more skilled-labour intensive than \( X_2 \)), \( |\theta| < 0 \).

We know from equations (4.16) and (4.17) that:

\[
\hat{w}_s - \hat{w}_1 = \frac{(\theta_{s_2} - 1)}{|\theta|} \hat{p} \quad (4.19)
\]

The impact of trade liberalization on income distribution changes depends on the relative factor intensities in the production of \( X_1 \) and \( X_2 \). If \( X_2 \) is more skilled-labour intensive than \( X_1 \), wage inequality will be widening. If \( X_2 \) is more unskilled-labour intensive than \( X_1 \), wage inequality will be reduced. We are now going to examine both cases separately.
Proposition 4.1: if $|\theta| > 0$, then $\hat{w}_s - \hat{w}_1 > 0$.

As long as the exported commodity $X_2$ is more skilled-labour intensive than the imported commodity $X_1$, implying that $|\theta| > 0$ (since $s_2 > s_1$), then the differences between wage rate of skilled-labour and that of unskilled-labour will widen when the relative price of the imported good $P$ falls after trade liberalization. When $|\theta| > 0$, it is always true that $\hat{w}_s > 0 > \hat{P} > \hat{w}_1$ and $\hat{w}_s > 0 > \hat{P}_N > \hat{w}_1$. The magnification effects are still valid. The real wages of unskilled labour will decrease and those of skilled labour increase after trade liberalization.

Hence, assuming that $X_1$ is non-skilled labour intensive vis-à-vis the $X_2$ sector in this model, wage inequality will increase as a result of trade liberalization. Unskilled workers become worse off relative to skilled workers. This condition aligns with reality in developed countries since generally these countries import unskilled labour intensive commodities and export skilled-labour intensive products. The fall in $w_1$ will also result in a decrease in the relative price of the non-traded good, $P_N$, since there is a reduction in the cost of production.

The results can be easily illustrated in Figure 4.3. Since relative price of imported good $P$ declines, the net unit price curve for commodity $X_1$ in quadrant I will shift leftward, while the net unit price curve for commodity $X_2$ is constant. As a result, $w_1$ will decrease and $w_s$ will rise. The new equilibrium wage rate of unskilled-labour is $w_{1''}$, and that of skilled-labour is $w_{s''}$. With a new lower $w_1$ and a fixed $r^*$, the unit price curve $P_N$ in quadrant II will shift downward to $P_N'$.

---

34 The positions of those two net unit price curves have already implied that commodity $X_1$ is more unskilled-labour intensive than commodity $X_2$.
Proposition 4.2: if $|\theta| < 0$, then $\hat{w}_s - \hat{w}_i < 0$.

In the case that the imported commodity $X_1$ is skilled-labour intensive vis-à-vis the exported commodity $X_2$, $\hat{w}_i > \hat{P}_i > 0 > \hat{P} > \hat{w}_s$, and then wage inequality will be reduced after trade liberalization. The condition for $|\theta| < 0$ may be more representative of reality in the developing countries since generally these countries import skilled-labour intensive commodities and export unskilled-labour intensive commodities. On the other hand, these countries are poorly endowed with skilled-labour and frequently face serious outflow of skilled-labour problem, which does not tallied with the situation we assumed in the economy in question.

Having examined both cases, we find that the impacts of trade liberalization on factor prices are related to the relative factor intensities in the production of $X_1$ and $X_2$. This is an interesting application of the S-S theorem in this 3-factor, 3-good model. According to S-S theorem, trade liberalization necessarily lowers the real reward of the factor used intensively in the import sector and increases the real reward of the other factor. Our result is consistent with what the S-S theorem predicts. The reason for the consistency is because that, capital, the third factor of
production in our model, is internationally mobile. In the presence of a given rental rate of capital \( r^* \), changes in relative good prices are totally born by changes in the other two factor prices: wage rate of skilled-labour and that of unskilled-labour. In the H-O model with two-factor, changes in good prices are also borne by changes in the two factor prices.

The next step is to examine the national income consequence of trade liberalization. By assuming utility-maximisation, solution for national income can also be derived from equation (4.15):

\[
\dot{I} = -\alpha_i \dot{P} > 0
\]  

(4.20)

where \( \alpha_i = \frac{M_i P}{I} \) denotes the relative share of national income represented by the value of imported goods.

Therefore, in the presence of perfectly mobile capital, national income in the economy always improves after trade liberalization since a movement towards free trade leads to a fall in imported prices. However, the group of skilled-labour will gain from trade liberalization at the expense of unskilled-labour on the condition that the imported good is more non-skilled labour intensive than the exported good.

4.4. Consequences of technical progress:

(a) The impact of Hicks-neutral technical progress in the export sector

In this section, we turn our attention to analyzing the consequences of technological progress on distribution of income in the economy. We first consider that neutral technical progress takes place in the export industry \( X_2 \). We assume that the \( X_2 \) sector is more skilled-labour intensive than \( X_1 \) sector, and therefore, technical progress will most likely take place in the production of \( X_2 \). This analysis is undertaken at constant commodity prices.
Equation (4.2) is now replaced by
\[ X_2 = F_2[L_2, S_2, K_2, t_2] = t_2 F_2[L_2, S_2, K_2] \] (4.21)
where \( t_2 \) is a parameter representing the level of technology and is assumed to be equal to unity initially.

An alternative expression of Hicks-neutral technological change is also given below:
\[ \hat{B}_2 = b_{l2} = b_{k2} = b_{ls2} = \hat{t}_2 \] (4.22)
where \( \hat{b}_j \), as defined as \(- \frac{1}{a_{ij}} \frac{\partial a_{ij}}{\partial t}\), a measure of technological change on the input-output coefficient that would take place at constant factor prices.

The national income equation is revised by adding the expressions for the total value of domestic factors of production:
\[ PD_i + D_2 + P_N D_N = P X_1 + X_2 + P_N X_N - r^* K' = w_l \bar{L} + w_s \bar{S} + r^* \bar{K}^d \] (4.23)
Other equations making up the model are unchanged.

Suppose that now technological change occurs in the production of \( X_2 \), implying that:
\[ \hat{t}_2 > 0 \] (4.24)

To examine the impact of technological changes in the \( Y \) sector on endogenous variables, we totally differentiate the equations of our system to the change in technology \( t_2 \).

Solutions for the changes in factor prices are derived from equations (4.7) – (4.9):
\[ \hat{w}_1 = \frac{-\theta_{s1}}{\theta} \hat{t}_2 < 0 \] (4.25)
\[ \hat{w}_s = \frac{\theta_{li}}{\theta} \hat{t}_2 > 0 \]  
(4.26)

\[ \hat{P}_N = -\frac{\theta_{ln} \theta_{i1}}{\theta} \hat{t}_2 < 0 \]  
(4.27)

where \(|\theta| = \theta_{li} \theta_{i2} \frac{w_s}{w_1} (s_2 - s_1)\). Having assumed the economy in question imports unskilled labour intensive goods \(X_1\) and exports skilled-labour intensive goods \(X_2\), \(|\theta|\) is positive. It is also true that \(\hat{w}_s > 0 > \hat{P}_N > \hat{w}_1\).

Since technical progress takes place in the production of \(X_2\), cost of production will decrease in this sector. In order to keep the commodity prices constant, the cost-reducing impact must be removed, and there must be an increase in the factor prices. Meanwhile, there should be no change in the cost of production of \(X_1\), which is produced with the help of the same factors as used in \(X_2\). This can be done by raising the price of the intensive factor used in the production of \(X_2\) while the price of the other factor fall. So the wage rate of skilled labour \(w_s\) will increase and that of unskilled labour \(w_1\) will be lower. And the fall of the wage rate of non-skilled labour indirectly reduces labour cost in the production of \(X_N\). Correspondently, the price of the non-traded good \(P_N\) will fall.

By subtracting equation (4.26) from (4.27), we obtain:

\[ \hat{w}_s - \hat{w}_1 = \frac{\theta_{li} + \theta_{i1}}{\theta} \hat{t}_2 > 0 \]  
(4.28)

It is clear that wage inequality widens as a result of technical progress in the production of \(X_2\).

The solution for national income can be derived from equation (4.23):

\[ \hat{i} = \alpha_i \hat{t}_2 > 0 \]  
(4.29)

Therefore, national income for the economy in question will always improve, regardless of relative factor intensity in the production of traded goods. It rises proportionally less to the extent of the technical progress in the production of \(X_2\).
(b) Impact of Hicks-neutral technical progress in the non-traded sector

The next case to be considered is that of technological progress incurred in the non-traded goods sector - $X_N$.

Suppose that neutral technological progress takes place in the $X_N$ sector. Equation (4.3) is then replaced by

$$X_N = F_N[K_N, L_N, t_N] = t_N F_N[K_N, L_N]$$

(4.30)

where $t_N$ is a parameter representing the level of neutral technology in the $X_N$ sector.

And now

$$\hat{t}_N > 0$$

(4.31)

Other equations in the previous model are unchanged.

Technological progress in the non-traded sector leads to much simpler consequences than it does in the previous case. The reason is because the wage rates to skilled-labor and unskilled labour are solely determined by the price of internationally traded goods in this case. The internationally determined commodity price is fixed, so that nominal factor prices remain unchanged. We come to the conclusion that income distribution between skilled labour and unskilled labour remains undisturbed by technical progress in the non-traded goods sector.

The price of non-traded goods will change since technical progress takes place in this sector. The solution is given from differentiating equation (4.9):

$$\hat{p}_N = -\hat{t}_N < 0$$

(4.32)

Technical progress in the non-traded sector is going to reduce the cost of production of $X_N$. So at given factor prices, the good price $P_N$ will fall proportionally to the same extent of technical progress. Since $\hat{w}_S = \hat{w}_L = 0 > \hat{P}_N$, both real wage rate of skilled-labour and that of unskilled-labour actually rise after technical progress takes place in the production of $X_N$. 
By totally differentiating the national income equation (4.24) in our system, we obtain the following result:

\[ \dot{I} = \alpha_N \dot{Y}_N > 0 \]  \hspace{1cm} (4.33)

where \( \alpha_N = \frac{t_N X_N P_N}{I} \) denotes the relative share of national income represented by the value of the non-traded good.

National income will inevitably improve, since there is an induced reduction in the price of the non-traded good. It rises proportionally less to the extent of technical progress in the production of the non-traded good. Income distribution between the skilled labour and unskilled labour remains undisturbed.
Appendix 4.1: The derivation of equations of change – section 4.3

Equations (4.4) to (4.9) and (4.11), (4.15) give us a system of 8 equations in 8 unknowns. Given $P, \bar{L}, \bar{S}, \bar{K}$ and $r^*$, we can determine the eight unknowns $w_1, w_s, P_N, X_1, X_2, X_N, K'$ and $I$.

By totally differentiating equations (4.7) - (4.9) using the cost minimising condition, we can derive the following equations:

\[ \theta_{L1} \dot{w}_1 + \theta_{S1} \dot{w}_s = \dot{\hat{P}} \]  
(A4.1.1)

\[ \theta_{L2} \dot{w}_1 + \theta_{S2} \dot{w}_s = 0 \]  
(A4.1.2)

\[ \theta_{LN} \dot{w}_1 = \dot{\hat{P}}_N \]  
(A4.1.3)

These equations give us solutions to three unknowns $w_1, w_s$ and $P_N$, as shown in equation (4.16) – (4.18) of the text.

Differentiating equation (4.15), we obtain solution for national income:

\[ \dot{I} = -\alpha_i \dot{\hat{P}} > 0 \]  
(A4.1.4)

National income will improve since price of the imported goods fall, as a consequence of trade liberalization.

We can also derive from equation (4.11) that:

\[ \dot{K}' = \varepsilon_{K'P} \dot{\hat{P}} \]  
(A4.1.5)

Since $\varepsilon_{K'P}$ is negative in this case, the amount of foreign capital in the economy will increase.

The solutions for the output of $X_j$ can be obtained from the following matrix:

\[
\begin{vmatrix}
\lambda_{L1} & \lambda_{L2} & \lambda_{LN} & \dot{\hat{x}}_1 \\
\lambda_{S1} & \lambda_{S2} & 0 & \dot{\hat{x}}_2 \\
\lambda_{K1} & \lambda_{K2} & \lambda_{KN} & \dot{\hat{x}}_N \\
\end{vmatrix} =
\begin{vmatrix}
\beta_{SL} \theta_{L2} - \beta_{LL} \theta_{S2} \\
\beta_{SS} \theta_{L2} - \beta_{SL} \theta_{S2} \\
\varepsilon_{K'P} \lambda_{K'} \theta + \beta_{KL} \theta_{L2} + \beta_{KL} \theta_{S2} \\
\end{vmatrix}
\dot{\hat{P}} / \theta
\]

60
where \( \beta_{ik} = \sum \lambda_{ij} \theta_{ij} \sigma_{ik} \)

with \( \sigma_{ik} \) denoting the partial elasticity of substitution between factors of production \( i \) and \( k \) (\( i, k = L, S, K \)) in the \( j^{th} \) sector.

We have the determinant of the above matrix. This turns out to be:
\[
D = \lambda_{K_N} (\lambda_{S_1} - \lambda_{L_1}) + \lambda_{L_N} (\lambda_{K_1} - \lambda_{S_1}).
\]
And \( D > 0 \) if \( \lambda_{K_1} > \lambda_{S_1} > \lambda_{L_1} \).

The results for \( \hat{X}_1, \hat{X}_2 \) and \( \hat{X}_N \) are determined by many elements: factor intensities, relative factor shares in production, substitution or complementary relationship between factors of productions. Whether the results are positive or negative depend on which determining element dominates.

**Appendix 4.2: Relationship between \( r \) and \( P \) in a model with non-traded good**

In equation (4.10.1), \( \mathcal{E}_{K^r}P \) is assumed to be negative. It is derived from the relationship between \( P \) and \( r \) in a three-factor and three-good model without foreign capital.

From the assumption of perfect competition, price equations follow that:
\[
\begin{align*}
a_{L_1}w_1 + a_{S_1}w_s + a_{K_1}r &= P \quad (A4.2.1) \\
a_{L_2}w_1 + a_{S_2}w_s + a_{K_2}r &= 1 \quad (A4.2.2) \\
a_{L_N}w_1 + a_{K_N}r &= P_N \quad (A4.2.3)
\end{align*}
\]

The market clearing condition requires that:
\[
D_N(P, P_N, I) = X_N(P_N) \quad (A4.2.4)
\]
The national income equation are defined as follows:

\[ PD_1 + D_2 + P_N D_N = PX_1 + X_2 + P_N X_N \]  \hfill (A4.2.5)

With five unknowns - \( w_1, w_2, r, P_N \) and \( I \) in five equations, we can get the relationship between \( P \) and \( r \) by simply differentiating the above equations:

\[ \theta_{L_1} \hat{w}_{L_1} + \theta_{S_1} \hat{w}_{S} + \theta_{K_1} \hat{r} = \hat{P} \]  \hfill (A4.2.6)

\[ \theta_{L_2} \hat{w}_{L_2} + \theta_{S_2} \hat{w}_{S} + \theta_{K_2} \hat{r} = 0 \]  \hfill (A4.2.7)

\[ \theta_{LN} \hat{w}_{L} + \theta_{KN} \hat{r} = \hat{P}_N \]  \hfill (A4.2.8)

\[ A \hat{P}_N + \varepsilon_n \hat{i} = -\varepsilon_{NP} \hat{P} \]  \hfill (A4.2.9)

\[ \hat{i} = -\alpha_i \hat{P} \]  \hfill (A4.2.10)

where \( A = \frac{P_N}{D_N} \frac{\partial D_N}{\partial P_N} - \frac{P_N}{X_N} \frac{\partial X_N}{\partial P_N} < 0 \)

- Difference between price elasticity of demand for and supply of non-traded good

\[ \varepsilon_n \equiv \frac{I}{D_N} \frac{\partial D_N}{\partial i} > 0 \]

- Income elasticity of demand for non-traded goods

\[ \varepsilon_{NP} \equiv \frac{P}{D_N} \frac{\partial D_N}{\partial P} = 0 \]

- Cross-elasticity of demand for non-traded goods: assumed to be zero for simplicity

This yields that:

\[ \hat{r} = \frac{D_1}{D} \hat{P} \]  \hfill (A4.2.11)

where \( D_1 = \left[ \varepsilon_n \alpha_i (\theta_{L_1} \theta_{S_2} - \theta_{S_1} \theta_{L_2}) - A \theta_{S_2} \theta_{LN} \right] / A < 0 \), on the condition that \( s_2 > s_1 \).

And \( D = \theta_{S_2} (\theta_{L_1} - \theta_{LN}) + \theta_{S_1} (\theta_{LN} - \theta_{L_2}) > 0 \), as long as \( \theta_{L_1} > \theta_{LN} > \theta_{L_2} \).

If \( D_1 < 0 \) and \( D > 0 \), then we have that \( \frac{\hat{r}}{\hat{P}} < 0 \). Thus, \( \varepsilon_{K'} P < 0 \).
For $\varepsilon_{K^l} p < 0$, we need to assume that: firstly, the non-traded good $X_N$ is more non-skilled labour intensive in the value sense than the export good $X_2$ but less non-skilled labour intensive in the value sense than the import goods $X_1$. Secondly, $X_2$ has a higher ratio of skilled labour to unskilled labour than $X_1$.

In the previous 3-factor, 2-good model, the sign of $\varepsilon_{K^l} p$ has been given as positive, showing that a fall in the relative price of imported good will lead to a fall in domestic rental rate of capital. Now in the 3-factor, 3-good model, we find that $\varepsilon_{K^l} p$ is negative, directly contradicting the result derived from the 2-good model. The reason lying the inconsistency is that in the present model without foreign capital, the non-traded good is introduced and its price will rise after trade liberalization. The rise in the price of the non-traded good can only be neutralized by the rise in domestic rental rate of capital, when the price of the other factor, unskilled-labour, is destined to fall.

Appendix 4.3: The derivation of equations of change – section 4.4.a

Equations (4.4) to (4.9) and equations (4.14) and (4.23) give us a system of 8 equations in 8 unknowns. Given $t_2, P_{\bar{L}, \bar{S}}, \bar{K}^d$ and $r^*$, we can determine the eight unknowns - $w_1, w_5, P_N, X_1, X_2, X_N, K^l$ and $I$.

By totally differentiating equations (4.7) - (4.9) using the cost minimising condition, we can derive the following equations:

\[ \theta_{t_1} \dot{w}_1 + \theta_{t_2} \dot{w}_2 = 0 \]  
\[ \theta_{t_2} \dot{w}_1 + \theta_{t_5} \dot{w}_5 = \ddot{r}_2 \]  
\[ \theta_{t_N} \dot{w}_1 = \ddot{P}_N \]

Those equations give us solutions to three unknowns $w_1, w_5$ and $P_N$, as shown in equation (4.25) –(4.27) of the text.
Differentiating equation (4.23), we obtain the solution for national income:

\[ \dot{I} = \alpha_2 \dot{I}_t > 0 \]  

(A4.3.4)

National income will improve with technical progress in the exported sector.

We can also derive from equation (4.14) that:

\[ \dot{X}_N = \varepsilon_{np} \ddot{P}_N + \varepsilon_n \dot{I} \]  

(A4.3.5)

where

\[ \varepsilon_{np} = \frac{P_N}{D_N} \frac{\partial D_N}{\partial P_N} < 0 \]  

Price elasticity of demand for non-traded goods

Substituting the solutions for \( \ddot{P}_N, \dot{I} \) into equation (A4.3.5), we obtain:

\[ \dot{X}_N = \dot{I}_2 (-\varepsilon_{np} \frac{\theta_{LN} \theta_{S1}}{\theta} + \varepsilon_n \alpha_2) > 0 \]  

(A4.3.6)

\( \dot{X}_N \) will increase as a consequence of the fall in the price of the non-traded good and the rise in the national welfare.

The solutions for the output \( X_j (j=1,2) \) and the amount of foreign capital \( K^f \) can be obtained from the following matrix:

\[
\begin{bmatrix}
\lambda_{L1} & \lambda_{L2} & 0 \\
\lambda_{S1} & \lambda_{S2} & 0 \\
\lambda_{K1} & \lambda_{K2} & -\lambda_{k^f}
\end{bmatrix}
\begin{bmatrix}
\dot{X}_1 \\
\dot{X}_2 \\
\dot{K}^f
\end{bmatrix} =
\begin{bmatrix}
\beta_{L1} \theta_{L12} - \beta_{L5} \theta_{L1} + \lambda_{25} \theta \\
\beta_{S1} \theta_{S1} - \beta_{S3} \theta_{L1} + \lambda_{S2} \theta \\
\beta_{K1} \theta_{S1} - \beta_{K3} \theta_{L1} + \lambda_{K2} \theta
\end{bmatrix}
\begin{bmatrix}
\dot{I}_2 \\
0 \\
\lambda_{k^f}
\end{bmatrix}
\]

where \( D = \lambda_{k^f} (\lambda_{L2} \lambda_{S1} - \lambda_{L1} \lambda_{S2}) \). And \( D < 0 \), if \( s_2 > s_1 \).

The results for \( \dot{X}_1, \dot{X}_2 \) and \( \dot{K}^f \) are determined by many elements, including factor intensities, relative factor shares in production, substitution or complementary relationship between factors of productions, income effect and price effect. Whether the results are positive or negative depend on which determining element dominates.
Appendix 4.4: The derivation of equations of change – section 4.4.b

Equations (4.4) to (4.9) and equations (4.14), (4.23) give us a system of 8 equations in 8 unknowns. Given \( t_N, P_L, S, K^d \) and \( r^* \), we can determine the eight unknowns - \( w_1, w_2, P_N, X_1, X_2, X_N, K^f \) and \( I \).

By totally differentiating equations (4.7) - (4.9) to change in technological level in the non-traded good sector, using the cost minimising condition, we can derive the following equations:

\[
\theta_{11}\hat{w}_1 + \theta_{12}\hat{w}_2 = 0 \tag{A4.4.1}
\]

\[
\theta_{21}\hat{w}_1 + \theta_{22}\hat{w}_2 = 0 \tag{A4.4.2}
\]

\[
\hat{P}_N = -\hat{I}_N \tag{A4.4.3}
\]

From those equations, we find that the wage rate of both skilled-labour and unskilled-labour will be unchanged. The price of the non-traded good will decrease due to the technical progress having taken place in this sector.

Differentiating equation (4.23), we obtain the solution for national income:

\[
\hat{I} = \alpha_N\hat{I}_N > 0 \tag{A4.4.4}
\]

National income will improve with technical progress in the non-traded good sector.

We can also derive from equation (4.14) that:

\[
\hat{X}_N = \nu_{NPx}\hat{P}_N + \nu_{x}\hat{I} \tag{A4.3.5}
\]

Substituting the solution for \( \hat{P}_N, \hat{I} \) into equation (A4.3.5), we obtain the result for \( \hat{X}_N \):

\[
\hat{X}_N = \hat{I}_N (-\nu_{NPx} + \nu_N\alpha_N) > 0 \tag{A4.3.6}
\]

\( \hat{X}_N \) will increase as a consequence of the fall in the price of non-traded good and the rise in national income.

65
The solutions for output of $X_j$ ($j = 1, 2$) and the amount of foreign capital $K^f$ can be obtained from the following matrix:

$$
\begin{vmatrix}
\lambda_{L1} & \lambda_{L2} & 0 & \hat{X}_1 \\
\lambda_{S1} & \lambda_{S2} & 0 & \hat{X}_2 \\
\lambda_{K1} & \lambda_{K2} & -\lambda_{K'} & \hat{K}'
\end{vmatrix}
= 
\begin{vmatrix}
\lambda_{LN} & \lambda_{LN} \\
\hat{i}_N & 0 & 0 & \hat{X}_N
\end{vmatrix}
$$

where $D = \lambda_{K'} (\lambda_{L2} \lambda_{S1} - \lambda_{L1} \lambda_{S2})$. And $D < 0$, if $s_2 > s_1$.

The results for $\hat{X}_1, \hat{X}_2$ and $\hat{K}'$ depend on many factors, including relative factor intensities between sectors, relative factor shares in sectoral production, income effect and price effect. Whether the results are positive or negative depend on which determining element dominates.
Chapter 5. Trade, Technical Progress and Income Inequality
- a Model with Illegal Immigrants

5.1. Introduction
So far, we have studied the effect of trade liberalization and technical progress in the presence of internationally mobile capital in a simple 2-good, 3-factor model and in an extended model with a non-traded good. In this chapter, our attention turns to the case involving the international flow of unskilled labour.

International labour flows can be roughly divided into two categories: legal migration and illegal migration. Since all countries impose immigration policies which not only limit the quantity of immigrants, but prescribe what special characteristics these people must have to be granted immigrant status, the former category has been effectively controlled by recipient countries. Such immigrants thus become homogenous with domestic labour. The influx of legal migrants has a similar effect on the host economy to an increase in the domestic labour force. However, the nature and behaviour of illegal immigrants is different and much more complicated to explain. Hence, in this chapter we confine our attention to the latter category of labour inflow, which is also heavily controlled by the recipient country but is extraordinarily difficult to administer in practice.

Many developed countries receive illegal migrants from developing countries. Illegal migration from the poor countries typically involving unskilled workers, who receive below average wages even in their home countries. Illegal migrants move mainly in response to wage differentials between the recipient countries and their home countries\(^\text{35}\). And usually they can only receive a wage less than that of local workers, although much higher than what they can earn in their home country\(^\text{36}\).

\(^{35}\) Although a large proportion of labour flows is motivated by purely economic reasons, some may be motivated by political or war reason;

\(^{36}\) According to Kemp (1969), international wage differentials can emerge only a. if trade is impeded artificially (for example, by tariffs), or b. if at least one country is completely specialised in production, or c. if, given the terms of trade, the two trading countries are characterised by different factor rental
Lured by the chance of higher earnings, more and more migrants from poor countries have illegally entered the relatively rich countries during recent years. Even if illegal migration generally is policed in recipient countries and there is always a substantial threat of being deported, they are willing to take the risk. Illegal immigrants are also willing to take any hard physical job. They will work for low wages and long hours, with or without any security or other benefits in the recipient countries. Being illegal and without many work skills, they are mainly employed in the non-traded sector, particularly non-tradable service industries. For instance, a larger number of unskilled labour used in smaller-established hotel and restaurant trade are illegal workers who are willing to work for a wage less than that of local workers. These unskilled illegal migrants help in lowering the relative prices of non-traded goods.

Traditional economic theory suggests that the migration of unskilled labour from developing countries to developed countries should be expected to narrow the wage gap between those countries and thereby reduce the incentive for further migration. In the developed countries themselves, the competition between unskilled illegal immigrants and local workers lower the wage rate paid to unskilled labour, and therefore income gap between skilled labour and unskilled labour is widening. This result shows that the surge of illegal migrants has similar effect on income distribution in the developed countries to what trade liberalisation process and technical progress have done. But so far, no attempt is made to establish the linkage between trade liberalisation, technical progress and labour migration.

We continue to employ a three-good model to study the impact of trade liberalization and technological progress on the economy in question, while taking into consideration of the presence of illegal immigrants in the labour market. National income effects are also examined. In the model, two internationally traded

ratios, or d. if the same production functions do not prevail in both countries, or e. if returns to scale are variable. In our model, we assume that the developed country and developing country do not have the same production functions in at least one sector, so as to ensure that wage differential is in existence between the two trading countries;
goods and one non-traded good are produced with the help of domestic skilled-labour and unskilled-labour, internationally mobile capital and illegal immigrants. The illegal immigrants are assumed to be sector specific and thus only work in the non-traded goods sector. The international flow of illegal migrants and capital is endogenously determined. Our main interest will be in the analysis of the impact of trade liberalisation and technological progress on the wage rate of domestic skilled-labour and unskilled-labour, the inflow of illegal migrants and their wages, relative prices of non-trade goods and resident welfare.

5.2. Model

We consider a small open country producing three goods: the importable $X_1$, the exportable $X_2$ and a third non-traded good, $X_N$. Both commodity $X_1$ and $X_2$ are produced with the help of capital $K$, domestic non-skilled labour $L$ and domestic skilled labour $S$, by using the following neoclassical production functions:

$$X_1 = F_1(L_1, S_1, K_1)$$  
(5.1)

$$X_2 = F_2(L_2, S_2, K_2)$$  
(5.2)

where $L_j$, $S_j$, $K_j$ ($j=1,2$) denote the allocation of unskilled-labour, skilled-labour and capital to the traded sectors respectively.

The non-traded commodity $X_N$ is produced with the help of capital $K$, domestic non-skilled labours $L_N$ and illegal migrants $L_I$ by a neoclassical production function:

$$X_N = F_N(L_N + L_I, K_N)$$  
(5.3)

where $L_N$ and $K_N$ are the allocation of domestic unskilled labour and capital to the non-traded sector and $L_I$ represents the illegal immigrants used in the production of the non-traded good. We assume that domestic non-skilled labour and illegal immigrant are actually perfectly substitutes in producing the non-traded good.

Note that skilled labour is specific to the production of commodities $X_1$ and $X_2$ with non-skilled labour and capital perfectly mobile across all the three sectors. It is assumed that illegal immigrants are only used in the production of commodities $X_N$. 

69
It is also assumed that capital is internationally mobile and the country in question is assumed to be an importer of capital at the initial equilibrium situation. The amount of foreign capital imported is denoted by $K'$. The factor utilisation equations require that:

$$a_{L1} X_1 + a_{L2} X_2 + a_{LN} X_N = \bar{L} \quad (5.4)$$

$$a_{S1} X_1 + a_{S2} X_2 = \bar{S} \quad (5.5)$$

$$a_{K1} X_1 + a_{K2} X_2 + a_{KN} X_N = \bar{K}^d + K' = K \quad (5.6)$$

$$a_{Ii} X_N = L_I \quad (5.7)$$

where $\bar{L}, \bar{S}$ and $\bar{K}^d$ are the inelastic supplies of the factors, skilled labour, non-skilled labour and domestic capital. The term $K'$ and $L_I$ represent the endogenously determined supply of foreign capital and illegal immigrants respectively. And $a_{ij}$'s denote variable input coefficients, which are functions of the factor prices and the state of technology $t_j$. The expressions are given as below:

$$a_{ij} = a_{ij}(w_1, w_2, r^*, t_j) \quad (5.8)$$

where $i = L, S, K, I$ and $j = 1, 2, N$.

We have assumed that the economy in question employs illegal workers in the production of the non-traded good. Then we need to introduce a migration function for the endogenously determined illegal migrants:

$$L_I = f\left[U(\alpha w_i) - U(w^*)\right] \quad (5.9)$$

where $f$ is an increasing function of the difference between the expected utility of the wage rate received by illegal immigrants in the economy in question and the utility of the wage rate in their home country. $\alpha$ is assumed to be between (0,1) and $\alpha w_i$ denotes the wage received by illegal workers in the economy in question which is less than the wage received by domestic unskilled workers. $w^*$ represents the wage in the rest of the world which is exogenously given and assumed less than $\alpha w_i$.

The term $\alpha$ has two features. Firstly, it captures the aspect of illegality – the difficulty for the illegal immigrants with language and cultural problem to find a job
and the threat that they may be reported to local authorities and deported. Secondly, it shows the degree of exploitation by employers – employers have their own way to distinguish illegal workers from legal workers before they decide whether to use an illegal worker for a small wage, or employ a legal worker for higher wage and, therefore without the threat of being fined. In this case, for simplicity, we assume that $\alpha$ is fixed and has a value between $(0,1)$.

We shall assume that the country is small in both the commodity market and the factor market for foreign capital, and hence, the two commodity prices $P_1, P_2$ and the return to foreign capital $r^*$ are given from outside. The pricing equations for the system are:

$$a_{L_1} w_1 + a_{S_1} w_s + a_{K_1} r^* = P \quad (5.10)$$
$$a_{L_2} w_1 + a_{S_2} w_s + a_{K_2} r^* = 1 \quad (5.11)$$
$$a_{L_N} w_1 + a_{L_F} \alpha w_i + a_{K_N} r^* = P_N \quad (5.12)$$

where the terms $w_1$ and $w_s$ denote the wages of skilled and non-skilled workers in the economy, $P = P_1 / P_2$ represents the relative price of good $X_1$ and $P_N$ relative price of good $X_N$. It is interesting to note that the non-traded goods sector pays a lower average wage rate than the traded sector to the unskilled labour. The wage rate of unskilled labour in the non-traded goods sector is the weighted average of the wage paid to domestic unskilled labour and the wage paid to illegal immigrants, which is defined below:

$$\bar{w}_N = \left( \frac{L_N}{L_N + L_F} \right) w_1 + \left( \frac{L_F}{L_N + L_F} \right) \alpha w_i \quad (5.13)$$

Since $\alpha < 1$, then $\bar{w}_N < w_1$. It is also assumed that $L_N > L_1$.

In figure 5.1, the price determination procedure for this model is shown. A small open economy takes the rental on capital as given, and hence, $r$ is specified exogenously. Given the values of $r^*, P_1$ and $P_2$, the net unit price curves for commodity $X_1$ and commodity $X_2$ are drawn in quadrant I. The intersection of the
two unit-price curves determines the equilibrium factor prices. The values of \( w_i \) and \( w_s \) are attained as shown by \( w_i^* \) and \( w_s^* \) in quadrant I. We shall assume that \( w_s^* > w_i^* \). Once \( w_i^* \) is attained, \( \alpha w_i^* \) is determined, and then the position of the net unit price curve for commodity \( X_N \) can be drawn in quadrant II.

![Figure 5.1](image)

We still assume that the economy in question imports \( X_1 \) and exports \( X_2 \). Market clearing condition requires that:

\[
D_1 = X_1 + M_1 \quad (5.14)
\]
\[
D_2 = X_2 - E_2 \quad (5.15)
\]
\[
D_N(P, P_N, I) = X_N \quad (5.16)
\]

We shall note that the non-traded good market is locally stable. The price of the non-traded good, \( P_N \), is determined endogenously, through the influence of domestic demand and supply for non-traded good. To achieve the equilibrium in the domestic market, the demand for non-traded goods \( D_N \), which is a function of relative prices and income, must be equal to the supply of non-traded good \( X_N \).
The national income equation can be given as:

\[ P_1 D_1 + D_2 + P_N D_N = I = P_1 X_1 + X_2 + P_N X_N - r^* K^f - \alpha \nu_i L_i = w_1 \bar{L} + w_2 \bar{L}_s + r^* \bar{K}^d \]  

(5.17)

This completes the specification of the present model. The model with illegal immigrants comprises twenty equations [(5.4) – (5.7), eight equations implied by (5.8), and (5.9) – (5.13), three equations out of (5.14) to (5.17)], in twenty unknowns (eight of \( a_{ij}, X_1, X_2, X_N, w_1, w_s, \bar{w}_N, P_N, K^f, L_i, D_1, D_2, \) and \( I \)) with several exogenous variables (\( P_1, P_2, I, \alpha, K^d, L \) and \( S \)).

We then follow the same approach used in previous chapter to examine the effects of trade liberalisation and technical progress on income distribution and national welfare, in the presence of international mobile capital and illegal inflow of unskilled-labour. It is also in our interest to analyse such impact on the flow of illegal migrants and their wages.

5.3. Consequences of Trade liberalisation:

In this section of the paper we examine the consequence of trade liberalisation on factor returns and welfare as captured by the above twenty-one equations. To simplify the question, we still assume that trade liberalisation will result into lower prices of imported goods while prices of exported goods are constant.

By totally differentiating equations (5.10) - (5.12), we can obtain the solutions for the following variables:

\[ \hat{w}_1 = \frac{\theta_{s_1}}{|\theta|} \hat{p} \]  

(5.18)

\[ \hat{w}_s = \frac{-\theta_{L_2}}{|\theta|} \hat{p} \]  

(5.19)
\[ \hat{P}_N = \frac{(\theta_{Lw} + \theta_{Lw})\theta_{s_2}}{|\theta|} \hat{p} \]  \hspace{2cm} (5.20)

where \(|\theta| = \theta_{L_1}\theta_{L_2} \frac{w_s}{w_1} (s_2 - s_1)\) and \(s_j = S_j/L_j\). The sign of \(|\theta|\) depends on relative factor intensity in the traded sectors. \(|\theta| > 0\) if \(s_2 > s_1\).

We know from equations (5.18) and (5.19) that:

\[ \hat{w}_s - \hat{w}_1 = \frac{(\theta_{K_2} - 1)}{|\theta|} \hat{p} \]  \hspace{2cm} (5.21)

The impact of commodity price changes on the factor price changes depends on the relative factor intensities in the production of \(X_1\) and \(X_2\). As long as the imported commodity \(X_1\) is more non-skilled labour intensive than the exported commodity \(X_2\), \(|\theta|\) is positive, and thus wages of non-skilled labour will decrease and those of skilled labour will increase if \(P\) fall after trade liberalisation. This is consistent with S-S theorem in the H-O model since in our model, capital is internationally mobile and thus, rental rate of capital is fixed.

Under the same assumption, trade liberalisation will also result in a decrease in the price of the non-traded good, \(P_N\). From equation (5.20), the relationship between the price of imported good and the price of non-traded good is determined by \(|\theta|\). As long as the economy in question is assumed to be relatively non-skilled labour intensive in the imported sector than the exported sector, there will be a fall in the wage rate of unskilled-labour, and hence a further reduction in the labour cost of producing non-traded good. With a fixed rental rate to capital, the price of non-traded good \(P_N\) will fall. And when \(|\theta| > 0\), it is always true that \(\hat{w}_s > 0 > \hat{P} > \hat{w}_1\) and \(\hat{w}_s > 0 > \hat{P}_N > \hat{w}_1\). The real wages of unskilled labour decrease and those of skilled labour increase as a result of trade liberalization.
Hence, By assuming that \( X_1 \) is non-skilled labour intensive vis-à-vis the \( X_2 \) sector in this model, wage inequality will increase as a result of trade liberalisation. Unskilled workers become worse off relative to skilled workers. This condition is representative of reality in the developed country since generally those countries import non-skilled labour intensive commodities and export skilled-labour intensive products.

The previous results are illustrated in figure 5.2. Taking the graph shown in figure 5.1 as the initial equilibrium situation, the net unit price curve for \( X_1 \) in quadrant I will shift leftward after trade liberalisation. As a result, \( w_i^* \) will fall to \( w_i^{**} \) while \( w_s^* \) rises to \( w_s^{**} \). Correspondingly, \( \omega \omega_i^* \) will fall. The net unit price curve for \( X_N \) will shift rightward, coming across a new equilibrium point \( (\omega \omega_i^{**}, w_i^{**}) \). The price of non-traded good \( P_N \) will decrease.

Figure 5.2
By totally differentiating equation (5.9), we obtain the relationship between the inflow of illegal immigrants and the wage rate of unskilled-labour. It turns out to be:

\[ L_t \dot{L}_t = f U' \alpha \dot{\nu}_i \]  \hspace{1cm} (5.22)

Since both \( f \) and \( U \) are increasing functions, \( f' > 0 \) and \( U' > 0 \). Equation (5.22) shows the positive relationship between the changes in illegal immigrants and the changes in the wage rate of unskilled-labour in the recipient country. \( w_i < 0 \), so we have \( \dot{L}_t < 0 \). Hence, trade liberalisation leads to an outflow of illegal immigrants.

By assuming utility-maximisation, national income consequences can also be derived from equation (5.17):

\[ \dot{I} = -q \left[ \alpha_1 + \frac{\theta \alpha_2}{\theta} \alpha_2 \right] \]  \hspace{1cm} (5.23)

where

\[ \alpha_1 = \frac{M_i^d P_1}{I} \] \hspace{1cm} Share of national income used for imports

\[ \alpha_2 = \frac{\alpha \nu_i L_F}{I} \] \hspace{1cm} Share of national income earned by illegal workers

In equation (5.23), if \( |\theta| > 0 \), the second term in the bracket \( \frac{\theta \alpha_2}{|\theta|} \alpha_2 \) is positive. Therefore, if \( |\theta| > 0 \), \( \dot{I} > 0 \). By assuming that that the economy in question imports relative unskilled labour intensive products and exports relative skilled labour intensive products, national income for this economy will improve after trade liberalization.

The economic intuition lying the above result is that, trade liberalization will result in the following consequences. Firstly, trade liberalization will directly lead to a fall in imported price relative to the exported good. Domestic consumers benefit from the lower price of imported goods. Secondly, consider the income distribution effect
of trade liberalization in the economy in question. Under certain condition, the wage rate of unskilled labour relative to that of skilled-labour will fall. However, domestic skilled-labours gain at the expense of the losers, which include not only domestic unskilled-labours, but illegal immigrants. The losses of illegal immigrants from trade should be deducted, when we calculate domestic resident income.

As a result, the net effect on national income is that, if the economy in question imports relative non-skilled intensive products and exports relative skilled-labour products, it will inevitably benefit from trade liberalization. This condition is applicable to the fact in the developed countries.

5.4. Consequences of Technical Progress:
(a) Impact of Hicks-neutral Technical Progress in Export Sector
In this section, we turn our attention to the adjustment process of technological progress incurred in the economy. We first consider that neutral technological progress takes place in the export-industry $X_2$. We assume that the $X_2$ sector is more skilled labour intensive than $X_1$ sector, and therefore, technical progress is more likely to take place in the production of $X_2$. We also assume that the prices of traded goods are determined in the international market and for the moment the prices remain unchanged.

Equation (5.2) is now replaced by

$$X_2 = F_2[L_2, S_2, K_2, t_2] = t_2 F_2[L_2, S_2, K_2]$$

(5.24)

where $t_2$ is a parameter representing the level of technology and is assumed to be equal to unity initially.

An alternative expression of Hicks-neutral technological change is also given below:

$$\hat{b}_2 = b_{t_2} = b_{K_2} = b_{S_2} = t_2$$

(5.25)
where \( \hat{b}_j \), as defined as \(-\frac{1}{a_{ij}} \frac{\partial a_{ij}}{\partial t}\) is a measure of technological change on the input-output coefficient that would take place at constant factor prices.

Other equations making up the model are unchanged.

Suppose that now technological changes occur in the production of \( X_2 \), implying that:
\[
\hat{i}_2 > 0 
\] (5.26)

To examine the impact of technological changes in \( X_2 \) sector on endogenous variables, we shall totally differentiate the equations of our system.

Solutions for the changes in factor prices are derived from equations (5.10) – (5.11):
\[
\hat{w}_1 = \frac{-\theta_{11}}{|\theta|} \hat{i}_2 < 0 
\] (5.27)
\[
\hat{w}_2 = \frac{\theta_{21}}{|\theta|} \hat{i}_2 > 0 
\] (5.28)

Having assumed that the economy in question imports unskilled labour intensive good \( X_1 \) and exports skilled labour intensive good \( X_2 \), \(|\theta| > 0\). So the wages of skilled labour increase and those of unskilled labour decline as a result of technical progress in the production of \( X_2 \).

Subtracting equation (5.27) from (5.28), we get:
\[
\hat{w}_2 - \hat{w}_1 = \frac{(1-\theta_{11})}{|\theta|} \hat{i}_2 > 0 
\] (5.29)

It is clear that wage inequality widens after technical progress takes place.

Totally differentiating equation (5.12), we obtain:
\[
\hat{p}_N = -\frac{(\theta_{LN} + \theta_{IN})\theta_{s1}}{|\theta|} \hat{i}_2
\]  

(5.30)

Technical progress in \(X_2\) has an influence on determining the price of non-traded good. The price of non-traded good will fall, since there is a reduction in the labour cost in production of non-traded good resulted from technical progress in the production of exported good. It is also true that \(\hat{w}_2 > 0 > \hat{p}_N > \hat{w}_1\). Therefore, not only nominal wages, but real wages of skilled labour rise, while real wages of unskilled labour fall.

Substituting equation (5.27) into (5.22), we have the solution for the flow of illegal immigrants:

\[
L_t \hat{L}_t = -\frac{\theta_{s1}}{|\theta|} f U' \alpha w_{1} \hat{i}_2
\]  

(5.31)

Technical progress in \(X_2\) also plays a role in determining the flow of illegal immigrants. Since wage rate of unskilled-labour in the economy in question lowers after technical progress taking place in the production of \(X_2\), less foreigners will be interested in working illegally in this economy. And some illegal workers may choose to be back to their home countries. There will be an outflow of illegal immigrants.

Finally, considering the national income consequence, we have the following result derived from equation (5.17):

\[
\hat{i} = \hat{i}_2 \left[ \alpha_4 + \frac{\theta_{s1}}{|\theta|} \alpha_2 \right]
\]  

(5.32)

where

\[
\alpha_4 = \frac{X_2 t_2}{l} \quad \text{Share of national income represented by value of commodity } X_2.
\]

Following the same explanation for the case of trade liberalisation, we come to the conclusion that if the economy imports unskilled-labour intensive product and
exports skilled-labour intensive product, its resident welfare will definitely improve after technical progress taking place in the production of exported good.

(b) Impact of Hicks-neutral Technical Progress in Non-traded Goods Sector
The next case to be considered is that of technological progress incurred in the non-traded goods sector - \( X_N \). We still assume that for the moment there is no change in the price of traded goods.

Suppose that neutral technological progress takes place in the \( X_N \) sector. Equation (5.3) is then replaced by

\[
X_N = F_N[K_N, L_N, I_N, t_N] = t_N F_N[K_N, L_N, I_N] \tag{5.33}
\]

where \( t_N \) is a parameter representing the level of neutral technology in \( X_N \) sector. And now

\[
\hat{t}_N > 0 \tag{5.34}
\]

Other equations in the previous model are unchanged.

Technological progress in the non-traded sector leads to a much simpler consequences than it does in the previous case. The reason is that wage rate to skilled-labor and non-skilled labour is determined by two factors: one is the price of internationally traded goods and the other, the level of technology in the trade sectors. In this case, both factors are fixed, so that factor price remains unchanged. We come to the conclusion that income distribution between skilled labour and unskilled labour remains undisturbed by technical progress in the non-traded goods sector. Meanwhile, the flow of illegal immigrants, which is solely determined by the income level of unskilled-labour in the economy, will be unchanged.

The price of non-traded goods will change since technical progress takes place in this sector. The solution is given from differentiating equation (5.12):

\[
\hat{p}_N = -\hat{t}_N < 0 \tag{5.35}
\]
Technical progress in the non-traded sector is going to reduce the cost of production of $X_N$. So at given factor prices, the good price $P_N$ will decrease. Since $\hat{w}_s = \hat{w}_l = 0 > \hat{P}_N$, both real wage rate of skilled labour and that of unskilled labour actually rise as a result of technical progress in the production of $X_N$.

By totally differentiating equation (5.17) in our system, we obtain the following result:

$$\dot{I} = \alpha_s \dot{I}_N$$

(5.36)

where

$$\alpha_s = \frac{P_N X_N t_N}{I}$$

Share of national income represented by value of commodity $X_N$.

We come to the conclusion that national income for the economy in question will always improve, since there is an induced reduction in the price of the non-traded good.
Appendix 5.1: The Equation of Change – Section 5.3

Equations (5.4) to (5.7) and equations (5.9) - (5.12) and (5.17) give us a system of nine equations in nine unknowns. Given \( P, \bar{L}, \bar{S}, \bar{K}^d \) and \( r^* \), we can determine the nine unknowns \(- w_i, w_s, P_N, X_1, X_2, X_N, K', L_l \) and \( I \).

By totally differentiating equation (5.10) - (5.12) using the cost minimising condition, we can derive the following equations:

\[
\theta_{I1}\hat{w}_1 + \theta_{S1}\hat{w}_s = \hat{p} \quad \text{(A5.1.1)}
\]

\[
\theta_{I2}\hat{w}_1 + \theta_{S2}\hat{w}_s = 0 \quad \text{(A5.1.2)}
\]

\[
(\theta_{IN} + \theta_{LN})\hat{w}_1 = \hat{p}_N \quad \text{(A5.1.3)}
\]

Those equations give us solutions to three unknowns \( w_i, w_s \) and \( P_N \), as shown in equation (5.18) — (5.20) of the text.

By totally differentiating equation (5.7) and using equation (5.18), we have the result for the output of \( X_N \):

\[
\hat{X}_N = \frac{\theta_{s2}}{|\theta|} \left[ \frac{f U'Caw_i}{L_l} - (\theta_{IN} + \theta_{LN})\beta_{NL}^N \right] \hat{p} \quad \text{(A5.1.4)}
\]

Since \( |\theta| > 0, \beta_{NL}^N > 0 \), \( \hat{X}_N < 0 \).

Totally differentiating equations (5.4) — (5.6), the solutions for the output \( X_j \) (j = 1, 2) and the amount of foreign capital in the economy can be obtained from the following matrix:

\[
\begin{vmatrix}
\lambda_{L1} & \lambda_{L2} & 0 & \hat{X}_1 & \beta_{Ls}\theta_{L2} - \beta_{LL}\theta_{S2} & \hat{p} \\
\lambda_{S1} & \lambda_{S2} & 0 & \hat{X}_2 & \beta_{SS}\theta_{L2} - \beta_{SL}\theta_{S2} & 0 \\
\lambda_{K1} & \lambda_{K2} & -\lambda_{K1} & \hat{K}' & \beta_{KL}\theta_{S2} - \beta_{KS}\theta_{L2} & \lambda_{KN}
\end{vmatrix}
\]

where
\[ \beta_{ik} = \sum \lambda_{ij} \theta_{ij} \sigma_{ik} \] with \( \sigma_{ik} \) denoting the partial elasticity of substitution between factors of production \( i \) and \( k \) \( (i, k = L, S, I, K) \) in the \( j \)th sector.

We have the determinant of the above matrix. This turns out to be:

\[ D = \lambda_{K'} (\lambda_{L1} \lambda_{S1} - \lambda_{L1} \lambda_{S2}) \text{. And } D < 0, \text{ if } s_2 > s_1. \]

The results for \( \hat{X}_1, \hat{X}_2 \) and \( K' \) are determined by many elements: factor intensities, relative factor shares in production, substitution or complementary relationship between factors of productions. Whether the results are positive or negative depend on which determining element dominates.

Appendix 5.2: The Equation of Change – Section 5.4.a

Equations (5.4) to (5.7) and equations (5.9) - (5.12) and (5.17) give us a system of 9 equations in 9 unknowns. Given \( P, L, S, K^d, \hat{r}_2 \) and \( r^* \), we can determine the nine unknowns - \( w_1, w_5, P_N, X_1, X_2, X_N, K', L_I \) and \( I \).

By totally differentiating equation (5.10) - (5.12) to change in technological level in the exported sector \( t_2 \), using the cost minimising condition, we can derive the following equations:

\[ \theta_{L1} \hat{w}_1 + \theta_{S1} \hat{w}_5 = 0 \]  \hspace{1cm} (A5.2.1)

\[ \theta_{L2} \hat{w}_1 + \theta_{S2} \hat{w}_5 = \hat{r}_2 \]  \hspace{1cm} (A5.2.2)

\[ (\theta_{LN} + \theta_{IN}) \hat{w}_1 = \hat{P}_N \]  \hspace{1cm} (A5.2.3)

Those equations can be converted into solutions to three unknowns \( w_1, w_5, \text{and } P_N \), as shown in equation (5.27), (5.28), and (5.30) of the text.

We can also derive from equation (5.7) that:
\[ \hat{X}_N = -\frac{\theta_{s1}}{\theta} \left[ \frac{fU'\alpha w_i}{L_i} - (\theta_{in} + \theta_{ln}) \beta_{sl}^n \right] \hat{t}_2 \] (A5.2.4)

Output of non-traded good will decrease as a result of technical progress in the production of \( X_2 \).

The solutions for the output \( X_j \) (j= 1,2) and the amount of foreign capital \( K^f \) can be obtained from the following matrix:

\[
\begin{bmatrix}
\lambda_{L1} & \lambda_{L2} & 0 \\
\lambda_{S1} & \lambda_{S2} & 0 \\
\lambda_{K1} & \lambda_{K2} & -\lambda_{Kf}
\end{bmatrix}
\begin{bmatrix}
\hat{X}_1 \\
\hat{X}_2 \\
\hat{K}^f
\end{bmatrix} =
\begin{bmatrix}
\beta_{LL} \theta_{S1} - \beta_{LS} \theta_{L1} + \lambda_{L2} \theta \\
\beta_{SL} \theta_{S1} - \beta_{SS} \theta_{L1} + \lambda_{S2} \theta \\
\beta_{KL} \theta_{S1} - \beta_{KS} \theta_{L1} + \lambda_{K2} \theta
\end{bmatrix}
\begin{bmatrix}
\hat{t}_2 \\
\hat{t}_2 \\
\hat{t}_2
\end{bmatrix} -
\begin{bmatrix}
\lambda_{LN} \\
\lambda_{KN}
\end{bmatrix}
\]

where \( D = \lambda_{Kf} (\lambda_{L2} \lambda_{S1} - \lambda_{L1} \lambda_{S2}) \). And \( D < 0 \) if \( s_2 > s_1 \). Similarly, the results for \( \hat{X}_1, \hat{X}_2 \) and \( K^f \) depend on many elements, such as the factor intensities, relative share of factors in productions, factor relationships and so on.

**Appendix 5.3: The Equation of Change – Section 5.4.b**

Equations (5.4) to (5.7) and equations (5.9) - (5.12) and (5.17) give us a system of 9 equations in 9 unknowns. Given \( P, \bar{L}, \bar{S}, \bar{K}^d, \hat{r}_N \) and \( r^* \), we can determine the nine unknowns - \( w_1, w_s, P_N, X_1, X_2, X_N, K^f, L_i \) and \( I \).

By totally differentiating equation (5.10) - (5.12) to the change in the technology level in the production of \( X_N \), using the cost minimising condition, we can derive the following equations:

\[ \theta_{l1}\hat{w}_1 + \theta_{s1}\hat{w}_s = 0 \] (A5.3.1)
\[ \theta_{l2}\hat{w}_1 + \theta_{s2}\hat{w}_s = 0 \] (A5.3.2)
\[ \hat{p}_n = -\hat{r}_N \] (A5.3.3)
From those equations, we find that the wage rate of both skilled-labour and unskilled-labour will be unchanged. The price of non-traded good will decrease due to the technical progress having taken place in this sector.

Differentiating equation (5.9), we obtain the solution for the flow of illegal immigrants:

\[ L_t \dot{L}_t = f U' \alpha \dot{\omega}_t = 0 \tag{A5.3.4} \]

The flow of illegal immigrants, which is solely determined by the wage rate of unskilled labour in the economy, remains unchanged.

We can also derive from equation (5.7) that:

\[ \dot{X}_N = \dot{\tilde{N}}_N > 0 \tag{A5.3.5} \]

Output of non-traded goods increase as technical progress taking place in this sector.

The solutions for the output \( X_j \) (j = 1, 2) and the amount of foreign capital \( K^f \) can be obtained from the following matrix:

\[
\begin{bmatrix}
\lambda_{L1} & \lambda_{L2} & 0 \\
\lambda_{S1} & \lambda_{S2} & 0 \\
\lambda_{K1} & \lambda_{K2} & -\lambda'_{Kf}
\end{bmatrix}
\begin{bmatrix}
\dot{X}_1 \\
\dot{X}_2 \\
\dot{K}'
\end{bmatrix}
= 
\begin{bmatrix}
-\lambda_{LN} \\
0 \\
-\lambda_{KN}
\end{bmatrix}
\]

where \( D = \lambda'_{Kf} (\lambda_{L2} \lambda_{S1} - \lambda_{L1} \lambda_{S2}) \). And \( D < 0 \) if \( s_2 > s_1 \). The results for \( \dot{X}_1, \dot{X}_2 \) and \( K' \) is determined by the relative factor intensities and relative factor share used in the production.
Chapter 6: Summary

6.1 Conclusion:
This thesis considers the effects of trade liberalization and technical progress on the income distribution between skilled-labour and unskilled-labour in a general equilibrium framework. Previous researches have only concentrated on commodity trade, not in this area. This thesis aims to develop a model integrating commodity trade, international capital mobility and illegal immigrants to analyze the problem of income inequality and trade liberalization.

The effects of trade liberalization and technical progress are first examined in a two-good, three-factor model allowing for the presence of international capital mobility. Such model is extended from simple specific-factor model by adding some features which are characteristic of modern international trade. Our comparative static analysis shows that, in the presence of internationally mobile capital, both trade liberalization and technical progress in the exported sector will work in favour of skilled-labour and against unskilled-labour and therefore increases the income inequality between the two groups. However, national income necessarily improves.

Next, the analysis extends to a three-good, three-factor model, by adding a non-traded good. Non-traded good market is cleared locally and thus, prices of non-traded goods are determined solely by local demand and supply. However, exogenous changes in international market or traded sector will still have an indirect influence on the non-traded sector via changing the factor prices. In this model, both trade liberalization and technical progress in the exported sector will contribute to the widening income inequality problem, on the condition that the economy in question exports skill-labour intensive products and imports unskilled-labour intensive products. Such a condition is representative of reality in the developed countries. On the contrary, technical progress in the non-traded sector has no effect
on the wage rate of both skilled-labour and unskilled-labour. National income in all the three cases will inevitably improve.

Finally, when international illegal immigrants are taken into consideration, the model becomes striking in the sense that the setting closely resembles the situation of many developed countries and captures the growing significance of international capital and labour mobility. It is assumed that, the illegal immigrants, mostly unskilled-labour, are used in the non-traded sector. We come to the same conclusion as that in the three-good, three-factor model without internationally mobile labour. More importantly, it is interesting to find that the worsening economic situation for domestic unskilled-labour in developed countries will reduce the incentive for further illegal migration.

As a conclusion, in the presence of internationally mobile capital and labour, both trade liberalization and technical progress will lead to a widening income inequality between skilled labour and unskilled labour in the developed countries. However, national income inevitably improves, at the expenses of unskilled labour being worse off. Therefore, it is still in the interest of the whole nation for the government to implement the free trade policies or the policies intended to stimulate technical progress in the domestic economy, as long as the government can redistribute income so that everyone can be made better off.
References:


Lerner A.P. (1932), "The diagrammatical representation of cost conditions in international trade", *Economica* 12, pp.246-561;
Masao Oda (1995), "Trade, technical progress, and wage inequality", Unpublished, Department of Economics, Kansai University, Osaka;
Samuelson, P.A. (1971), “Ohlin was right”, *Swedish Journal of Economics* 73, pp.365-84;
Viner, J. (1950), “Supplementary note to ‘cost curves and supply curves’”, in R.V. Clemence eds. Readings in Economic Analysis 2, pp.31-5, Addison-Wesley, Cambridge, MA;