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Microanalysis of excerpts of video-stimulated post-lesson student interviews illuminated the nature and role of queries that sustained creative problem solving activity. These queries led to student realisation that they ‘did not yet know’, and the impetus to explore further. In this study, using Seligman’s (1995) construct of ‘optimism’, ‘not knowing’ is considered ‘failure’ and ‘finding out’, ‘success’. By responding optimistically to identified ‘failure’, these students achieved success—developed new mathematical understandings. The intertwined nature of problem solving, and optimistic activity subsequent to such queries is elaborated. This study contributes to the body of knowledge on sustaining autonomous problem solving activity.

Introduction

Mathematics learning associated with developing ‘deep understanding’ (‘relational understanding’, Skemp, 1976) differs from predominant teaching practices where:

... doing mathematics means following the rules laid down by the teacher; knowing mathematics means remembering and applying the correct rule when the teacher asks the question, and mathematical truth is determined when the answer is ratified by the teacher (Lampert, 1990, p. 29).

The pedagogical approach employed in this study promotes student activity consistent with the title to Lampert’s article: “When the problem is not the question and the solution is not the answer”. The questions that students in this study focused upon were not those in the teacher’s problem, and the results of student explorations were not explicit answers to the problem questions. Students focused their own questions, and developed their own pathways. During such activity, students are “not only choosing the cues and concepts—and often unexpected cues and concepts—but even the very question” (Chick, 1998, p. 17), and are making “not so much direct attempts at solving the problem ... [but] thoroughly investigating it, with auxiliary information being extracted from each trial” (Krutetskii, 1976, p. 292). Relational understanding (Skemp, 1976) develops through such exploration—a connected form of understanding where students know why mathematics is relevant and can select and use it in unfamiliar situations. The social element query (Schwarz, Dreyfus, & Hershkowitz, 2009) became a focus when it preceded shifts in thinking about mathematics generated. Seligman’s (1995) ‘optimism’ was employed as a lens because it has been linked with learning gains in mathematics (Yates, 2002), and problem solving (e.g., Williams, 2005).
Theoretically framing this study

Optimism (Seligman, 1995) is an explanatory style associated with how people respond to successes and failures. Sawyer (2007) described problem-solving activity of innovative design teams in industry as: “mak[ing] more mistakes, ... [with] as many misses as hits” (p. 16). Similarly, mathematical problem solving leading to insights generally includes ‘failures’ ‘on the way to’ attaining ‘successes’ (Williams, 2006a, b). Thus, for the purposes of this study, mathematical problem solving is considered a situation of adversity where ‘failure’ is ‘not knowing’ and ‘success’ is ‘finding out’. An optimistic child perceives successes as ‘permanent’, ‘pervasive’, and ‘personal’ and failures as ‘temporary’, ‘specific’, and ‘external’. These terms are elaborated later through illustrations. Indicators of optimism were displayed in interviews with students who creatively solved self-set problems (Williams, 2005). For example, Dean struggled to attain a passing grade in mathematics (See Williams, 2005, p. 284), but showed he considered ‘not knowing’ as temporary and able to be overcome: “it always takes me a long time to understand when we first start a new topic”. And, he employed personal effort to help overcome this temporary state: “I go over and over it until it makes sense”. He perceived his successes (finding out) as a characteristic of self (someone overcoming ‘not knowing’ through personal effort): “and then I get it” [Success as Pervasive]. Kerri (Williams, 2005, p. 321), a high achieving girl in the same study, spontaneously constructed new knowledge on several occasions during the research period (see Williams, 2007). Her comment: “last year I did not do as well in maths; the teacher took too big a leaps showed she limited her ‘failure’ in mathematics to a particular time frame [Failure as Specific], and identified an external factor that contributed [Failure as External]. Despite their differences in mathematical performances, Dean and Kerri both displayed indicators of optimism, and no indicators of lack of optimism, and each was willingly ‘stepped into unfamiliar territory’ to explore to develop new mathematical understandings. They did not refer to ‘not yet knowing’ as ‘failure’. The term ‘failure’ is used in this paper to link to optimism.

In common language usage, ‘optimism’ is taken to mean feelings of hopefulness and confidence. Seligman’s construct includes the perception that personal effort is required to attain successes (as well), not just a hope that it will happen. Martin (2003) and Williams (2003) identified similar constructs associated with student capacity to overcome adversities during learning: ‘academic resilience’, and ‘optimistic exploratory style’ respectively. Martin’s construct relates to learning in general, and Williams’ construct (Seligman, 1995; Williams, 2006a, b) relates to student inclination to explore: enter ‘flow’ situations.

Flow (Csikszenmihalyi, 1992), a state of high positive affect during creative activity, occurs when a person/group spontaneously develops new skills in response to self-set challenges. Such activity is ‘signalled’ by intense engagement and loss of all sense of time, self, and the world around as all energies are focused on the task at hand (e.g., Williams, 2006a). During mathematical problem solving, flow conditions occur when a group, or individual student, spontaneously and idiosyncratically identifies an unfamiliar mathematical complexity that was not apparent at the commencement of the task, and decides to explore it (e.g., Williams, 2007). The term spontaneous refers to student learning not caused by the teacher (or another ‘who knows’):
We do not use spontaneous in the context of learning to indicate the absence of elements with which the student interacts. Rather we use the term to refer to the non-causality of teaching actions, to the self regulation of the students when interacting ... we regard learning as a spontaneous process in the student's frame of reference. (Steffe & Thompson, 2000, p. 291)

Steffe and Thompson’s expression “in the student’s frame of reference” is crucial to spontaneity. Social elements (Schwarz et al., 2009)—control, elaboration, explanation, query, affirmation, and attention—have been linked to spontaneity (Williams, 2005). Where activity is spontaneous, control, elaboration, explanation, and affirmation are internal to the student because the student controls the pathway taken, explains and elaborates the mathematical ideas involved, and works out, for themselves, whether the mathematics they generate is reasonable. The role of attention and query in relation to spontaneity still need further investigation.

‘Observable cognitive elements’ during the process of critical inquiry (Schwarz et al., 2009), were integrated with Krutetskii’s (1976) ‘mental activities’ to analyse knowledge construction (Williams, 2005). Recognizing involves identifying a context in which a previously constructed mathematical entity applies, or identifying mathematics relevant to a context (Schwarz et al., 2009). Building-with involves using a mathematical procedure that has been recognized, in a context in which it has previously been used (non-spontaneous) or in a new context (spontaneous). Krutetskii’s ‘mental activities’ form subcategories of building-with associated with spontaneity: element-analysis (examining a problem element by element), synthetic-analysis (simultaneous analysis of several elements), and evaluative-analysis (synthetic-analysis for the purpose of judgement). Constructing involves integrating previously constructed knowledge to develop new insight (Schwarz et al., 2009; Krutetskii, 1976), checking internal and external consistency, and recognizing its usefulness in other situations (Krutetskii, 1976).

The research question for this study was “What are the nature and role of queries that sustained spontaneous problem solving activity in this study?”

Research design

This section describes the context (schools and students), data sources, and excerpts selected, and the pedagogical approach employed (including the tasks, composition of groups, and the types of interactions intended to support spontaneous student thinking).

Context, data sources, excerpts selected

Two Grade 5 students were the focus of excerpts in this study: Tom [Excerpt 1] and Lenny [Excerpt 2, 3]. They attended either a Northern Suburbs Government Primary School, or a Southern Suburbs Catholic Primary School in Melbourne. The broader study from which this data was selected captured problem-solving activity in upper elementary school classes with three tasks undertaken each year over a two-year period. Six 80-minute sessions were undertaken each year with the researcher as teacher implementing the task, and the classroom teacher participating. Four cameras in the classroom captured the activity of each group of 3-4 students in the class as they worked with the task, and briefly reported group findings to the class every 10-15 minutes. Work generated by groups during these sessions was collected and used to support student discussion during interviews. Video-stimulated post-lesson student interviews were undertaken individually with four students after each lesson. Students were asked
questions including whether they learnt anything new, how they learnt it, and to find parts of the lesson that were important to them (including, if possible, anything that influenced their process of learning something new). Tom and Lenny were each interviewed after the lessons from which these excerpts were taken. Three excerpts of video data and associated student interviews were selected for microanalysis. Each excerpt included at least one query (self-query or external), and the student continued to control the pathways they explored, and the questions they focused upon after these queries, and simultaneously displayed high positive affect. Tom, in Excerpt 1, was working with the final task in the first year: “The Fours Task”. Lenny, in Excerpts 2 and 3, was working with the sixth task (at the end of the second year): “Marketing Through Blue Smarties”. Queries from different sources were the focus of each excerpt.

Engaged to Learn pedagogy

This approach was developed (see Williams, 2009) to provide opportunities for flow situations. The class undertook three to four cycles of group work followed by reports to the class. Questions asked by other students, the teacher, or the teacher-researcher were intended to be non-confrontational (no contradicting). Rather, they were expected to be requests for elaboration or explanation that were not focused beyond the content presented. For the purpose of sustaining spontaneous activity, the teacher-researcher and classroom teacher did not provide mathematical input or hints, or agree with or dispute pathways taken during these cycles. Instead, they tried to ask questions to elicit further thinking. Such questions are illustrated herein by the type of interviewer queries in Excerpt 3.

Group composition

Groups (3-4 students) were composed by the researcher-teacher informed by video data from group interactions during previous tasks, and teacher background knowledge. Students with similar paces of thinking (differs to student performance) were grouped together so they were more likely to develop new ideas at the same rate. Such composition was intended to reduce possibility of some members ‘falling out of flow’ because the challenge became too great, or not entering flow because the challenge was insufficient. A group member likely to buffer negative influences was included in each group where possible. I had developed these grouping strategies as a teacher before I knew about flow, and before I realised that the ‘positive group member’ was optimistic.

Tasks

Each task was accessible through a variety of representations and levels of mathematical sophistication, and included concrete materials to support student experimentation. The two tasks undertaken during the excerpts selected were:

The Fours task (Tom)

Make each of the whole numbers from one to twenty inclusive using four of the digit four and as many of the following operations and symbols as required:

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Develop strategies to generate these integers faster than other groups. Groups spent three minutes with individuals generating possibilities alone. Then they shared their
findings, and ideas, in their group. During reporting sessions, groups could focus on any of the following:

- Two numbers they had generated;
- Something they had found;
- Something that was not working that other groups might be able to help with;
- A ‘big picture idea’ that helped generate numbers faster; or
- Anything else they thought could be useful to other groups.

Reports later in the task tended to focus more on the later dot points.

*Marketing through Blue Smarties (Lenny)*

Design an advertising slogan by constructing a Blue Smartie Promise to attract lovers of blue Smarties to buy. Remember broken promises are not good for the company. Each group starts with a small-unopened box of Smarties (coloured candy), predicts the number of blue Smarties in their box (giving reasons for their predictions), opens the box, counts, and discusses their findings with their group. Groups then report to the class, and add a tally to the board (See Figure 1). Each class member then predicts, opens, and counts blue Smarties from a new box, and adds their data to Figure 1. Groups analyse the data and report on their analysis. Groups then try to develop a Blue Smartie Promise. The feasibility of keeping each promise is then discussed.

**Results and discussion**

**Excerpt 1: Queries from member of Tom’s Group**

Tom participated in the cycles of group work and reporting on ‘The Fours Task’. Alf, another group’s reporter explained how his group made 17. In doing so he expressed four divided by four as a mathematical object:

... we did four times four to get sixteen and we needed one more ... we had two extra fours ... then we did four times four plus ... four over four ... so it would be like saying four times four plus one ... four over four is one whole, so that is just like saying one, and four times four plus one you get seventeen\(^1\)

Tom excitedly realised he could apply this idea more generally: “when he [Alf] said four over four ... is the same as one just that sentence just flung me like quickly in my mind *ahhh I could use that*”. When group work recommenced Tom stated:

... we need ... a strategy to figure out every single one ... like what Alf and Ken’s group did because four over four... one could come in handy for everything that is a not multiple of four- so ... from sixteen you need one to get to seventeen ... umm- something minus four over four to get to fifteen\(^1\)

Tom undertook element-analysis in identifying the structure of Alf’s calculation (stem plus four over four). He could see he could vary the stem and use either plus, or minus, four over four for different integers. Tom elaborated his idea further to his group:

For four you can get three and five using four over four and for eight you can get six and seven and for twelve you can get eleven and 13 and for 16 you can get 15 and 17 and for 20 you can get 19.

\(^{1}\) Key to Transcripts: ‘...’ text omitted that does not alter meaning; ‘-’ changed direction to comment; ‘[text]’ explanatory text added by researcher; ‘/’ cut across another’s statement.
In doing so, he continued to focus on the part after the stem (plus or minus one) and did not elaborate on making stems. Gabrielle (another group member) requested further explanation and Tom focused again on the part after the stem. Eventually, Gabrielle took the pencil from Tom, shifted the paper towards him, and tapped on the sheet: “How how how?” Tom elaborated the end part again. Gabrielle did not give up though; she queried in a different way: “... so if somebody asked you to ... give answers to every single number ...?” This query did not draw attention to the stem, nor contradict Tom’s idea that the stem could be any multiple of four. In responding to Gabrielle’s query, Tom realised his idea was only partially correct: “... if you are going to do like 12- you can’t do 12- you won’t be able to do it because four plus four plus four is the only way to get 12”. Gabrielle’s queries captured Tom’s attention with her repeated emphasis on the word ‘why’ and her tapping. Her queries did not contain mathematical input, and did not draw specific attention to what was problematic. Yet Tom identified what he did not yet know [failure] as he responded, and spontaneously tried to find out more as a result.

Tom’s subsequent cognitive activity was simultaneously optimistic [synthetic-analysis; Failure as Temporary]. In simultaneously considering the structure and its usefulness for generating different integers, he perceived not knowing as temporary: “... so the only one that you can do it for is 16 ... oh unless you do eight- so four plus four- yeah so four eight- so four plus four minus four over four ...”. By continuing to vary parts of the calculation as he tried to see what was possible, and considering the outcome each time, Tom made decisions about whether he needed to explore further (evaluative-analysis: synthetic-analysis for the purpose of judgement). He simultaneously demonstrated that he perceived not knowing could be overcome by the personal effort of looking into the situation to see what could be changed to find out more [Failure as Specific, Success as Personal]. Thus, in response to Gabrielle’s queries, Tom’s initial ‘not fully correct construct’ became ‘more correct’.

Excerpt 2: Lenny focused his own query

During the “Marketing Through Blue Smarties Task”, students added tally marks to record how many blue Smarties were in their box of Smarties (See Figure 1). For example, the five tallies beside the top box in the second column of boxes represented five boxes containing six blue Smarties.

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Figure 1. Diagram on board: tallies of numbers of blue Smarties in boxes.
Lenny silently wondered why some groups had so few blue Smarties in their boxes: “I found that really really surprising … even the four (pause) because that is half (pause) what I thought it would be”. This was the start of Lenny’s thinking about chance in this situation. He recognized ‘average’ as relevant for examining this: “I was trying to think (pause) what the (pause) average was” but he was not sure how to find it. He explained one way he tried to find the average and why he did not think it was right:

I think I did it wrong but … I added it all up and I think it added to twenty four and then … I forget what we we – I – was supposed to do then so I just counted all the ones that had (pause) … the ah numbers next to them and then I think there was nine and then I divided it by nine.

Lenny could not recall the procedure for finding an average “I can’t remember how (pause) to do it properly” so tried a possibility using what he did remember (an add, and a divide). He knew the result was not reasonable (evaluative-analysis, identified ‘failure’): “I counted the fifteen as one”. When asked what he did, he elaborated: “I added all of them [the tally marks] up like (pause) so I added this one- three three eight” and considered the size of the answer was not reasonable: “It was two and a bit … so I did it wrong [confident voice]”. When asked how he knew it was wrong, Lenny showed some conceptual understanding of the term ‘average’: “I knew the (pause) it’s- there’s- if there was eight (pause) six and five each (pause) more of them are over five so how is it under [around?] two?” He also knew a low number of blue Smarties in a box would bring down the average: “And I knew probably it would be around five six because the one would bring it down a fair bit …”. Lenny’s evaluative-analysis undertaken from more than one perspective helped him identify his ‘failure’ to work out how to find an average. He continued to puzzle over this [Failure as Temporary]. Lenny’s personal effort expended on trying to work this out was reflected in his lack of awareness of group interactions around him: “Yeah I didn’t really put that much into our … promise because I was [soft laugh] trying to figure out the average”. Across the time of the interview, Lenny became more articulate in expressing why he considered he was not correct. It is unclear whether this happened because he had worked out ways to express himself more clearly, or because he had extended his thinking: “I just went one (pause) two and stuff but I didn’t count like (pause) … yeah I didn’t count all of them as 15 (pause) I just counted them as one each”.

In this excerpt, Lenny queried what he saw on the board based initially on the prediction he had made. As a result, he spontaneously posed a question to help them consider this further: “What is the average?” As he did not remember how to work this out, that became his focus. He used what he did remember to develop possible calculations (synthetic-analysis) and made judgements about the reasonableness of the answers he generated (evaluative-analysis). In doing so, he simultaneously displayed an optimistic enactment [Failure as Temporary, Failure as Specific, Success as Personal].

Excerpt 3: Interviewer queries support Lenny’s puzzling about table

Lenny knew there was something the matter with the procedure he was using to find the average, because he was counting each box as containing one Smartie. He had not found a way to overcome his problem though. This could have been because he did not understanding the table in Figure 1. Lenny continued to display intense interest when he had decided his answer was wrong. As interviewer, I was asking questions in a
'wondering' rather than confrontational way, and softly as though I was not expecting an answer: “so I wonder how many blue Smarties are there altogether?” Lenny responded with intensity and immediately began again to try to find this number (by counting tallies): 

Lenny: *I don't know* its two (pause) three (pause) oh eight (pause) mmm thirteen ...? [Failure Identified, exploration continued].
Interviewer: [Using language used previously by Lenny]: So are you counting the number of Smarties there, or are you counting the number of boxes?
Lenny: I am counting the number of (pause) how many lines.
Interviewer: [softly] I wonder what those lines stand for (pause) whether they stand for four?"
Lenny: [Cut off query with excited reply] /They stan- that st- that one stands for one and that one [one of the tallies beside the four] stands for (pause) four”.
Interviewer: [soft wondering] “Four what?”
Lenny: Four [long pause then confidence answer] blue Smarties.

Lenny was excited at the result of his synthetic-analysis. He realised he could use the numbers in the boxes together with the tallies to find the number of Smarties. He demonstrated failure was temporary and success personal as he began to interpret the table to answer the question he focused on (how many blue smarties?): “I’d have to count one ... add four [correcting himself to]- and then I would have to add eight which would be nine and then I’d have to add five fives ….”. The queries from myself as interviewer used language Lenny had introduced to encourage Lenny to elaborate his thinking. They were generally soft questions that were not necessarily intended for Lenny but could have just been me wondering about ideas that were developing. The more specific question about whether Lenny knew how to find the number of blue Smarties now, was based on what Lenny had been talking about in the interview. Lenny’s intense interest in this question was demonstrated by the emphasis in his response. He focused on this identified lack of knowing and continued to think about it. Lenny’s use of the term ‘lines’ and the interviewer’s attention to this in the subsequent query was almost immediately followed by Lenny’s excited realisation of how to interpret the table.

Conclusions

These excerpts illustrate queries from different sources (group member, self, expert other). Their role in each case was the same. They led to the student identifying a failure (not yet knowing) and intently continuing their spontaneous exploration. These queries did not contain mathematical input or hints or affirmations or contradictions, so did not eliminate spontaneity. They drew attention (in some way) to something that required further elaboration. In each case, evaluative-analysis occurred as a result as the student began to construct new understandings (mathematical structure, concept of average, meaning of table), and the activity was simultaneously optimistic. This study begins to make transparent some of the links between optimistic activity and learning gains:

- Exploring further when finding something is ‘not yet known’ [Failure as Temporary]
- Persevering by experimenting to find a way to find out [Success as Personal, Failure as Specific].

This research adds to the body of knowledge about increasing relational understanding through problem solving. It illustrates the nature of queries that sustained exploration.
Further study of problem solving activity in other contexts would be useful to see whether queries of the same nature perform the same role elsewhere, and to find other types of questions that achieve this. This study should be useful to teachers and teacher educators interested in developing questions that promote problem solving activity.

Acknowledgement

Thanks to the International Centre for Classroom Research (ICCR, University of Melbourne) for hosting my ARC fellowship DP0986955. Their technical support and collegiality is appreciated.

References


