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Identification of Nonlinear Systems Using Hybrid Functions

R. Dosthosseini, F. Sheikholeslam, *Member, IEEE*, and A. Z. Kouzani, *Member, IEEE*

Abstract—**Most real systems have nonlinear behavior and thus model linearization may not produce an accurate representation of them. This paper presents a method based on hybrid functions to identify the parameters of nonlinear real systems. A hybrid function is a combination of two groups of orthogonal functions: piecewise orthogonal functions (e.g. Block-Pulse) and continuous orthogonal functions (e.g. Legendre polynomials). These functions are completed with an operational matrix of integration and a product matrix. Therefore, it is possible to convert nonlinear differential and integration equations into algebraic equations. After mathematical manipulation, the unknown linear and nonlinear parameters are identified. As an example, a mechanical system with single degree of freedom is simulated using the proposed method and the results are compared against those of an existing approach.**

I. Introduction

NATURALLY, the structures of real systems are nonlinear and have nonlinear dynamics. Although there are some methods to linearize these dynamics, usually parameter identification errors increase in these systems. In such situations, proper methods for nonlinear systems should be developed.

Regression techniques in conjunction with two dimensional orthogonal functions are used in [1] to determine an approximation for a nonlinear one degree of freedom (d.o.f.) dynamic model. The model matrix of the system was assumed to be a known mass matrix. Also, this approach was used to handle the special case of chain like multi degree of freedom nonlinear dynamic systems [2].

Chen and Tomlinson [3] presented a parametric identification method. They used time series to identify the dynamical parameters of the system and predict the time response.

There are some methods to identify the parameters of discrete non-linear systems. A wavelet based procedure was developed to identify the mechanical parameters of a discrete nonlinear system [4]. Volterra series [5] is presented to estimate non-linear systems employing multi dimensional kernels.

Recently, orthogonal functions that are a well known method for identification of dynamic and optimal control, are applied in estimating parameters [6-9]. The important characteristic of this approach is the reduction of the differential and integration equations to a system of algebraic equations by introducing the operational matrix of integration and product matrix. Consequently, these approximation algorithms are known as Direct Method [10]. This method is used for identification of linear multi degree of freedom mechanical systems [11-13]. It is also used to estimate the parameters of non-linear mechanical systems [14, 15]. For this, a methodology was developed to identify physical parameters of non-linear systems through orthogonal functions. Also, numerical simulations were used to testify the efficiency of the orthogonal functions and show their applicability for single and multi degree of freedom systems. Although the produced results are reasonable, many bases were used in the simulations taking a large amount of calculations.

Hybrid functions have received considerable attention in dealing with various problems of dynamic systems. The main benefit of using hybrid functions in identification and control problems is that differential equations can be reduced to a set of algebraic equations. In this paper, hybrid functions method is employed to identify the parameters of a nonlinear system. This method involves reducing the optimal control problem to a set of algebraic equations utilized to evaluate unknown coefficients [16]. In order to demonstrate the accuracy of the proposed numerical method and compare it with the results of direct method, the example that is solved in [15] using orthogonal functions is evaluated with the proposed hybrid functions.

This paper is organized as follows. Section II provides an overview of hybrid functions and their properties. Section III describes a nonlinear motion problem using hybrid functions. Section IV explains how an example is solved to demonstrate the accuracy of the presented numerical approach. The results are compared against those reported in [15]. Finally, conclusions are given in Section V.

II. Hybrid functions and their properties

Hybrid functions are based on two groups of orthogonal functions, first block-pulse functions as piecewise orthogonal functions, and second continuous orthogonal polynomials such as Chebyshev or Legendre. They are defined between $[0, t_f)$ as follows [17]

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R. Dosthosseini is with the School of Engineering, Deakin University, Geelong, Victoria 3217, Australia, and the Electrical and Computer Engineering Department, Isfahan University of Technology, Isfahan, Iran, 84156, (phone: +61352272183; fax: +61 352272167; e-mail: rdost@deakin.edu.au & doosthoseyni@ec.iut.ac.ir).

F. Sheikholeslam, is with the Electrical and Computer Engineering Department, Isfahan University of Technology, Isfahan, Iran, 84156 (e-mail: sheikh@cc.iut.ac.ir).

A. Z. Kouzani is with the School of Engineering, Deakin University, Geelong, Victoria 3217, Australia, (e-mail: kouzani@deakin.edu.au).

$$b_{nm}(t) = \begin{cases} P_m\big((2N/t_f)\,t - 2n + m\big)\,, & t \in \big[((n-1)/N)t_f, (n/N)t_f\big] \\ 0\,, & otherwise \end{cases} \quad (1)$$

where $n = 1,2, \dots, N$ is the order of block-pulse functions, t is the normalized time, and $P_m(t)$ are orthogonal polynomials such as Legendre or Chebyshev polynomials of order m, $m = 1,2, \dots, M-1$. $P_m(t)$ should satisfy the following recursive formula:

$$\begin{cases} P_0(t) = 1\ ,\quad P_1(t) = t \\ P_n(t) = 2tP_{n-1}(t) - P_{n-2}(t)\,, \end{cases} \quad t \in [-1,1] \quad (2)$$

for Chebyshev polynomials and

$$\begin{cases} P_0(t) = 1, P_1(t) = t, \quad t \in [-1,1] \\ P_n(t) = ((2n-1)/n)tP_{n-1}(t) - ((n-1)/n)P_{n-2}(t) \end{cases} \quad (3)$$

for Legendre polynomials.

Assume $f(t)$ is a function in the interval $[0, t_f]$. It can be approximated using hybrid functions as follows:

$$f(t) \cong \sum_{n=1}^{N} \sum_{m=0}^{M-1} c_{nm} b_{nm}(t) = \boldsymbol{C}^T \boldsymbol{B} \quad (4)$$

where

$$\boldsymbol{C} = [c_{10}, \dots, c_{1(M-1)}, c_{20}, \dots, c_{2(M-1)}, \dots, c_{N0}, \dots, c_{N(M-1)}]^T \quad (5)$$

is coefficients vector for $f(t)$ and

$$\boldsymbol{B} = [b_{10}(t), \dots, b_{1(M-1)}(t), \dots, b_{N0}(t), \dots, b_{N(M-1)}(t)]^T \quad (6)$$

is the bases vector. The integration of the vector $\boldsymbol{B}$ can be approximated by

$$\int_0^t \boldsymbol{B} d\tau \cong \boldsymbol{P}\boldsymbol{B} \quad (7)$$

where $\boldsymbol{P}$ is a $MN \times MN$ operational matrix for integration and given [7] by:

$$\boldsymbol{P} = \begin{bmatrix} \boldsymbol{E} & \boldsymbol{H} & \boldsymbol{H} \dots & \boldsymbol{H} \\ \boldsymbol{0} & \boldsymbol{E} & \boldsymbol{H} \dots & \boldsymbol{H} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{E} \dots & \boldsymbol{H} \\ \vdots & \vdots & \vdots \ddots & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \dots & \boldsymbol{E} \end{bmatrix} \quad (8)$$

where

$$\boldsymbol{H} = \frac{t_f}{N} \begin{bmatrix} 1 & 0 & 0 \dots & 0 \\ 0 & 0 & 0 \dots & 0 \\ 0 & 0 & 0 \dots & 0 \\ \vdots & \vdots & \vdots \ddots & \vdots \\ 0 & 0 & 0 \dots & 0 \end{bmatrix} \quad (9)$$

and $\boldsymbol{E}$ is a $M \times M$ matrix that is defined as:

$$\begin{cases} \boldsymbol{E}(1,1) = t_f/N\,, \\ \boldsymbol{E}(1,2) = t_f/N\,, \\ \boldsymbol{E}(i, i-1) = -t_f/(N(2i-1))\,, & i = 2, \dots, M-1 \\ \boldsymbol{E}(i, i+1) = t_f/(N(2i-1))\,, \\ \boldsymbol{E}(M, M-1) = t_f/(N(2M-1))\,, \\ 0, & otherwise \end{cases} \quad (10)$$

The product operational matrix is another property of hybrid functions to approximate the product of two hybrid functions vectors. Suppose

$$\boldsymbol{B}\boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{C} \cong \widetilde{\boldsymbol{C}}\boldsymbol{B} \quad (11)$$

where $\widetilde{\boldsymbol{C}}$ is a $MN \times MN$ product operational matrix. As an example, let $t_f = 1$, $M = 3$, and $N = 4$. Therefore

$$\boldsymbol{C} = [c_{10}, c_{11}, c_{12}, \dots, c_{40}, c_{41}, c_{42}]^T, \quad (12)$$

$$\boldsymbol{B} = [b_{10}(t), b_{11}(t), b_{12}(t), \dots, b_{40}(t), b_{41}(t), b_{42}(t)]^T, \quad (13)$$

and

$$\widetilde{\boldsymbol{C}} = diagonal\ matrix(\widetilde{\boldsymbol{C}}_{\boldsymbol{1}}, \widetilde{\boldsymbol{C}}_{\boldsymbol{2}}, \widetilde{\boldsymbol{C}}_{\boldsymbol{3}}, \widetilde{\boldsymbol{C}}_{\boldsymbol{4}}) \quad (14)$$

where $\widetilde{\boldsymbol{C}}_i, i = 1,2,3,4$ are 3×3 matrices defined as follows

$$\widetilde{\boldsymbol{C}}_i = \begin{bmatrix} c_{i0} & c_{i1} & c_{i2} \\ (1/3)\,c_{i1} & c_{i0} + (2/5)\,c_{i2} & (2/3)\,c_{i1} \\ (1/5)\,c_{i2} & (2/5)\,c_{i1} & c_{i0} + (2/7)\,c_{i2} \end{bmatrix} \quad (15)$$

when Legendre polynomials are used and when Chebyshev polynomials are used $\widetilde{\boldsymbol{C}}_i$ is defined as

$$\widetilde{\boldsymbol{C}}_i = \begin{bmatrix} c_{i0} & c_{i1} & c_{i2} \\ (1/2)\,c_{i1} & c_{i0} + (1/2)\,c_{i2} & (1/2)\,c_{i1} \\ (1/2)\,c_{i2} & (1/2)\,c_{i1} & c_{i0} \end{bmatrix}. \quad (16)$$

III. PROBLEM STATEMENT

The equation of motion of a non-linear system with n degree of freedom with respect to an external force $f(t)$ can be described as follows [15]

$$M\ddot{x} + C\dot{x} + Kx + g(x, \dot{x}) = f(t) \quad (17)$$

where M, C, and K are mass, damping and stiffness parameters, x is the displacement variable, $g(x, \dot{x})$ is a function of the displacement and velocity, and $f(t)$ is the excitation force vector. Although $g(x, \dot{x})$ is generally nonlinear, it can be considered as a linear term depending on the nature and magnitude of the nonlinear forces and the vibration level of the system [15]. In this work, $g(x, \dot{x})$ is assumed to be a nonlinear function. A general formulation is developed for a d.o.f. system with cubic stiffness and viscous and dry friction damping. Thus

$$M\ddot{x} + C\dot{x} + Kx + K_3x^3 + f_d sign(\dot{x}) = f(t) \quad (18)$$

where K_3 is the cubic stiffness coefficient, f_d is the dry friction force, and $sign(\dot{x})$ is defined as follows:

$$sign(\dot{x}) = \begin{cases} 1, & for\ \dot{x} > 0 \\ 0, & for\ \dot{x} = 0 \\ -1, & for\ \dot{x} < 0 \end{cases} \quad (19)$$

Integrating (18) twice in the interval $[0, \mathrm{t}]$, it becomes:

$$\begin{aligned} &M(x - x_0 - \dot{x}_0 t) + C\left(\int_0^t x(\tau) d\tau - x_0 t\right) \\ &+K \int_0^t \int_0^t x(\tau)\, d\tau^2 + K_3 \int_0^t \int_0^t x(\tau)^3\, d\tau^2 \\ &+f_d \int_0^t \int_0^t sign(x(\tau))\, d\tau^2 = \int_0^t \int_0^t f(\tau)\, d\tau^2 \end{aligned} \quad (20)$$

where x_0 and $\dot{x}_0$ are the initial conditions of displacement and velocity. x, x^3, $sign(\dot{x})$, and $f(t)$ are approximated with hybrid functions as:

$$\begin{aligned} x &= \boldsymbol{B}^T \boldsymbol{X} \\ x^3 &= \boldsymbol{B}^T \boldsymbol{X}_1 \\ sign(\dot{x}) &= \boldsymbol{B}^T \boldsymbol{S} \\ f(t) &= \boldsymbol{B}^T \boldsymbol{F} \end{aligned} \quad (21)$$

where $\boldsymbol{X}$, $\boldsymbol{X}_1$, $\boldsymbol{S}$, and $\boldsymbol{F}$ are known vectors of order $MN \times 1$.

Similarly, t, x_0, and $\dot{x}_0$ can be expanded as follows:

$$t = \boldsymbol{B}^T\boldsymbol{T}$$
$$x_0 = \boldsymbol{B}^T\boldsymbol{X}_0 \tag{22}$$
$$\dot{x}_0 = \boldsymbol{B}^T\boldsymbol{X}_{d0}.$$

By substituting (21) and (22) in (20), the one-d.o.f. system with cubic stiffness and viscous and dry friction damping is approximated as:

$$\begin{aligned} &M(\boldsymbol{B}^T\boldsymbol{X} - \boldsymbol{B}^T\boldsymbol{X}_0 - \boldsymbol{X}_{d0}^T\boldsymbol{B}\boldsymbol{B}^T\boldsymbol{T}) \\ &+C\left(\int_0^t \boldsymbol{B}^T\boldsymbol{X}d\tau - \boldsymbol{X}_0^T\boldsymbol{B}\boldsymbol{B}^T\boldsymbol{T}\right) + K\int_0^t\int_0^t \boldsymbol{B}^T\boldsymbol{X}\,d\tau^2 \\ &+K_3\int_0^t\int_0^t \boldsymbol{B}^T\boldsymbol{X}_1\,d\tau^2 + f_d\int_0^t\int_0^t \boldsymbol{B}^T\boldsymbol{S}\,d\tau^2 \\ &\qquad = \int_0^t\int_0^t \boldsymbol{B}^T\boldsymbol{F}\,d\tau^2 \end{aligned} \tag{23}$$

Applying the property for the integration of hybrid functions (7) and product operational matrix (11), (23) is converted to

$$\begin{aligned} &M\left(\boldsymbol{X}^T\boldsymbol{B} - \boldsymbol{X}_0^T\boldsymbol{B} - \boldsymbol{X}_{d0}^T\widetilde{\boldsymbol{T}}\boldsymbol{B}\right) + C\left(\boldsymbol{X}^T\boldsymbol{P}\boldsymbol{B} - \boldsymbol{X}_0^T\widetilde{\boldsymbol{T}}\boldsymbol{B}\right) \\ &+K\boldsymbol{X}^T\boldsymbol{P}^T\boldsymbol{P}\boldsymbol{B} + K_3\boldsymbol{X}_1^T\boldsymbol{P}^T\boldsymbol{P}\boldsymbol{B} + f_d\boldsymbol{S}^T\boldsymbol{P}^T\boldsymbol{P}\boldsymbol{B} \\ &\qquad = \boldsymbol{F}^T\boldsymbol{P}^T\boldsymbol{P}\boldsymbol{B} \end{aligned} \tag{24}$$

therefore:

$$\begin{aligned} &\boldsymbol{M}\boldsymbol{X}^T - \boldsymbol{M}\boldsymbol{X}_0^T - \boldsymbol{M}\boldsymbol{X}_{d0}^T\widetilde{\boldsymbol{T}} + C\boldsymbol{X}^T\boldsymbol{P} - C\boldsymbol{X}_0^T\widetilde{\boldsymbol{T}} \\ &+K\boldsymbol{X}^T\boldsymbol{P}^T\boldsymbol{P} + K_3\boldsymbol{X}_1^T\boldsymbol{P}^T\boldsymbol{P} + f_d\boldsymbol{S}^T\boldsymbol{P}^T\boldsymbol{P} = \boldsymbol{F}^T\boldsymbol{P}^T\boldsymbol{P} \end{aligned} \tag{25}$$

Suppose:

$$\widehat{\boldsymbol{C}} = [M, -M, -M, C, -C, K, K_3, f_d\,] \tag{26}$$

$$\widehat{\boldsymbol{X}} = \left[\boldsymbol{X}^T, \boldsymbol{X}_0^T, \boldsymbol{X}_{d0}^T\widetilde{\boldsymbol{T}}, \boldsymbol{X}^T\boldsymbol{P}, \boldsymbol{X}_0^T\widetilde{\boldsymbol{T}}, \boldsymbol{X}^T\boldsymbol{P}^T\boldsymbol{P}, \boldsymbol{X}_1^T\boldsymbol{P}^T\boldsymbol{P}, \boldsymbol{S}^T\boldsymbol{P}^T\boldsymbol{P}\right]^T \tag{27}$$

and

$$\widehat{\boldsymbol{F}} = \boldsymbol{F}^T\boldsymbol{P}^T\boldsymbol{P} \tag{28}$$

where $\widehat{\boldsymbol{C}}$ is a 1×8 vector, $\widehat{\boldsymbol{X}}$ is a $8 \times MN$ matrix, and $\widehat{\boldsymbol{F}}$ is a $1 \times MN$ vector. Thus, (25) is converted as:

$$\widehat{\boldsymbol{C}}\widehat{\boldsymbol{X}} = \widehat{\boldsymbol{F}} \tag{29}$$

Using the least-squares method, an estimate of vector $\widehat{\boldsymbol{C}}$ where its coefficients are unknown is given as follow [15]:

$$\widehat{\boldsymbol{C}} = \widehat{\boldsymbol{F}}\widehat{\boldsymbol{X}}^T\left(\widehat{\boldsymbol{X}}\widehat{\boldsymbol{X}}^T\right)^{-1} \tag{30}$$

Equation (30) can be solved using the singular value decomposition method to determine the unknown parameters. If the displacement and velocity initial conditions are unknown, they can be solved with the same rule.

It is important to point out that, in using (30), there is no need for any information about physical or model parameters concerning the mechanical system. However, other identification techniques such as those mentioned in this paper require such information. This means that force identification problems can be studied on a straightforward manner by using our method. Also, it is shown in the following section that the errors for the example solved are less than those reported in [15].

IV. One-d.o.f. Mechanical System with Mixed Damping

Let

$$M = 1\,kg, C = 20\,N\,s/m\,, K = 10000\,N/m, \tag{31}$$
$$f_d = 1\;and\;3\;N$$

are the known parameters for a nonlinear one-d.o.f. mechanical system. In this example for simplicity, it is assumed that K_3 is equal to zero. A swept-sine excitation with $F_{rms} = 10\;N$ from 10 to 20 Hz was used [15]. The response, considering $f_d = 1$ and $3\;N$ was sampled at a frequency of 1700 Hz using the fourth-order Runge–Kutta method as shown in Fig. 1 and Fig. 2.

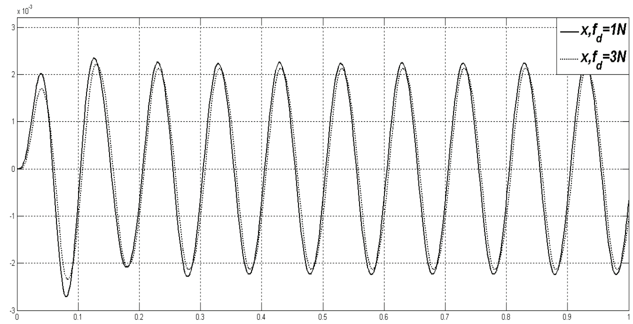

Fig. 1. $x(t)$ of the system using a swept-sine excitation with $F_{rms} = 10\;N$

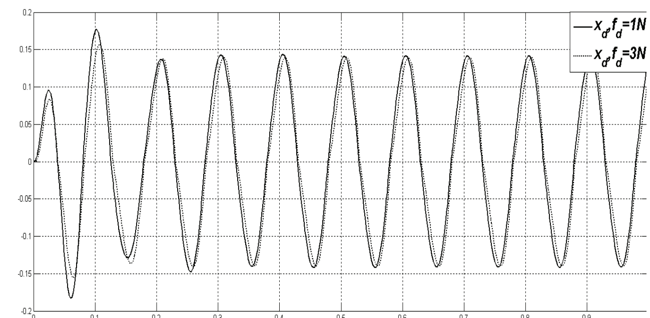

Fig. 2. $x_d(t)$ of the system using a swept-sine excitation with $F_{rms} = 10\;N$

The parameters identified using the hybrid functions method and the orthogonal functions [15] are shown in Table I.

Also, Table II represents that when f_d/F_{rms} is increased, the errors in the identified parameters increases in comparison with the first case. According to Tables I and II, the parameters identified by hybrid functions are more accurate than those identified by orthogonal functions. Also the number of bases used in this approach is less than those reported in [15].

A harmonic excitation is used to identify the unknown parameters. In this case, the excitation force is:

$$f(t) = F_0\sin\,(2\pi f_0 t) \tag{32}$$

with $f_0 = 21\,Hz$, $F_{rms} = 10\,N$, and the corresponding sampling frequency is 10240 [15]. The results are shown in Tables III and IV. Considering these tables, it is clear that the errors using Chebyshev polynomials have increased for both hybrid and orthogonal functions.

TABLE I
SWEPT-SINE EXCITATION FOR ONE-D.O.F. SYSTEM, $f_d = 1\,N$

Hybrid functions	$M\ (kg)$	$C\ (N\,s/m)$	$K(N/m)$	$f_d\ (N)$
Block-pulse& Legendre $N = 4\,, M = 5$	0.995	20.17	10001.7	0.996
Block-pulse& Chebyshev $N = 4, M = 5$	0.997	20.12	10006.8	0.997
Orthogonal functions [15]				
Legendre $M = 35$	0.993	20.40	10003	0.993
Chebyshev $M = 30$	0.995	20.10	10019	1.020
Block-pulse $M = 512$	0.993	20.18	10045	1.004

TABLE II
SWEPT-SINE EXCITATION FOR ONE-D.O.F. SYSTEM, $f_d = 3\,N$

Hybrid functions	$M\ (kg)$	$C\ (N\,s/m)$	$K(N/m)$	$f_d\ (N)$
Block-pulse& Legendre $N = 4\,, M = 5$	0.992	20.20	10002.9	3.021
Block-pulse& Chebyshev $N = 4, M = 5$	0.993	20.53	10011.5	3.014
Orthogonal functions [15]				
Legendre $M = 35$	0.987	20.13	9957	3.103
Chebyshev $M = 30$	0.987	20.82	9974	2.914
Block-pulse $M = 512$	0.987	20.47	10012	2.980

In addition, if a parameter is known, the results are considerably improved. If the dry friction force and the stiffness parameters used in the identification procedure are assumed as 1 N and 9997 N/m, the errors for identified parameters decrease as shown in Table V.

On the other hand, let the dry friction force and the stiffness parameter be 3 N and 9995 N/m. Table VI presents the identified parameters.

Clearly, Tables V and VI show that when the number of unknown parameters decreases, the results using orthogonal and hybrid functions become close.

Also, if all the parameters in (18) are considered, it can be observe that the magnitude of the errors in the identification of mass, damping and stiffness parameters were similar for all kinds of excitation forces. These parameters are given by

$$\mathrm{M} = 1\ \mathrm{kg}, \mathrm{C} = 20\ \mathrm{N\,s/m}\,, \mathrm{K} = 10000\ \mathrm{N/m}, \quad (33)$$
$$\mathrm{f_d} = 1\ , \text{and } \mathrm{K_3} = 5 \times 10^9\ \ \mathrm{N/m^3}$$

Similarly, a harmonic excitation is used to identify the unknown parameters. In this case, the excitation force is:

TABLE III
HARMONIC EXCITATION FOR ONE-D.O.F. SYSTEM, $f_d = 1\,N$

Hybrid functions	$M\ (kg)$	$C\ (N\,s/m)$	$K(N/m)$	$f_d\ (N)$
Block-pulse& Legendre $N = 4\,, M = 5$	1.000	20.16	10003.1	0.991
Block-pulse& Chebyshev $N = 4, M = 5$	0.989	20.19	9987.3	1.112
Orthogonal functions [15]				
Legendre $M = 25$	1.000	20.28	10029	0.982
Chebyshev $M = 21$	0.986	19.77	9830	1.161
Block-pulse $M = 512$	1.000	20.29	10054	0.973

TABLE IV
HARMONIC EXCITATION FOR ONE-D.O.F. SYSTEM, $f_d = 3\,N$

Hybrid functions	$M\ (kg)$	$C\ (N\,s/m)$	$K(N/m)$	$f_d\ (N)$
Block-pulse& Legendre $N = 4\,, M = 5$	0.999	20.25	10007.4	3.033
Block-pulse& Chebyshev $N = 4, M = 5$	0.999	20.28	10021.9	3.012
Orthogonal functions [15]				
Legendre $M = 35$	1.002	20.43	10065	2.959
Chebyshev $M = 30$	1.000	20.65	10036	2.945
Block-pulse $M = 512$	1.004	20.56	10120	2.925

TABLE V
FREE RESPONSE FOR ONE-D.O.F. SYSTEM, $f_d = 1\,N$

Hybrid functions	$M\ (kg)$	$C\ (N\,s/m)$	$f_d\ (N)$
Block-pulse& Legendre $N = 4\,, M = 5$	1.001	20.01	0.993
Block-pulse& Chebyshev $N = 4, M = 5$	1.005	19.91	0.978
Orthogonal functions [15]			
Legendre $M = 33$	1.002	20.01	0.991
Chebyshev $M = 20$	1.008	19.88	0.971
Block-pulse $M = 512$	0.995	19.96	1.001

$$f(t) = F_0 \sin\,(2\pi f_0 t) \quad (34)$$

with $f_0 = 21\,Hz$, $F_{rms} = 20\,N$, and the corresponding sampling frequency is 5115 [15]. The results are shown in Tables VII. Since increasing the number of unknown parameters will lead to worse results, in this case to decrease errors occurred, $\mathrm{M} = 50$ for orthogonal functions and $\mathrm{M} = 6$ for hybrid functions are considered.

TABLE VI
FREE RESPONSE FOR ONE-D.O.F. SYSTEM, $f_d = 3\,N$

Hybrid functions	$M\,(kg)$	$C\,(N\,s/m)$	$f_d\,(N)$
Block-pulse& Legendre $N = 4\,, M = 5$	1.000	20.00	3.004
Block-pulse& Chebyshev $N = 4, M = 5$	1.006	19.92	2.953
Orthogonal functions [15]			
Legendre $M = 33$	1.000	20.00	3.005
Chebyshev $M = 20$	1.007	19.88	2.941
Block-pulse $M = 512$	0.995	19.97	2.978

TABLE VII
HARMONIC EXCITATION FOR ONE-D.O.F. SYSTEM, $f_d = 1\,N$

Hybrid functions	$M(kg)$	$C(N\,s/m)$	$K(N/m)$	$K_3.10^{-9}$ N/m^3	$f_d\,(N)$
Block-pulse& Legendre $N = 4\,, M = 6$	1.000	18.99	10009.7	5.003	1.006
Block-pulse& Chebyshev $N = 4, M = 6$	0.991	18.81	10011.6	5.003	1.023
Orthogonal functions [15]					
Legendre $M = 50$	1.000	18.94	10012	5.004	1.008
Chebyshev $M = 50$	0.998	18.79	10013	4.997	1.031
Block-pulse $M = 512$	0.997	18.87	10012	5.002	1.013

V. CONCLUSIONS

A numeric approach for parameter identification of one-d.o.f nonlinear system using hybrid functions has been presented. The important property of hybrid functions is their conversion of differential or integral equations to algebraic equations. An example of nonlinear mechanical system has been solved using Chebyshev and Legendre polynomials with a combination of block-pulse bases. The results show that the number of bases used for the estimation has decreased. In addition, the errors become less than those produced by orthogonal functions.

REFERENCES

[1] S. F. Masri, and T.K. Caughey, "A nonparametric identification technique for nonlinear dynamic problems," *Journal of Applied Mechanics Transaction, ASME*, vol. 46, 1979, pp. 433–447.

[2] P. F. Pai, and A. N. Palazotto, "HHT-based nonlinear signal processing method for parametric and non-parametric identification of dynamical systems," *International Journal of Mechanical Sciences*, vol. 50, 2008, pp. 1619–1635.

[3] Q. Chen, and G. R. Tomlinson, "Parametric identification of systems with dry friction and nonlinear stiffness using a time series model," *ASME, Journal of Vibration and Acoustics*, vol. 118, 1996, pp. 252–263.

[4] R. Ghanem, and F. Romeo, "A wavelet-based approach for model and parameter identification of non-linear systems," *International Journal of Non-linear Mechanics*, vol. 36, 2001, pp. 835–859.

[5] A. Chatterjee, and N. Vyas, "Non-linear parameter estimation using Volterra series with multi-tone excitation," *XX IMAC Proceedings of the 20th International Modal Analysis Conference*, Los Angeles, CA, 2002, pp. 880–885.

[6] D. Remond, J. Neyrand, G. Aridon, and R. Dufour, "On the improved use of Chebyshev expansion for mechanical system identification," *Mechanical Systems and Signal Processing*, vol. 22, 2008, pp. 390–407.

[7] M. Razzaghi, and H. R. Marzban, "A hybrid analysis direct method in the calculus of variations," *International Journal of Computer Mathematics*, vol. 75, 2000, pp. 259–269.

[8] M. Razzaghi, "Optimal control of linear delay systems via combined piecewise and continuous basis functions," *Automation Congress, 2008. WAC 2008. World*, vol. 2, 2008, pp. 1–6

[9] B. M. Mohan, and S. K. Kar, "Comments on Optimal control via Fourier series of operational matrix of integration," *IEEE Transactions on Automatic Control*, vol. 50, no. 9, 2005, pp. 1466–1467.

[10] H. Yen, F. Fahroo, and I. M. Roos, "Optimal feedback control laws by Legendre pseudo spectral approximations," *Proceeding American Control Conference*, Arlington, VA, vol. 3, 2001, pp. 2388–2393.

[11] R.P. Pacheco, "Mechanical systems identification through time domain methods," Ph.D. Thesis, Federal University of Uberlandia, 2001.

[12] R. P. Pacheco, and V. Steffen Jr., "Using orthogonal functions for identification and sensitivity analysis of mechanical systems", *Journal* of *Vibration* and *Control*, vol. 8, no. 7, 2002, pp. 993–1021.

[13] R. P. Pacheco, V. Steffen Jr., and D. A. Rade, "Orthogonal functions techniques for the identification of mechanical systems," *XIX IMAC Proceedings (in CD) of the 19th International Modal Analysis Conference*, Orlando, FL, vol. 4359, no. 1, 2001, pp. 768–774.

[14] R. P. Pacheco, and V. Steffen Jr., "Nonlinear identification of mechanical systems using orthogonal functions," *Proceedings (in CD) of the Fourth International Conference on Inverse Problems in Engineering: Theory and Practice*, Brazil, 2002, pp. 26–31.

[15] R. P. Pacheco, and V. Steffen Jr., "On the identification of non-linear mechanical systems using orthogonal functions," *International Journal of Non-Linear Mechanics*, vol. 39, 2004, pp. 1147–1159.

[16] V. Yen, and M. Nagurka, "Linear quadratic optimal control via Fourier-based state parameterization," *Journal of Dynamic Systems, Measurement and Control*, vol. 113, 1991, pp. 206–215.

[17] H.R. Marzban, and M. Razzaghi, "Hybrid functions approach for linearly constrained quadratic optimal control problems," *Applied Mathematical Modeling*, vol. 27, 2003, pp. 471–485.