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# Wavelets/Multiwavelets Bases and Correspondence Estimation Problem: An Analytic Study

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**Abstract**—Correspondence estimation in one of the most active research areas in the field of computer vision and number of techniques has been proposed, possessing both advantages and shortcomings. Among the techniques reported, multiresolution analysis based stereo correspondence estimation has gained lot of research focus in recent years. Although, the most widely employed medium for multiresolution analysis is wavelets and multiwavelets bases, however, relatively little work has been reported in this context. In this work we have tried to address some of the issues regarding the work done in this domain and the inherited shortcomings. In the light of these shortcomings, we propose a new technique to overcome some of the flaws that could have significantly impact on the algorithm performance and has not been addressed in the earlier propositions. Proposed algorithm uses multiresolution analysis enforced with wavelets/multiwavelets transform modulus maxima to establish correspondences between the stereo pair of images. Variety of wavelets and multiwavelets bases, possessing distinct properties such as orthogonality, approximation order, short support and shape are employed to analyse their effect on the performance of correspondence estimation. The idea is to provide knowledge base to understand and establish relationships between wavelets and multiwavelets properties and their effect on the quality of stereo correspondence estimation.

**Index Terms**—Wavelets, Multiwavelets, Multiresolution analysis, correspondence estimation

## I. INTRODUCTION

Finding correct corresponding points from more than one perspective views in stereo vision is subject to number of potential problems, such as occlusion, ambiguity, illuminative variations and radial distortions. A number of algorithms has been proposed to address the problems as well as the solutions, in the context of stereo correspondence estimation. The majority of them can be categorized into three broad classes i.e. local search algorithms (LA) [1], [2], [20], global search algorithms (GA) [3], [4] and hierarchical iterative search algorithms (HA) [6], [21]. The algorithms belonging to the LA class try to establish correspondences over locally defined regions within the image space. Correlations techniques are commonly employed to estimate the similarities between the stereo image pair using pixel intensities, sensitive to illuminative variations. LA perform well in the presence of rich textured areas but have tendency of relatively lower performance in the featureless regions. Furthermore, local search using correlation windows usually lead to poor performance across the boundaries of

image regions. On the other hand, algorithms belonging to GA group deals with the stereo correspondence estimation as a global cost-function optimization problem. These algorithms usually do not perform local search but rather try to find a correspondence assignment that minimizes a global objective function. GA group algorithms are generally considered to possess better performance over the rest of the algorithms. Despite of the fact of their overall better performance, these algorithms are not free of shortcomings and are dependent on how well the cost function represents the relationship between the disparity and some of its properties like smoothness, regularity. Moreover, how close that cost function representation is to the real world scenarios. Furthermore, the smoothness parameters makes disparity map smooth everywhere which may lead to poor performance at image discontinuities. Another disadvantage of these algorithms is their computational complexity, which makes them unsuitable for real-time and close-to-realtime applications. Third group of algorithms uses the concept of multi-resolution analysis [8] in addressing the problem of stereo correspondence. In multi-resolution analysis, as is obvious from the name, the input signal (*image*) is divided into different resolutions, i.e. *scales and spaces* [8], [35], before estimation of the correspondence. This group of algorithms do not explicitly state a global function that is to be minimized, but rather try to establishes correspondences in a hierarchical manner [36], [19], similar to iterative optimization algorithms [21]. Generally, stereo correspondences established in lower resolutions are propagated to higher resolutions in an iterative manner with mechanisms to estimate and correct errors along the way. This iterative error correction minimizes the requirements for explicit post processing of the estimated outcomes.

In this work, the goal is to provide brief overview of the techniques reported within the context of stereo correspondence estimation and wavelets/multiwavelets theory. In the light of aforementioned inherited shortcomings; we propose a comprehensive algorithm addressing the these issues in detailed manner. The presented work also focuses on the use of multiwavelets basis that simultaneously posses properties of orthogonality, symmetry, high approximation order and short support, which is not possible in the scaler wavelets case [8], [7]. A table is compiled to provide an insight of the

comparative performance of wavelets and multiwavelets in estimating stereo correspondences.

## II. WAVELETS/MULTIWAVELETS ANALYSIS IN STEREO VISION: BACKGROUND

The multi-resolution analysis is usually performed by either Wavelets or Fourier analysis [22], [23]. Wavelets analysis is a newer way of scale space representation of the signals and considered to be as fundamental as Fourier and a better alternative. One of the reasons that makes wavelet analysis more attractive to researchers is the availability and simultaneous involvement of a number of compactly supported bases for scale-space representation of signals, rather than infinitely long sine and cosine bases as in Fourier analysis [22]. In addition, approximation order of the scaling and wavelet filters provide better approximation capabilities and can be adjusted according to input signal and image by selecting the appropriate bases. Other features of wavelet bases that play an important role in signal/ image processing application are their shape parameters, such as symmetric and asymmetric, and orthogonality and orthonormality. All these parameters can be enforced at the same time in multiwavelets bases however is not possible in scalar wavelets case [7].

Wavelet theory has been explored very little up to now in the context of stereo vision. Some work has been reported in applying wavelet theory for addressing correspondence problem. To the best of author's knowledge, Mallat [23], [24] was the first who used wavelet theory concept for image matching by using the zero-crossings of the wavelet transform response to seek correspondence in image pairs. In [24] he also explored the the signal decomposition into linear waveforms and signal energy distribution in time-frequency plane. Afterwards, Unser [25] used the concept of multi-resolution (coarse to fine) for image pattern registration using orthogonal wavelet pyramids with spline bases. Olive-Deubler-Boulin [26] introduced a block matching method using orthogonal wavelet transform coefficients whereas [27] performed image matching using orthogonal Haar wavelet bases. Haar wavelet bases are one of the first and simplest wavelet basis and posses very basic properties in terms of smoothness and approximation order [11], therefore are not well adapted for correspondence estimation problem.

In aforementioned algorithms, the common methodology adopted for stereo-correspondence cost aggregation was based on the difference between the wavelet coefficients in the perspective views. This correspondence estimation suffers due to inherent problem of translation variance with the discrete wavelet transform. This means that wavelet transform coefficients of two shifted versions of the same image may not exhibit exactly similar pattern [30], [31]. On the other hand, Mallat's concept of zero-crossing is considered to be translation invariant and a complete representation [23], however Mallat didn't pursue his idea deep into stereo correspondence estimation.

A more comprehensive use of wavelet theory based multi-resolution analysis for image matching was done by He-Pan in

1996 [28], [29]. He took the application of wavelet theory bit further by introducing a complete stereo image matching algorithm using complex wavelet basis. In [28] He-Pan explored many different properties of wavelet basis that can be well suited and adaptive to the stereo matching problem. A number of real and complex wavelet bases were used in both [28], [29] and transformation is performed using wavelet pyramid, commonly known by the name Mallat's dyadic wavelet filter tree (DWFT) [8]. Common problem with DWFT is the lack of translation and rotation invariance [30], [31] especially in real valued wavelet basis inherited due to the involvement of factor 2 as in expression 1.

$$\begin{aligned} S_A[n] &= \sum_{-\infty}^{\infty} x[k]L[2n - k] \\ S_W[n] &= \sum_{-\infty}^{\infty} x[k]H[2n + 1 - k] \end{aligned} \quad (1)$$

Where  $L$  and  $H$  represent filters based on scaling function and wavelet coefficients [8], [42]. Furthermore similarity measures were applied on individual wavelet coefficients which is very sensitive to noise. In [29], conjugate pairs of complex wavelet basis were used to address the issue of translation variance. Conjugate pairs of complex wavelet coefficients are reported to provide translation invariant outcome, however increases the search space by twofold. Similarly, Magarey [32], [33] introduced algorithms for motion estimation and image matching, respectively, using complex discrete *Gabor-like* quadrature mirror filters. Afterwards, Shi [33] applied *sum of squared difference* technique on wavelet coefficients to estimate stereo correspondences. He uses translation invariant wavelet transformation for matching purposes, which is a step forward in the context of stereo vision and applications of wavelet.

More to the wavelet theory, multi-wavelet theory evolved [34] in early 1990s from wavelet theory and enhanced for more than a decade. Their success, over scalar wavelet bases, stems from the fact that they can simultaneously posses the good properties of orthogonality, symmetry, high approximation order and short support, which is not possible in the scalar case [8], [7]. Being a new theoretical evolution, multi-wavelets are still new and are not yet applied in many applications. In this work we will devise a new and generalized correspondence estimation technique based wavelets and multiwavelets analysis to provide a framework for further research in this particular context.

## III. WAVELETS/MULTIWAVELETS FUNDAMENTALS AND WAVELET TRANSFORM MODULUS MAXIMA

Classical wavelet theory is based on the dilation equations as

$$\phi(t) = \sum_k c_h \phi(Mt - h) \quad (2)$$

$$\psi(t) = \sum_k w_h \phi(Mt - h) \quad (3)$$

where  $c_h$  and  $w_h$  represents the scaling and wavelet coefficients. Similarly, multiscaling functions satisfy the matrix dilation equation as

$$\Phi(t) = \sum_h C_h \Phi(Mt - h) \quad (4)$$

Similarly for the multi-wavelets the matrix dilation equation can be expressed as

$$\Psi(t) = \sum_h W_h \Phi(Mt - h) \quad (5)$$

where

$$\Phi(t) = \begin{bmatrix} \phi_0(t) \\ \phi_1(t) \\ \vdots \\ \phi_{r-1}(t) \end{bmatrix}, \quad \Psi(t) = \begin{bmatrix} \psi_0(t) \\ \psi_1(t) \\ \vdots \\ \psi_{r-1}(t) \end{bmatrix} \quad (6)$$

In equations 4 and 5,  $C_h$  and  $W_h$  are real and matrices of multi-filter coefficients. Generally only two band multi-wavelets, i.e.  $M = 2$ , defining equal number of multi-wavelets as multi-scaling functions, are used for simplicity. For more information, about the generation and applications of multi-wavelets with, desired approximation order and orthogonality, interested readers are referred to [7], [8].

The Wavelet Transform Modulus, in general vector representation, can be expressed as

$$\Xi_s = W_s \angle \Theta_{W_s} \quad (7)$$

Where  $W_s$  can further be defined as

$$W_s = \sqrt{|D_{h,s}|^2 + |D_{v,s}|^2 + |D_{g,s}|^2} \quad (8)$$

where  $D_{h,s}$ ,  $D_{v,s}$  and  $D_{g,s}$  are the horizontal, vertical and diagonal detail components at scale  $s$ . The phase of Wavelet Transform Modulus ( $\Theta_{W_s}$ ) can be expressed as

$$\Theta_{W_s} = \begin{cases} \alpha & \text{if } D_{h,s} > 0 \\ \pi - \alpha & \text{if } D_{h,s} < 0 \end{cases} \quad (9)$$

where

$$\alpha = \tan^{-1} \left( \frac{D_{v,s}}{D_{h,s}} \right) \quad (10)$$

The vector  $\vec{n}_s$  points to the direction normal to the edge surface as

$$\vec{n}_s = [\cos(\Theta_{W_s}), \sin(\Theta_{W_s})] \quad (11)$$

An edge point is the point  $p$  at some scale  $s$  such that  $\Xi_s$  is locally maximum at any point  $k$  if  $k = p$  and  $k = p + \varepsilon \vec{n}_s$  for  $|\varepsilon|$  small enough. These points are known as *wavelet transform modulus maxima* (WTMM), and are shift invariant through the wavelet transform. For further details in reference to wavelet modulus maxima and its translation invariance, reader is kindly referred to [8].

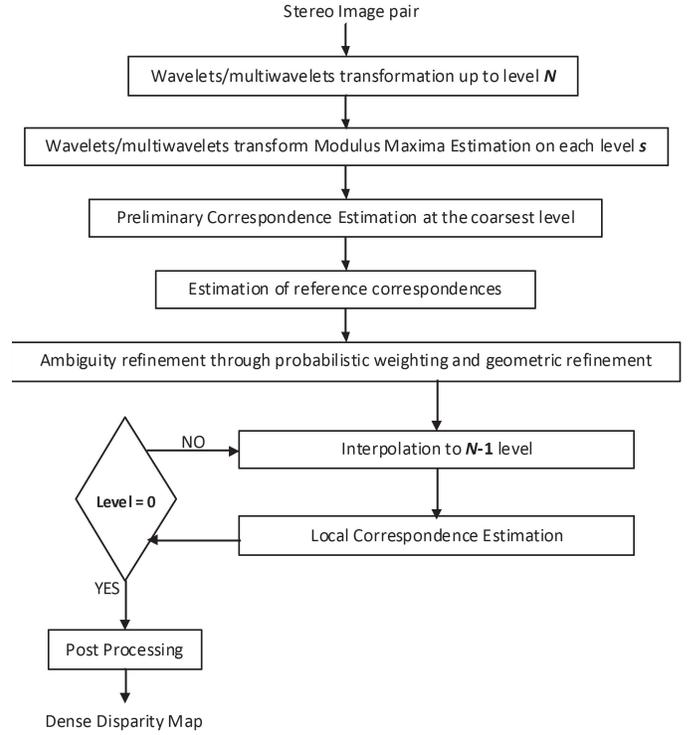


Fig. 1. A simple block representation of the proposed algorithm

#### IV. CORRESPONDENCE ESTIMATION

In the light of aforementioned background, we propose a novel wavelets and multiwavelets analysis based stereo correspondence estimation algorithm. Following the already established mechanism of coarse-to-fine search generally employed in multiresolution analysis, the matching process of the proposed algorithm is divided into two major steps. First part defines the correspondence estimation at the coarsest transformation level, whereas second part defines the iterative matching process from finer to finest wavelets/multiwavelets transformation levels. Correspondence estimation at the coarsest level is the most important part of the proposed algorithm due to its hierarchical nature. The authenticity of correspondences at finer levels are subject to the quality of outcomes at the coarser level. Estimation at finer levels, use local search methodology, searching only the locations where correspondences have already been established in the coarser level.

The matching process initiates with wavelets/multiwavelets decomposition up to level  $N$ , usually taken within the range of [4 5] depending on the size of the input image. To minimize the effect of illuminative variations that could exist in between the perspective views, scale normalization is performed as:

$$Wn_s = \frac{W_{s,i}}{|A_s|} \quad \forall i \in \{h, v, d\} \quad (12)$$

where  $\{h, v, d\}$  represents horizontal, vertical and diagonal details, respectively and  $s$  represents the scale of decomposition, where as  $A$  and  $W$  represents the approximation

and wavelet spaces, respectively. The benefit of wavelets and multiwavelets scale normalization is two fold. Firstly, it normalizes the variations in coefficients, at each transformation level, either introduced due to illuminative variations or by filters gains. Secondly, if the wavelets and multiwavelets filters are perfectly orthogonal, the features in the detail space become more prominent. Scale normalization is followed by the calculation of transform modulus maxima using expressions 7 to 11.

To establish initial correspondences, similarity measure is performed on wavelets transform modulus maxima ( $\Xi_s$ ) using normalized correlation [38] measure enforced by multi-window approach [9] as

$$C_{\Xi} = C_{\Xi, W_0} + \sum_{i=1}^{n_W/2} C_{\Xi, W_i} \quad (13)$$

Where  $C_{\Xi}$  represents the correlation score of wavelets transform modulus maxima, under investigation and  $n_W$  represents the number of surrounding windows, usually taken as 9, without considering  $W_0$ . The second summation term in (13) represent the summation of best  $n_W/2$  windows out of  $n_W$ . An average of the correlation scores from these windows is taken to keep the score normalized i.e. within the range of [0 1]. A block diagram representing the process of the proposed algorithm is shown in Figure 1.

#### A. Probabilistic Weighting

Wavelets and multiwavelets transformation using Mallat's filter-bank [8] produces  $r^2$  spaces for each bank at each scale.  $r$  is the multiplicity of filter coefficients [7], [18], which is one in case of wavelets, i.e.  $r = 1$  and  $r > 1$  in case of multiwavelets. To ensure the contribution of wavelets/multiwavelets coefficients from all  $r^2$  search spaces, in correspondence estimation process, probabilistic weighting is introduced for correlation measure of (13). It is the probability of selection of any coefficient, say  $C$  as a correspondence to the reference coefficient from any of the search space, out of  $r^2$  spaces as

$$P_C = n_C/r^2 \quad \text{where } 1 \leq n_C \leq r^2 \quad (14)$$

where  $n_C$  is the number of times a coefficient  $c$  is selected. It is obvious from expression (14) that the  $P_C$  lies between the range of  $[1/r^2 \ 1]$ . The correlation score in expression (13) is then weighted with  $P_C$  as

$$\aleph_C = P_C \sum_{n_C} C_{\Xi_C} \quad \forall n_C \in \mathbb{Z} : n_C \leq r^2 \quad (15)$$

$\aleph_C$  defines the potential of the corresponding coefficient to be considered for further processing. For some candidate  $C$ , with  $P_C = 1$  and  $C_{\Xi_C}$  above a certain threshold, makes it suitable to be used as reference in the geometric refinement for the estimation of other potential candidates with ambiguity. These aforementioned candidates are tagged with  $O_p$  and facilitate the extraction of other potential candidates out of the pool of ambiguous corresponding pairs.

#### B. Geometric Refinement

Due to the involvement of  $r^2$  search spaces, for any reference coefficient, there is a possibility of selection of different wavelet transform modulus maxima from different search spaces, commonly referred to as ambiguity. To address this issue, a simple geometric topological refinement is employed in order to extract the optimal corresponding coefficients out of the pool of ambiguous ones. For this purpose, the geometric relationship of the ambiguous coefficients in reference to the  $O_p$  coefficients is checked. The pairs having the closest geometric topology are selected as optimal correspondences. Three geometric features, that are relative distance difference (*RDD*) 16, absolute distance difference (*ADD*) 17 and relative slope difference (*RSD*) 18 are selected to check the geometric orientational of the candidate pairs with respect to the  $O_p$ .

$$d_{AC_j} = \left( \frac{d_{Ca_j} - d_{Op}}{d_{Ca_j} + d_{Op}} \right)_n \quad (16)$$

$$d_{RC_j} = \left( \min_i \left| \frac{d_{RC_{1,i}} - d_{RC_{2,i,j}}}{d_{RC_{1,i}} + d_{RC_{2,i,j}}} \right| \right)_n \quad (17)$$

$$d_{SC_j} = \left( \min_i \left| \frac{S_{C_{1,i}} - S_{C_{2,i,j}}}{S_{C_{1,i}} + S_{C_{2,i,j}}} \right| \right)_n \quad (18)$$

The reason of selecting these geometric features for addressing the problem of ambiguity is there invariance through many geometric transformations, such as Projective, Affine, Matric and Euclidean [10]. The geometric measurement is then weighted with the correlation score to keep the previous achievements of the candidates in the consideration as

$$\wp_{C_j} = \aleph_{C_j} \left( \frac{e^{-d_{AC_j}} + e^{-d_{RC_j}} + e^{-d_{SC_j}}}{3} \right) \quad (19)$$

Corresponding pair with maximum geometric refinement score will be selected as optimal correspondences.

#### V. EFFECT OF DIFFERENT WAVELET AND MULTI-WAVELET BASES

To address the influence of wavelets and multiwavelets bases on the quality of correspondence estimation, 16 wavelets and multiwavelets bases are employed. These bases are carefully chosen to cover range of properties such as orthogonality, bi-orthogonality, symmetry, asymmetry, multiplicity and approximation order.

Using proposed algorithm, 13 different wavelet and multi-wavelet bases are chosen to address the influence on the quality of the correspondence estimation. Wavelets and multiwavelets bases are selected encompassing the variety of different properties such as orthogonality and bi-orthogonality, symmetry and a-symmetry, approximation orders and different multiplicities. The coefficients related to all wavelet and multi-wavelet bases presented in Table I can be found in [41].

Statistics are collected using *root mean squared error* and *percentage of bad estimated disparities* to perform the qualitative comparison between estimated and

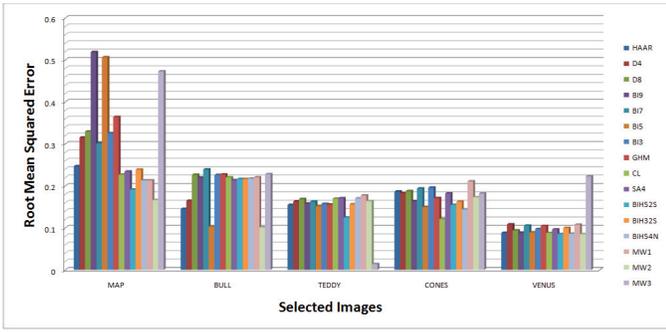


Fig. 2. Root Mean Square Error (R) for number of images

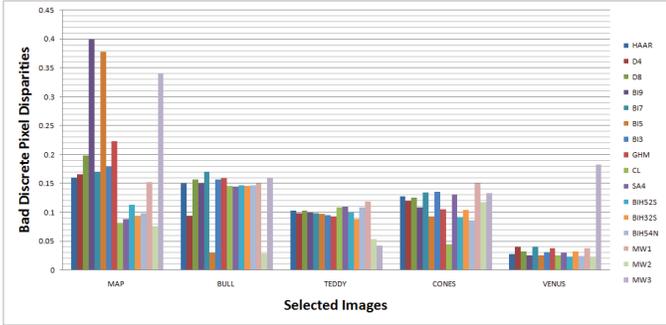


Fig. 3. % of Bad Pixel Disparity (B) for number of images

the ground truth disparity maps. The expression used for the calculation of these statistics are taken from [5] as  $R = \sqrt{\frac{1}{N} \sum_{x,y} |d_E(x,y) - d_G(x,y)|^2}$  and  $B = \frac{1}{N} \sum_{(x,y)} |d_E(x,y) - d_G(x,y)|^2 > \xi$ , where  $d_E$  and  $d_G$  are the estimated and ground truth disparity maps.  $N$  is the total number of pixels in an image whereas  $\xi$  represents the disparity error tolerance and is usually taken as 1. In other words any difference greater than 1 between ground truth disparity maps and the estimated disparity is considered as bad disparity value. These statistics are given in Table I related to the images *MAP*, *BULL*, *TEDDY*, *CONES* and *VENUS*.

Referring to Table I and related visual representation in Figures 2 and 3, a distinguished higher performance of multiwavelets bases can be observed throughout the set of employed images. This statistical behavior of the estimated data strengthens earlier established understanding about the superior performance of multiwavelets bases over the scalar ones. Their success stems from the fact that they can simultaneously possess the good properties of orthogonality, symmetry, high approximation order and short support [39], [18], which is not possible in the scalar wavelets case [18], [40].

Out of 9 multiwavelets bases presented in Table I, CL, MW2 and MW3 has outperformed rest of the bases with major contribution from MW2. Within multiwavelets bases, it is hard to define a clear pattern between the performance and the relevant influential properties. In case of Map, Bull and Venus images, MW2 has performed best possessing properties of orthogonality, asymmetry and approximation order of 2, whereas in case of Teddy MW3 and for image Cones CL

TABLE I  
PERFORMANCE OF ALGORITHM IN TERMS OF  $R$ ,  $B$  W.R.T THE IMAGES  
*Map, Bull, Teddy, Cones AND Venus*

Basis	MAP		BULL		TEDDY		CONES		VENUS	
	R	B	R	B	R	B	R	B	R	B
HAAR [11]										
D4 [12]										
D8 [13], [37]										
B9 [14]										
B3 [14]										
GHM [15]										
CL [16]										
SA4 [14]										
BIH52S[14]										
BIH32S [14]										
MW1 [7]										
MW2 [17]										
MW3 [17]										
	0.4728	0.3395	0.2278	0.1594	0.01370	0.04190	0.1817	0.1329	0.2230	0.1830
	0.2469	0.1603	0.1452	0.1497	0.1548	0.1029	0.1863	0.1276	0.0880	0.0281
	0.3151	0.1654	0.1646	0.0934	0.1632	0.0987	0.1828	0.1195	0.1082	0.0395
	0.3295	0.1983	0.2264	0.1572	0.1685	0.1026	0.1877	0.1251	0.0938	0.0322
	0.5189	0.3991	0.2197	0.1496	0.1580	0.0997	0.1636	0.1083	0.0885	0.0252
	0.3256	0.1786	0.2258	0.1563	0.1571	0.0945	0.1958	0.1350	0.0972	0.0310
	0.3641	0.2232	0.2273	0.1587	0.1555	0.0928	0.1710	0.1046	0.1045	0.0373
	0.2267	0.0811	0.2201	0.1452	0.1696	0.1083	<b>0.1210</b>	<b>0.0439</b>	0.0879	0.0254
	0.2343	0.0883	0.2135	0.1443	0.1707	0.1093	0.1821	0.1304	0.0961	0.0301
	0.1912	0.1132	0.2163	0.1469	0.1245	0.0534	0.1547	0.0915	0.0852	0.0234
	0.2389	0.0943	0.2162	0.1456	0.1561	0.0887	0.1623	0.1041	0.0998	0.0319
	0.2132	0.1524	0.2204	0.1497	0.1770	0.1181	0.2110	0.1505	0.1076	0.0378
	<b>0.1662</b>	<b>0.0754</b>	<b>0.1011</b>	<b>0.0289</b>	0.1638	0.0992	0.1731	0.1178	<b>0.0851</b>	<b>0.0231</b>

has performed best. It is hard, at this stage, to draw some conclusion that could provide evidence of the bases' properties that could lead to higher performance while estimating stereo correspondences. In future work we do intend to establish some relationships between the wavelets and multiwavelets filters properties and the quality of correspondence estimation.

## VI. CONCLUSION

In this research work we have tried to explore the relationship of wavelets and multiwavelets theory with the estimation of stereo correspondence, one of the most active research streams of computer vision. Motivated by the shortcomings involved in the existing techniques, we presented a new algorithm that employs the use of wavelets/multiwavelets transform modulus maxima as corresponding features that are translation invariant. In addition we introduced the contribution of probabilistic weighted normalized correlation and geometric refinement. Probabilistic weighting involves the contribution of more than one search spaces, whereas, geometric refinement addresses the problem of geometric distortion between the perspective views and the ambiguity problem.

A qualitative comparison of 13 wavelets and multiwavelets basis is also presented, clearly showing the superior performance of the multiwavelets basis over the wavelets ones. This study is also intended to initiate a research debate that could help in understanding the relationship of the wavelets/multiwavelets filter properties and the quality of the correspondence estimation.

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