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Noughts and Crosses on a Draughts board

Use an ordinary Draughts (Checkers) board, with the standard set of 12 pieces each, red versus black. Play Noughts and Crosses in the usual way, taking turns to place counters in any of the empty squares. As usual the winner will be the first player to achieve three-in-a-row. Of course, like a drawn 0 or X, the pieces are not allowed to move once they have been placed. However if no one has won by the time 24 pieces have been placed, allow players to move one of their pieces to any adjacent (horizontally, or vertically—and what about diagonally?) empty square.

When a three-in-a-row-win becomes too easy (is there a fool-proof winning strategy?), try requiring four-in-a-row to win. If that becomes too easy, try five-in-a-row.

Game analysis for a do-it-yourself computer

An interesting activity that extends flow-chart analysis is to make a “Learning Robot” that uses matchboxes to represent each possible branching move. Each matchbox contains as many coloured beads as there are next moves, a different colour for each different move. The “Robot” plays the game, move by move, by a human (the “Robot’s” minder) shaking the appropriate matchbox, and selecting one of the beads it contains, and then following to the next move with the same colour as the one selected. By playing successive games, and noting which pathway of nodes, or matchbox moves, is traced through the flow-chart, it is possible to “reward” the nodes that lead to a WIN by placing an extra coloured bead in each box, to increase the probability of that successful move being made in the next game. Martin Gardner (1962, always an outstanding resource for mathematics classrooms) describes a simple “Learning Robot” that plays Hexapawn, a micro-chess game that uses SIX pawns on a $3 \times 3$ board. The game is won by advancing a pawn to the opposite back row, or by standard chess pawn moves, capturing all the opponent’s pieces (en passant capture is not allowed), or by totally blocking the opponent. The “Robot” for Hexapawn uses 24 matchboxes and coloured beads or Smarties. He suggests making a similar “Robot” for mini-draughts, on a $4 \times 4$ board, and notes that Donald Machie created a Noughts and Crosses “Robot” that uses 300 matchboxes!

$3 \times 3 \times 3$ three-dimensional Noughts and Crosses

As well as the 3-D Noughts and Crosses variants described by Clarkson, it is possible to play $3 \times 3 \times 3$ 3-D Noughts and Crosses, using a special 3-D board that resembles a multi-level car-park, with a ground floor, a middle floor, and a top-floor.

Players can play using pencil-and-paper with drawn or printed grids, such as this, or can use counters. Perspex or other rigid transparent plastic is perfect for a make-it-yourself board, using glued pieces of dowel to support the upper-floor. Play is identical to standard $3 \times 3$ Noughts and Crosses. Any straight line of three of the same pieces wins, whether this is on one of the flat floor, or is a vertical line, or even a diagonal line. Some diagonals can be tricky! Consider this line of As, and the other line of Bs.
There is a perfect strategy for \(3 \times 3 \times 3\) Noughts and Crosses. As soon as players discover this, the game has no further interest. Therefore, alter the rules to ban the first and second move using this perfect strategy, or use a moving-counter variant.

4 \(\times\) 4 \(\times\) 4 three-dimensional Noughts and Crosses

Most games shops sell versions of 3-D Noughts and crosses, with no perfect strategy. To achieve this, a further variant is used, namely Noughts and Crosses on a \(4 \times 4 \times 4\) board. This is a four-storey multi-story car-park with each floor like this.

Despite the obvious four-ness of this variant, this is not four-dimensional Noughts and Crosses!

Nine-Men’s-Morris

Another ancient (possibly dating back to Egypt!) and outstanding strategy board game with a Noughts and Crosses flavour is Nine-Men’s-Morris, or Mills or Merelles, described in many games books, but not well known in Australia. It is an interesting mixture of three-in-a-row and capture: like Noughts and Crosses, plus draughts. The winner is the player who captures all the opponent’s pieces (or the losing opponent has only two pieces left), or the opponent is blocked and unable to make a move.

Excellent versions, with sturdy metal cases, and magnetic counters (ideal for playing) are cheaply available in S2 and Reject shops, and more expensive commercial versions can be bought in toy and games shops, often with very attractive wooden boards and peg-counters or marbles. This is the special board.

Players have nine counters each, usually black versus white. Players take turns to place one of their counters in any empty circle on the board. Players aim to get three counters in a row (known as a “Mill”). When a player makes a Mill, the player can remove any one of the opponent’s pieces. During this placement stage, apart from capture, once a counter is placed it cannot be moved. Play continues until all the counters have been placed. During the second stage of play, players take turns to move one of their counters to an adjacent empty circle (moving along the lines on the board, a “sliding” move), aiming, as before, to make a Mill, and capture an enemy piece. During this stage, when a Mill is made and an enemy piece captured, the player who made the Mill is not allowed in his or her next turn, to break that Mill, and then in his or her next turn, move the appropriate counter back to reform the same Mill. However, when that player has had more than two further moves after making a Mill, that same Mill can be reformed. In the end stages, if a player is reduced to only three men, that player can move any of his or her counters to any empty circle on the board (this is known as “hopping”).

Beginner’s might like to practise more simply, using smaller variants: Three-Men’s-Morris on a board made of four joined squares, with counters placed on the vertices.

Or try Six-Men’s-Morris, with a Nine-Men’s board that lacks the inner square (or the outer square).
Finally, look for other Noughts and Crosses variants, such as Derrah (Noughts and Crosses meets Nine-Men’s Morris; in Gough, 2008; Arnold, 1975, p. 226) which uses a $6 \times 6$ board, and 12 counters of one colour and 12 counters of another.

**Noughts and Crosses in disguise**

**The Fifteens Game**

Nine playing cards from Ace to 9 are placed face up.

Two players take turns to pick up one of the cards.

The winner is the first player to have, in the cards he or she has collected, a set of three cards that add to 15.

**Leo Moser’s HOT Game**

Nine cards are used, each of which has one of the words HOT, HEAR, TIED, FORM, WASP, BRIM, TANK, SHIP and WOES written on it.

Two players take turns to take a card not yet chosen.

The first player to obtain three cards (among the other cards he or she has picked up) with the same letter is the winner.

These two unusual games were described by Martin Gardner (1975, chapter 16).

**Challenge:** How are these two games equivalent to Noughts and Crosses?

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**References and further reading**


