SYMBIOSIS BETWEEN CREATIVE MATHEMATICAL THINKING ACCOMPANIED BY HIGH POSITIVE AFFECT, AND OPTIMISM

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Video-stimulated post-lesson interviews captured changes in a Grade 5 elementary school student, Tom's, orientation to problem solving. Whilst participating in small group problem solving including reports to the class ('Engaged to Learn' pedagogy), Tom changed from self-focused (Task 1), to group focused (Task 2), and task focused (Task 3). He experienced surprise as complexities became apparent in what had appeared to be simple (Task 2), and displayed positive affect during his creative thinking leading to insight (Task 3). Consistent with Seligman's (1995) findings, 'flow' (Csikszentmihalyi, 1992), a state of high positive affect accompanying creative activity was associated with optimism building. Instead of needing to be valued by others to feel successful, Tom began to internalise his successes as attributes of self.

INTRODUCTION

This study is part of a broader longitudinal study of building psychological characteristics to increase student inclination to engage intellectually with challenging mathematical problem solving. Students 'recognizing' and 'building-with' mathematics to overcome challenges they set themselves are realising what mathematics they have known previously ('cognitive artefacts') might be relevant and using this mathematics in unfamiliar sequences and/or combinations (Dreyfus, Hershkowitz, & Schwarz, 2001). Students who are 'optimistic' (Seligman, 1995) are inclined to make a personal effort and struggle to 'come to know' ('success') when they encounter situations that they are unfamiliar with. They look for what they could change to increase their likelihood of 'success' or 'find ways to proceed'. In doing so, they overcome 'not knowing' ('failure'). Students who are not optimistic are not inclined to engage in such activity because they see 'not knowing' as permanent unless they are 'told' or given hints about what to use and how to use it by an 'expert'. We may be able to improve the problem solving capacity of such students by increasing their optimistic characteristics. Optimism is an orientation to successes and failures that influences how people respond in subsequent situations. Optimistic children see not knowing as temporary, specific to the situation, and able to be overcome by personal effort by looking into situations to see what they can change. They perceive their successes as permanent 'I know how to do this' and due to attributes of self (or pervasive): 'I can do this, I am a good at this'. Children who are not optimistic perceive failures as permanent, and pervasive and personal: 'I failed, I am stupid?'. They perceive their successes as temporary, specific, and due to external influences.

Williams

The research questions are: Can optimism change over time as a result of participation in problem solving activity? If so, what is the nature of these changes and what contributed to them? And, are there accompanying changes to the problem solving activity of the student?

THEORETICAL FRAMEWORK

Flow (Csikszentmihalyi, 1992) is a state of high positive affect associated with creative activity. People in this state lose all sense of time, self, and the world around as they focus all of their energies on the task at hand. Conditions for flow include a spontaneously focused challenge that is almost out of reach that involves the development of new skills to overcome it. Flow during mathematical problem solving (Williams, 2002, 2006) may occur when a student or group become aware of a mathematical complexity that was not apparent to them at the commencement of the task, and they decide that they want to pursue it and in doing so develop new conceptual understandings. To do so, they spontaneously ask themselves a question about this complexity that results in an intellectual mathematical challenge that requires the use of mathematics in unfamiliar ways to achieve this.

For the purposes of this study, ‘collaboration’ occurs when students working as part of a small group spontaneously, and creatively (see Dreyfus, Hershkowitz, & Schwarz, 2001; Williams, 2002) building new ideas together. In elaborating the meaning of abstracting in context, Schwarz, Dreyfus, and Hershkowitz’s (2009) research included descriptions that fit collaboration (key: ... text omitted without changing meaning; [text] researcher-teacher (RT)) comment:

- ‘Explanations underwent a transformation that appeared to support ... reaching a mathematically valuable understanding’ (pp. 16).
- ‘Reorganisation within mathematics, ... finding shortcuts and discovering connections between concepts and strategies (pp. 17)
- Prior constructs ... are ... reorganised ... [and] ideally, ... also integrated and interwoven (pp. 17).

Recognizing of mathematics that is relevant, findings new ways to combine this mathematics, and integrating mathematical ideas to develop mathematical insights are aspects of collaboration that are associated with high positive affect. The process of abstracting and processes associated with collaboration both include student interactions in which students draw upon their past knowledge as it appears to be relevant to the new context (‘recognizing’) and ‘build-with’ this mathematics in unfamiliar ways. They clarify, justify, and evaluate and refine ideas presented in their groups and through this process, and synthesise ideas to reveal insights that are new to the student/group. The process of collaboration requires the collaborative interval to be spontaneous, not assisted by input of mathematics from an external source. Although challenges associated with the mathematical tasks presented in this research could be overcome through assistance from an external source, this does not occur
because that limits flow opportunities and flow experiences build optimism. It is under these conditions that flow can occur and these are ‘optimism building’ situations (Seligman, 1995). The teachers ask questions that assist students to clarify their thinking.

As student-student interaction supports students making meaning of mathematics (Clarke, 2001) and students developing mathematical insights (Schwarz, Dreyfus, & Hershkowitz, 2009), the Engaged to Learn approach is based around small group work with regular reporting to the class as a whole. The reporting sessions serve a variety of purposes. They provide opportunities for consolidation (Schwarz, Dreyfus, & Hershkowitz, 2009) as groups reflect on what they have found and extend their thinking. Secondly, ‘priming’ the reporter is intended to make the group, not the individual reporter, accountable for the report and add security for reporters.

The RT designs ‘conceptual tasks’ (Lampert, 2001) to provide opportunities for students to discover mathematical complexities. New complexities become apparent during work with the task and differ depending on the aspects of the task a group focuses on. There are ‘twists’/surprises to increase student engagement and entice students into the task. This contributes to flow opportunities.

**RESEARCH DESIGN**

Classroom video and video stimulated post-lesson student interviews were employed to study links between creative mathematical thinking (high level cognition), high positive affect, and psychological factors associated with mathematical problem solving in situations relevant to the teaching and learning of mathematics in classrooms. Four videos were used to capture the six to seven groups and the reporting sessions. The videos were mixed and the student had the remote control to a video of his group, and the reporting session so he could find parts of the lesson that were interesting to him to talk about. The RT was the primary implementer of the problem solving activities and the classroom teacher participated. Only those aspects of the design of the broader study that are crucial to Tom’s case are elaborated herein.

Task 1 groups were formed by the classroom teacher (T) using the RT’s criteria. They contained 3-4 students with similar paces of thinking, and a student who was likely to be able to keep their group on task and encourage all students to participate. Students were interviewed no more than twice in a year (ethics decision). Tom was interviewed after Task 1 and Task 3. He displayed indicators of optimism and indicators of lack of optimism. To try to increase Tom’s flow opportunities, he was grouped differently in the second and third tasks. Student reports of affect, their body language, and their exclamations provided indicators. For coding scheme to identify high positive affect, see Williams (2002). Optimistic indicators were displayed in interviews in response to questions asked about: how students learnt mathematics. This provided information about whether they saw ‘not knowing’ as temporary, or whether they perceived ‘not knowing’ as permanent without mathematical assistance. How students made decisions on how good or not they were at mathematics provided
information about whether they judged success using only external sources or included self-reflection in decisions. Each group works together for 10-12 minutes before a group member from each group reports to the class and students moved back to their groups. A different reporter from each group reported each time. Students took on different roles between one reporting session and the next. The ‘priming’ the reporter provided video data that summarized student findings.

Decisions on who to interview are made during or straight after the lesson and are based upon previous activity, whether creative thinking occurred or was inhibited, which students had been interviewed, and whether there appeared likelihood of change in optimistic indicators (as Tom did).

Brief Summary Task 1: using all 14 tiles (each time), make as many different flat ‘filled’ rectangles as you can. Repeat using 12 tiles. Have you found all possibilities? Make an argument for how you know you have them all. Do more tiles make more rectangles? Why or why not? Select a number of tiles between 16 and 45 to make as many rectangles as possible using all each time. Explain your process of thinking.

Brief summary of Task 2: A large fish drawn on the board has an arrow pointing to the head stating ‘the head is as heavy as four tails’ and an arrow pointing to the body stating ‘the body is as heavy as the head and the tail together’. Find all you can about the maths of such fish.

Brief summary of Task 3: use four of the digit four, and any number of the following ‘+ - × ÷ ( ) √ ’ to make each of the whole numbers from 1-20. Then look for ways to find them all as fast as you can. Explain.

RESULTS AND ANALYSIS

As Rows 3 and 4 of Table 1 show, Tom displayed some indicators of lack of optimism during and after Task 1 [Row 3, Columns 3 and 4], and none during and after Task 3 [Row 4, Column 2 to 4]. Tom displayed more indicators of optimism during and after Task 3 than during and after Task 1 [Columns 5-7].

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Table 1: Changes in Tom’s orientation to success and failures over time

During Task 1, Tom was grouped with a Grade 4 and Grade 6 boy. As shown from the excerpt of lesson transcript before the last of four reporting sessions, Billy (Grade 6) has begun to develop an idea but Tom argued that it was his reporting session and he was going to report his own ideas not the ideas of the group. This was contrary to Engaged to Learn ‘rules’ and a disagreement ensued. The majority of the group...
discussion time was taken up arguing and the group did not have time to refine Billy’s ideas. Each made numerous attempts to control the paper, pen and materials throughout the task. Tom displayed Success as External in many ways. Some illustrations are provided. Success for Tom involved an expert other valuing him for what he said. This non-optimistic indicator of Success as External is consistent with Tom’s position that he wanted to report his own ideas rather than the ideas of the group in the final Task 1 report.

Billy had found (with the examples tried) that if you continued dividing a number and using the answer for the next step, you could not go any further once your answer would not divide by two. He thought the number of divides gave the number of rectangles. Billy had been discussing these ideas with Sammy and Tom and in Line 1 began to summarise for Tom. Lines 2-4 show the types of disagreement that continued until almost the end of the reporting session. Both Tom and Billy show they are aware of the expected group rules [Lines 6, 7] but Tom showed he is disregarding them [Line 8]. Tom was too focused on reporting his own ideas so they could be valued by the RT and T. He did not consider Billy’s idea and whether it always worked. Sammy did not have a chance to discuss it either. Tom’s interview comments fitted with his activity in class: ‘It’s my time to report and I am going to say what I want to say’. When asked in his interview what his ideas were, Tom showed he did not have a report in mind:

1 Tom Billy wanted a pattern for it to end up in three but I wanted to do what I was thinking and see if there was anything better to be made
2 Int Okay (pause) What were you thinking?
3 Tom Ah I was thinking-ah well (pause) I was thinking um- oh- well (pause) um I was thinking ah there must be other ways to do it than just following a pattern- and I know you should follow patterns but there has to be a different way and if there is not then I will just follow Billy’s way and then the timer rang.

Tom wanted to be the one to do something original. He was focused on himself and not the task. It did not matter that Billy had presented the start of some idea that could turn out to be very useful, to Tom that idea was around patterns [Line 1] and patterns
Williams

were something many focused on. He wanted to be different [Line 3] rather than solve the task. His focus was on form not function.

Tom showed on several occasions that his focus was not on the task, but rather on not being controlled by others. This showed in his argument with Billy [Line 6,8] and in the interview when he showed he was not focusing on the task but rather on the interactions: ‘Sammy was a bit- uh I don’t know if (pause) humble is the right word- he was um letting himself be taken control of by Billy’. On three occasions in interview Tom stated that the reports of Billy and Harry were embarrassing [Failure as Pervasive] instead of considering what was useful and what would require further clarification. Again, he focused on self not task.

Task 2: to increase Tom’s likelihood of gaining task focus, Tom was grouped with two Grade 6 students Natasha and Ken who had both collaborated to build insights in Task 1. They were both focused and calm and it was considered they would be able to stop Tom from exhibiting ‘take over’ activity and make him conform to group reporting rules. The group did calmly discuss ideas, and they quietly used body language to deter Tom’s attempts, initially, to ‘take over’. Tom listened and contributed by asking when he did not understand. The group primed Tom to present a clear report that contributed to class knowledge. Tom stated he was most interested in this task after Task 3 because:

You can just draw something so small with so little detail and find out so much about it and end up [small laugh] (pause) just like you have known it for ever.

Task 2 contained an element of surprise that engaged Tom with the mathematics.

Task 3: Tom was grouped with a student who should be able to ‘control’ any ‘take over’ activity and would also be able to think creatively but not at a fast pace (Gabrielle). This was expected to give Tom more time to think. It was hoped that Tom and Gabrielle would create new ideas together. The other two students in the group had not previously contributed creative ideas but had not disrupted their groups. Tom was the final reporter for his group. In the previous reports, Alf from another group explained: ‘Well four over four is one whole so that’s just like saying one (pause) and four times four plus one you would get seventeen’. Tom reported his excitement in his interview:

‘When he [Alf] said four over four and it is the same as one just that sentence just flung me [intense, twirls hand] like quickly in my mind ahh I could use that’.

Tom initially thought he would be able to apply this using each multiple of four as a stem. Gabrielle insistently requested elaboration and her actions appear to elicit deeper thinking about the detail of what he was saying from Tom:

1 Gabrielle how how how how are you going to get seventeen and nineteen twenty

2 Tom Four over four- … that’s one okay and or a whole (pause) so that means that using a whole you can go … minus 1 or plus one … we could get these numbers … (eleven) thirteen fifteen (seventeen and twenty).
Williams

3 Gabrielle so if you were going- if somebody asked you to make- to get answers for every single number [ruling up sheet] ...

Tom had excitedly talked without illustrating most statements. The group had not understood. Gabrielle [Line 1] may be aware that at present he has a 'partially correct construct' (Schwarz, Dreyfus, & Herschkowitz, 2009). Tom [Line 2] explains again without specifically answering Gabrielle’s question. Gabrielle makes another attempt [Line 3] by requesting Tom show the calculation for each number using his ideas. Gabrielle’s request for elaboration led to Tom’s revised ideas which limited the stem to 8, (4+4) and 16, (4x4). He elaborated on the reason for his changed position: ‘something like 12 (pause) you won’t be able to do it because four plus four plus four is one of the only ways to get to twelve and ... [there] will be five fours’. Tom’s ideas were refined through Gabrielle’s insistence for justification and elaboration. His ideas became closer to a correct construct, Tom located some correct possibilities but did not see new possible opportunities using other operations.

After Task 3: In his final interview, Tom showed he was considering himself as a problem solver after ‘seeing’ his activity over time:

Mum, she said ... ‘[students ask you] because you are smart Tom’. ... I don’t know whether I believed it or not ... but when I have gone through with these [tasks] (pause) I have believed that I am a bit smart.

Tom has begun to shift from perceiving success as external to making his own assessment about his maths ability by reflecting on his own activity (Success as Personal) and taking on successes as characteristics of self (Table 1).

DISCUSSION AND CONCLUSIONS

Tom’s interviews and classroom activity illustrated his high positive affect in the form of surprise (Task 2) and excitement and intensity (Task 3) in relation to mathematical activity associated with these two tasks. In Task 2, mathematical complexities became apparent as the other two group members developed mathematical ideas. In this task, it was Tom’s recognising of relevant mathematics that he was not initially aware of that led to high positive affect. In Task 3, Tom drew upon cognitive artefacts presented by Alf in another group when he recognised 4/4 could be used more generally as one to make several numbers using two of the fours, and adding or subtracting the 4/4. His high positive affect showed as excitement and intensity as he recognised relevance, gained insight about mathematical structure, was surprised to realise part of his idea did not work but persisted to work out more. Gabrielle’s insistent ‘push’ for elaboration was crucial to Tom extending his understanding of the detail of the ideas he was putting forward. Without this, it is possible Tom may not have thought further. Tom’s activity has added to the construct of partially constructed constructs by showing an instance of how such constructs can come closer to ‘correct constructs’ over time (Schwarz, Dreyfus, & Herschkowitz, 2009). In this instance the student has recognised what was incorrect and why but not yet how to overcome this problem. In Tom’s case, creative mathematical activity
accompanied by high positive affect was associated with increased indicators of optimism as expected (Seligman, 1995). Indicators were tentative not strong and further research to see whether they stabilize would be a fruitful area for further research. This study highlights the important role group composition played in changing Tom’s orientation to problem solving. This student who had required external affirmation to perceive himself as successful inhibited opportunities for his group in Task 1 to think creatively. The indicator ‘Success as External’ appears to have created an unproductive interaction because Tom was not focused on the task but on ‘showcasing’ himself. Further research is required to find more cases to study change in optimistic indicators.

Acknowledgements

Research funded by the Australian Research Council Grant DP0986955 and hosted by the International Centre for Classroom Research at the University of Melbourne.

References


Williams