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Analytic Study on Pure Bending of Metal Sheets

Jeerachai Supasuthakula, Peter D. Hodgsona, and Chunhui Yangb

aCentre for Material and Fibre Innovation, Deakin University, Waurn Ponds, VIC 3217, Australia
bSchool of Engineering, Deakin University, Waurn Ponds, VIC 3217, Australia

Abstract. In this work, analytical models of pure bending are developed to simulate a particular type of bend test and to determine possible errors arising from approximations used in analyzing experimental data. Analytical models proposed for steels include a theoretical solution of pure bending and a series of finite element models, based on the von Mises yield function, are subjected to different stress and strain conditions. The results show that for steel sheets the difference between measured and calculated results of the moment-curvature behaviour is small and the numerical results from the finite element models indicate that experimental results obtained from the test are acceptable in the range of the pure bending operation. Further for magnesium alloys, which exhibit unsymmetrical yielding, the algorithm of the yield function with a linear isotropic hardening model is implemented by programming a user subroutine in Abaqus for bending simulations of magnesium. The simulations using the proposed user subroutine extract better results than those using the von Mises yield function.

Keywords: Pure bending, metal sheets, theoretical solution, finite element modelling, and yield function.
PACS: 46.15.-x

INTRODUCTION

Sheet-metal bending is a widely-used sheet forming operation. Although bending is technically a simple operation, it causes a complex stress history and material flow in a sheet. For isotropic materials Ludwik [1] firstly established a theory of the plastic bending of a non-strain-hardening sheet subjected to only bending moments. Hill [2] extended Ludwik’s work by defining a plane-strain condition for the case where the radius of curvature of a bent sheet is large compared to the sheet thickness and the through-thickness stress is negligible. He then considered the movement of the neutral surface for small radius bends. Marciniak et al [3] suggested that the large radius bend approximations are suitable for a bent sheet having a curvature greater than five times the curvature in elastic bending. Lange [4] proposed a theoretical model under elastic-perfectly plastic deformation to determine the total bending moment. In reality, sheet metals, generally manufactured by cold rolling, have anisotropic mechanical properties varying with direction. Tan et al [5] presented a model of the plastic, plane-strain bending of anisotropic sheet metals incorporating the Bauschinger effect to consider such effects on material thinning. A model of the elastic-plastic bending of anisotropic sheets was introduced by Chakrabarty et al [6] to determine a moment-curvature relation. They accounted for the elastic-plastic bending, and showed a relation of the bending moment to the curvature. However, this model did not consider deformation of fibers away from the middle surface as Marciniak et al did where they considered the movement of the neutral surface.

For magnesium alloy sheets which exhibit asymmetrical yielding, an analytical model of bending with tension was proposed by Lee et al [7], based on the classical beam theory, linear strain hardening, and observations from experimental stress-strain tests. For numerical solutions of sheet metal forming of magnesium alloys, the finite element method has been applied widely. Palumbo et al [8] developed a fully coupled thermo-mechanical finite element model for the stretch-bending operation and studied the bend radius and temperatures. Kim et al [9] proposed a hardening law based on the orthotropic yield criterion of Cazacu et al [10] for shell-element models on three-point bending of magnesium alloy sheets. Lee et al [11] proposed an asymmetric hardening model for magnesium alloys using the modified anisotropic Drucker-Prager yield criterion. Lee et al [12] extended their own
work to prediction of springback of magnesium alloy AZ31B using shell-element models of the draw bend test. However, when the thickness stress plays an important role on forming operations, solid-element models using such yield function should be considered. When considering the yield criteria for magnesium alloys, the yield function of Cazacu and Barlat [13] is attractive because the yield function is derived from the test and includes the third stress invariant $J_3'$ describing the difference between compressive and tensile stresses.

In the current work, considering the elastic-plastic bending and the deformation of fibers through the thickness, a theoretical bending model for isotropic materials, such as stainless steels, is firstly developed. We deal with two loading conditions: plane strain and uniaxial tension, and study which condition is more suitable for the modelling of pure bending. Secondly, a series of finite element models for stainless steels are developed, and the bending condition of the apparatus is studied to see whether or not these two conditions in the test rig can be approximated to pure bending. Thirdly, an algorithm for the yield function of Cazacu and Barlat is implemented in the user subroutine UMAT for Abaqus [14, 15], and it is further employed in the finite element model to simulate forming operations of magnesium alloy sheets without heat involved. Lastly, the results from both the analytical modelling and experimental work are compared and verified.

THEORETICAL SOLUTION OF PURE BENDING OF A SHEET OF ISOTROPIC MATERIAL

For a thin sheet which is bent by end couples, the maximum bend radius in the sheet is not less than three or four times the sheet thickness. According to the classic bending theory, a plane normal section in the sheet will remain plane and normal and converge on the centre of curvature [3]. In addition, the material is assumed as homogeneous and isotropic, and the sheet thickness is assumed to be constant throughout the process. In this study, let the 1-, 2- and 3- directions be the circumferential, width and radial directions, respectively.

It is assumed that the neutral surface is located at the mid-surface of the sheet and the axial strain of a fiber is,

$$\varepsilon_1 = \varepsilon_b \approx y / R,$$

where $\varepsilon_b$ is the bending strain, $y$ is the distance measured from the middle surface, and $R$ is the radius of curvature at the middle surface.

In elastic bending, the relation of axial stress to axial strain is

$$\sigma_1 = E' \varepsilon_1,$$

where $E'$ is the modulus of elasticity of bending in the circumferential direction.

Hooke’s law under uniaxial-tension condition ($\sigma_1; \sigma_2 = 0; \sigma_3 = 0$) indicates that

$$E' = E,$$

In the plastic bending, assuming uniaxial deformation, the process is

$$\varepsilon_1^p; \varepsilon_2^p = -(1/2) \varepsilon_1^p; \varepsilon_3^p = -(1/2) \varepsilon_1^p.$$

Under such loading condition, the von Mises yield function is

$$f = \sigma_{ef} - \sigma_j(\varepsilon_{ef}) = \sigma_1 - \sigma_j(\varepsilon_{ef}),$$

and the effective strain derived from Drucker’s postulate is

$$\varepsilon_{ef} = \varepsilon_1^p.$$

For a cold-worked material with a work-hardening behaviour, the following equation can be used to the data:

542
\[ \sigma_y = K_s (\varepsilon_0 + \varepsilon_e)^n, \]  
\hspace{1cm} \text{(7)}

where \( K_s \) is the strength coefficient, \( \varepsilon_0 \) is the pre-strain constant and \( n \) is the strain hardening index.

The axial stress can thus be expressed as follows:

\[ \sigma_1 = K_s (\varepsilon_0 + \varepsilon_p)^n. \]  
\hspace{1cm} \text{(8)}

In the case of elastic-plastic bending, for unit width, the equilibrium condition becomes

\[ M = 2 \left( \int_0^{\gamma_y} E (y / R) y \, dy + \int_{y_e}^{b/2} K_s \left[ \varepsilon_0 + \left( y / R \right) - \left( \sigma_y / E \right) \right] y \, dy \right), \]  
\hspace{1cm} \text{(9)}

where \( y_e \) is the distance to the interface of elastic and plastic zones, \( y_e = R \sigma_y / E \).

Considering the pure bending of a sheet under plane-strain condition \((\sigma_1; \sigma_2 = \sigma_1 / 2; \sigma_3 = 0)\), Hooke’s law indicates that

\[ E' = E / \left( 1 - \nu^2 \right). \]  
\hspace{1cm} \text{(10)}

In the plastic bending under the plane-strain condition, the deformation process is

\[ \varepsilon_p^1; \varepsilon_p^2 = 0; \varepsilon_p^3 = -\varepsilon_1. \]  
\hspace{1cm} \text{(11)}

Under this loading condition, the von Mises yield function is

\[ f = \sigma_{ef} - \sigma_y = \left( 3^{1/2} / 2 \right) \sigma_1 - \sigma_y, \]  
\hspace{1cm} \text{(12)}

and the effective strain derived from Drucker’s postulate is

\[ \varepsilon_{ef} = \left( 2 / 3^{1/2} \right) \varepsilon_1^p. \]  
\hspace{1cm} \text{(13)}

The axial stress is therefore

\[ \sigma_1 = \left( 2 / 3^{1/2} \right) K_s \left[ \varepsilon_0 + \left( 2 / 3^{1/2} \right) (\varepsilon_1 - \varepsilon_e) \right]^n, \]  
\hspace{1cm} \text{(14)}

In the case of elastic-plastic bending, the equilibrium condition is

\[ M = 2 \left( \int_0^{\gamma_y} E' (y / R) y \, dy + \int_{y_e}^{b/2} \left( 2 / \sqrt{3} \right) K_s \left[ \varepsilon_0 + \left( 2 / \sqrt{3} \right) \left( y / R \right) - \left( 2 / \sqrt{3} \right) \left( \sigma_y / E' \right) \right] y \, dy \right), \]  
\hspace{1cm} \text{(15)}

where the distance \( y_e = \left( 2 / 3^{1/2} \right) \sigma_y R / E' \).

**FINITE ELEMENT MODELLING ON PURE BENDING OF METAL SHEETS**

In the study, two-dimensional finite element models were developed by using the commercial finite element analysis package–Abaqus. The bend equipment prototype for pure bending designed by Weiss et al [16] was used to generate the experimental data as shown in Fig. 1. The bend test can be performed in a standard tensile testing
machine to produce appropriate results of nearly pure bending. Bending and reverse bending can be performed when the crosshead of the tensile testing machine is displaced either upwards or downwards.

![Diagram](image)

**FIGURE 1.** (a) Bending apparatus and (b) finite element modelling with translation of the ram [7]

In the 2-D finite element models for both isotropic and anisotropic materials, since the bending operation is symmetric about a plane along the centreline of the sheet, only half of the sheet was modelled, as shown in Fig 1 (b). Two finite element models using two types of solid elements: plane-strain and plane-stress elements, to model the sheet are devised, respectively. The clamp and ram are modelled as discrete rigid parts whilst the surfaces of the clamp that contact the top and bottom surfaces of the strip are modelled as discretized rigid surfaces. As for interactions between two parts, the regions in which the surfaces of the sheet and clamp contact each other are fused using tie constraints, and a connection between two points of the clamp and ram is modelled using a joint type connector.

In the bend tests of isotropic materials, a commercial stainless steel sheet of 200mm length, 19.84mm width and 5mm thickness was used. Its elastic properties are $\nu$ of 0.3 and $E$ of 190 GPa, and its plastic data is obtained from fitting Eq. (7) with the true stress-strain curve, so $K_s = 690$ MPa, $\sigma_0 = 0.0009$, and $n = 0.095$. The von Mises yield function was employed.

In the bend tests of anisotropic materials, Magnesium alloy strip (AZ31B) of 150mm length, 18mm width and 2mm thickness was used in the bending experiment. Its elastic properties are defined by a Poisson ratio of 0.35 and Young’s modulus of 45 GPa. Fitting the stress-strain equation to the true stress-strain curve derived from a tensile test gave the following parameters: $K_s = 450 \times 10^6$ Pa, $\sigma_0 = 0.002$, and $n = 0.14$. The yield function of Cazacu and Barlat that describes the asymmetry in yielding was then used for the magnesium alloy:

$$f = \left( J_{2}^{3/2} - c J_{3}^{'} ight)^{1/3} - \tau_y,$$

where $J_{2}^{'}$ is the second stress invariant, $J_{3}^{'}$ is the third stress invariant, $c$ is the material parameter of unsymmetrical yielding and $\tau_y$ is the yield stress in pure shear.

In addition to the yield function of Cazacu and Barlat, Hooke’s law and Drucker’s postulate were applied. The implicit integration technique, called the radial return method [14], was utilized and improved to solve the constitutive equations. For plastic data in the user subroutine UMAT [15], since linear hardening is defined, only parameter $H$ is specified; a suitable value of $H$ is 2200 MPa. Other parameters that need to be specified in the user subroutine are the initial tensile stress and initial compressive stress; the initial tensile stress of AZ31B is 210 MPa and the initial compressive stress is 120 MPa.
RESULTS AND DISCUSSION

The proposed theoretical solution of the bending of isotropic materials based on the von Mises yield criterion leads to a relation of the bending moment to curvature. Using the proposed solutions, the values of bending moment and curvature were predicted. Finite element modelling of the bending operation was also devised to study two loading modes: plane strain and plane stress, and two bending approaches – by applying a rotation to a reference point of the clamp and applying a translation to the ram making the clamp rotate. For the validation of the theoretical and simulation models, experimental work was also employed.

In Fig. 2, the theoretical solution under plane-strain yields higher results than the experimental data, but under uniaxial-tension conditions the curve is lower.

When considering the stress state of both solutions, the intermediate principal stress $\sigma_2$ is likely to have an important effect. Under uniaxial tension $\sigma_2$ is zero and the bending moment is significantly underestimated. The solution under plane strain provides results closer to the experimental data. In the experiments the width to thickness ratio of the test-piece is 9; the greater it is, the more closely the bend process would be expected to be plane strain.

![Graph of Moment-curvature from theoretical solutions and experimental data](image)

FIGURE 2. Moment-curvature diagram from theoretical solutions and experimental data

To obtain better simulation results other conditions may be considered. For two-dimensional problems without regard to heat transfer, Abaqus offers two major kinds of solid elements – plane-strain elements and plane-stress elements – which characterize the condition of deformation for the process. When the plane-strain condition is defined, the simulation probably overestimates results; hence, the plane-stress condition may be more appropriate for this problem.

Fig. 3 shows a difference between simulation results under the plane-strain and plane-stress conditions. Besides selecting proper elements for a sheet, the characteristics of loading in the bending operation are examined. Two simulations whose loadings are different: the one with applying a translation at the ram, similar to the experiment, as shown in Fig. 1 (b), and the other with applying a rotation at the clamp at its reference point (called pure-bending model), produce close results.
To investigate the magnitude of errors of the experimental data, simulations are performed at further displacement of crosshead. In Fig. 4, bending moments obtained from the simulation having translation of the clamp begin to drop at a curvature of about 15.708 /m or at a displacement of the crosshead of 0.095 m. This indicates that for the experiments in this work, where the curvature did not exceed about 5 /m, either mode of analysis will give similar results.

The user subroutine was firstly examined by reducing the form of the yield function of Cazacu and Barlat into the form of the von Mises yield function by assuming the tensile and compressive strengths are equal. Results obtained from the user subroutine were compared with those from the model following the definition of linear kinematic hardening available in Abaqus and from the model containing pairs of plastic data of stresses and strains.
Fig. 5 shows results of the relationship between moment and curvature from all the three models. In Fig. 5, under the plane strain condition, the simulation results are higher than the experimental result, and the model using the linear kinematic hardening in Abaqus predicted the same result as the model using linear isotropic hardening in the proposed user subroutine. This verifies the user subroutine is reliable.

In the bending of AZ31B magnesium alloy sheet, a variation in the material constant $c$ for the yield function of Cazacu and Barlat is studied, as shown in Fig. 6. When the initial compressive stress is less than the initial tensile stress, the moment-curvature result is lower and closer to the experimental result. This shows that the yield function of Cazacu and Barlat improves the simulation results.
CONCLUSIONS

In this paper, a theoretical solution and a series of finite element models for the pure bending of metal sheets have been developed for steels and magnesium. For steels, the theoretical solution of pure bending under the plane-strain condition gives better agreement to the experiment than that under the uniaxial-tension condition but overestimates the moment by an appreciable amount. A finite element model of the process using a plane-stress model gives a better agreement with experimental results. It reveals that the numerical model developed by using the plane-stress condition and the von Mises yield criterion is apparently an optimal choice for bending simulations of stainless steel sheet. For the finite element modelling of magnesium alloys, the user subroutine UMAT for Abaqus/Standard has been developed, verified and implemented. When the material constant c for the asymmetry in yielding is incorporated, the simulations using the yield function of Cazacu and Barlat provides a better agreement to the experimental data than the simulations using the von Mises yield function approximately. It therefore shows the differential strength effect on magnesium alloys.

In the experimental work, the mechanism of the bending apparatus is considered to ensure that the apparatus produces proper results of pure bending. The simulation under the translation of the crosshead, representing the experiment, predicts results that are different to those from the simulation under the rotation at the end of strip, representing the pure bending. It shows an increase of the error in the bending progression of experiment. The error can be neglected at an admissible value to ensure the experiment operates as pure bending.

ACKNOWLEDGEMENTS

Authors would like to acknowledge the financial support from the ARC Centre of Excellence for the Design of Light Metals and the Australian Laureate Fellowship for Peter D. Hodgson and the kind assistance of Dr Matthias Weiss in the bending experiments and analysis. The valuable discussions with Prof John L. Duncan are also appreciated.

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548