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Low Complexity MMSE Based Channel Estimation Technique for LTE OFDMA Systems

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Abstract—Long term evolution (LTE) is designed for high speed data rate, higher spectral efficiency, and lower latency as well as high-capacity voice support. LTE uses single carrier-frequency division multiple access (SC-FDMA) scheme for the uplink transmission and orthogonal frequency division multiple access (OFDMA) in downlink. The one of the most important challenges for a terminal implementation are channel estimation (CE) and equalization. In this paper, a minimum mean square error (MMSE) based channel estimator is proposed for an OFDMA systems that can avoid the ill-conditioned least square (LS) problem with lower computational complexity. This channel estimation technique uses knowledge of channel properties to estimate the unknown channel transfer function at non-pilot sub-carriers.

Index Terms—Channel estimation, LTE, least-square, OFDMA, SC-FDMA.

I. INTRODUCTION

The 3rd generation partnership project (3GPP) members started a feasibility study on the enhancement of the universal terrestrial radio access (UTRA) in December 2004, to improve the mobile phone standard to cope with future requirements. This project was called evolved-UTRAN or long term evolution [1], [22]. The main purposes of the LTE is substantially improved end-user throughputs, low latency, sector capacity, simplified lower network cost, high radio efficiency, reduced user equipment (UE) complexity, high data rate, and significantly improved user experience with full mobility [2].

3GPP LTE uses orthogonal frequency division multiplexing access (OFDMA) for downlink and single carrier-frequency division multiple access (SC-FDMA) for uplink. SC-FDMA is a promising technique for high data rate transmission that utilizes single carrier modulation and frequency domain equalization. Single carrier transmitter structure leads to keep the peak-to average power ratio (PAPR) as low as possible that will reduced the energy consumption. SC-FDMA has similar throughput performance and essentially the same overall complexity as OFDMA [1]. A highly efficient way to cope with the frequency selectivity of wideband channel is OFDMA. It is an effective technique for combating multipath fading and high bit rate transmission over mobile wireless channels. In OFDMA system, the entire channel is divided into many narrow subchannels, which are transmitted in parallel, thereby increasing the symbol duration and reducing the intersymbol-interference (ISI) [2], [4]. Channel estimation (CE) plays an important part in LTE OFDMA systems. It can be employed for the purpose of detecting received signal, improving the capacity of OFDMA systems by cross-layer design, and improving the system performance in terms of symbol error probability (SEP) [4], [5].

A key aspect of the wireless communication system is the estimation of the channel and channel parameters. CE has been successfully used to improve the performance of LTE OFDMA systems. It is crucial for diversity combination, coherent detection, and space-time coding. Improved channel estimation can result: improved signal-to-noise ratio, channel equalization, co-channel interference (CCI) rejection, mobile localization, and improved network performance [1], [2], [3], [18].

Many CE techniques have been proposed to mitigate inter-channel interference (ICI) in the downlink direction of LTE systems. In [3], the LS CE has been proposed to minimize the squared differences between the receive signal and estimation signal. The LS algorithm, which is independent of the channel model, is commonly used in equalization and filtering applications. But the radio channel is varying with time and the inversion of the large dimensional square matrix turns out to be ill-conditioned. In [19], Wiener filtering based two-dimensional pilot-symbol aided channel estimation has been proposed. Although it exhibits the best performance among the existing linear algorithms in literature, it requires accurate knowledge of second order channel statistics, which is not always feasible at a mobile receiver. This estimator gives almost the same result as 1D estimators, but it requires higher complexity. To further improve the accuracy of the estimator, Wiener filtering based iterative channel estimation has been investigated [4]. However, this scheme also require high complexity.

In this paper we proposed a channel estimation method in the downlink direction of LTE systems. This proposed method uses knowledge of channel properties to estimate the unknown channel transfer function at non-pilot sub-carriers. These properties are assumed to be known at the receiver for the estimator to perform optimally. The following advantages
will be gained by using this proposed method. Firstly, the proposed method avoids ill-conditioned problem in the inversion operation of a large dimensional matrix. Secondly, the proposed method can track the changes of channel parameters, that is, the channel autocorrelation matrix and SNR. However, the conventional LS method cannot track the channel. Once the channel parameters change, the performance of the conventional LS method will degrade due to the parameter mismatch. Finally, the computational complexity of the proposed method is significantly lower than existing LS and Wiener CE method.

We use the following notations throughout this paper: bold face lower and upper case letters are used to represent vectors and matrices, respectively. Superscripts $x^T$ denote the conjugate transpose of the complex vector $x$, $\text{diag}(x)$ is the diagonal matrix that its diagonal is vector $x$; and the symbol $E(\cdot)$ denotes expectation.

The remainder of the paper is organized as follows: section II describes LTE OFDMA system model. The proposed channel estimation scheme is presented in section III, and its performance is analyzed in section IV. Section V concludes the work.

II. SYSTEM DESCRIPTION

A. System model

A simplified block diagram of the LTE OFDMA transceiver is shown in Fig.1. At the transmitter side, a baseband modulator transmits the binary input to a multilevel sequences of complex number $m(n)$ in one of several possible modulation formats including binary phase shift keying (BPSK), quandary PSK (QPSK), 8 level PSK (8PSK), 16-QAM, and 64-QAM [1]. CE usually needs some kind of pilot information as a point of reference. CE is often achieved by multiplexing known symbols, so called, pilot symbols into data sequence [15]. These modulated symbols, both pilots and data, are perform a N-point inverse discrete Fourier transform (IDFT) to produce a time domain representation [1]:

$$s(m) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} m(n)e^{j\frac{2\pi nm}{N}}, \quad (1)$$

where $m$ is the discrete symbols, $n$ is the sample index, and $m(n)$ is the data symbol. The IDFT module output is followed by a cyclic prefix (CP) insertion that completes the digital stage of the signal flow. A cyclic extension is used to eliminate intersymbol-interference (ISI) and preserve the orthogonality of the tones.

B. Channel model

Channel model is a mathematical representation of the transfer characteristics of the physical medium. These models are formulated by observing the characteristics of the received signal. According to the documents from 3GPP [15], in the mobile environment, a radio wave propagation can be described by multipaths which arise from reflection and scattering. If there are $L$ distinct paths from transmitter to receiver, the impulse response of the wide-sense stationary uncorrelated scattering (WSSUS) fading channel can be represented as [4]:

$$w(t, \tau) = \sum_{l=0}^{L-1} w_l(t)\delta(t - \tau_l), \quad (2)$$

where fading channel coefficients $w_l(t)$ are the wide sense stationary i.e. $w_l(t) = w(m, l)$, uncorrelated complex Gaussian random paths gains at time instant $t$ with their respective delays $\tau_l$, where $w(m, l)$ is the sample spaced channel response of the $l$th path during the time $m$, and $\delta(\cdot)$ is the Dirac delta function. Based on the WSSUS assumption, the fading channel coefficients in different delay taps are statistically independent. Fading channel coefficient is determined by the cyclic equivalent of sinc-functions [7]. In time domain fading coefficients are correlated and have Doppler power spectrum density modeled in Jakes [13] and has an autocorrelation function given by [5]:

$$E[w(m, l)w(n, l)^\dagger] = \sigma_w^2(l)r_l(m - n)$$
$$= \sigma_w^2(l)J_0[2\pi f_dT_f(m - n)], \quad (3)$$

where $w(n, l)$ is a response of the $l$th propagation path measured at time $n$, $\sigma_w^2(l)$ denotes the power of the channel coefficients, $f_d$ is the Doppler frequency in Hertz, $T_f$ is the OFDMA symbol duration in seconds, and $J_0(\cdot)$ is the zero order Bessel function of the first kind. The term $f_dT_f$ represents the normalized Doppler frequency [5].

C. Received signal model

At the receiver, the opposite set of the operation is performed. We assume that the synchronization is perfect. Then, the cyclic prefix samples are discarded and the remaining $N$ samples are processed by the DFT to retrieve the complex constellation symbols transmitted over the orthogonal subchannels. The received signal can be expressed as [5]:

$$r(m) = \sum_{l=0}^{L-1} w(m, l)s(m - l) + z(m), \quad (4)$$

where $s(m - l)$ is the complex symbol drawn from a constellation $s$ of the $l$th paths at time $m - l$, and $z(m)$ is the additive white Gaussian noise (AWGN) with zero mean and variance $x$. After DFT operation, the received signal at pilot

![Fig. 1. OFDM transceiver system model.](image-url)
locations is extracted from signal and the corresponding output is represented as follows:

\[
R(k) = \sum_{m=0}^{M-1} r(m)e^{-j2\pi m k}
\]

\[
= \sum_{m=0}^{M-1} [u(m, l)s(m - l) + z(m)]e^{-j2\pi m k}
\]

The received signals are demodulated and soft or hard values of the corresponding bits are passed to the decoder. The decoder analyzes the structure of received bit pattern and tries to reconstruct the original signal. In order to achieve good performance the receiver has to know the impact of the channel.

**D. OFDMA waveform**

The frequencies (sub-carriers) are orthogonal, meaning the peak of one sub-carrier coincides with the null of an adjacent sub-carrier. With the orthogonality, each sub-carrier can be demodulated independently without ICI. In OFDM system, the entire channel is divided into many narrow sub-channels, which are transmitted in parallel, thereby increasing the symbol duration and reducing the ISI.

Like OFDM, OFDMA employs multiple closely spaced sub-carriers, but the sub-carriers are divided into groups of sub-carriers. Each group is named a sub-channel. The sub-carriers that form a sub-channel need not be adjacent. In the downlink, a sub-channel may be intended for different receivers. Finally, OFDMA is a multi-user OFDM (single user) that allows multiple access on the same channel. Despite many benefits of OFDMA for high speed data rate services, they suffer from high envelope fluctuation in the time domain, leading to large of OFDMA for high speed data rate services, they suffer from multiple access on the same channel. Despite many benefits of OFDMA for high speed data rate services, they suffer from high envelope fluctuation in the time domain, leading to large

\[
\text{One radio frame} = 20 \text{ Slots} = 10 \text{ Sub-frames} = 10 \text{ ms}
\]

\[
1 \text{ Slot} = 7 \text{ OFDM symbols} = 0.5 \text{ ms}
\]

\[
2 \text{ Slots} = 1 \text{ Sub-frame} = 10 \text{ ms}
\]

\[
\begin{array}{ccccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & \ldots \ldots & 15 & 16 & 17 & 18 & 19 & 20
\end{array}
\]

\[1 \text{ slot} = 7 \text{ OFDM symbols} = 0.5 \text{ ms} \]

\[2 \text{ slots} = 1 \text{ sub-frame} = 10 \text{ ms} \]

**III. CE procedure**

CE is the process of characterizing the effect of the physical medium on the input sequence. The aim of most CE algorithm is to minimize the mean squared error (MSE), while utilizing as little computational resources as possible in the estimation process [2], [4]. CE algorithms allow the receiver to approximate the impulse response of the channel and explain the behavior of the channel. This knowledge of the channel’s behavior is well-utilized in modern mobile radio communications. One of the most important benefits of channel estimation is that it allows the implementation of coherent demodulation. Coherent demodulation requires the knowledge the phase of the signal. This can be accomplished by using channel estimation techniques. Once a model has been established, its parameters need to be estimated in order to minimize the error as the channel changes. If the receiver has a priori knowledge of the information being sent over the channel, it can utilize this knowledge to obtain an accurate estimate of the impulse response of the channel.

In LTE, like many OFDMA systems, known symbols called training sequence, are inserted at specific locations in the time frequency grid in order to facilitate channel estimation [10], [15]. As shown in Fig. 3, each slot in LTE downlink has a pilot symbol in its seventh symbol [6] and LTE radio frames are 10 msec long. They are divided into 10 subframes, each subframe 1 msec long. Each subframe is further divided into two slots, each of 0.5 msec duration. The subcarrier spacing in the frequency domain is 15 kHz. Twelve of these subcarriers together (per slot) is called a physical resource block (PRB) therefore one resource block is 180 kHz [2], [3], [6]. Six resource blocks fit in a carrier of 1.4 MHz and 100 resource blocks fit in a carrier of 20 MHz. Slots consist of either 6 or 7 ODFM symbols, depending on whether the normal or extended cyclic prefix is employed [10], [15], [17].

Channel estimates are often achieved by multiplexing training sequence into the data sequence [18]. These training symbols allow the receiver to extract channel attenuations and phase rotation estimates for each received symbol, facilitating the compensation of channel fading envelope and phase. General channel estimation procedure for LTE OFDMA system is shown in Fig. 4. The signal S is transmitted via a time-varying channel w, and corrupted by an additive white Gaussian noise (AWGN) z before being detected in a receiver. The reference signal w_{est} is estimated using LS , Wiener based, or proposed method. In the channel estimator, transmitted signal S is convolved with an estimate of the channel w_{est}. The error between the received signal and its estimate is

\[
e = (r - r_1)
\]
The aim of most channel estimation algorithms is to minimize the mean squared error (MMSE), while utilizing as little computational resources as possible in the estimation process. The equation (4) can be written as vector notation as [1]:

\[ r = Sw + z, \]  

(7)

where \( r = (r_0, r_1, \ldots, r_{L-1})^\top, S = \text{diag}(s_0, s_1, \ldots, s_{L-1}) \), \( w = (w_0, w_1, \ldots, w_{L-1})^\top \), and \( z = (z_0, z_1, \ldots, z_{L-1})^\top \).

The least-square estimate of such a system is obtained by minimizing square distance between the received signal and its estimate as [3]:

\[ J = (Sr - w)^2 = (r - Sw)(r - Sw)^\top. \]  

(8)

We differentiate this with respect to \( w \), and set the results equal to zero to produce [3]:

\[ w_{LS} = (\alpha I + SS^\top)^{-1}S^\top r, \]  

(9)

where \( \alpha \) is the regularization parameter and has to be chosen such that the resulting eigenvalues are all defined and the matrix \( (\alpha I + SS^\top)^{-1} \) is the least perturbed. Where the channel is considered as a deterministic parameter and no knowledge on its statistics and on the noise is needed. The LS estimator is computationally simple but problem that is encountered in the straight application of the LS estimator is that the inversion of the square matrix turns out to be ill-conditioned. So, we need to regularize the eigenvalues of the matrix to be inverted by adding a small constant term to the diagonal [3]. If the transmitted signal is more random, the performance of the LS method is significantly decrease. Also the LS estimate of \( w_{est} \) is susceptible to Gaussian noise and inter-carrier interference (ICI). Because the channel responses of data subcarriers are obtained by interpolating the channel responses of pilot subcarriers, the performance of OFDM system based on comb-type pilot arrangement is highly dependent on the rigorously of estimate of pilot signals. The successful implementation of the LS estimator depends on the existence of the inverse matrix \( (SS^\top)^{-1} \). If the matrix \( (SS^\top) \) is singular (or close to singular), then the LS solution does not exist (or is not reliable).

To improve the accuracy of the estimator, Wiener filtering based iterative channel estimation has been investigated [4], [7]:

\[ w_{est} = R_{ww}F^\top(SFR_{ww}F^\top + \varepsilon I)^{-1}w_{LS}, \]  

(10)

where \( R_{ww} \) is the autocovariance matrix of \( w \), \( F \) is the DFT matrix, and \( x \) denotes the noise variance. However, this scheme also requires higher complexity.

IV. PROPOSED MMSE BASED CE TECHNIQUE

The equation (7) can be rewritten as [22]:

\[ w_1 = \frac{r}{S} + \frac{z}{S} = w_2 + z_1, \]  

(11)

where actual channel value is \( w_2 = r/S \), noise contribution \( z_1 = z/S \), and \( w_1 \) is the result of direct estimated channel. The proposed channel estimation is

\[ w_{prop} = \sum_{k=0}^{L-1} a_k^1 w_1(k) \]

\[ = \sum_{k=0}^{L-1} a_k^1 [w_2(k) + z_1(k)] \]

\[ w_{prop} = a_1^1 w_3, \]  

(12)

where \( a_k = (a_0, a_1, \ldots, a_{L-1})^\top \) is the column vector filter coefficients, and \( w_3 = \sum_{k=0}^{L-1} [w_2(k) + z_1(k)] \). The mean square error (MSE) for the proposed LTE channel estimation is

\[ J = (w - w_{prop})^2. \]  

In order to calculate the optimal coefficient, taking the expectation of MSE and partial derivative with respect to channel coefficient:

\[ \frac{\partial E(J)}{\partial a_1} = \frac{\partial}{\partial a_1} (E[(w - w_{prop})(w - w_{prop})^\top]). \]  

(13)

Now putting the value of \( w_{prop} = a_1^1 w_3 \) into the above equation to produce:

\[ \frac{\partial E(J)}{\partial a_1} = \frac{\partial}{\partial a_1} (E[(w - a_1^1 w_3)(w - a_1^1 w_3)^\top]) \]

\[ = \frac{\partial}{\partial a_1} (E[(w - a_1^1 w_3)(w - a_1^1 w_3)^\top]) \]

\[ = \frac{\partial}{\partial a_1} (E[ww^\top - a_1^1 w_3 w^\top - a_1^1 w_3 w^\top a_1^1 w_3 + a_1^1 w_3 w^\top a_1^1]) \]

\[ = E[ww^\top + w_3 w_3^\top a_1^1]. \]  

(14)

Now putting the partial derivative equal to zero in the above equation and after some manipulations we get the coefficient as:

\[ a = E(ww^\top)[E((w_3 w_3^\top))^{-1}] \]

\[ = E((w_2 + z_1)w^\top)[E((w_2 + z_1)(w_2 + z_1)^\top))^{-1}] \]

\[ = E((w_2 w_2^\top + z_1 w_1^\top)[E((w_2 + z_1)(w_2 + z_1)^\top)]^{-1}) \]

\[ a = E((w_2 w_2^\top + z_1 z_1^\top)[E((w_2 + z_1)(w_2 + z_1)^\top)]^{-1}). \]  

(15)

In this paper we assume that mean of the AWGN is zero i.e. \( E(z) = 0 \) and variance is \( x \) i.e. \( E(zz^\top) = x \). So, the above equation is simplified as:

\[ a = E(w_2 w_2^\top)[E(w_2 w_2^\top) + x^{-1}] \]

\[ = E(w_2 w_2^\top) + x^{-1} \]

\[ = w_{cross} * (W_{auto} + x^{-1}), \]  

(16)
where \( w_{\text{cross}} = E(w_3 w_2^\dagger) \) and \( w_{\text{auto}} = E(w_2 w_2^\dagger) \) are the channel cross-correlation vector and autocorrelation matrix respectively. Now putting this filter coefficient value in equation (12), we get the final channel estimation formula as:

\[
w_{\text{prop}} = [w_{\text{cross}} \ast (w_{\text{auto}} + x)^{-1}] w_3. \tag{17}
\]

V. COMPLEXITY COMPARISON

The complexity of CE is of crucial importance especially for time varying wireless channels, where it has to be performed periodically or even continuously. For this proposed estimator, the main contribution to the complexity comes from the term \( [w_{\text{cross}} \ast (w_{\text{auto}} + x)^{-1}] \). The variance of the AWGN is pre-calculated and added with the autocorrelation matrix. Thus, only one run-time matrix inversion is required. Also \( w_3 \) is pre-calculated column vector. Table I summarizes the computational complexity of the proposed and existing channel estimation methods. It shows that the proposed CE algorithm has lower complexity than existing methods.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>COMPUTATIONAL COMPLEXITY OF ALGORITHMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operation</td>
<td>LS method</td>
</tr>
<tr>
<td>Matrix inversion</td>
<td>1</td>
</tr>
<tr>
<td>Multiplication</td>
<td>3</td>
</tr>
<tr>
<td>Addition</td>
<td>1</td>
</tr>
</tbody>
</table>

VI. CONCLUSION

In this paper, we present a MMSE based channel estimation method for LTE OFDMA systems and compared the performance with the LS and Wiener based filtering method. This proposed channel estimation method uses knowledge of channel properties to estimate the unknown channel transfer function at non-pilot sub-carriers. It can well solve the ill-conditioned least square (LS) problem and track the changes of channel parameters with low complexity.

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REFERENCES


