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Development of Material Models to Predict the Crashworthiness of Tubes

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Abstract

Metallic tubes have been extensively studied for their crashworthiness as they closely resemble automotive crash rails. Recently, the demand to produce light weight yet safer vehicles has led to the need to understand the behaviour of novel materials such as composites, metallic foams and sandwich structures during a crash. This paper presents a method to predict the crashworthiness of structural components using material models. The material factors that most affect the crushing response are determined and quantified by developing and validating the crushing of a square tube model in Abaqus. The inputs from the model are used to construct a simple, physically realistic constitutive model and new test methods for predicting the material behaviour at high strain rates using low test speeds. These material models enable a designer to predict the crash behaviour of a structure without the need to perform extensive physical tests, thus reducing the time and cost of development.

Keywords: Metallic Tubes, Composites, strain-rate, Crashworthiness

Introduction

The development of predictive material models to evaluate the crashworthiness of a vehicle structure has evolved from simple analytical models to complex finite element models via the increase in computational speed and accuracy. Researchers have evaluated the application of such models to predict the crash behaviour of vehicle structures by idealizing the crash rails as tubes so as to minimise the computational time. Predicting the final response of the structure after taking all potential failure modes into consideration is a very complicated and difficult task. Hence, fine tuning of these material models needs to be done based on experimental results to get the correct response. Yet another problem in using finite element codes is that the tensile response of the material is usually given as an input, where the data has been obtained from static tests conducted at very low strain rates.

During crash, the flow stresses in steels are strain-rate sensitive. In some multiphase steels the work hardening rate increases with strain as the strain rate increases but in some low alloy steels the work hardening rate decreases. It is unclear whether existing physically based models can account for these trends, so further investigation is required. To overcome these issues, this paper highlights the application of an inverse technique to identify the constitutive parameters of a rate sensitive plastic material model in Abaqus based on axial impact test results. This technique has been verified for its accuracy in predicting the structural response for mild steel tubes.

Inverse Analysis Technique

Markiewicz et al. (Eric Markiewicz 1998) applied this technique to determine the Cowper-Symonds parameters by axially crushing a thin-walled mild-steel square tube and comparing the response to the experimental results. Square tubes, idealized as crash rails have been analysed for crashworthiness by many researchers by conducting axial impact tests (W. Abramowicz 1984; Abramowicz W. 1986; Mamalis A. G. 1996; Mamalis A. G. 2004; Dorogoy A 2008). The prediction of this model is shown in Fig. 1 and the agreement between test and model is good, except at small displacements. The good correlation with experimental result shows that this technique can be successfully applied to predict a material response.
Inverse analysis relies upon good experimental data and realistic models of the deformation process and material behaviour. Additionally, an effective search method is needed. Therefore, the success of the proposed inverse analysis relies upon the quality of the models and test methods developed in this paper.

Inverse analysis works by running a model and comparing the predictions with experimental results. To improve the predictions a search method is used. To simplify a few general terms are introduced first. The objective function $f$ measures the performance of the model and, for the current problem; it is the error between sets of experimental and predicted quantities. The challenge is to find the set of material parameters $p = [p_1, p_2, p_3 ...]$ such that the value of $f$ is a minimum. The predicted crushing response is made to match experimental results by using a modified version of an inverse method proposed by (Eric Markiewicz 1998).

**Abaqus Model - Setup**

The dimensions of the tube modelled are same as Markiewicz's model as shown in Figure 2. More details of the modelling approach can be found in reference (Mullins 2006).
The material plasticity was assumed to be isotropic and its rate-sensitivity was initially modelled using the Cowper-Symonds relation shown below.

\[ \sigma = \left[ \sigma_y + K \dot{\varepsilon}_p^n \right] \left[ \left( \frac{\dot{\varepsilon}_p}{D} \right)^p + 1 \right] \]  

Where \( \dot{\varepsilon}_p \) and \( \dot{\varepsilon}_p \) are the equivalent plastic strain and strain rate, respectively and \( \sigma_y, K, n, D \) and \( p \) are material dependent parameters. The above equation multiplies the quasi-static flow stress by a factor determined by the magnitude of the plastic strain rate so that an increase in strain rate brings with it an increase in flow stress. The values of the material parameters used are listed in Table below.

<table>
<thead>
<tr>
<th>( \sigma_y ) (MPa)</th>
<th>K (MPa)</th>
<th>n</th>
<th>D (s(^{-1}))</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>301</td>
<td>337</td>
<td>0.52</td>
<td>1150</td>
<td>7.75</td>
</tr>
</tbody>
</table>

Table 1. Material parameters used in Cowper-Symonds Equation (Eric Markiewicz 1998)

Results from Simulation

The Load Vs Displacement curve for the initial model as shown in Fig. 3, did not match the experimental curve obtained by Markiewicz et al (Eric Markiewicz 1998).

![Load Vs Displacement curve](image_url)

Figure 3. Load Vs Displacement curve for the initial model
The Cowper-Symonds equation over predicted the experimental stress-strain values (Figure 5). Moreover, it was assumed that the flow stress at 0.002 s\(^{-1}\) is same as the value for zero strain which is not true for mild steel. Hence the Zerilli-Armstrong equation was used to predict the behaviour of the tube assuming that mild steel has a parallel behaviour with most steels.

The Zerilli-Armstrong equation has the following form when temperature effects are neglected:

\[
\sigma = A (\dot{\varepsilon})^m + B \varepsilon^n + C.
\]  

(2)

A, m, B, n and C are material dependent parameters. To simplify this equation, the athermal term C was omitted assuming its contribution to the result is very small. A quasi-static reference strain rate (\(\dot{\varepsilon}_0\)) of 0.002 is used. The resulting equation is:

\[
\sigma = A \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^m + B \varepsilon^n.
\]  

(3)
An inverse analysis method is adopted to derive the parameter $m$. Stresses are calculated at various strain rates and strain combinations and given as input in Abaqus material model. An initial random value of 0.024 is predicted for $m$ and the model is run in Abaqus Explicit.

The average load obtained from the initial model ($\bar{p}_m^{\text{exp}}$) is compared to the experimental value ($\bar{p}_m^{\text{exp}}$) and the static stress $\sigma_0$ is modified using the relation below to get an improved estimate of $m$:

\[
\sigma_{0,n+1} = \sigma_n \left( \frac{p_m^{\text{exp}}}{\bar{p}_m^{\text{exp}}} \right) ^{\frac{1}{m}}.
\]  

(4)

Where $\bar{p}_m^{\text{exp}}$ is the experimentally recorded mean crushing force and $\bar{p}_m^{\text{exp}}$ is the mean crushing force predicted by the FE model for $m = m_n$. To get an estimate for the value of $m$, the static stress $\sigma_0$ corresponding to a zero strain value is used. Hence, the second term, $(B \varepsilon^p)$ vanishes.

\[
\sigma_0 = A \left( \frac{1}{m} \right) ^{m}.
\]  

(5)

The value of $m$ is predicted by applying an iterative procedure as shown in Figure 6. The root mean square error was calculated from the average loads as follows:

\[
e = \sqrt{\frac{\left( \frac{p_m^{\text{exp}}}{\bar{p}_m^{\text{exp}}} \right) ^2 \left( \frac{\bar{p}_m^{\text{exp}}}{p_m^{\text{exp}}} \right) ^2}{2}}.
\]  

(6)

Figure 6. Procedure used to define the parameters of Zerilli-Armstrong Equation
Following average load values were obtained from these iterations.

<table>
<thead>
<tr>
<th>Iteration</th>
<th>m</th>
<th>Mean crushing force (kN)</th>
<th>Root mean square error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial estimate</td>
<td>0.0240</td>
<td>57.8</td>
<td>7.38</td>
</tr>
<tr>
<td>1</td>
<td>0.0323</td>
<td>61.0</td>
<td>2.26</td>
</tr>
<tr>
<td>2</td>
<td>0.0348</td>
<td>63</td>
<td>0.95</td>
</tr>
<tr>
<td>3</td>
<td>0.0337</td>
<td>64.5</td>
<td>3.34</td>
</tr>
</tbody>
</table>

Table 2. Results of iterations

The value of m was estimated to be 0.0348 as it results in least error value. The load-displacement curve for this m value is shown in Figure 7. The maximum load and mean crushing force were accurately predicted to be 195 kN and 62.6 kN, respectively.

Figure 7. Comparison of predicted (left) and experimental (right) load Vs displacement curve for m=0.0348.
**Effect of Stress Ratios**

The optimised axial tube-crushing model was used to study the effect of stress ratios on the distribution of plastic work. Existing analytical models (Wierzbicki 1994) suggest that plane strain tension and compression equally influence the axial crushing of a tube due to bending along the dynamic plastic hinges. Since the flow over a toroidal surface produces compression and not tension, there was a need to resolve this issue using the viscoplastic, strain-hardening model as it captures the local deformation well. The element formulation allows bending, in-plane deformation and transverse shear and so it provides a more general solution to the deformation problem. Figure 8 shows the distribution of plastic work with stress ratio for the model.

![Figure 8. Distribution of plastic work with stress ratio](image)

The peaks in the graph above indicate that the elements undergoing compression contribute more to plastic work than tension. This implies that the tube thickens during deformation along the corners due to a majority of elements undergoing compression. However, the tube mid-sides experience very little thickening, consistent with deformation by pure bending. This is verified by inspecting the tube corners for section thickness as shown in Figure 9. The variation of plastic strain along the thickness of an element located on the tube corner as shown in Figure 10 shows the linear trend and the strain at the neutral axis is interpolated to be -0.35 which is consistent with plane strain conditions. Therefore, results from the finite element model support the notion of plane strain compressive, rather than tensile forces when material flows around the tube corners.
The plane strain compression also has a peak in Figure 8 and this was thought to be due to the bulk compression of the tube prior to folding. This was verified by simulating the model without the initial bulk compression and Figure 11 shows that this peak disappears, as predicted. Inspection of the finite element model confirms that plane strain conditions are the most important.
Effect of strains and strain rates

The distribution of plastic work, with strain and strain rate, over the entire tube crushing process was quantified to understand at what conditions they influence the plastic work significantly. It can be seen from the plot below that major contributions were made at strains between 0.16 and 0.32, and strain rates between 31 and 100/s.

Figure 11. Distribution of plastic work after initiation of plastic folding.

Figure 12. Distribution of plastic work with strain and strain rate
Sensitivity to material factors

In order to determine which material factors most affect the mechanical response, the factors were individually scaled up to study their effect on the crushing response.

Yield Stress

The term \( A \) in Eq. (3) was increased by a factor of 1.5. This resulted in the increase of peak load by a factor of 1.53 and the mean crushing force by a factor of 1.44 (Fig. 18). The mode of folding and the folding wavelength remained the same. This confirms previous observations (Tarigopula V. 2006) that the crushing load depends strongly upon the yield strength.

![Figure 13. The effect of increasing the yield stress by a factor of 1.5.](image)

Strain Rate Sensitivity

The strain rate sensitivity was modified by increasing the term \( m \) in Eq. (3) by a factor of two. The peak load increased by a factor of 1.41 (Fig. 14) and the mean crushing force by a factor of 1.32, but the mode of folding and the folding wavelength didn't change. This confirms previous observations (W. Abramowicz 1984; Abramowicz W. 1986) that the crushing load depends upon the strain rate.
Figure 14. The effect of increasing the strain rate sensitivity by a factor of 2.

**Work Hardening Rate**

The work hardening rate was modified by increasing the term in Eq. (3) from 0.52 to 0.72. There was little change in the peak load (Fig. 15) and the effect was very little on the crushing response.

Figure 15. The effect of increasing the n-value from 0.52 to 0.72.

**Conclusion**

The actual load-displacement response obtained from experiments by Markiewicz et al was predicted accurately by optimizing the material parameters in a rate sensitive constitutive equation. The stress-strain curves obtained at different strain rates ranging from 0.002 to 1000 s\(^{-1}\), as calculated from the optimized constitutive equation were given as inputs in the Abaqus model. After iteration, the next value of m was predicted by matching it with the experimental
values. As this method relies on experimental data to optimize the parameters, it closely represents the actual material behaviour. The validity of the optimised model can be easily verified by studying its sensitivity to various important material factors like strain-rate, yield stress etc and comparing its response to actual experiments. This is very much beneficial to the automotive industry to predict the material response at very high strain rates of the order of $10^3$ /s, wherein physical crash tests would be very expensive to conduct.

References


