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How many variants of Noughts and Crosses do you know?

Last issue you were left with the promise that variants of the traditional noughts and crosses game would be explored. Here are just a few.

Investigate misère Noughts and Crosses versions in which the player forced to complete a line of 3 is the loser. Martin Gardner calls this “Toetacktick”, of course (Gardner, 1959, p. 44).

Try Connect Four (four in a row with gravity using a $6 \times 6$ board)—widely available in sturdy affordable versions from many $82$ and reject shops, and more expensively as a brand-name product in toy and games shops. As with Clarkson’s 3-D Stacking Cube Noughts and Crosses, a piece (or a drawn O or X) cannot be left suspended in mid-air. Hence the placing of pieces (or marks) works upwards from the ground-floor.

Go-Moku or Five-In-A-Row is an excellent extension of Noughts and Crosses. Go-Moku (five in a row on a large board, such as $20 \times 20$) is based on the materials of the outstanding and classic Chinese territory-capturing game Go, or Wei-chi—a game that is well worth investigating! (see Gough, 2000, pp. 74–75). As a “make-it-yourself” or pencil-and-paper game, players use squared paper and counters of different color, or one player draws Xs and the other draws Os. Players take turns to place one of their counters or draw their mark in any empty square. The winner is the first player to complete a row of 5 of his or her counters or signs. The row may be horizontal, vertical or diagonal. Players should recognise that a row of 4 counters or marks with an empty square at each end cannot be blocked. So a row of 3 counters with an empty square at each end is dangerous!

Maurice Kraitchik (1953, p. 280) describes Orthogonal Five-in-a-Row as a two-player spatial variant of Go-Moku, using a $16 \times 16$, or $18 \times 18$ or $20 \times 20$ board, with the winner being the first to get five of his or her pieces or markings in a horizontal (or vertical) line in consecutive squares. Kraitchik does not specify how many counters each player has, but both have the same number of counters. (We may assume that 20 to 30 counters would be enough.) Initially players take turns placing one of their counters on any empty square, aiming to make five in a horizontal or vertical line (but, in this stage of playing, not being allowed to move a placed counter). Kraitchik adds that if no-one has won by the time all the counters have been placed, players continue taking turns, still aiming to make five in a horizontal or vertical line, but in each turn a player moves one of his or her counters one space (horizontally, vertically, or diagonally, as in the move of a chess King), to any adjacent empty space.

A Five-in-a-Row variant—Quasi Teeko—is featured in the primary teachers’ mathematics resource Guidelines in Number (1985, p. 51) which describes a moving-counter variant of Noughts and Crosses that begins with pieces in a standard starting position, Black versus White.

Players take turns to move one of their counters, one space, horizontally, diagonally, or vertically, to an adjacent empty square (no jumping, of course), aiming to get their five counters aligned, diagonally, vertically, or horizontally.

The original game, Teeko, was invented and published by John Scarne in 1945. It uses the $5 \times 5$ board, but each player has only four counters, and initial play involves taking turns to place one counter at a time in any empty square. The aim, as a variant of Noughts and Crosses, is to get all four counters of one color in a straight line, horizontal, vertical or diagonal, or in a $2 \times 2$ square arrangement. In the description of Teeko in Guidelines in Number (1985, p. 146), moving is only allowed one space, horizontally, or vertically, into an adjacent empty square. The Wikipedia description of Teeko does not rule out a possible diagonal move, but notes that a perfect strategy exists.
**Fifteens** is a game for two players using pencil and paper, or counters on a standard 3 × 3 Noughts and Crosses board.

Players take turns to write one of the digits from 1 to 9 (or place a counter with that digit) in any empty square on the board. Each digit may be used only once. The loser is the first player who makes a line of three digits (horizontally, vertically, or diagonally) which do not add up to fifteen.

Note that the game is a draw if the players succeed in making a Magic Square which totals fifteen.

Magic Squares, 3 × 3, 4 × 4, and beyond, are mathematically interesting. (Gardner, 1961, Chapter 12: “Magic Squares”, is a good starting point.)

Note: a similar game, Tot-Ten by Spears, uses a larger board and the idea of trying to build a complete row which adds to 10: the counters which add to 10 are then removed and that player scores one point. The Parker game Hi-Spot uses the same idea, adding to 9, using transparent plastic tiles.

**Moving Counter Three-Counter Noughts and Crosses** is a simple variant of the traditional game, widely played in ancient China, Greece and Rome (Gardner, 1959, pp. 43–44).

Players use the standard 3 × 3 board, but instead of drawing a O or a X, players use two kinds of counters. For example, one player could use ten-cent coins and the other could use five-cent coins. Each player is allowed three counters. Players take turns to place a counter in any empty square, trying to get three in a row, as usual. If no-one has won when all the counters have been placed, players may take turns to move one of their counters to an empty square. Two alternative rules may apply: either the empty square must be adjacent (horizontally, vertically or diagonally) to the counter being moved; or else a counter may be moved to any empty cell.

Experienced players may try a 4 × 4 board, with five counters each, trying to get four in a row.

Maurice Kraitchik calls Moving Counter Three-Counter Noughts and Crosses **Hopscotch** (1953, p. 280), and uses a board where players place counters on “points” (circles), and later move them along lines. It is easy to see that mathematically (topologically, according to the playing connections of one position to another) this is equivalent to playing on the standard Noughts and Crosses 3 × 3 grid.

Such topological or geometric equivalences, where a square is replaced by a point, and the adjacency of squares is replaced by a joining line, are said to be *duals* of one another. What would the dual of a Draughts board look like?

Kraitchik extends Hopscotch further, using a quadruple board: players aim to get five in a line. However Kraitchik neglects to specify the number of counters each player uses. The minimum would be five, of course, and the maximum would be 12.

**Guidelines in Number** (1985, p. 102) uses three counters for each player on the Quadruple Hopscotch board, and calls the three-in-a-row game **Picaria**.

**Guidelines in Number** (1985, p. 145) has a further variant on Hopscotch, called **Tee-Tah-Toe**. Two players, each using three counters, take turns placing their counters, and, if necessary, when all counters have been placed, moving one counter at a time to an adjacent empty “point” or circle, aiming to block the opponent from being able to move (the opponent will have no available adjacent circle).

**References**


