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PLENARY ADDRESS 3

Children’s Informal Reasoning: Concerns and Contradictions
CHILDREN’S INFORMAL REASONING: CONCERNS AND CONTRADICTIONS

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In this paper an attempt is made to answer two of Steen’s questions about students’ mathematical reasoning. Definitions of informal reasoning are provided, and from these bases, some examples of informal reasoning in action are given. These examples come from both mathematics and science, and from a range of prior-to-school and school settings. It is argued that these examples support both an affirmative and a negative response to these questions respectively.

Introduction

In this paper we look at children’s informal reasoning, and therefore this will not concern ourselves with the more formal aspects that lead to formal ideas of logic and proof. Thus, the learning and practice of logic and proof, although interesting, will be left to other, more erudite, authors.

In terms of reasoning per se, it has been said that ‘[R]easoning is an ancient subject but an everyday practice' and that '[W]e are all able to reason.' (Munson, Conway, & Black, 2004 p. 1). Scriven suggested, somewhat presciently, that the importance of reasoning lies in the fact that '[R]easoning is the only ability that makes it possible for humans to rule the earth and to ruin it' (1976, p. 2), and at a practical level, that ‘reasoning helps you to work out correct answers for yourself' (p. 3).

In the present case, therefore, it is not the ‘correct answer’, to the likes of the following problem, which concerns us:

1. Babies are illogical.
2. Nobody is despised who can manage a crocodile.
3. Illogical persons are despised.

(Carroll L. (Charles Lutwidge Dodgson), 1939, p. 1119)

(The solution to this problem is at the end of the references).

This paper will explore how informal reasoning is exhibited by children, and draws on both research and the author’s own experiences, particularly in mathematics. While considering a range of sources, the main intention is to re-visit two of Steen’s (1999) twenty questions about mathematical reasoning in general: Question 19, Is mathematical reasoning innate? and, Question 20, Is school too late?

Doig

Why reasoning?
Historically, Plato's notion of knowledge, as justified belief, endorses the primacy of reasoning in the acquisition of knowledge in general. More recently, the importance of reasoning in mathematics was highlighted in *Adding it up: Helping Children Learn Mathematics*, the report to the United States National Academies (Kilpatrick, Swafford, & Findell, 2001). This report stated that ‘In a recent study comparing schools participating in state initiatives in mathematics and science with schools not involved in such initiatives, elementary school teachers in the initiatives schools spent significantly more time than their counterparts on reasoning and problem-solving activities’ (p. 45).

Consequently, Kilpatrick and his colleagues placed reasoning as one of their five strands of mathematical proficiency, defining *adaptive reasoning* as the capacity for logical thought, reflection, explanation, and justification (p, 5). This, in turn, influenced the authors of the proposed Australian Curriculum, in mathematics, to define four proficiencies: *Understanding, Fluency, Problem solving, and Reasoning*. Of these four proficiencies, *Reasoning* is described as ‘the capacity for logical thought and actions, such as analysing, proving, evaluating, explaining, inferring, justifying, and generalising’ (p.6). Further, it is argued that ‘[E]ngaging students in reasoning and thinking about solutions to problems, and the strategies needed to find these solutions, are core parts of the mathematics curriculum’ (p. 9). However, at the prior-to-school and primary school levels of mathematical education, *explanation* appears to be the major aspect of this proficiency, leaving the remaining aspects to the later years of school.

The Victorian state Early Years Learning and Development Framework (VEYLDF) in the prior-to-school years (Department of Education and Early Childhood Development, 2009), argues for strengthening learning opportunities for young children, particularly in terms of explaining and reasoning, by emphasizing that '[W]ith support, children expect to learn and … begin to develop simple explanations for observed phenomena' (p. 25).

This resonates with Piaget’s ‘[E]xperience fashions reason, and reason fashions experience. Thus between the real and the rational there is a mutual dependence joined to a relative independence, and the problem is a singularly arduous one to know how much of the growth and elaboration of knowledge is due to the pressure of external things, and how much to the exigencies of the mind’ (1930, p. 301), This relative independence was, perhaps, more succinctly put by Eliot (1925) as ‘Between the idea and the reality … falls the shadow’.

Mason (1998) argues from a self-development point of view, rather than linking the usefulness of reasoning only to the natural sciences when he says that '[T]o develop a sense of self, however socially embedded, it must surely be of assistance to have discovered that confidence can come from personal reasoning and not just from some higher authority. It is important therefore that students have experience of domains in which … there are general sources and principles rather than a maze of particulars
from which truth can be decided. Mathematics provides an ideal domain for such
discovery’ (p. 4).
Which brings us to Steen’s questions.
**Is reasoning innate?**
To answer Steen’s Question 19, Is mathematical reasoning innate? this section offers
some examples of children’s informal reasoning that I believe support the notion that
reasoning may be an innate capability.
Hunting and Mousley, reporting on some of the findings of the *Mathematical
taking of preschool children in rural and regional Australia: Research and practice*
project (Hunting et al., 2008) reported that ‘a total of 58 of the practitioners (88%)
thought that mathematical thinking starts before the age of 3, and many identified
mathematical activity in babies and toddlers’ This is confirmed by Hughes (2009).
Hughes records an instance of informal reasoning in the home. In this case, a toddler
(usually about 2 to 3 years of age), planned a series of related moves to achieve his
goal. Hughes recalls:

'I once observed a toddler named Tom, who wanted a cup from a high shelf in the
kitchen. His mother was not readily available so Tom dragged a chair towards the
worktop and proceeded to clamber on to the chair. He then climbed on to the surface
of the worktop, from where he could now successfully pick up his self-chosen cup. He then
put the cup on to the worktop and reversed the climbing process till he was back on the
floor. He could now reach the cup and looked deeply satisfied as he took it to his mother.'
(p. 14).

Tom’s reasoning was, in part physical, yet it is undeniable that the plan was a very
clever combination of understanding of basic measurement and informal logic.
However, this type of creative activity is denied to many children, who are kept ‘safe’
by ever-watchful parents, perhaps to the child’s mathematical detriment.

A more formal set of observations have been made by Doig and Ompok (2010) using
the Gumnut Game. Gumnuts are the seed-pods of the eucalyptus tree, and provide
cheap and easily found ‘counters’: other suitable counting objects would be shells,
buttons or pebbles, The game is played as shown in Figure 1. The Gumnut Game is
reproduced here so that interested researchers may trial the game for themselves, and
report on their experiences with it.

Note that although the game is ostensibly about counting, to play one must be able to
decompose numbers mentally. The reasoning aspect is mental too, as the How do you
know? part of the game shows (see Figure 1 below).

This, and other, games have been used with prior-to-school aged children in Hungary,
as well as Australia, and more recently, in in Sabah, Malaysian Borneo.
The Gumnut Game

If the child is very young try two gumnuts (counters) to start, but experience suggests that three is an ideal starting point from two-year-olds and upwards.

**Part 1**

Show the gumnuts (counters) to the child.
Can you tell me how many (counters) I have?
If the response is not three, ask them to count. If the child cannot agree that there are three (counters), then either try two (counters), or abandon this task.

**Part 2**

I am going to hide the (counters) in my hands.
Do this behind your back, or by turning away from the child.
Place some (counters) in each hand.
Show your closed hands to the child.
Which hand do you want to see (open)? Open that hand.
How many (counters) are there? Wait for their response.
How many (counters) are there in my other hand? After the child’s response, open that hand.
Were you correct?
Repeat the process, changing the number of (counters).

**Part 3**

Now it is your turn. Don’t let me see you hide the (counters).
After your say how many in each hand, ask:
Was I correct?
How do you know?

And so on …

Figure 1: The Gumnut Game

In the Gumnut Game, the child needs to know either a number fact, for example that 5 is 2 and 3, or must reason that starting with 5, seeing 2, leaves 3 still hidden. Seven 3-year-olds could play this game with up to 5 gumnuts, while three 4-year-olds could play with up to 10 gumnuts. None of these children had attended any formal schooling.

Figure 2: Playing the Gumnut Game in Sabah
An example drawn from science is the response given to the question on Figure 3 below, taken from Tapping Students’ Science Beliefs (TSSB) (B Doig & Adams, 1993). Students were required to write a reason for agreeing, or disagreeing, with Jenny’s statement that we are animals.

![Figure 3: A question from Tapping Students’ Science Beliefs](image)

A number of students responded that we are animals because we look like animals. Visual reasons we also encountered in trialling other items for the TSSB. One item, that showed a drawing of a bird sitting on an electricity transmission wire asked why the bird was not electrocuted. Responses included that birds have ‘special feet’, which indeed they do, in contrast to their otherwise feathered bodies. Apparently, bird feet tend to look different, and to those who know about such things, insulated by rubbery looking material.

Some years ago I observed some Year 3 children (about 8 years-old) complete a short mathematics assessment. I was standing beside Alice, who was attempting to solve the following:

Tom had 23 cents and Jo had 58 cents. How much did they have altogether?

Like others of her classmates, Alice used tally marks to calculate her answer, and began her calculation by drawing a series of pencil strokes as shown below.

![Figure 4: Alice’s original tally marks](image)

But before she had drawn 23 strokes, she murmured aloud “I can’t do that”, erased the strokes, and commenced to replace them with circles labelled with 1, representing one-cent coins as shown.
Alice’s reasoning was that “they had to be like cents to work”.

When Alice’s teacher was informed of this, she replied that many of the children use tally marks for calculations. She did not attach any importance to the circular ‘tally’ marks, but regarded them as equivalent to lines. I believe that this incident suggests that Alice’s reasoning shows that her mathematical development was not at the same level as her classmates, and that she was connected strongly to visual inscriptions that looked like the real object. How, then, could she work confidently with numbers represented by digits?

The Victorian Early Years Learning Development Framework (VEYLDF) emphasizes the importance of reasoning in the prior-to-school years insisting that '[C]hildren need many opportunities to generate and discuss ideas … reflect and give reasons for their choices' (p. 25). Clearly, these opportunities, either did not occur for Alice, or she was not able to grasp them. In either case, the VEYLDF statement remains relevant.

A concern here is that a curriculum that does not address reasoning does not provide the support needed by children to develop their informal reasoning skills, skills necessary for developing other mathematical knowledge.

These examples demonstrate the types of informal reasoning that occurs before reaching school, and continues informally unless the curriculum attends to its development, as in the case of Alice and the TSSB ‘animals’ question.

Is school too late?

While reasoning, both informal and formal has been the subject of educational research for many years (for example, Hughes, 2009; Voss, 1988; Voss, Perkins, & Segal, 1991) the assessment of reasoning reveals the difficulties that this proficiency causes within education. Yet, there do exist successful implementations.

Reasoning, at the school level, appears, often, in the guise of ‘explanation’. Doig, Groves and Fujii (2011) reported an example of reasoning in a first year of school class. The class of five- and six-year-old children had heard the story of *Snow White and the Seven Dwarfs*. The teacher said that Snow White always sat at the head of the table, while the dwarfs sat at the two long sides, with a different number of dwarfs sitting on each side each day. The teacher asked the children ‘How might the 7 dwarfs sit at the table?’ The children had counters to represent the dwarfs and re-arranged these around a coloured paper rectangle representing the table.

Children were told that their job was to find as many ways seating the dwarfs as possible. The children worked on this problem for several minutes, and then, after asking for several children’s answers, the teacher asked ‘Have we found all the
ways?’ to which the children chorused ‘Yes!’ The teacher responded to this by asking ‘How do we know we’ve got all the ways?’ After some (to the teacher) unsatisfactory attempts, a boy said: ‘We’ve used all the numbers’.

This example demonstrates how very young children can engage with mathematical reasoning, in this case by using a pattern shown by the use of concrete materials (the counters).

Open-ended questions have become popular in both teaching and assessment, and these often require students to give their reasoning for their response. For example, Stacey and her colleagues (Stacey, Groves, Bourke, & Doig, 1993) developed a test of problem-solving, Profiles of Problem Solving (POPS) that contained a sub-scale, ‘quality of explanation’. This was an early attempt to both quantify and promote reasoning as an aspect of mathematics learning.

In the POPS Profile Level Descriptions three levels are specified. Students are categorized as being either:

1. beginning problem solvers, who give unclear explanations using numbers but not words. In this case, reasoning has to be inferred from symbols only;

2. developing problem solvers, who give explanations of only part of their reasoning (e.g. only numerical). In this case, the student’s reasoning is incomplete; or

3. advanced problem solvers, who give clear and complete explanations using both words and numbers. In this case, the student’s reasoning is made visible.

In the POPS item Ladders, students are asked to find the pattern for the number of matches needed to make ‘ladders’ of varying number of steps (see Figure 6).

In the ultimate step, students needed to explain, that is, give their reasoning, for the number of matches required for a ladder with a 1000 rungs. The example, given for a high quality explanation for this part of the Ladders problem, is a ‘Clear explanation of the pattern used — e.g. *there are 1000 rungs and 1001 matches down each side.*’ The reasoning displayed by this level of explanation is clearly well-developed.
Figure 6: POPS Ladders problem

Explanations were assessed, also, in the Third International Mathematics and Science Study (TIMSS), where open-ended questions generally asked for a solution followed by an explanation of how this solution was correct. However, despite the application of partial credit scoring and the so-called Viking rubrics (see Dossey, Jones, & Martin, 2002), for these solutions, the published TIMSS results were based on a simple dichotomous (correct or incorrect) scoring basis (Martin, Gregory, & Stemler, 2000). This was surely a lost opportunity.

Doig and Groves (2008) reported on upper primary and junior secondary students’ explanations to mathematics questions in a large-scale project in Victoria, Australia¹ and some of these questions were drawn from TIMSS. These questions asked students to write, or draw, reasons for their answers. These questions were believed to be susceptible to analysis with the tools described by researchers such as Draper (1988) and Doig and Groves (2007).

An example of one of these items is the Odds and Evens item shown below.

¹ Improving Middle Years Mathematics and Science: The role of subject cultures in school and teacher change (IMYMS) is funded by an Australian Research Council Linkage Grant, with Industry Partner the Victorian Department of Education and Training. The Chief Investigators are Russell Tytler, Susie Groves and Annette Gough, and Associate Investigator Brian Doig.
The following diagram represents the first seven positive whole numbers:

1  2  3  4  5  6  7

Amy says that the sum of any two ODD numbers is always an EVEN number.

Is Amy correct?
Explain how you know.

Figure 7: The open-response item Odds and Evens

While responses varied, many showed ingenious use of reasoning based on a visual approach. For example, one child wrote that:

‘An odd number always has one number that is left out, so if you add 2 together, the two numbers that are left out will join into a pair. This creates an even number’

The ‘one number left out’ refers to the fact that a common approach to learning small numbers uses the Montessori visual arrangement of quantities as pairs objects. On the other hand, there were responses that simply appealed to a higher authority, such as: ‘Because it is a maths rule’.

Figure 8: Example response to the Odds and Evens item

Figure 8 shows the use of demonstration as the basis of reasoning, supporting Hersh’s view that ‘convincing is no problem. Students are all too easily convinced. Two special cases will do it. (1993, p. 396). In this instance, convincing is taken to be a synonym for reasoning.

These examples show that, whether taught or caught, students possess a range of reasoning skills. As Carlson (2009) points out, in a paper considering the use of inscriptions, that 'they [inscriptions] provide foci for the successive coordination [sic] of reasoning’ (p. 56).

Further, Carlson’s research shows that it is possible to foster students’ mathematical reasoning through the use of inscriptions as 'Inscriptions, when integrated into the flow of activities, thus contribute to the structuring the ways users will reason about a problem' (p. 58), and that it is '[A]lso evident … that the students literally are reasoning through the inscriptions' (p. 65).
In answer to the question, Is school too late? the best answer is ‘No’, but schools need to introduce more opportunities for students to use their reasoning from the earliest years. That is, to build on children’s informal reasoning and guide them through to ideas of formal justification and proof.

**Discussion**

The examples described in this paper illustrate a range of issues. Some of which are concerns, others of which are contradictions. The concerns, I believe are that the implemented curriculum in mathematics, by-and-large, does not include reasoning, except incidentally, which means that the reasoning is essentially students’ informal reasoning. This is in contradiction to many mathematics curricula, which include reasoning (and many of its synonyms) in their statements of mathematical skills to be developed.

A further concern is that students come to prior-to-school, and school settings, with reasoning already in their ‘tool kit’. It does not seem sensible to me that we ignore this fact, and let the informal reasoning remain as the only tool available, when it would seem a simple matter to employ tasks and questions, such as the Snow White and the Seven Dwarfs example, from the beginning of school. As this example showed, clearly, informal mathematical reasoning was part of the children’s mathematical tool kit on entry to school.

Steen’s two questions, therefore could be answered by ‘Yes’ there is an innate reasoning capability that children appear to have, and to use, and ‘No’ school is not too late. At the moment, it can be argued that schools provide too little, too late, but if educators do not take this capability as important, and attempt to develop it from the informal to the formal, then we could claim, reasonably, that school is too late!

I present two examples that show why it is important to develop mathematical, and general, reasoning skills. The first of these concerns a common, in Australia, experiment conducted in prior-to-school centres called ‘floating and sinking’. My pre-school teacher trainees tell me that this experience is fun and children find it highly enjoyable. So, what does this experience involve, and what are the outcomes? Objects of different mass (weight) are placed in a tub of water and which objects float and which sink are noted. The outcome of this experience for children and for my teacher trainees is identical: light objects float and heavy objects sink. If I object to this finding, I am accused of being unreasonable: commonly I hear ‘We saw it happen’. Usually it takes counter-examples, such as: How does a large, metal ship float, while a small, comparatively light, orange sinks?, to disturb the teachers’ perspective. Some teachers then remember ‘density’ from secondary school, but the damage, to the children, has been done.

The second example is familiar to most of us: the longer the decimal the larger the number. That is. 0.1234 is larger than 0.2. This can be reasoned from the fact that
longer whole numbers are larger than smaller whole numbers. Clearly, this is a case of fallacious reasoning based on stretching an analogy beyond its limits.

A school curriculum that included reasoning would, one would hope, raise teacher and student awareness of looking beyond simple observations, of looking for counter-examples, of being cautious in using analogies and so on. The place that this aspect of mathematics (and science?) should occupy in the curriculum must be central. Eradicating misconceptions based on poor reasoning requires good reasoning skills, which can develop from students’ prior-to-school informal reasoning.

References


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Doig


The answer from Lewis Carroll is: Babies cannot manage crocodiles.