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DESIGNING PROBLEMS: WHAT KINDS OF OPEN-ENDED PROBLEMS DO PRESERVICE TEACHERSPOSE?

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This paper describes preservice teachers’ reported experience of problem posing based on self-selected original digital images. The 176 participants from Australia and Canada designed open-ended problems as part of their mathematics education course. Their 444 problems and accompanying photos have been analysed to explore the types of problems posed and the focus of the mathematical connections. Findings indicate that preservice teachers are challenged when posing open-ended problems however, this process enables them to develop strategies for problem posing and to become more critically aware of the mathematical potential within their environment.

INTRODUCTION

Learning to pose mathematical problems to students is a significant aspect of mathematics teaching. Teachers select problems to assess their students’ understanding of mathematics. They decide on appropriate problems as examples to illustrate a mathematical concept. And they select, adapt and extend mathematical problems to provide a context for learning mathematical skills, concepts and inquiry.

Deciding on what counts as an appropriate problem or worthwhile problem to pose is a complex and important task. It is a significant aspect of planning. Problems or tasks selected give students implicit images about what counts as mathematical inquiry or what it means to do mathematics (Schoenfeld, 1992). Problems contextualize, provide possibilities for inquiry, and can pedagogically frame students’ attention toward noticing mathematical ideas. Some problems more than others may be better-quality exemplars for learning specific concepts (Watson & Mason, 2005). Other problems and how they are varied might be better at inviting abstraction and generalization or help students in seeing mathematical ideas (Marton & Booth, 1997; Marton & Tsai, 2001).

How teachers use mathematical problems and tasks in the classroom is receiving increased attention. Stein, Grover, and Henningsen (1996) reported that it is extremely difficult for teachers to maintain with students the high cognitive demand of potentially high-level tasks that were initially research-informed. Teachers adapt tasks based on what they know about their students, their understanding of the mathematical topic, teaching goals, and classroom environment. How and why teachers change and adapt tasks was the topic of a research forum at the 2008 International Group for the Psychology of Mathematics Education meeting. Herbst (2008) examined the stakes for teachers of investing class time on certain tasks and how accountability, management and institutional obligations might play into teachers’ decisions to change a task while teaching. Herbst stated that the teacher “is responsible for the task as a representation of the mathematics to be learned and for

the task as an opportunity to study and learn mathematics” (2008, p. 126). How might teachers learn to use tasks in this way?

Sullivan (2008) offered a research-based model for developing task-based lessons particularly to address barriers to mathematics learning for some students. The model includes: a) teachers selecting tasks and deciding on their sequence, b) enabling prompts to support students experiencing difficulty, c) extending prompts for those who complete the initial task readily, d) making implicit teaching strategies more explicit so that all students have access the intended goals and expectations, and e) developing a learning community. Sullivan’s model is important for providing possibilities for how to support teachers incorporating designed tasks into their teaching. Yet, in this case the task is given. Although the model emphasizes making the pedagogical practices explicit it leaves hidden the task design practices. Watson (2008) instead suggested that “[a]nother way to engage teachers with tasks is to involve them in the design process” (p. 152). Given that a task is both a representation of mathematics to learn and an opportunity to learn Watson further stated “it makes sense, therefore, to work with teachers on task design rather than only on task implementation” (2008, p. 153).

Over the past few years we have been working in the spirit of Watson’s call of task design but in our case with preservice teachers. In this paper we build on our previous research in which we examined preservice teachers’ responses to the experience of posing mathematical problems (Bragg & Nicol, 2008; Nicol, 2006; Nicol & Bragg, forthcoming). Specifically, we examine the types of problems preservice teachers create, what they notice and attend to and the challenges they experience when designing mathematical problems within the context of a teacher education course.

THEORETICAL CONSIDERATIONS

Our current research is informed by a theory of variation (Marton & Booth, 1997; Marton & Tsui, 2004; Runesson, 2006) and conceptualization of learning and awareness (Watson & Mason, 2006). A theory of variation posits that learning involves the development of a capability to discern or notice critical aspects of a phenomenon while at the same time being focally aware of these aspects. It is assumed that learners only discern that which varies and so discerning requires experiencing variation. Thus the critical features of a phenomenon are brought to the fore of our awareness when we experience variation in those features and are at the same time able to compare the current instance with our past experience of the feature. Watson and Mason (2006) argued that awareness of discernment is more likely if it is experienced against a background of relative invariance. For example, if students have only experienced addition number sentences of the form \(a + b = c\) then it is less likely that they will be aware that \(c = a + b\) is a different way of writing the same equation. Comparing these two situations and systematically varying the placement of the equals sign, or the number of terms to be added can help direct students’ attention to critical features of a number sentence or algebraic equation.
Thus experiencing a phenomenon in a new or different way can change students' awareness of its structure.

Marton and Booth (1997) referred to the something or phenomenon to be learned as the *object of learning*. The object of learning has, according to Marton and Booth (1997) and Marton and Tsui (2004), different characteristics depending on the perspective of different actors throughout the teaching and learning experience. From the teacher's viewpoint the object is referred to as the *intended object of learning*, from the researcher's perspective it is the *enacted object of learning*, and from the student's perspective it is the *lived object of learning*. The use of variation theory for developing pedagogical problems and using these problems with students in mathematics classroom situations is documented in various recent studies. The effectiveness of a pattern of simultaneous variation was demonstrated in studies by Pang, Linder and Fraser (2006) where economic principles of supply and demand were simultaneously varied. In addition, Al-Marani (2007) documented how deliberate and systematic use of dimensions of variation had some influence on students' learning of algebra concepts. Our study adds to this research and focuses on the experiences of preservice teachers learning to pose open-ended mathematical problems within the context of a mathematics teacher education course for elementary teachers.

Variation theory was used to inform and develop adaptations to the task posed to preservice teachers. Our intended object of learning was to help preservice teachers broaden the common ground or space of learning between themselves and their future students by learning to pose mathematical problems that were open-ended inspired by a set of digital images collected by preservice teachers. As with other studies using variation theory, this study could be described as action research - Author B is one of the researchers and also the teacher educator. Our study explored the lived object of learning of preservice teachers: the kinds of open-ended problems they posed, what they noticed, and what they found challenging in the process.

**METHODOLOGY**

The participants were given the task of creating a set of Problem Pictures during their mathematics education course taught by Author B. The task required that the students capture four original photos and develop a set of 3 to 4 accompanying open-ended problems for each photo. The photos and problems were to have some connection to and be suitable for elementary aged students. Sullivan and Lilburn's (2004) definition of a "good" problem was employed to assist the preservice teachers in developing open-ended questions. The three main features of a good question are; 1) it requires more than remembering a fact or reproducing a skill, 2) students learn by doing the task and teachers learn from the students' attempts, and 3) there are several acceptable answers (p. 2). The following is an example of an open-ended problem given by Anna a participant in this study with an accompanying photo of Dakota her dog; "Dakota has gained weight recently. The vet recommends that everyday Dakota
walk 10% further than she did the day before. What are some possible distances that Dakota could walk for 8 days? Show your work”.

The participants in this research were from one Australian and one Canadian university. The data were collected from three cohorts over two years. The two Canadian cohorts (C2007 n=33 and C2008 n=23) were engaged in a 13 week mathematics education course as part of a post-graduate teacher education program. The mathematics education course was in the first semester of their teacher education program. They engaged in one day a week teaching practicum experience running concurrently with the course. The Australian cohort (A2008 n=120) were in the final semester of a four year under-graduate Bachelor of education program. They had accrued 90 days of teaching practicum and completed two mathematics education courses prior to this final 10 week mathematics education course.

The data collected consisted of students’ work samples in the form of the Problem Pictures they had developed (as described above), researcher field notes, and a written response survey completed by participants at the conclusion of the course. For the purpose of this paper we draw upon the students’ work samples and their written survey responses.

A written survey of 15 open response questions was developed to understand the creation of Problem Pictures from the preservice teachers' perspective and was administered through an online survey program (SurveyMonkey). This paper specifically explores the participants’ responses to the strategies the students employed and the challenges faced in the creation of open-ended questions based on original photos.

The researchers met on several occasions to develop and cross check a coding system for the student work samples and survey responses. The data were coded independently and the researchers met again to cross check for consistency and themes that arose from the data. A statistical computer programme, SPSS, was employed to collate and analyse the data gathered from the student work samples. The statistical methods employed were an examination of frequency and percentage of the open-endedness of the problems, the focus and the appropriateness of the mathematics to the problems, and the use of the photos. These data are presented in the form of tables in this paper. A qualitative computer program, Nvivo, was employed for analysis of the online survey data. The survey data are presented in a narrative form and are typical of the views articulated by the many of the participants.

RESULTS AND DISCUSSION
This section presents work samples and survey data to illustrate the types of Problem Pictures preservice teachers design. At the time of writing this paper, an analysis of the data from the Canadian 2007 cohort was completed and is presented. The survey data suggested that the preservice teachers found designing problems of an open-ended nature difficult. Their experience as problem solvers was in finding one correct
answer and the Problem Pictures task was their first formal experience in creating problems. Andrea’s experience was typical of those in this group,

"For most [of the problems], I thought of a question for the picture, and then tried to turn it into an open ended question. It worked sometimes, but wasn’t the most efficient method. However, it was difficult to think open-endedly as so much of what we learn is about exact answers etc. I think this open-ended theory is something that needs to be further explored in the classroom.”

Despite their recent induction to this process, an analysis of the 444 problems revealed that 97% of the problems were open-ended in nature. It was clear from the work samples that the process of creating open-ended problems was achievable for novice teachers despite the initial uncertainty and challenge of the task.

The local curriculum standards had an impact on the preservice teachers’ selection of the intended object of learning in their problems. As noted by Alice, “I used the IRPs [curriculum document] as a guide and tried to cover a variety of the Prescribed Learning Outcomes with the questions”. For coding purposes the mathematical focus of the problems were categorised in line with the provincial curriculum document for (see table 1 below). The data indicated that the mathematical focus of the Problem Pictures (n=444) were: Shape and Space (38%); Number (37%); Pattern (12%), and; Statistics and Probability (12%). Whilst the traditional preferred focus of Number in mathematical problems is popular in the context of these Problem Pictures, a high percentage of problems focused on Shape and Space. It is possible that the context of the photo appeared to lend itself more towards shape and space type problems, it was noted that a relatively high number of photos of buildings were featured. A common shape related question was, “Identify 3 symmetrical shapes in this photo. Draw these shapes and show the lines of symmetry.” Most of the Statistics and Probability questions were focused on data analysis in the form of creating surveys or plotting charts based on data from the photos. For example, “Which fruit do you think is the most popular in your classroom? Create a survey, record and chart your results.” Pattern problems were strongly linked to lower grade levels in repeating or extending patterns in the photo rather than linking with more algebraic related problems.

Accompanying each problem was a statement of the intended object of learning to clarify the mathematical focus for the reader. The mathematical statement was assessed for its strength of relationship to the problem by the researchers. A three point scale was devised for coding; 0 = no link, 1 = partial link, 2 = strong link. The data indicated that the strength of the relationship was: No link (32%); Partial link (42%), and; Strong link (26%). It appears that linking the intended object of learning to the problem was a more challenging task than creating an open-ended question for these preservice teachers. However, this was not articulated in the survey responses. The result is not surprising given the minimal knowledge these participants had with the local curriculum framework and limited classroom experience. However, with nearly a third of the problems not linked to the stated intended object of learning it is
an important consideration when assisting preservice teachers with problem posing to ensure that meaningful links are made to the intended object of learning.

The nature of the use of the photo was explored to determine the relationship with the content of the problem. A problem was coded as Interactive if the photo was necessary to complete the problem and Illustrative if the photo was a motivational device or visual enhancement to the problem but unnecessary for solving the task. Figure 1 depicts a photo with an interactive and illustrative problem. A larger proportion of the problems were considered Interactive (59%) versus Illustrative (41%). The preservice teachers attempted to engage the students with the context of the photo in a meaningful way. The preservice teachers stated that designing interactive questions was extremely challenging for all questions. However, their awareness of the potential for mathematics in the environment had elevated as reported in Authors (2008).

**Figure 1.** Problem Picture with Interactive and Illustrative Problems.

Survey results indicate that preservice teachers appreciated the opportunity to explore mathematics through taking and analysing digital images. Most preservice teachers (95%) stated that the task was challenging. Sarah’s comment is representative of others when she stated: “It took me more than an hour to generate one question.” Others found it easier to develop a single question for an image but found it extremely difficult to create more for that same image. They did, however develop different strategies that helped them design open-ended problems. These included: 1) thinking of a closed question then removing some information; 2) looking at the photo and thinking about major math topic areas; 3) forming questions around the curricular topics then fitting these with the photo; 4) imagining themselves as young children; 5) playing with the language of the problem to make it more open. Of those surveyed only one preservice teacher stated her main strategy for creating a problem was asking herself if the problem made sense.

**CONCLUSION**

Our study explores elementary preservice teachers’ experiences and the types of problem they posed during a mathematics teacher education course. Our results indicate that preservice teachers can pose open-ended mathematics problems and that posing these problems within the context of collecting digital images broadens their awareness of what is possible in mathematics teaching and learning. Nonetheless
preservice teachers indicate that posing open-ended problems inspired by the world around them is a challenging task. Posing one open-ended problem from an image was challenging but achievable, posing more than one moved preservice teachers to develop strategies for creating open-ended problems that could be used across images. Thus keeping the photo invariant, that is, requiring that preservice teachers pose more than one problem for each photo increased their awareness of problem posing practices. They developed strategies for creating open-ended problems that were then used across the various images and they compared these to the process of designing closed problems. Opportunities to develop mathematical problems with images increased their awareness of what counts as a possible mathematics problem.

Our results also indicate that in developing open-ended problems inspired by images, preservice teachers were concerned with attending to the appropriateness of the problem for children related to the intended object of learning and to what they thought would be an interesting context for students. At the same time few preservice teachers mentioned creating math problems as exemplars of big mathematical ideas or as problems they personally were inspired to solve. The problems preservice teachers posed were thus created from a pedagogical perspective (for students to solve) rather than a personal perspective (for them to solve).

The challenge preservice teachers experienced in posing open-ended problems is shared with practicing teachers. Gerofsky’s (2004) analysis of teachers’ use and development of word problems indicated that even experienced teachers who may see the world with “calculus eyes” may have difficulty seeing the world with other concepts such as fractions. Teachers and preservice teachers could be supported with strategies for creating and adapting problems. The work of Prestage and Perks (2007) provided such support for adapting and extending math problems given an initial task. What might these strategies look like in the context of developing problems from collected images? Might these strategies help preservice teachers shift their attention to explore mathematics for themselves or to create questions that encourage their students to generalize? Might they provide more explicit opportunities for preservice teachers to observe variation or regularities in creating problems and thus become more familiar and experienced with the practice of problem posing? These questions are important to consider as we continue to explore pedagogical strategies for developing a space of learning that supports preservice teachers in learning to pose good problems that may contribute to their future students’ mathematical sense-making.

References
Nicol, Bragg


