# Fault Tolerance Force for Redundant Manipulators

Hamid Abdi, IEEE Member,
Centre for Intelligent Systems Research (CISR)
Deakin University
Waurn Ponds, VIC 3217, Australia
Email: Hamid.Abdi@deakin.com.au

Saeid Nahavandi, IEEE SM
Centre for Intelligent Systems Research (CISR)
Deakin University
Waurn Ponds, VIC 3217, Australia
Email: Saeid.Nahavandi@deakin.edu.au

*Abstract*—**Fault tolerant manipulators maintain their trajectory even if their joint/s fails. Assuming that the manipulator is fault tolerant on its trajectory, fault tolerant compliance manipulators provide required force at their end-effector even when a joint fails. To achieve this, the contributions of the faulty joints for the force of the end-effector are required to be mapped into the proper compensating joint torques of the healthy joints to maintain the force. This paper addresses the optimal mapping to minimize the force jump due to a fault, which is the maximum effort to maintain the force when a fault occurs. The paper studies the locked joint fault/s of the redundant manipulators and it relates the force jump at the end-effector to the faults within the joints. Adding on a previous study to maintain the trajectory, in here the objective is to providing fault tolerant force at the end-effector of the redundant manipulators. This optimal mapping with minimum force jump is presented using matrix perturbation model. And the force jump is calculated through this model for single and multiple joints fault. The proposed optimal mapping is used in different fault scenarios for a 5-DOF manipulator; also it is deployed to compensate the force at the end-effector for the 5-DOF manipulator through simulation study and the results are presented.**

***Keywords-component; Redundant Serial Link Manipulator, Fault Tolerance Compliance, Minimum Force Jump.***

## I. INTRODUCTION

Fault tolerant manipulators are essential where high available robots are required such as robotic manipulators working in hazardous environments, nuclear disposal, exploring deep sea and space [2] handling of hazardous or explosive materials [1,3].

The literature surrounding the fault tolerant manipulator has focused on the design level or the control level. The design level addresses fault tolerant serial link redundant manipulator [4,6] or some other structures such as parallel [5]. The control level includes Fault analysis and fault tolerant motion planning or control such as Fault Detection, Fault Isolation and Identification [7,8,20], Fault Recovery [9,12,13,14] and safety issues [3].

Serial link redundant manipulators (SLRM) have received great attention in the robotics community. Fault tolerant design of the SLRMs can be achieved by adding extra kinematic redundancy [4,9,10,11]. Using this, the manipulator maintains its dependability to perform the required [12] or prioritized tasks [13] even if one or more joint fails. In addition it has been observed that adding kinematic redundancy not only improves the fault tolerance specifications of the manipulators, but also it promotes other static or dynamic advantages. Including higher dexterous movements [11], lower maintenance and repair costs, more capability for obstacle avoidance [3] or essentially motion planning and control with multiple criteria [14,15]. However only having kinematic redundancy, does not guarantee the fault tolerant operation of the redundant manipulators [16].

The fault tolerant compliance manipulator should continue their task with minimum velocity and force jump at the end effector (EEF) when a fault occurs into the joint/s of the manipulator (however depends to the task the velocity jump can be released when there is no constraints on the trajectory) . This means that the contribution of the locked joint for the force and velocity of the EEF just prior to fault time has to be compensated through proper command for the torque and velocity of the healthy joints. The velocity of the EEF has been studied previously and the solution has been provided in [12,15], our focus in this paper is about fault tolerant compliance of the manipulator. Fault tolerant force control for parallel manipulator has been addressed in [23] based on D'Ambert principle and equivalent force method, the minimum force jump has been achieved for a 2-DOF parallel manipulator. Robust adaptive control of the redundant manipulator was addressed [21] where it was designed for a 9-DOF manipulator. Cooperative manipulators are proposed for fault tolerant force control at their EEF. The second manipulator is used to carry out the loss of the capacity of the first manipulator due to the fault for handling a load [22]. The work in [17] provides a hybrid solution by dividing the manipulation tasks into primary and secondary tasks, and the fault tolerant was achieved by maintaining the only primary manipulation task under fault. In [24] the problem is called torque redistribution but the work addresses the PID controllers for the joints of the faulty robot.

Fault tolerant properties of SLRM and their motion planning have been addressed extensively by Maciejewski et al [10,11,14,16,17], but fault tolerant force at the EEF for SLRM has not investigated as much as the manipulator motion. To achieve fault tolerant manipulating force at the EEF, an optimal mapping is required to map the jump of the force due to the locked joint/s into the proper torque commands for the healthy joints. Similar approach has been studied for the joint velocities by the authors and the optimal joint velocities to resolve the locked joint fault has been provided as was indicated in Appendix A. In this study the focus is on force jump.

**978-1-4244-5848-6/10/$26.00 ©2010 IEEE**

The rest of the paper is organized as follows; the Jacobian for SLRMs with some immobilized joints will be introduced. Then joint torque and force at EEF is studied under locked joint fault through matrix perturbation model. Then the force jump is calculated and minimum force jump is provided. The extension form single fault to multiple faults is provided. Finally different fault scenarios are studied and the optimal mapping with minimum force jump is calculated, the result has been used for fault tolerant force of a 5-DOF spatial manipulator through a simulation study and the result are presented.

## II. Kinematic of Redundant Manipulators

### A. *Kinematic and Self motion manifolds*

The forward kinematics of a manipulator relates joint angles for revolute joints (joint displacements for prismatic joints) to the EEF position/orientation (1). The joint variables (2) define the configuration space and position/orientation variables (3) define the workspace.

$$X = f(q) \quad (1)$$

$$q = [q_1 \quad q_2 \quad .... \quad q_n]^T \quad (2)$$

$$X = [x_1 \quad x_2 \quad ... \quad x_m]^T \quad (3)$$

$$q = f^{-1}(X) \quad (4)$$

The manipulator is n-DOF, and m is the work space dimension. The degree of kinematic redundancy (DOR) in non-singular configurations is n-m. In [3], the number of required redundancy was investigated by applying joint fault possibility and total reliability of the manipulator. The upper limit of optimal fault tolerant configuration for redundancy is studied [16].

### B. *Jacobian of Kinematic Redundant Manipulator under locked fault/s*

The Jacobian of manipulators is indicated by (5). Jacobian relates the EFF translation and orientation velocities to the joint variable speeds (6).

$$J = \left[\frac{\partial f}{\partial q}\right]_{mn} \quad (5)$$

$$\dot{X} = J\dot{q} \quad (6)$$

If Jk (7) is the kth columns of J, each column indicates the contribution of the corresponding joint velocity in the velocity of the EEF.

$$J = [J_1 \quad J_2 \quad ... \quad . \quad J_{i-1} \quad J_i \quad J_{i+1} \quad ... \quad J_n \quad ]_{mn} \quad (7)$$

When manipulator has a fault in the ith joint, this joint does not contribute into EFF velocity, so the Jacobian of the manipulator under an immobilized joint fault can be introduced by replacing a zero vector in the ith column of the manipulator Jacobian, this Jacobian is called reduced Jacobian (8) which, can be rewritten as (9)

$$[J_1 \quad J_2 \quad . \quad . \quad J_{i-1} \quad 0 \quad J_{i+1} \quad . \quad J_n \quad ] \quad (8)$$

$$\dot{X} = {}^iJ\,{}^i\dot{q} \quad (9)$$

where

$${}^iJ = [J_1 \quad J_2 \quad . \quad . \quad J_{i-1} \quad J_{i+1} \quad . \quad . \quad J_n \quad ] \quad (10)$$

$${}^i\dot{q} = [\dot{q}_1 \quad \dot{q}_2 \quad . \quad . \quad \dot{q}_{i-1} \quad \dot{q}_{i+1} \quad . \quad . \quad \dot{q}_n \quad ]^T \quad (11)$$

For single fault, there are n reduced Jacobian matrices (12)

$$\{{}^1J, {}^2J ..., {}^nJ\} \quad (12)$$

With this approach, if the manipulator has f faults, then the reduced Jacobian matrices will be indicated with permutation of f zero vectors in the original Jacobian matrix. In general if there are f (f=1..n), there will be $\binom{n}{f} = \frac{n!}{f!(n-f)!}$ different possible reduced Jacobian matrices. As an instance for two faults, there will be $\frac{n(n-1)}{2}$ reduced Jacobian matrices (13).

$$\{{}^{k,i}J = [J_1 \quad . \quad J_{k-1} \quad 0 \quad J_{k+1} \quad ... \quad J_{i-1} \quad 0 \quad J_{i+1} \quad .. \quad J_n]\} \quad (13)$$

where $k, i = 1..n, i > k$

These matrices can be used to rewrite (6) as (14) which results in (15) and (16).

$$\dot{X} = {}^{k,i}J\,{}^{k,i}\dot{q} \quad (14)$$

$$\{{}^{k,i}J = [J_1 \quad . \quad J_{k-1} \quad J_{k+1} \quad ... \quad J_{i-1} \quad J_{i+1} \quad .. \quad J_n]\} \quad (15)$$

$$\{{}^{k,i}\dot{q} = [\dot{q}_1 \quad . \quad \dot{q}_{k-1} \quad \dot{q}_{k+1} \quad ... \quad \dot{q}_{i-1} \quad \dot{q}_{i+1} \quad .. \quad \dot{q}_n]^T\} \quad (16)$$

## III. Optimal Force Jump in Manipulators

### A. *EEF Force and joint torque*

For a SLRM and in a given configuration, applying a force into the EEF results into corresponding torques within the joints as (17). In duality the torque in the joints results into dynamic motion of the manipulator and force at the EEF. (18).

$$\tau = J^T F \quad (17)$$

$$F = (J^T)^{\dagger}\tau + (I - J^T(J^T)^{\dagger})z \quad (18)$$

$(J^T)^{\dagger}$ is pseudo inverse (Penrose-Moore inverse) of the $J^T$ and can be defined by (19). While $z$ is an arbitrary joint torque vector and $\{I - (J^T)(J^T)^{\dagger}\}$ is a transformation which maps $z$ into null space of $(J^T)^{\dagger}$. $z$ Normally is calculated through desired potential function.

$$(J^T)^{\dagger} = (JJ^T)^{-1}J \quad (19)$$

The general dynamic equation of the SLRM is

$$M(q)\ddot{q} + V(q,\dot{q})\dot{q} + G(q) = \tau \quad (20)$$

$M(q)$ is mass matrix

$V(q,\dot{q})$ Coriolis and centrifugal forces

$G(q)$ Gravity forces

$\tau_d$ torque to provide the motion profile

If the manipulator is required to provide a force at EEF or a force is applied to the EEF then (20) is changed into (21)

$$M(q)\ddot{q} + V(q,\dot{q})\dot{q} + G(q) = \tau_d + J^T F \quad (21)$$

and $J^T F$ is the joint torque map of the force is applied into or provided by the EEF to the joint torque space.

### B. *Modelling of joint fault trough perturbation*

If the kth joint is a locked joint then the kth reduced Jacobian is ${}^{k}J$ . From perturbation model if this fault is modelled by variation in Jacobian then force at the EEF and torque equation is changed into (22).

$$\tau + \Delta\tau = (J + \Delta J)^T (F + \Delta F) \quad (22)$$

$\Delta\tau$ is the change into the torques at fault time.

$\Delta J$ Jacobian matrix perturbation due to fault

$\Delta F$ force jump at EEF

For fault tolerant force at EEF it is required find a new joint's torque to minimize the force jump. The new joint's torque is called $U$ and

$$U = (J + \Delta J)^T (F + \Delta F) \quad (23)$$

If the fault occurs into the kth joint then from previous section, the perturbation model is turned into

$${}^{k}U = {}^{k}J^T (F + {}^{i}\Delta F) \quad (24)$$

Where

$$\Delta J = [0 \quad \ldots \quad -J_k \quad \ldots \quad 0] \quad (25)$$

$$J + \Delta J = {}^{k}J \quad (26)$$

${}^{k}U$ indicates the new joint velocities when the kth is locked. Substitution of (18) into (24) results in (27) and (28)

$${}^{k}U = {}^{k}J^T \left\{ \left(J^T\right)^{\dagger} \tau + \left(I - J^T \left(J^T\right)^{\dagger}\right) z + {}^{k}\Delta F \right\} \quad (27)$$

$${}^{k}\Delta F = \left\{ \left({}^{k}J^T\right)^{\dagger k} U - \left\{\left(J^T\right)^{\dagger}\right\} \tau - \left(I - J^T \left(J^T\right)^{\dagger}\right) z \right\} \quad (28)$$

The minimum norm of ${}^{k}\Delta F$ will result into minimum force jump for the EEF, to achieve this it requires ${}^{k}\Delta F = 0$ (29).

$${}^{k}U = \left({}^{k}J^T\right) \left\{ \left\{\left(J^T\right)^{\dagger}\right\} \tau - \left(I - J^T \left(J^T\right)^{\dagger}\right) z \right\} \quad (29)$$

as $z$ is an arbitrary vector, therefore it is assumed zero. And (29) results into ${}^{k}U = \left({}^{k}J^T\right) \left\{ \left(J^T\right)^{\dagger} \tau \right\}$.

So the mapping of the torque vector into the torque for the faulty manipulator can be defined by ${}^{k}J^T \left(J^T\right)^{\dagger}$ . When ${}^{k}\Delta F$ is not zero then the force jump can be calculated by a series of calculation results in (33).

$${}^{k}\Delta F = \left(J^T\right)^{\dagger} \tau - \left({}^{k}J^T\right)^{\dagger} {}^{k}U \quad (30)$$

$${}^{k}\Delta F = \left(J^T\right)^{\dagger} \tau - \left({}^{k}J^T\right)^{\dagger} {}^{k}J^T \left\{ \left(J^T\right)^{\dagger} \right\} \tau \quad (31)$$

$${}^{k}\Delta F = \left\{ I - \left({}^{k}J^T\right)^{\dagger} {}^{k}J^T \right\} \left(J^T\right)^{\dagger} \tau \quad (32)$$

$${}^{k}\Delta F = \left\{ I - \left({}^{k}J^T\right)^{\dagger} {}^{k}J^T \right\} F = N({}^{k}J^T) F \quad (33)$$

where $N({}^{k}J^T) = I - \left({}^{k}J^T\right)^{\dagger} {}^{k}J^T$ is the null space projection of $\left({}^{k}J^T\right)^{\dagger}$ (33) shows the force jump due to the kth fault (${}^{k}\Delta F$) is the map of the force at the EEF into the null space of the kth reduced Jacobian matrix. It provides the frame work to calculate force jump for a single joint fault. The necessary conditions for no force jump as follows.

The necessary conditions for no force jump are

The null space of the ${}^{k}J^T$ is zero means that ${}^{k}J^T$ is full row ranked matrix.

F belongs to null space of ${}^{k}J^T$ .

The second condition releases the full rank for ${}^{k}J^T$ but it limits the force at EEF to do not have projection into null space of ${}^{k}J^T$ .

Minimum force jump due to multi-joint faults

If two joint fail then from (23), say kth and ith joints then

$${}^{k,i}U = {}^{k,i}J^T (F + {}^{i,k}\Delta F) \quad (34)$$

where $\Delta J = [0 \quad \ldots \quad -J_k \quad \ldots \quad -J_i \quad \ldots \quad 0]$ (35)

$$J + \Delta J = {}^{k,i}J \quad (36)$$

This model results in

$${}^{k,i}U = \left({}^{k,i}J^T\right) \left\{ \left(J^T\right)^{\dagger} \tau \right\} \quad (37)$$

$${}^{k,i}\Delta F = \left\{ I - \left({}^{k,i}J^T\right)^{\dagger} {}^{k,i}J^T \right\} F = N({}^{k,i}J^T) F \quad (38)$$

${}^{k,i}\Delta F$ force jump due to fault into kth and ith joints.

${}^{k,i}J$ reduced Jacobian matrix, fault in kth and ith joints

$N({}^{k,i}J^T) = I - \left({}^{k,i}J^T\right)^{\dagger} {}^{k,i}J^T$ is the null space of the reduced Jacobian matrix

In general if $S_f = \{ i_1 \quad i_2 \quad \ldots \quad i_f \mid i_1 < i_2 < \ldots < i_f \quad 1 \le i_1, i_f \le n \}$ is the set of faulty joints then the minimum force jump can be achieved through

$${}^{S_f}U = \left({}^{S_f}J^T\right) \left\{ \left(J^T\right)^{\dagger} \tau \right\} \quad (39)$$

$${}^{S_f}\Delta F = \left\{ I - \left({}^{S_f}J^T\right)^{\dagger} {}^{S_f}J^T \right\} F = N({}^{S_f}J^T) F \quad (40)$$

${}^{S_f}\Delta F$ force jump due to fault $S_f$ .

${}^{S_f}J$ reduced Jacobian matrix, fault $S_f$ joints

$N({}^{S_f}J^T) = I - \left({}^{S_f}J^T\right)^{\dagger} {}^{S_f}J^T$ is the null space of the reduced Jacobian matrix due to $S_f$ faults .

## IV. – Case Study- Mapping for 5DOF Manipulator

A 5-DOF spatial manipulator, with D-H parameters indicated in Table 1 is modelled using Matlab Robotics Toolbox [19]. Table 2 indicates the manipulator parameters prior to fault time, the velocity of the EEF of the manipulator at the indicated configuration in Fig.2 is $\dot{X} = [0.033 \quad 0.01 \quad -0.23]^T$ *cm/sec*, and the force at EEF is assumed as $F = [80_N \quad 50_N \quad 10_N]^T$ .

Three different fault scenarios have been tested through this model including single joint fault using (29) and (33) and multiple joint failure using (37),(38).

The last two columns are computed by $\frac{\|J_k \dot{Q}_k\|}{\|J \dot{Q}\|}$ and $\frac{\|(J^T)_k^{\dagger} \tau_k\|}{\|F\|}$ respectively and they are indicating how much the joint velocity or torque contributes to the velocity or force of the EEF at the given configuration.

### A. First Scenario: Single Joint Fault

In this scenario, there are five possible selections, first assume the fault is in the first joint. The compensation torque mapping and force jump are calculated as follows. For the fault in the second, third, fourth and the fifth joints with the same approach as the first joint, the results are presented as follows.

TABLE 1

D-H PARAMETERS OF MODELLED 5-DOF MANIPULATOR

| Joint No | $S_{i(m)}$ | $D_{i\,(m)}$ | $\alpha_i$ | $\theta_i$ |
|---|---|---|---|---|
| 1 | 0.05 | 0.45 | 0 | $\theta_1$ |
| 2 | 0.05 | 0.32 | 90 Deg | $\theta_2$ |
| 3 | 0.05 | 0.18 | 0 | $\theta_3$ |
| 4 | 0.05 | 0.12 | 0 | $\theta_4$ |
| 5 | 0.05 | 0.08 | 0 | $\theta_5$ |

TABLE 2

CONFIGURATION AND PARAMETERS OF THE MANIPULATOR AT FAULT TIME

| Joint No | Angle $Q$ | Speed $\dot{Q}$ | Torque $\tau$ | Contribution of joint torques into force of EEF | Contribution of joint into the velocity of EEF |
|---|---|---|---|---|---|
| 1 | 10 | 0.05 | -35.01 | 3.18 | 0.19 |
| 2 | 70 | 0.40 | -12.33 | 1.12 | 0.89 |
| 3 | 25 | 0.20 | 26.85 | 2.44 | 0.28 |
| 4 | 65 | 0.10 | 10.00 | 0.91 | 0.09 |
| 5 | 0 | 0.30 | 4.00 | 0.36 | 0.11 |

I. First joint fault

In this case, using the provided D-H parameters in Table 1, and the given configuration in Table 2, the Jacobian of the manipulator is

$$J=\begin{bmatrix}-0.00 & -0.00 & 0.16 & 0.00 & 0.00\\ -0.57 & -0.15 & 0.28 & 0.20 & 0.08\\ -0.64 & -0.48 & 0.00 & 0.00 & 0.00\end{bmatrix} \quad (34)$$

When first joint fails then

$$^{1}J=\begin{bmatrix}0.00 & 0.16 & 0.00 & 0.00\\ -0.15 & 0.28 & 0.20 & 0.08\\ -0.48 & 0.00 & 0.00 & 0.00\end{bmatrix} \quad (35)$$

and the optimal torque map for faulty robot is

$$^{1}U={}^{1}J^{T}\left\{(J^{T})^{\dagger}\right\}\tau \Rightarrow {}^{1}U=[-12.33 \quad 26.85 \quad 10.00 \quad 4.00]^{T}\,N.m \quad (42)$$

The required torque to compensate the fault is calculated as $^{1}\tau=[0 \quad -12.33 \quad 26.85 \quad 10.00 \quad 4.00]^{T}$ N.m . In the next step the error of this mapping can be determined though (33). Substitution of the parameters results into zero force jump at the EEF.

II. 2nd, 3rd, 4th and 5th single joint fault

For other selection of single joint faults, Table 3 indicates the compensating joint torques similar to the first joint. This study indicated that at the giver configuration only the fault at the third joint cannot be tolerated so there is force jump.

### A. Second Scenario: Two Joint Faults

For two joint faults there are ten possible selections. The result of the minimum force jump mapping for two of these selections are provided in Table 4.

I. Faults in the 2nd and 3rd joints

In the second column of Table 4, provides the optimal mapping of torques when fault this fault occurs. The minimum force jump is 80N

II. Faults in the 2nd and 4th joints

The third column of Table 4 presents the mapping and associated force jump at EEF due to this fault and it indicates that the fault in the 2nd and 4th joints is completely tolerated.

TABLE 3

TORQUES WITH MINIMUM FORCE JUMP PROVIDED BY THE SAME CALCULATION FOR THE FIRST JOINT (N.M.)

| Joint No | 2nd Joint Fault N.m | 3rd Joint Fault N.m | 4th Joint Fault N.m | 5th Joint Fault N.m |
|---|---|---|---|---|
| 1 | -35.01 | -35.01 | -35.01 | -35.01 |
| 2 | 0 | -12.33 | -12.33 | -12.33 |
| 3 | 26.85 | 0 | 26.85 | 26.85 |
| 4 | 10.00 | 10.00 | 0 | 10.00 |
| 5 | 4.00 | 4.00 | 4.00 | 0 |
| Force Jump | 0.00 | $80.00_N$ | 0.00 | 0.00 |

TABLE 4

JOINT VELOCITIES WITH ZERO SPEED JUMP FOR TWO FAULTY JOINTS (RAD/SEC)

| Joint No | 2nd and 3rd Joint Fault N.m | 2nd and 4rd Joint Fault N.m |
|---|---|---|
| 1 | -35.01 | -35.01 |
| 2 | 0 | 0 |
| 3 | 0 | 26.85 |
| 4 | 10.00 | 0 |
| 5 | 4.00 | 4.00 |
| Force Jump | $80.00_N$ | 0.00 |

### B. Third Scenario: Three Joint Faults

Three joints fault can be studied for ten selections. The results for two selections have been presented. Table 5.

TABLE 5

JOINT VELOCITIES WITH ZERO SPEED JUMP FOR TWO FAULTY JOINTS (RAD/SEC)

| Joint | 2nd ,3rd and 4th Joint Fault N.m | 2nd ,4th and 5th Joint Fault N.m |
|---|---|---|
| 1 | -35.01 | -35.01 |
| 2 | 0 | 0 |
| 3 | 0 | 26.85 |
| 4 | 0 | 0 |
| 5 | 4.00 | 0 |
| Force Jump | $80_N$ | $43.66_N$ |

## V. FAULT TOLERANT FORCE AT EEF

### A. Simulation

To validate the fault tolerant force at the EEF for a 5DOF spatial manipulator, with D-H parameters in Table.1, a simulation study has been developed using Matlab Robotic Toolbox. The indicated trajectory in Fig.1, is a fault tolerant zero velocity jump trajectory. The trajectory starts form $R_s = [1.150 \quad -0.134 \quad 0.034]^T m$ and it ends at $R_d = [0.781 \quad 0.509 \quad 0.280]^T m$. The focus on this study has been the force jump at the EEF.

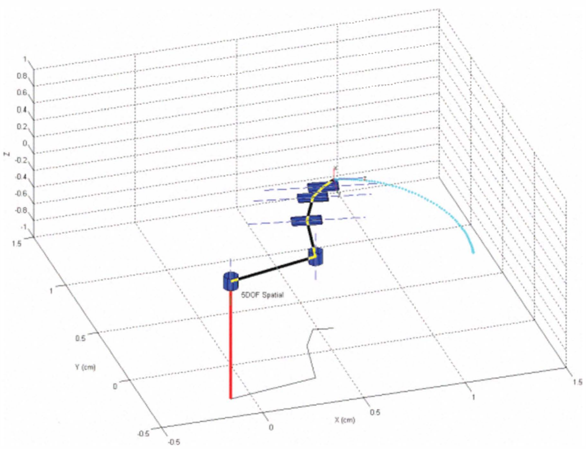


Fig. 1. Manipulator trajectory, the trajectory is provided by a maximum velocity of 15mm/sec. The 3rd joint supposed to have a fault after 50sec, for the simulation the manipulator will not have any velocity jump. The focus is to study the force jump.

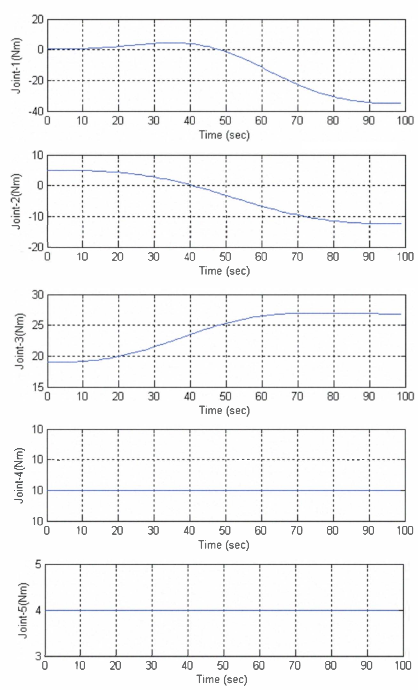


Fig. 2. Manipulator joint torques for the indicated trajectory

The main objective of this simulation is to provide fault tolerant operation for the force at EEF and compare the provided force by the healthy and faulty manipulators in a dynamic problem. As the result of this simulation joint torques for the healthy and faulty manipulator are computed to maintain a fault tolerant force $F = [80_N \quad 50_N \quad 10_N]^T$ at the EEF. Fig.2. indicates joint torques for healthy manipulator while the next figure shoes the torques for faulty manipulator (3rd joint locked). As it has been assumed the trajectory have been maintained using minimum velocity jump techniques Appendix A.

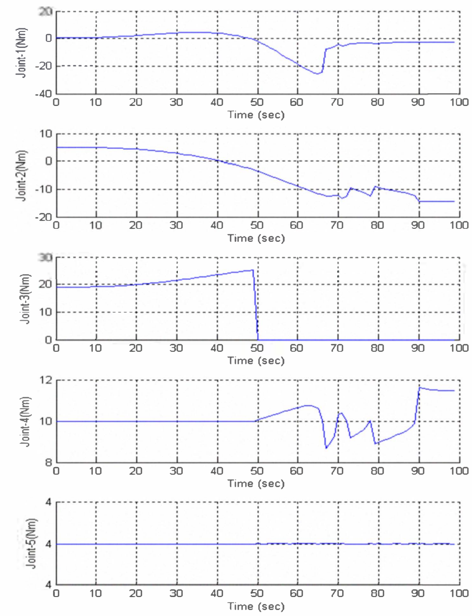


Fig. 3. Manipulator new joint torque profiles when the 3rd joint locked after 50sec,

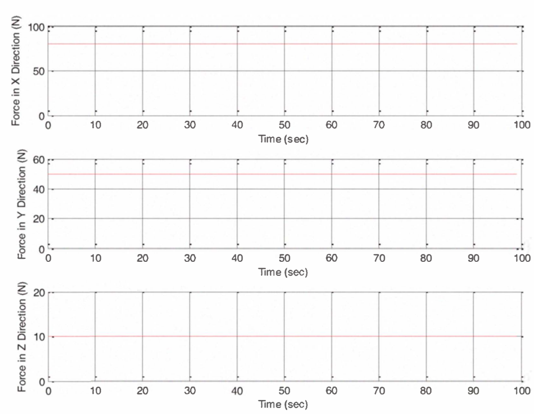


Fig. 4. Compare the Force at EEF for healthy and faulty manipulator.
Blue line is force for the healthy manipulator (solid line),
Red line is for the faulty manipulator (dash line)

### *B. Discussion*

Fig.3 in compare to Fig.1 indicates how much lack of the contribution of the torque of the 3rd joint is mapped into other joint torques. As it is expected the 3rd joint force will mapped to zero after fault (50th second of the simulation scenario). Other joint forces have been changed to provide the required force at the EEF.

## VI. CONCLUSION

Locked joint fault of the redundant manipulator has been addressed and the end-effector force jump due the faults has been studied. Optimal mapping of joint torque to compensate the effect of one locked joint faults for serial link redundant manipulators has been derived though the minimization of force jump at fault.

Three different fault scenarios is studied with the proposed mapping and the force jump is calculated. Then through the simulation study for the 5DOF manipulator, the proposed fault tolerant compliance method is used to maintain the force at EEF under faulty joint. And the result justified the proposed mapping based on matrix perturbation model.

**Appendix-A**

The optimal joint velocities to minimise the EEF velocity jump due to the f joint faults when $S_f = \{i_1 \quad i_2 \quad \dots \quad i_f \,|\, i_1 < i_2 < \dots < i_f \quad 1 \le i_1, i_f \le n\}$ is

$${}^{S_f}U = ({}^{S_f}J)^{\dagger}\left\{\sum_{k\in S_f} J_k \dot{Q}_k\right\}$$

Using this optimal mapping the minimum EEF velocity jump is

$$\Delta\dot{X} = \left\{{}^{S_f}J\left\{({}^{S_f}J)^T\,{}^{S_f}J\right\}^{-1}({}^{S_f}J)^T - I\right\}\left\{\sum_{k\in S_f} J_k\dot{Q}_k\right\}$$

${}^{S_f}U$ : change in joint velocities to resolve the fault

$\dot{Q}_k$ : velocity of the faulty joint prior to fault time

$\Delta\dot{X}$ : minimum velocity jump at the EEF

${}^{S_f}J$ : reduced Jacobian of the manipulator with into $S_f$

$S_f$ : set of faulty joint

## ACKNOWLEDGEMENT

The Authors would like to acknowledge Dr. Zoran Najdovski for his comments on the previous work as this work is a part of series of work on fault tolerant robotics.

## REFERENCES

[1] R. Colbaugh and M. Jamshidi, "Robot manipulator control for hazardous waste-handling applications," J. Robot. Syst., vol. 9, no. 2, pp. 215–250, 1992.

[2] E. C. Wu, J. C. Hwang, and J. T. Chladek, "Fault-tolerant joint development for the space shuttle remote manipulator system: Analysis and experiment," IEEE Trans. Robot. Autom., vol. 9, no. 5, pp. 675–684, Oct. 1993.

[3] M. L. Leuschen, I. D. Walker, and J. R. Cavallaro, "Investigation of reliability of hydraulic robots for hazardous environment using analytic redundancy," in Proc. Annu. Rel. Maintain. Symp., Washington, DC, Jan. 18–21 ,pp. 122–128, 1999.

[4] C. J. J. Paredis and P. K. Khosla, "Designing fault-tolerant manipulators: How many degrees of freedom?," Int. J. Robot. Res., vol. 15, no. 6, pp. 611–628, Dec. 1996.

[5] L. Notash and L. Huang, "On the design of fault tolerant parallel manipulators," Mech. Mach. Theory, vol. 38, pp. 85–101, 2003.

[6] Y. Yi, J. E. McInroy, and F. Jafari, "Optimum design of a class of fault tolerant isotropic Gough-Stewart platforms," in Proc. IEEE Int. Conf. Robot. Autom., New Orleans, LA, pp. 4963–4968, 2004.

[7] L. Notash, "Joint sensor fault detection for fault tolerant 7parallel manipulators," J. Robot. Syst., vol. 17, no. 3, pp. 149–157, 2000.

[8] I. D. Walker and J. R. Cavallaro, "The use of fault trees for the design of robots for hazardous environments," in Proc. Annu. Rel. Maintain. Symp., Las Vegas, NV, pp. 229–235, 1996.

[9] J. Park, W.-K. Chung, and Y. Youm, "Failure recovery by exploiting kinematic redundancy," in 5th Int. Workshop Robot Human Commun., Tsukuba, Japan, pp. 298–305, Nov. 11–14, 1996.

[10] J. D. English and A. A. Maciejewski, "Fault tolerance for kinematically redundant manipulators: Anticipating free-swinging joint failures," IEEE Trans. Robot. Autom., vol. 14, no. 4, pp. 566–575, 1998.

[11] A. A. Maciejewski, "Fault tolerant properties of kinematically redundant manipulators," in Proc. IEEE Int. Conf. Robot. Autom., Cincinnati, OH, May 13–18, pp. 638–642, 1990.

[12] K.N.Groom,A.A. Maciejewski, andV. Balakrishnan, "Real-time failure tolerant control of kinematically redundant manipulators," IEEE Trans Robot. Autom., vol. 15, no. 6, pp. 1109–1116, Dec. 1999.

[13] Y. Chen, J. E. McInroy, and Y. Yi, "Optimal, fault-tolerant mappings to achieve secondary goals without compromising primary performance" IEEE Trans. Robot., vol. 19, no. 4, pp. 680–691, Aug. 2003.

[14] AA Maciejewski, CA Klein; "Obstacle avoidance for kinematically redundant manipulators in dynamically environment", The International Journal of Robotics Research, pp.109-116, 1985

[15] Zhao Jing, Fang Cheng; "On the joint velocity jump during fault tolerant operations for manipulators with multiple degrees of redundancy", Mechanism and Machine Theory, Vol 44, Issue 6, Pp 1201-1210, 2009.

[16] RG Roberts, HG Yu, AA Maciejewski Fundamental limitations on designing optimally fault-tolerant redundant manipulators ,IEEE Trans. Robot, pp1224-1237, 2008

[17] R. S. Jamisola, Jr., A. A. Maciejewski, and R. G. Roberts, "Failure-tolerant path planning for kinematically redundant manipulators anticipating locked-joint failures", IEEE Transactions on Robotics, Vol 22, p. 603-612, Aug. 2006.

[18] M. L. Visinsky, J. R. Cavallaro, and I. D. Walker, "A dynamic fault tolerance framework for remote robots," IEEE Trans. Robot. Autom., vol. 11, no. 4, pp. 477–490, Aug. 1995.

[19] P.I. Corke, "A Robotics Toolbox for Matlab", CSIRO ,Australia

[20] M. Dev Anand , T. Selvaraj, S. Kumanan , J. Janarthanan , "Fault diagnosis system for a robot manipulator through neuro fuzzy approach", International Journal of Modelling, Identification and Control, Volume 3, Number 2,pp 181 – 192, 2008

[21] R. Colbaugh and K. Glass, "Robust adaptive control of redundant manipulators", Journal of Intelligent and Robotic Systems, Volume 14, Number 1 / September,,pp 69-88, 1995

[22] Tinos Renato; Henrique Terra Marco ; Yoshiyaki Ishihara Joao; "Motion and force control of cooperative robotic manipulators with passive joints" , IEEE Trans. On Control System Technology, VOL. 14, NO. 4, Pp. 725-734, JULY 2006

[23] Yan Caixia He Guangping, "Over-actuated Parallel Manipulator Fault Tolerant Control Based On Minimum Difference Oriented Smoot