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RELATIONSHIPS BETWEEN ELEMENTS OF COGNITIVE, SOCIAL, AND OPTIMISTIC MATHEMATICAL PROBLEM SOLVING ACTIVITY

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The mathematical problem solving activity of an elementary school student (Lenny) was studied to find relationships between cognitive, social and psychological activity. Lenny was selected because he changed from non-optimistic to optimistic orientation providing opportunity to contrast activities. Four cameras captured activity during six tasks over a two-year period. ‘Two up’ video images of group discussion, and reports to the class stimulated student reconstruction of lesson activity in individual post-lesson interviews. Data collection techniques enabled retrospective analyses of videos and interviews to study activity of students who changed in optimistic orientation. Findings illuminate the simultaneity of some cognitive, social, and optimistic activity raising awareness of the need for further study of optimism.

INTRODUCTION

Focus on deep learning of mathematical concepts rather than rules and procedures is emphasised by Sawyer (2008) as valued by the “knowledge economy” (p. 48):

... deep conceptual understanding of complex concepts, and the ability to work with them creatively to generate new ideas, new theories, new products, and new knowledge [is needed]. ... [people] need to learn integrated and useable knowledge, rather than the sets of compartmentalised and decontextualised facts” (Sawyer, 2008, p. 49).

Mathematics education research has found many aspects of problem solving activity associated with deep learning including: ‘cognitive’ and ‘social’ elements of the process of abstracting new knowledge (e.g., Dreyfus, Hershkowitz, & Schwarz, 2001), subcategories of these processes during ‘spontaneous abstracting’ (Williams, 2007a), and high positive affect associated with such processes (e.g., Liljedahl, 2002; Williams, 2010). This study extends research into links between cognitive, social, and psychological activity during problem solving. Constructs are elaborated later.

THEORETICAL FRAMEWORK

‘Problem solving’, for the purposes of this study, is activity of groups and / or individuals working on unfamiliar problems (problems for which they do not know a rule or procedure). The creative development of new knowledge (spontaneous abstracting, Williams, 2007a) involves high positive affect during ‘flow’ situations (Csikszentmihalyi, 1992). Conditions for flow during mathematical problem solving (Williams, 2010) include a spontaneous self-set or group-set challenge that can be overcome by developing new skills and conceptual knowledge. In other words, the
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student’s are not solving routine problems but rather finding ways to answer a question they have posed for themselves. To do so, they need to ‘move into unknown territory’ by working with mathematics in unfamiliar ways. During flow, students lose all sense of time, self, and the world around as all their energy is focused on the task at hand. Epstein, Schorr, Goldin, and colleagues (2007) identified interactions during group work that they linked to students’ ways of ‘surviving’ in an urban culture. These types of interactions: “Don’t Disrespect Me” (p. 651) and “Stay Out of Trouble” (p. 653) inhibited student engagement with mathematical ideas because the focus was on ‘not losing face’ rather than on mathematics as the authority in the task at hand. In other words, students’ sense of self inhibited their problem solving activity. Had these students entered a flow situation, sense of self would not have been operating. This may be what happened during the other type of behaviour identified by Epstein and colleagues: “Check This Out” (p. 653) where the interactions resulted from interest and curiosity and students were inclined to try to work out something they did not know. This happened in the case in this study.

Seligman’s (1995) construct ‘optimism’ is employed as an analysis tool to study psychological aspects of problem solving in this study because students who creatively develop mathematical ideas have been found to be optimistic and enact optimism during problem solving (Williams, 2010). Optimistic people perceive successes (S) as permanent, pervasive, and personal (SPal), and failures (F) as temporary (FTemp), specific (FSpec), and external (Seligman, 1995). Success during problem solving, for this study, is ‘knowing’ (finding something out), and failure is ‘not knowing’. Solving unfamiliar problems requires optimism because not knowing (failure) is perceived as temporary and able to be overcome (success) through personal effort by looking in to the situation and working out what can be changed (failure as specific) / cannot be changed (as failure externally controlled).

The construct of ‘cognitive elements of the process of spontaneous abstracting’ involves abstracting without mathematical input of an expert other during the interval of spontaneous abstracting. The RBC Model, Recognizing, Building-with, and Constructing (Dreyfus, Hershkowitz, and Schwarz, 2001) (DHS), and Krutetskii’s (1978) ‘mental activities’ were used in combination to develop the thinking framework employed (Williams, 2007a). Cognitive elements of problem solving processes are made visible through discussion. In this case, Lenny’s group work in class was studied for the first task, and his discussion in the post lesson interview for the second task. The thinking framework includes the following elements. 1) Recognizing (R) which includes realizing certain mathematical constructs were applicable to the task at hand (DHS, 2001). 2) Building-with (B) (DHS, 2001) previously known rules and procedures in familiar and unfamiliar ways has been subcategorised into (a) Simple analysis (Bsa): identifying and using rules and procedures in routine ways possibly in new sequences and or combinations; (b) Element-analysis (Ba): isolating parts of something unfamiliar and examining them one by one; (c) Synthetic-analysis (Bs-a): simultaneously examining and building with several
elements; and (d) Evaluative-analysis (Be-a): synthetic-analysis for purposes of judgment. 3) Constructing (C) (DHS. 2001): the process of developing insight or seeing something mathematically profound that the student was not previously aware of has been subcategorised into (a) Synthesis (Csyn): insight illuminating mathematics not previously known which could occur through identification of generality; and (b) Evaluation (Cev): reflection on mathematics progressively developed and the results obtained for the purpose of checking internal and external consistency and considering the usefulness of the insight for other purposes.

Social elements of the process of abstraction (DHS. 2001) are ‘control’ (Co), ‘elaboration’ (El), ‘explanation’ (Ex), ‘query’ (Qu), ‘agreement’ (Ag), and ‘attention’ (At). During spontaneous abstracting, these elements are generally internal to those taking part in the abstracting process. If the source is external and spontaneous abstracting continues, mathematical input into the abstracting process was not given (see Williams, 2004). This sometimes happens with external queries and attention. Meanings of social elements are elaborated in interpreting the data.

Research Question: What empirical evidence is there of links between cognitive and social elements of the process of spontaneous abstracting and optimistic activity?

RESEARCH DESIGN

This data was drawn from a broader study on the role of optimism in collaborative problem solving. Lenny was a Grade 4 (2009), Grade 5 (2010) student in an Australian elementary school who was selected as a case for this study because he reported changing from not perseverant to perseverant in his problem solving over the two-year period. In his interview after Task 3, 2010 Lenny reflected on changes to his problem solving activity over time: “Instead of just going (pause) ‘I don’t know’, I (pause) [now] sit there and I really really (pause) think about it- even if it is [only] a sum or an angle [learned procedure] …”. When asked how that happened, Lenny said: “I don’t really know … it just made me think more (pause) It was probably actually doing the [research] tasks …”. Lenny’s contrasting activities were expected to help elaborate links between optimistic, cognitive, and social problem solving activity. Lenny performed at average level in mathematics classes.

Classroom video and video stimulated post-lesson student interviews were employed to identify links between cognitive and social elements of problem solving activity, and the enactment of optimism. Four videos were used to capture these three types of activity in class, and individual video-stimulated student interviews were undertaken after each lesson to identify where such activity occurred, and gain further information about it. Students were questioned about what they had learnt, and asked to find the parts of the lesson that were important to them and their learning. The video images they viewed were a ‘two up’ showing the activity of their group, and the group reports to the class. Students were also asked additional questions about how they thought they were going in maths, whether they enjoyed maths, and to describe a good lesson for them. These questions elicited indicators of optimism or
lack thereof. The data collection methods allowed the retrospective analysis of the behaviour of any student identified (like Lenny) to have changed in their optimistic orientation.

The researcher (author) was the primary implementer of the problem solving activities and the interviewer. The classroom teacher participated as the class worked in groups with the tasks, and groups reported at intervals to the class as a whole. For more detailed descriptions of the pedagogy employed see Williams, (2007b). Three tasks were undertaken each year (Task 1, 3 x 80 Min. Task 2, 2 x 80 Min. and Task 3, 1 x 80 Min. session). T1, 2009 and T3, 2010 are briefly described:

Task 1, 2009: Using all 14 tiles (each time), make as many different flat ‘filled’ rectangles as you can (using top surface of tiles). Repeat using 12 tiles. Have you found all possibilities? Make an argument that justifies that you have them all. Do more tiles make more rectangles? Why or why not? Select a number of tiles between 16 and 45 to make as many rectangles as possible. Explain your thought process.

Task 3, 2010: Design an advertising slogan by constructing a Blue Smartie Promise to attract lovers of blue Smarties to buy. Remember broken promises are not good for the company. Each group starts with a small-unopened box of Smarties (coloured candy), predicts the number of blue Smarties in their box giving reasons for their predictions, opens the box, counts, and discuss their findings. This is followed by reports to the class with the reporter adding a tally to the board (See Figure 1). The procedure is repeated with each student opening a box after predicting. Each student adds their data to Figure 1. Groups analyse the data, then report to the class on their Blue Smartie Promise. The feasibility of keeping each promise is discussed.

ANALYSIS AND RESULTS

Lenny’s activity during Task 1 2009 fitted his descriptions. Where previous knowledge could be easily be applied, he participated. When his group worked with the 12-tile case, Lenny quickly realised the two remainder (reported by another group in 14-tile case) could be removed giving a three by four rectangle. He took time to make sure the group report included this. As his group did not use factors in the 12, 14, or 16-tile parts (even though some other group reports mentioned factors) they did not have an easy procedure to apply to find solutions for the final task part. Lenny did not ‘step’ into this unfamiliar territory to try to work this out. Instead, he spent considerable time off task—sometimes distracting others by poking them playfully.

During Task 3 2010, the following diagram (Figure 1) was generated on the board by students adding tally marks to record their findings. For example, if a box had 6 blue Smarties, a tally mark was added beside the box with 6 in it [top of Box Column 2]. The five tally marks beside this box show five boxes with six blue Smarties in each.
Figure 1: Diagram on board: tallies of numbers of blue Smarties in boxes.

Lenny considered the diagram as he puzzled about how to answer his spontaneously posed question. He continued to puzzle about this after the group had begun writing their promise thus displaying his interest: “Yeah I didn’t really put that much into our promise because I was [soft laugh] trying to figure out the average”.

The following excerpt of Lenny’s interview shows what led to his spontaneous challenge and how he began to overcome it. It includes interpretations of lines of transcript (cognitive, social, and optimistic activity), and short notes to support understanding. The transcript is to be considered in conjunction with Figure 1.

Transcript Key: ‘T’: interviewer, ‘L’ Lenny, ‘(pause)’ pause; ‘/’ cut across statement of another; ‘*’ emphasis; ‘[text]’ researcher comments, ‘...’ part of transcript line not essential to meaning omitted; ‘—’ transcript lines not essential to meaning omitted. Interviewer’s soft word or two to encourage further discussion, omitted without the inclusion of ‘—’ is evident from line numbering.

Cognitive, social, and optimistic activity symbols used were defined in the theoretical framework.

1. L … [a] group had (pause) one (pause) … I found that really really surprising (pause) … even the four (pause) because that is half (pause) what I thought it would be (pause) ——

2. I And why do you think that happens? [External Qo]

3. L It is probably (pause) when they were putting them in the boxes it is just (pause) random [R, possibly B, commencing C, as considers meaning of his term]

4. I … Had you thought that out at that time? Or (pause)? [Qo]

5. L [intense] Yeah I was trying to think (pause) what the (pause) average was [R; FTemp, SPal, FSpec, inclined to shift into ‘unknown territory’]

6. L And I think I did it wrong but–I added all of them up [Bsa] so I counted the fifteen as one [Be-a, At has identified a problem, FSpec looks into situation]

7. L I added it all up and I think it added to twenty four and then … I forget what we we– I was supposed to do then so I just counted all the ones that had (pause) … the ah numbers next to them and then I think there was mine
and then I divided it by nine and then it was like [Bsa, partially correct: need to divide totals; total smarties/boxes found incorrectly. FSpec] —

12. I Oh it was two (pause) was it? [surprise] [Qu]
13. L Yeah (pause) two and a bit [Ag, El]
14. I Oh did you add the numbers? [Qu]
15. L Yeah I added all of them up like (pause) so I added this one three 555- eight [no. of tallies not no. of Smarties represented by tallies] [Bsa, El, FSpec]
17. L So I did it wrong [confident voice no Qu] [El, Perceived ‘not knowing’ (F)]
18. I And so when you are trying to average it you were trying to work out how many you are sharing amongst?
19. L/No! h- what the average of blue was in each packet (pause) so like in between twenty no! (pause) four- five packets (pause) yeah [Ex] —
22. I … do you … know something was the matter with what you did do you? [Qu]
23. L Yeah but I can’t remember how (pause) to do it properly [Trying to recall rules and procedures for finding an average rather than making meaning]
24. L How did you know there was something the matter with what you did? (Qu)
25. L Because I knew the (pause) it’s there’s if there was eight (pause) six and five each (pause) more of them are over five so how is it under two? [R (average partially correct) Be-a partially correct; perceived S]
27. L And I knew probably it would be around five six because the one would bring it down a fair bit … [R (average partially correct) Be-a; perceived S]
35. L … But I-I like di- I just went one (pause) two and stuff but I didn’t count like (pause) … yeah I didn’t count all of them as 15 (pause) I just counted them as one each [B-e-a, Qu, At, El (Line 15). FSpec, identified F]
36. L … so I wonder how many blue Smarties are there altogether? [External Qu]
38. L [soft exclamation] I don’t know- its two (pause) three (pause) oh eight (pause) mm thirteen (pause) what’s that- is that two or three (pause) on the sseven — [Bsa still counting tallies only (partially correct); El; FSpec]
41. I So are you counting the number of Smarties there or are you counting the number of boxes? [External Qu, External At]
42. L I am counting the number of (pause) how many times [Bsa; El shows not B-s-a yet]
43. I … I wonder what those lines stand for (pause) whether they stand for? [Qu]
44. L … [excited] /They stand- that st- that one stands for one [one tally beside 1-box] and that one stands for (pause) four [two tallies beside 4-box] [Bsa commenced, probably Csym commenced; El; FSpec; S perceived (beginning to realise need tallies and number in box to count no. blue Smarties so constructing an understanding of nature of frequency tables]
45. I Four what [soft questioning/wondering voice]? [External Q requesting further El]
46. L Four (pause) blue Smarties [confident] [El, perceived S]
47. I Okay (pause) so to find how many blue smarties (pause)? [Qu eliciting El]
48. L I’d have to count one … add four [then corrected] … add eight which would be nine and then I’d have to add five fives … [Ex, B-s-a, S perceived] —
DISCUSSION AND CONCLUSIONS

Lenny changed from ‘giving up’ (2009) to looking to see what he could find out (developed Failure as Specific) which was integral to achieving successes within his problem solving as he persevered (Failure as Temporary, Success as Personal). He displayed this behaviour when class findings surprised him [see Line 1 (L1)] and he spontaneously asked “What's the average?” recognizing the relevance of this construct (R, At, FSpec) [L5]. He first tried just adding the tally marks to find the number of blue Smarties [L6], and counting the number of boxes with tallies beside them to find the number of boxes and dividing these [L8] (Bsa, FSpec). He considered whether his answer was reasonable [Line 25, 27] displaying some understanding of average as he ‘justified’ that it was not (Be-a, El, FSpec). He considered that he did not yet know—F, so looked further to see if he could identify what was the matter, and realised he had counted (for example) 15 Smarties as one [L6, 35] (Be-a, Qu, FSpec). He had realised what needed to change to find the number of Smarties (At) but not how to change it [L38, 42]. The interviewer's queries [L36, 43] led to his realising how to simultaneously consider two ‘columns’ of Figure 1 (boxes and tallies). He demonstrated this [L44, 48] (Bsa-a, At, El, FSpec).

Lenny’s activity showed the evaluative-analysis subcategory of the cognitive element building-with, and social elements query and attention, occurred simultaneously with the enactment of the optimistic activity Failure as Specific. The ability to identify failures and successes through enactment of these activities guided the problem solving. Perseverance (perceiving Failure as Temporary, Success as Personal) was not sufficient to achieve problem-solving successes. Perception of Failure as Specific was also required to identify failures to be reconsidered, find what could change, make changes, consider the results, and recognise when success was achieved. This study highlights optimism as a productive area for study to further enhance student problem solving. Studying more cases should elaborate and extend identified links and inform the process of partially correct constructs becoming more correct.

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