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Designing Optimal Fault Tolerant Jacobian for Robotic Manipulators

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***Abstract*—Fault tolerance of robotic manipulators is determined based on the fault tolerance measures. In this study a Jacobian of a 7DOF optimal fault tolerant manipulator is designed based on optimality of worse case relative manipulability and worse case dexterity from geometric perspective instead of numerical solution of constrained optimisation problem or construction of optimal Jacobian through a desired null space. The proposed Jacobian matrix is optimal and equally fault tolerant for a single joint failure within any joint of the manipulators.**

Keywords:
Serial link manipulator, Optimal fault tolerant, Optimal Jacobian, Relative manipulability, Dexterity.

I. INTRODUCTION

Fault tolerant manipulators are essential where highly available robots are required such as robotic manipulators working in medical applications, hazardous environments nuclear disposal, exploring deep sea and outer space. The design or control of fault tolerant manipulators aims to maintain the availability of the robot even under partial faults if they occur within their actuators or sensors [10,21].

The literature surrounding fault tolerant robotics has focused on design level. Design of fault tolerant manipulators includes fault tolerant structures or control level which includes fault analysis, fault tolerant motion planning and control.

Within the design level, different structures such as serial or parallel manipulators have been studied [11-14],or a manipulator with a specific fault tolerant property has been designed or its optimal configurations have been determined [1-6]. Literature within the control level has studied the Fault Detection, Fault Isolation and Identification and Fault Recovery techniques as in [15-16]. Various strategies such as model based, robust or AI based techniques (neural and fuzzy) for fault tolerant control have been proposed. This study is on the design of fault tolerant manipulators.
Serial link manipulators (SLM) have received significant attention in the robotics community. Fault tolerant design of the SLMs is achieved by adding extra kinematic redundancy [7],[8] named serial link redundant manipulators (SLRM). By adding a kinematic redundancy, a SLRM maintains its availability to perform the required [9] or prioritised tasks [17], even if one or more joint fails. While it has been observed that adding kinematic redundancy improves the fault tolerance of the manipulators, it also promotes other static or dynamic properties, such as higher dexterous movement [24], lower maintenance and repair cost and more capability for obstacle avoidance [9]. However, simply having kinematic redundancy does not ensure fault tolerant operation, as the kinematic redundancy has to be used efficiently to maintain the availability of the manipulator [7],[8].

This work is adding up to a series of work from Robert and Maciejewski mainly in their latest work in [1-3] all are about the design of optimal fault tolerant manipulators. But the main distinction between this study and the method provided in their work is that we are designing on optimal Jacobian based on desired geometric properties compare to the numerical solution of optimisation of measures [5] or using the properties of desired null space of the Jacobian matrix [1-3]. Recently the design of optimal fault tolerant Jacobian is shifted from numerical optimisation methods to construction of optimal Jacobian based on desired null space. As it has been proven that null space has a close relationship to the fault tolerance measure [3], the optimal Jacobian matrices proposed in [1-2] are based on this fact.

The first step to design and optimal fault tolerant manipulators is to obtain the optimal fault tolerant Jacobian. This should design a Jacobian matrix which provides maximum fault tolerance measures. The problem is still challenging both in mathematics and robotics communities as it is shown that only limited number of the cases of these matrices have been identifies. In this study based on the geometric properties of the optimal Jacobian a class of optimal fault tolerant Jacobian matrixes is provided.

This leads to introducing an optimal fault tolerant Jacobian for a 7DOF manipulator which is a minimal redundant manipulator.

The remainder of this paper is organized as follows. At first the Jacobian of SLRM is defined when a fault occurs into the joints. Then the fault tolerance indices are presented and optimal fault tolerance is defined. Through this, the optimal Jacobian is introduced from symmetric geometries. Then optimal Jacobian for a 7DOF SLRM introduced using the symmetric geometries in R6.

II. LOCKED JOINT FAULT IN REDUNDANT MANIPULATORS

A. Forward Kinematic

The forward kinematics of a manipulator relates the revolute joint angles (position for prismatic joints) to the end effector (EFF) position and orientation (1). The joint variables (q) define the configuration space (2) and position/ orientation variables (X) define the work space of the manipulator (3).

$$X = f(q) \quad (1)$$
$$q = [q_1 \quad q_2 \quad .... \quad q_n]^T \quad (2)$$
$$X = [x_1 \quad x_2 \quad ... \quad x_m]^T \quad (2)$$
$$q = f^{-1}(X) \quad (4)$$

Where the manipulator is a *n*DOF manipulator and *m* is the dimension of the workspace.

The degree of kinematic redundancy (DOR) is obtained by *n-m* when $\{n \geq m\}$. In [18] the number of required redundancy was investigated by applying joint fault possibility and total reliability and the economic cost of the manipulator.

B. Jacobian of SLRM under locked joint fault/s

The Jacobian matrix (5) relates the EFF translational and orientation velocities to the joint velocities (6).

$$J = \left[\frac{\partial f}{\partial q}\right]_{mn} \quad (5)$$
$$\dot{X} = J\dot{q} \quad (6)$$

Fault analysis is possible not only in forward kinematic domain as [19] but also through the Jacobian matrix [7],[8]. The first approach results in global analysis of fault, but due to its nonlinearity the second approach which uses Jacobian (7) is more common.

Without loss of generality, let assume the manipulator is revolute joint. If J_k in (7) is the k^{th} columns of *J*, this column indicates the contribution of the corresponding joint (k^{th} joint) velocity in the translational and orientation velocity of the EEF.

$$J = [J_1 \quad J_2 \quad ... \quad . \quad J_{k-1} \quad J_k \quad J_{k+1} \quad ... \quad J_n \quad]_{mn} \quad (7)$$

When manipulator has a fault in its k^{th} joint, this joint does not contribute into EFF velocity. Thus the Jacobian of the manipulator under an immobilized joint can be introduced by replacing a *0* vector in the k^{th} column of the original manipulator Jacobian. This Jacobian is called reduced Jacobian (8) and can be rewritten as (9,10)

$${}^kJ = [J_1 \quad J_2 \quad . \quad . \quad J_{k-1} \quad 0 \quad J_{k+1} \quad . \quad J_n \quad] \quad (8)$$
$${}^kJ = [J_1 \quad J_2 \quad . \quad . \quad J_{k-1} \quad J_{k+1} \quad . \quad . \quad J_n \quad] \quad (9)$$
$$\dot{X} = {}^kJ\,{}^k\dot{q} \quad (10)$$
$${}^k\dot{q} = [\dot{q}_1 \quad \dot{q}_2 \quad . \quad . \quad \dot{q}_{k-1} \quad \dot{q}_{k+1} \quad . \quad . \quad \dot{q}_n \quad]^T \quad (11)$$

For one possible fault there are *n* reduced Jacobian matrices as $\{{}^1J, {}^2J, ..., {}^nJ\}$.

III. FAULT TOLERANT MANIPULATORS

A. Fault tolerant Indices

It is promising that in the last two decades, the fault tolerance of serial link redundant manipulators has been well discussed by A.A. Maciejewski and R.G. Roberts et al. [1-9]. In the literature global and local measures have been used for fault tolerant analysis of the redundant manipulator.

TABLE I

COMMON FAULT TOLERANT MEASURES

Index Name	Definition	Description
Manipulability	$\sqrt{\det({}^iJ\,{}^iJ^T)}$ (12)	${}^iJ$ Jacobian matrix of manipulator with fault in i^{th} joint
Relative Manipulability	$\frac{\sqrt{\det({}^iJ\,{}^iJ^T)}}{\sqrt{\det(JJ^T)}}$ (13)	${}^iJ$ Jacobian matrix of manipulator with fault in i^{th} joint J Jacobian matrix of manipulator
Condition Number	$\frac{\sigma_{\max}}{\sigma_{\min}}$ (14)	$\sigma_{\max}, \sigma_{\min}$ maximum and minimum singular values of the Jacobian matrix
Dynamic Manipulability	$\sqrt{\det(J(MM^T)^{-1}J^T)}$ (15)	M is mass matrix J is Jacobian matrix
Worst Case Dexterity	$\sigma_{\min}$ (16)	$\sigma_{\min}$ minimum Singular value of the Jacobian matrix
Null space basis	N (17)	Column of null space matrix are orthogonal bases of null space of Jacobian matrix, norm of these columns indicate the contribution of each join in self motion velocity of manipulator
Sum of singular values	$\sum_{i=1}^{i=n} {}^k\sigma_i$ (18)	This is not very common for fault tolerance but it has been mentioned in the literature

The global measures have limited performance as they are working in forward kinematic domain [20]. But the local measures are more suitable, however they need to be calculated for any configuration [7-8].
These measures are identified based on the Jacobian or null space of the Jacobian matrix. They have being extensively used for design and control of fault tolerant manipulators. The most common local fault tolerance measures of manipulators are shown in Table 1.

The measures such as manipulability (12), relative manipulability (23) or worst case dexterity measures (16) have been widely used for fault tolerance of the manipulator. for instance in [1],[2],[3] the optimal Jacobian for fault tolerant manipulators has been addressed based on relative manipulability and in [4],[5],[19] based on worse case dexterity. In [1-3], the relation between the null space of the Jacobian and the fault tolerance have been addressed resulted to design of optimal Jacobian based on desired null space. Also the author has deeply studied condition number for fault tolerance of manipulators [25].

The dexterity of under actuated manipulator has been discussed in [5] resulted into optimal fault tolerant Jacobian based on worst case dexterity through numerical approach.
To design an optimal manipulator, the optimal values for the measures of fault tolerance are required. Two methods are used to solve this problem. The first is solution of optimality problem with numeric approach [5],[19] and the second one is using some sort of properties of the optimal Jacobian and their relationship to fault tolerance measures [1-3]. It is shown that the fundamental limitations of designing optimal fault tolerant manipulator is exists as it is related to a class of generally unsolved geometric problem.

B. *Basic Definition and some properties*

If J_{mn} is a Jacobian matrix at a given EEF pose, then the singular value decomposition (SVD) of the Jacobian matrix is provided by $J_{mn} = U\Sigma V^T$ where

1- U_{mm} is an orthogonal matrix of J is in non-singular configuration in workspace.
2- Σ_{mn} is a diagonal matrix is composed by singular values of J, $\Sigma_{mn} = diag(\sigma_1, \sigma_2, ..., \sigma_l, 0, ..., 0)$ and $l \le \min(m,n)$.
3- V_{nn} is an orthogonal matrix composed by the Eigen vectors and the orthogonal base of null space of J.

If the manipulator is redundant then $n \ge m$ and if the configuration is not singular then $l = m$, which are assumed in this paper.

C. *Optimal fault tolerant manipulators*

The relative manipulability (${}^k\rho$) associated with a fault in the k^{th} joint is (19)

$$ {}^k\rho = \frac{\sqrt{\det({}^kJ\,{}^kJ^T)}}{\sqrt{\det(JJ^T)}} \qquad (19) $$

In [3] following properties have been observed for relative manipulability due to the fault in the k^{th} joint. They properties have been used in [1-2].

1- $0 \le {}^k\rho \le 1$ the zero relative manipulability means the k^{th} joint does not contribute into velocity of the EEF. (20)
2- $\sum_{k=1}^{n} {}^k\rho^2 = n - m$ (21)
3- $\min({}^k\rho) \le \sqrt{\frac{n-m}{n}}$ (22)
4- $\sqrt{\det(JJ^T)} = \prod_{k=1}^{m} \sigma_k$ (23)

All the parameters in above items have been introduced previously. The minimum relative manipulability over all joint faults is called worst case relative manipulability. The optimal fault tolerant manipulator is the manipulator which maximises the worse case relative manipulability of the manipulator. This requires equally fault tolerant manipulator which was discussed in [1-2]. Thus the manipulator is called equally fault tolerant manipulator. This requires the contribution of each joint into the velocity of the EEF to be equal for all the joints when a unit velocity is applied for the joints.

In [4],[5],[19] the worst case dexterity of the manipulator is used as fault tolerant measure and following properties have been reported. If ${}^k\sigma_{\min}$ is the minimum singular value of each reduced Jacobian matrix (${}^kJ$) then (24).

1- $0 \le {}^k\sigma_{\min} \le \sigma\sqrt{\frac{n-m}{n}}$ (24)

the σ is the norm of the original Jacobian matrix. The zero case means that the manipulator becomes singular when the corresponding joint fails.

2- The maximum worst case dexterity is archived when the manipulator is isotropic dexterous and ${}^k\sigma_{\min} = \sigma\sqrt{\frac{n-m}{n}}$ [5]

3- The isotropic dexterity requires equal norm for the column vectors of Jacobian matrix. [5]

D. Null space of the Jacobian and fault tolerant measures

Recently, design of the fault tolerant manipulator has been addressed through the relationship between relative manipulability and the null space orthogonal bases.
If the SVD of the Jacobian is given and the Jacobian is full rank column matrix (non-singular configuration) then *U* is orthogonal matrix which is considered as a rotation and *V* is an orthogonal matrix which is divided into two parts as (25).

$$V^T = \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix}_{nn} \quad (25)$$

While $[V_1]_{mn}$ are composed by the Eigen vectors associated with the *m* singular values providing a base for range space of J and $[V_2]_{(n-m)n}$ is made by the orthogonal base of the null space of the *J*. Each column of V indicates how much a joint contributes not only into the range space of *J* (through the columns of V_1) but also into the null space of *J* (through the columns of V_2).

Knowing the orthogonal base of the null space the theorem which relates the relative manipulability into null space orthogonal base has been provided in [3]. Due to this theorem if N_k is the k^{th} column vector of V_2^T , this column indicates contribution of the k^{th} joint into motion of the manipulator projected into base of the null space. Then the corresponding relative manipulability is calculated from ${}^k\rho = \sqrt{N_k^T N_k}$ which is the norm of the k^{th} column. Therefore the norm of the column vectors of V_2 determines the corresponding relative manipulability of the joints. Thus the equality in fault tolerance implies equal norm for the column of V_2^T. For example for a single fault of the optimal worse case relative manipulability factor of a 3DOF manipulator, each column of V_2^T is $\sqrt{\frac{1}{3}}$. And for the optimal Jacobian of a 7DOF manipulator each element of V_2^T the columns are required to be $\sqrt{\frac{1}{7}}$. And null space base of a 7DOF manipulator is $V_2^T = \left[\sqrt{\frac{1}{7}} \quad \sqrt{\frac{1}{7}} \quad \sqrt{\frac{1}{7}} \quad \sqrt{\frac{1}{7}} \quad \sqrt{\frac{1}{7}} \quad \sqrt{\frac{1}{7}} \quad \sqrt{\frac{1}{7}}\right]$.

As it was discussed earlier, having optimal Jacobian requires equal column for the Jacobian and for the relative manipulability factors [3]. Also equal Eigen vectors associated to equal singular values needs isotropic dexterity of the manipulator [5]. If the manipulator is an isotropic dexterous manipulator then the relative manipulability factors will be equal. This motivates using symmetric form for the Jacobian. It is shown that symmetric geometries result into introducing a class of Jacobian matrix which are maximally isotropic dexterous while they are equal relative manipulable.

E. Overview on the design of Optimal Jacobian

From the literature, two methods have been used to design optimal fault tolerant manipulator:

E.1- First Method- Direct optimisation

The Jacobian for spatial manipulator includes linear velocity and orientation velocity parts. For a spatial manipulator, the workspace is R^6 (J_{6n}). Following constraints are defined for the optimality problem for the desired Jacobian matrix. It is shown to find this Jacobian with optimal worse case dexterity the norm of the columns of *J* are required to be equal providing 6 constraints.
The rows of *J* are orthogonal provides *15* extra constraints. Then the constrained optimality problem is indicated through the optimisation of worse case dexterity of the manipulator needs to be optimized (26) under aforementioned constraints [5].

$$\max_{\rho} \; \min_{k,\rho} \; {}^k\sigma \quad (26)$$

In [5] this problem has been solved and the optimal Jacobian was introduced for the *3*DOF, *4*DOF, *7*DOF and *8*DOF. But because of the nonlinearity and locality of the measures the performance of this approach is strictly limited as well as it is computational burden. The similar approach can be used for optimisation of relative manipulability $\max_{\rho} \; \min_{k,\rho} \; {}^k\rho$. In [5] the optimality is obtained only for worse case dexterity, we couldn't find any work to optimise both measures to gather.

E.2- Second Method – Null space based or indirect optimisation

It is possible to find an optimal Jacobian for fault tolerance using null space properties [3]. If the desired null space is assumed then following constrained has been identified to obtain the optimal Jacobian from relative manipulability perspective. These constraints includes *2n* constraints for the column vectors of J and *6(n-6)* constraints associated to the as the rows of *J* are required to be orthogonal to columns of null space base [1],[2]. Then it starts with assuming the null space base of the Jacobian matrix thus the optimality is preserved. All the Jacobian matrices which satisfy the required constraints [1],[2] are optimal fault tolerant through worse case relative manipulability such as those Jacobian matrices were introduced in [1-3].

IV. Geometric Properties of Optimal Fault Tolerant Jacobin Matrix

In both mentioned cases, this has been noticed that the optimal Jacobian in R^2 resulted into symmetric structures. Physically this is a correct observation because the optimality is in the equally fault tolerance and in a symmetric geometries like limited number of polyhedrons losing one vertex the remained geometry is still symmetric.

A. *Optimal fault tolerant 3DOF Jacobian*

An optimal fault tolerant Jacobian in R^2 including minimal redundancy requires 3DOF planar manipulator. This Jacobian is made by three vectors as indicated in (27). In (27) δ is an arbitrary constant. This Jacobian has been noticed in [5], [7-8]. It is clear that all three column vectors have equal norms, all vectors have maximum possible angles/distance from each other (120 Deg and $\delta\sqrt{3}$) and all three vectors are placed on a circle Fig-1.

The simple calculation of the relative manipulability factors and worse case dexterities indicates them as $\sqrt{\frac{1}{3}}$ and $\delta\sqrt{\frac{1}{3}}$ respectively for relative manipulability and worst case dexterity. Therefore the optimality is achieved through both measures.

$$J = \delta\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \qquad (27)$$

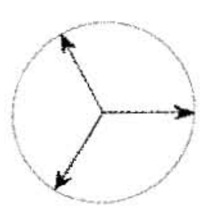

Fig-1 – Vectors of optimal Jacobian

B. *Optimal fault tolerant 7DOF Jacobian*

Motivated from optimal Jacobian in R^2 following the properties of a 7DOF manipulator is addressed. However some of those need to be proven, which will be the next research of the author. The optimal fault tolerant 7DOF Jacobian can be composed with 7 vectors in R^6 with following properties.

1- All the vectors are equal norm
2- The vectors have maximum distance/angles from each other
3- The vectors provide a complete set in R^6
4- The configuration of the vectors is symmetric

Thus the vectors belong to vectors form origin to vertices of hyper regular polyhedron. Finding a 7DOF spatial manipulator which is the minimal redundant manipulator in R^6 had been discussed in the literature and the optimal Jacobian was presented based on optimisation [5] or desired null space [1-2]. The approach in this paper stems on the extension previous observations. Therefore finding the optimal Jacobian is about localizing seven vectors in R^6 with aforementioned properties. This leads us to a challenging mathematic problem of symmetric distribution of n points on an *m*-dimensional hyper sphere (*m*-sphere), this problem has been discussed in mathematics, coding and graphics community specifically for R^3. [22-23]. We solved the problem based on charge distribution and stable configurations and from that the vectors of the optimal fault tolerant of a 7DOF manipulators were obtained.

The obtained Jacobian matrix is indicated in (30).

For this Jacobian all relative manipulability factors and worst case dexterities are equal and they are $\sqrt{\frac{1}{7}}$ and $\delta\sqrt{\frac{1}{7}}$ respectively. This ensures the Jacobian is optimally fault tolerant for single joint fault.

The result is consistent with the result of those matrices were introduced in [1] and [5], as it is clear that the null space vector and worst case dexterities satisfies the condition required for the optimality.

$$J = \delta\begin{bmatrix} -0.0078 & 0.1874 & 0.5751 & 0.3990 & 0.3990 & 0.5619 \\ -0.6083 & 0.6313 & 0.0206 & -0.4124 & 0.0055 & -0.2476 \\ -0.2430 & -0.0856 & -0.1920 & 0.6042 & -0.7290 & 0.0137 \\ -0.3117 & -0.7022 & 0.0090 & 0.0628 & 0.4562 & -0.4445 \\ 0.0445 & -0.3452 & -0.2926 & -0.5944 & -0.1491 & 0.6464 \\ 0.5178 & 0.3366 & -0.6541 & 0.2086 & 0.3591 & -0.1352 \\ 0.6085 & -0.0219 & 0.5339 & -0.2677 & -0.3416 & -0.3947 \end{bmatrix}^T \qquad (30)$$

where δ is an arbitrary scaling factor.

Compare to the result of the other optimal Jacobian for 7DOF manipulators in [1],[5], the proposed manipulator is optimal both in worst case dexterity and relative manipulability. The Jacobian matrices were introduced in those literature are only optimal form only relative manipulability [1] or only worse case dexterity [5]. However this class of the Jacobian needs more research to be fully understood.

V. CONCLUSION

This paper builds on the results of the series of work by Roberts and Maciejewski to provide an optimal fault tolerant Jacobian for redundant manipulators. The main difference is that instead of construction of the optimal Jacobian based on desired null space approach, we were proposing a method for obtaining this Jacobian based of the geometry properties of the Jacobian matrix. It was noticed that symmetric distribution of points in hyper spaces can be used to define the optimal Jacobian. And also it was illustrated that the proposed Jacobian is optimal fault tolerant both from worse case manipulability and worst case dexterity point of views.

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