The Octahedral Hexarot - a Novel 6-DOF Parallel Manipulator

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Abstract

A novel 6-DOF parallel kinematic manipulator named the Octahedral Hexarot is presented and analyzed. It is shown that this manipulator has the important benefits of combining a large positional workspace in relation to its footprint with a sizable range of platform rotations. These features are obtained by combining a rotation-symmetric actuating arm system with links in an octahedral-like configuration. Thus the manipulator consists of a central cylindrical column with six actuated rotating upper arms that can rotate indefinitely around the central column. Each upper arm is connected to a manipulated platform by one 5-DOF lower arm link. The link arrangement of the Octahedral Hexarot is inspired by the original Gough platform. The manipulated platform is an equilateral triangle and the joint positions on the upper arms approximately form an equilateral triangle. A task dependent optimization procedure for the structural parameters is proposed and the workspace of the resulting manipulator is analyzed in depth.

Keywords: Parallel manipulator, PKM, Rotation-symmetric, Stewart-Gough, Workspace

1. Introduction

Parallel kinematic mechanisms using six 5-DOF passive links to manipulate a platform have several advantages. The rod in a 5-DOF link is not susceptible to bending or torsion, and can be constructed lightweight with a high stiffness to mass ratio. As the six links together constrain exactly six DOFs of the manipulated platform, mechanical redundancy will not exist, meaning that high precision assembly of the arm system is uncomplicated. Links of this type are constructed using a rod with either two 3-DOF joints in each end, or by using one 2-DOF joint in one end and one 3-DOF joint in the other end. The joints used are typically universal joints or ball- and socket joints. The advantage of using universal joints is that they can be designed to have a larger working range.

The earliest proposed robot using six 5-DOF links was introduced by Gough as a tyre testing machine [1]. A somewhat similar manipulator was later reinvented by Stewart, proposed as a flight simulator [2]. The original platform suggested by Gough is an Octahedral Hexapod with actuated prismatic links, as shown in Fig. 1(a). These links have five DOFs when the actuators are locked. Hexapods are utilized in a large number of applications [3].

One variant of the Stewart mechanism proposed by Hunt is the 6-RUS Hexapod [4] shown in Fig. 1(b). Three pairs of actuated rotating arms are mounted 120° apart. They are connected by 2- or 3-DOF joints to fixed-length rods, which in turn are connected by spherical joints to the manipulated platform. The workspace of this variant is analyzed in [5].

The HEXA robot [6] is an extension of the well known DELTA mechanism [7] to six DOFs. A sketch of the HEXA robot is shown in Fig. 1(c). The difference compared to the manipulator
Figure 1: Examples of 6-DOF mechanisms utilizing six 5-DOF links to manipulate a platform (striped). Fig. (a) shows an Octahedral Hexapod, using six prismatic actuators. The 6-RUS variant of the Stewart platform in Fig. (b) and the HEXA robot in Fig. (c) are both using three pairs of symmetrically distributed rotating actuators. Fig. (d) shows the HexaGlide manipulator, using six 5-DOF links with constant length on linear guideways.

Another variant of the Hexapod is the HexaGlide [10], which utilizes six 5-DOF links of fixed length to manipulate a platform. The links are actuated by linear actuators, distributed pairwise on three parallel rails as shown in Fig. 1(d). The HexaM [11] developed by Toyoda is a similar manipulator utilizing three pairs of symmetrically placed rails that are angled downwards. Two other variants using six linearly actuated fixed length links are the Linapod III and the Paralix, which are both described in [12]. A comparison of the workspace and manipulability of different variants of HexaGlide is found in [13].

One limitation of all the manipulators discussed above is that the size of the usable workspace in relation to the footprint of the robot is small. In [14] the author derives a group of parallel manipulators inspired by the DELTA robot [7], but with a workspace similar to that of a serial type robot. One of the proposed manipulators is the SCARA-Tau robot, patented by ABB Robotics [15]. The SCARA-Tau is a 3-DOF robot with three actuated upper arms that rotate around a cylindrical base column. This manipulator is analyzed in [16, 17]. Manipulators with 3, 4 and 5 DOFs using a rotation-symmetric arm system have also been proposed in [18, 19, 20, 21]. However, none of these manipulators are exclusively using 5-DOF lower arm links.

The idea of allowing the actuated arms to rotate around a central column is attractive as it enables a large positional workspace to be established. Expanding the SCARA-Tau concept to include more than three actuated upper arms, one or more orientation DOFs of the manipulated platform can be controlled. The Octahedral Hexarot proposed in this paper utilizes six actuated rotating upper arms, connected to a manipulated platform by six 5-DOF lower arm links. With suitable values of the structural parameters this manipulator can achieve a large workspace, both with respect to platform position and orientation.

2. The Octahedral Hexarot

2.1. The proposed manipulator

The Octahedral Hexarot, shown in Fig. 2, is inspired by the Octahedral Hexapod originally proposed by Gough [1]. The name Octahedral Hexarot is based on the properties of the suggested manipulator. Six actuated rotating arms with coinciding axes of rotation are each connected by a 5-DOF link to a manipulated platform. The platform is triangular and the three pairs of joints
on the rotating arms approximately form a triangle. The two triangles compose two sides of an octahedron.

The bodies and joints of the Octahedral Hexarot are marked in Fig. 2(a). There are six actuated rotational joints \( E_i \) between the central cylindrical base column B and the upper arms \( A_i \). Each upper arm is connected to a rod \( R_i \) by a 3-DOF joint \( U_i \). There are six 2-DOF (or 3-DOF) joints \( P_i \) between the rods \( R_i \) and the manipulated platform \( P \). One of the six 5-DOF lower arm links \( L_i \) is plotted separate from the main mechanism. Each link \( L_i \) is composed of joints \( U_i \) and \( P_i \) connected by a rod \( R_i \). The actuators are placed on the base column and since the rods \( R_i \) are not susceptible to bending or torsion their design can be lightweight. Hence, the total moving mass of the Octahedral Hexarot is low. The proposed mechanism has six manipulated DOFs, and since the upper arms can rotate indefinitely around the base column its positional workspace is comparatively large for a parallel manipulator. The arrangement of the joints on the manipulated platform and the upper arms reduces the risk of collisions between the lower arm links and enables a sizable range of platform rotations. With careful choice of structural parameters this manipulator can also achieve favorable isotropic properties. The possibility of using identical drivelines, identical upper arms and identical lower arm links ensures that the number of different components can be kept low, which would reduce the cost of manufacturing the manipulator.

The Octahedral Hexarot could be useful in haptics or for work inside cylindrical spaces, such as repair work inside pipes or assembly tasks inside the body of an airplane.

2.2. Kinematic parameters

The kinematic parameters of the Octahedral Hexarot are indicated in Fig. 2(b). A fixed coordinate system \( F \) is defined in the bottom center of the cylindrical base column B, with its \( x \)-axis pointing outwards and its \( z \)-axis pointing upwards. The joints \( U_i \) on the upper arms \( A_i \) are placed at a distance \( h_i \) from the bottom of B, and at a distance \( a_i \) from the center of B. The angles of the upper arms \( q_i \) are measured from the \( x \)-axis of \( F \). The joint positions \( U_i \) can be expressed in \( F \) as

\[
F_{u_i} = [a_i \cos(q_i), a_i \sin(q_i), h_i]^T. \tag{1}
\]
The manipulated platform is an equilateral triangle with side length $s_1$. The distance between each corner of this triangle and the closest joint position is $s_2$. A tool coordinate system $\mathbf{M}$ is positioned in the center of the manipulated platform according to Fig. 2(b). The x-axis of $\mathbf{M}$ is defined by the direction between $P_4$ and $P_3$, while the z-axis is perpendicular to the plane formed by the joint positions, pointing out of the paper in Fig. 2(b). The expression for each joint position $P_i$ in the coordinate system $\mathbf{M}$ is $\mathbf{M}p_i = [p_{ix}, p_{iy}, p_{iz}]^T$:

\[
\begin{align*}
\mathbf{M}p_1 &= \left[\frac{s_1 - s_2}{2}, -\frac{s_1 + 3s_2}{2\sqrt{3}}, 0\right]^T, \\
\mathbf{M}p_2 &= \left[\frac{s_2}{2}, \frac{2s_1 - 3s_2}{2\sqrt{3}}, 0\right]^T, \\
\mathbf{M}p_3 &= \left[\frac{s_1 - 2s_1}{2}, -\frac{s_1 + s_2}{2\sqrt{3}}, 0\right]^T, \\
\mathbf{M}p_4 &= \left[\frac{s_1 - 2s_2}{2}, -\frac{s_1}{2\sqrt{3}}, 0\right]^T, \\
\mathbf{M}p_5 &= \left[\frac{s_2}{2}, \frac{2s_1 - 3s_2}{2\sqrt{3}}, 0\right]^T, \\
\mathbf{M}p_6 &= \left[\frac{s_1 - s_2}{2}, -\frac{s_1 + s_2}{2\sqrt{3}}, 0\right]^T.
\end{align*}
\]

The distance between each joint $U_i$ on the upper arms and the corresponding joint $P_i$ on the manipulated platform is $l_i$. The position of $\mathbf{M}$ in the fixed coordinate system $\mathbf{F}$ is given by the three translations $x, y, z$. For the orientation of $\mathbf{M}$ in $\mathbf{F}$ Euler ZYZ convention is chosen and the three successive rotations are given by the parameters $\phi, \theta$ and $\psi$. Using these conventions the platform joint positions in the fixed coordinate system are

\[
\mathbf{F}p_i = \mathbf{F}_{om} + \mathbf{R}_\mathbf{M} \mathbf{M}p_i,
\]

where

\[
\begin{align*}
\mathbf{F}_{om} &= [x, y, z]^T, \\
\mathbf{R}_\mathbf{M} &= R_z(\phi)R_y(\theta)R_z(\psi).
\end{align*}
\]

2.3. Inverse kinematics

To obtain expressions for the inverse kinematics, the starting point is the six length equations for the lower arm links $L_i$. Squaring those equations gives

\[
|\mathbf{F}p_i - \mathbf{F}u_i|^2 - l_i^2 = 0.
\]

Using (1) and (3) to expand the equations (5) leads to

\[
d_{i1} + d_{i2} \sin(q_i) + d_{i3} \cos(q_i) = 0,
\]

where parameters $d_{ij}$ have been introduced according to

\[
\begin{align*}
d_{i1} &= x^2 + y^2 + (z - h_i)^2 - l_i^2 + a_1^2 + p_{iz}^2 + p_{iz}^2 + 2(z - h_i)p_{iz}c_\theta - 2(z - h_i)p_{ix}s_\theta c_\psi \\
&+ 2(z - h_i)p_{iy}s_\theta s_\psi - 2x p_{ix}s_\theta s_\psi - 2x p_{iy}s_\theta c_\psi + 2x p_{ix}c_\theta s_\psi + 2y p_{ix}c_\theta c_\psi, \\
d_{i2} &= -2a_1y - 2a_1p_{iy}c_\psi c_\theta - 2a_1p_{ix}s_\theta s_\psi - 2a_1p_{ix}s_\theta c_\psi c_\theta - 2a_1p_{ix}s_\psi c_\theta c_\psi - 2a_1p_{iy}c_\theta s_\psi - 2a_1p_{iy}c_\theta c_\psi, \\
d_{i3} &= -2a_1x - 2a_1p_{ix}c_\psi s_\theta + 2a_1p_{iy}s_\theta c_\psi + 2a_1p_{ix}c_\theta c_\psi + 2a_1p_{iy}c_\theta s_\psi,
\end{align*}
\]

The length of the expressions (7) have been reduced by using the abbreviated forms $s_\theta = \sin(\theta), c_\theta = \cos(\theta)$ etc. Each of the six equations in (6) has two solutions; one with a smaller joint angle, $q_{iS}$, and one with a larger joint angle, $q_{iL}$:
The valid solutions are $q_{iL}$ for the upper arms $A_1$, $A_2$, $A_5$, $A_6$ and $q_{iS}$ for the upper arms $A_3$ and $A_4$.

2.4. Joints and joint limitations

One of the 3-DOF upper arm joints $U_i$ is shown in Fig. 3(a). It allows infinite rotation around the vector $n_{U_i}$. The angle $\alpha_{U_i}$ between the vectors $n_{U_i}$ and $m_{U_i}$ must be less than $90^\circ$. This angle is calculated according to

$$
\alpha_{U_i} = \left| \arccos \left( \frac{n_{U_i} \cdot m_{U_i}}{\|n_{U_i}\| \|m_{U_i}\|} \right) \right|
$$

One of the 2-DOF platform joints $P_i$ is shown in Fig. 3(b). It is similar to the upper arm joints except that it does not allow rotation around the vector $n_{P_i}$. During the workspace simulations the angle $\alpha_{P_i}$ between the vector $n_{P_i}$ and the platform normal $n_p$ is limited to $90^\circ$ even if a larger angle is possible in some directions. The angle $\alpha_{P_i}$ is calculated according to

$$
\alpha_{P_i} = \left| \arccos (n_{P_i} \cdot n_p) \right|
$$

The valid solutions are $q_{iL}$ for the upper arms $A_1$, $A_2$, $A_5$, $A_6$ and $q_{iS}$ for the upper arms $A_3$ and $A_4$.
2.5. Collisions

The workspace of the Octahedral Hexarot is limited by the risk of collisions between the bodies of the manipulator. It is sufficient to evaluate a total of 18 potential collisions for each configuration of the manipulator. Six potential collisions between the base column B and the lower arm links \( L_i \) are checked together with a total of 8 potential collisions between the upper arms \( A_i \) and the lower arm links \( L_i \). The latter potential collisions are between the arm pairs \( A_1 - L_2, A_2 - L_1, A_3 - L_4, A_4 - L_3, A_5 - L_6, A_6 - L_5, A_2 - L_5, A_3 - L_2 \). Four potential collisions between the lower arm links must also be evaluated. These are between the link pairs \( L_1 - L_2, L_3 - L_4, L_5 - L_6, L_2 - L_5 \). Potential collisions between the manipulated platform and any of the other bodies of the mechanism are less critical. For the Octahedral Hexarot, other limitations such as the working range of the joints and collisions between lower arm links, make evaluation of most platform collisions redundant.

All the identified 18 potential collisions can be evaluated in the same manner. It is assumed that the base column, the upper arms and the lower arm links are all cylindrical. Let each of these bodies be named \( B_i \) and let the vectors \( u_{B_i} \) and \( v_{B_i} \) point to the endpoints of the centerline of each body. All positions on the centerline of one of these bodies can then be described by

\[
c_{B_i} = u_{B_i} + s_{B_i} (v_{B_i} - u_{B_i}), \quad 0 \leq s_{B_i} \leq 1.
\]  

The vector between a position on the centerline of one body \( B_i \) and a position on the centerline of another body \( B_j \) is called \( w_{B_iB_j} \) and is defined by

\[
w_{B_iB_j} = c_{B_j} - c_{B_i} = u_{B_j} - u_{B_i} + s_{B_j} (v_{B_j} - u_{B_j}) - s_{B_i} (v_{B_i} - u_{B_i}), \quad 0 \leq s_{B_i}, s_{B_j} \leq 1.
\]

The minimum length of this vector \( |w_{B_iB_j}|_{\text{min}} \) is determined using an algorithm from [22]. To avoid a collision between two cylindrical bodies \( B_i \) and \( B_j \) the minimum distance between the centerline of the two bodies must always be larger than the sum of the radii of the bodies:

\[
|w_{B_iB_j}|_{\text{min}} > r_{B_i} + r_{B_j}.
\]

For the cylindrical base column \( B \), the two endpoints are given by \([0,0,0]^T \) and \([0,0,h_B]^T \), where \( h_B \) is the height of \( B \). For each upper arm \( A_i \), one endpoint is given by \( F_i u_i \) according to (1) while the other endpoint can be assumed to be in the center of the base column (i.e. it has the same z-value as \( F_i u_i \) but the x- and y-values are zero). The endpoints of each lower arm link \( L_i \) are \( F_i p_i \) according to (1) and \( F_i p_i \) according to (3).

When a robot configuration is evaluated, the minimum distance \( |w_{B_iB_j}|_{\text{min}} \) is calculated for each of the 18 body pairs with the potential to collide, and it is verified that this distance is larger than the sum of the two corresponding radii. If the distance between two evaluated configurations is large, the possibility of stepping over a collision must also be considered.

3. Structural parameters of the manipulator

3.1. Reduced set of parameters

The complete parameter set for the Octahedral Hexarot is \( P_C = \{l, a_i, h_i, s_1, s_2\} \) and the total number of parameters is 20. In this subsection, a reduced set of parameters is selected. The
lengths of all lower arm links $l_i$ are chosen to be the same and equal to $\lambda$. This length defines the scale of the manipulator and the size of most other parameters are defined relative to it. The value of $\lambda$ is set to 1 m. The lengths of all upper arms $a_i$ are chosen to be the same and equal to $a_\lambda \lambda$. Values of $a_\lambda$ less than 1.5 are evaluated.

To achieve a manipulator similar to an Octahedral Hexapod the vertical positions of the joints on the upper arms are chosen such that the six joints are grouped in three pairs. The vertical distance between the joints in a pair is set to be $2h_f$. The parameter $h_f$ is chosen to be independent of $\lambda$ (the scale of the manipulator). To avoid collisions, the value of $h_f$ must be larger than the radius of the upper arms, $r_A$. Values of $h_f$ between $r_A$ and 0.5 m are evaluated. The middle pair of upper arm joints is placed at height $\lambda$ and the distance from this pair to the upper and lower joint pairs is chosen to be the same and equal to $h_\lambda \lambda$. Values of $h_\lambda$ between $(h_f + r_A)/\lambda$ and 1 are evaluated.

The side length of the equilateral platform $s_1$ is set to $s_\lambda \lambda$. Values of $s_3$ between 0.15 and 1.5 are evaluated. To avoid collisions between the lower arm links, the distance from the corner of the triangular platform to the joint position on the platform, $s_2$, must be larger than twice the radius of the lower arm links $r_L$. The parameter $s_2$ is chosen to be independent of $\lambda$. Values of $s_2$ between $2r_L$ and $s_1/3$ are evaluated. The chosen parameters are summarized below:

$$
\begin{align*}
l_i &= \lambda = 1m, \\
a_i &= a_\lambda \lambda, \\
s_1 &= s_\lambda \lambda, \\
h_1 &= (1 - h_\lambda) \lambda - h_f, \\
h_2 &= (1 - h_\lambda) \lambda + h_f, \\
h_3 &= \lambda - h_f, \\
h_4 &= \lambda + h_f, \\
h_5 &= (1 + h_\lambda) \lambda - h_f, \\
h_6 &= (1 + h_\lambda) \lambda + h_f.
\end{align*}
$$

The reduced set of parameters that must be selected is $P_R = \{a_\lambda, h_\lambda, h_f, s_\lambda, s_2\}$. The smallest tested difference between consecutive values of $a_\lambda$, $h_\lambda$ and $s_\lambda$ is 0.01 and the smallest tested difference between consecutive values of $h_f$ and $s_2$ is 0.01 m. The dimensions of the cylindrical base column, the upper arms and the lower arm links are needed to evaluate collisions. In the simulations and CAD models the radius of the central cylindrical column $r_B$ is set to 0.1 m, the radius of all upper arms $r_A$ to 0.05 m and the radius of all lower arm links $r_L$ to 0.025 m.

### 3.2. Isotropy

It has been observed that even if many different combinations of the parameters in Fig. 2(b) lead to a large range of both platform translations and platform rotations, the resulting manipulators for many of these choices are very anisotropic. For manipulators with large anisotropy the achievable speed, the manipulator stiffness, and other manipulator properties vary strongly for different task space directions. To avoid variants of this type some measure of the isotropic properties of a manipulator is necessary. The most commonly used index in the literature is the condition number of the Jacobian. However, for a 6-DOF manipulator the Jacobian is dimensionally non-homogenous and the singular values have different units. Hence, calculating ratios
between them has no meaning. Different approaches to this problem include first normalizing
the Jacobian with a characteristic length [23] or describing the task space configuration of the
manipulated platform by the positions of a set of points on the platform instead of a position
and an orientation [24]. Both approaches leads to a Jacobian where the elements have the same
units. These and other approaches are discussed in a recent overview [25] but the conclusion is
that most methods involve a large degree of arbitrariness and that there is no consensus in the
robot community on which indices are optimal. In this work separate indices are used for trans-
lational isotropy and rotational isotropy. The controversy of comparing translations and rotations
is therefore avoided. The drawback is that a manipulator could have very large (or very small)
translations in relation to the rotations. The relation between joint speed \( \dot{q} \) and the speed in task
space \( \dot{x} \) is

\[ \dot{x} = J\dot{q}. \]  

For the Octahedral Hexarot the Jacobian \( J \) can be written as

\[ J = \begin{bmatrix} J_A \\ J_B \end{bmatrix}, \]

where the elements in the 3x6 matrix \( J_A \) have the unit length and the elements in the 3x6 ma-
trix \( J_B \) are dimensionless. The singular values of \( J_A \) and \( J_B \) are the square roots of the eigenvalues
of \( J_A J_A^T \) and \( J_B J_B^T \). Condition numbers of \( J_A \) and \( J_B \) are calculated as

\[ \kappa_{\text{trans}} = \kappa(J_A) = \frac{\sigma_{\text{max}}(J_A)}{\sigma_{\text{min}}(J_A)}, \]

\[ \kappa_{\text{rot}} = \kappa(J_B) = \frac{\sigma_{\text{max}}(J_B)}{\sigma_{\text{min}}(J_B)}. \]
A low value of the condition number $\kappa_{\text{trans}}$ means that the maximum and minimum translational speeds achieved for joint speeds on the (six dimensional) unit sphere are similar. A low value of $\kappa_{\text{rot}}$ means that the maximum and minimum platform reorientation speed for joint speeds on the unit sphere are similar.

The center of the workspace in one radial direction ($x > 0, y = 0$) is given by

$$[x_c, y_c, z_c, \phi_c, \theta_c, \psi_c].$$

Due to the parameter choices (14) and symmetry, $y_c = 0 \text{m}, z_c = 1 \text{m}, \theta_c = 90^\circ, \psi_c = 0^\circ$. The values of $x_c$ and $\phi_c$ depend on the parameter choices and must be determined. Minimizing the condition number $\kappa_{\text{trans}}$ (17) in the center of the workspace leads to

$$\kappa_{\text{trans}} = 1.00. \quad (24)$$

A manipulator with these parameters is plotted in Fig. 4(a) and a cross-section of its workspace in Fig. 5(a). Minimizing the condition number $\kappa_{\text{rot}}$ (18) in the center of the workspace instead leads to

$$\kappa_{\text{rot}} = 2.35, \quad (23)$$

$$\kappa_{\text{trans}} = 1.00. \quad (24)$$

A manipulator with these parameters is plotted in Fig. 4(b) and a cross-section of its workspace in Fig. 5(b).

### 3.3. Task dependent parameter optimization

The workspace of the manipulators in Fig. 4(a) and 4(b) are illustrated in Fig. 5(a) and 5(b), where cross-sections ($x > 0, y = 0$) of the toroidal workspace for each manipulator are plotted. Each position in the xz-plane is colored according to the maximum platform rotation which is possible in all directions starting from the platform orientations (22) or (27). Different markers signifies the limiting factor for further rotation in the limiting direction. The workspace plots are created by evaluating poses where the position is in the xz-plane ($x > 0, y = 0$) and the platform Euler angles correspond to equal platform rotations in all directions starting from the central platform orientation. The evaluated Euler angles $[\phi_{uv}, \theta_{uv}, \psi_{uv}]$ are identified from

$$R_z(\phi_{uv})R_y(\theta_{uv})R_z(\psi_{uv}) = R_z(\phi_c)R_y(\theta_c)R_z(\psi_c)R_{v=\phi_{uv}}.$$
Figure 5: The plot (a) shows all reachable positions in the xz-plane ($x > 0, y = 0$) for an Octahedral Hexarot with the parameter set (20). Each position is colored according to the maximum platform rotation that is achievable in all directions starting from the platform orientation (22). If the achievable rotation angle from this pose is less than $30^\circ$ the platform position is colored red, if it is between $30^\circ$ and $45^\circ$ it is colored blue, while positions where a larger angle than $45^\circ$ can be achieved are colored green. Different markers are used to show the limiting factor for further rotations in each position. The use of a '+' signifies joint limitations, a square signifies collision and a filled circle that there is no solution due to reach. The plots (b) and (c) are similar plots for the parameter sets (25) and (32), where the central platform orientations are given by (27) and (34) respectively.

The angles $[\phi_c, \theta_c, \psi_c]$ are Euler angles for the central platform orientations (22) or (27) and the rotation matrix $R_{v_\upsilon\omega}$ corresponds to a positive rotation of $\xi_{v_\upsilon\omega}$ degrees around a unit vector $v_{\upsilon\omega}$. The unit vectors $v_{\upsilon\omega}$ have directions symmetrically distributed in three dimensional space:

$$v_{\upsilon\omega} = [\sin(\upsilon) \cos(\omega), \sin(\upsilon) \sin(\omega), \cos(\upsilon)]^T, \hspace{1cm} 0^\circ < \upsilon \leq 180^\circ, \hspace{0.5cm} 0^\circ < \omega \leq 360^\circ. \hspace{1cm} (31)$$

For the plots in Fig. 5 a total of 146 vectors are used where the angles $\upsilon$ and $\omega$ are spaced $20^\circ$ apart. The number of used vectors have been increased until the plots do not change. For each position the maximum possible positive rotation $\xi_{v_\upsilon\omega}$ in all 146 directions (31) is determined and the minimum of these rotation angles $\xi_{\text{all}}$ determines the color of each plotted position in Fig. 5. The used marker depends on the limiting factor for further rotation in the worst case direction.

The results (23), (24) and (28), (29) indicate that to achieve a low value of $\kappa_{\text{trans}}$ a large manipulated platform must be used. This result is also similar to the results for an Octahedral Hexapod where maximal isotropy is achieved when the side of the platform is half the side of the base and the distance between the platform and the base is the same as the side of the platform [26]. However, it is shown in Fig. 5(a) that for the Octahedral Hexarot manipulator a large platform limits the achievable workspace. For the manipulator with minimized $\kappa_{\text{rot}}$ shown in Fig. 4(b) both the translational and rotational workspaces are larger. However, as can be seen in Fig. 5(b) the achievable platform rotations are still limited. One of the reasons for this is the close vertical distance between the pairwise upper arms (small value of $h_f$ in (25)), which leads to collisions between upper arms and lower arms in a pair. Analysis of Fig. 5(a) and Fig. 5(b) makes it clear that workspace considerations must be included in the optimization criteria.

The proposed approach for optimizing the structural parameters for a particular task is to put a constraint on the translational volume by specifying a cross-sectional area (e.g. a rectangle or a circle) with its center point in the center of the cross-section of the workspace and its size according to the application needs. A second constraint is put on the minimum achievable rotation.
in any direction within the specified volume. Optimized parameters are determined by selecting the parameters that fulfill both constraints and maximize a measure of isotropy; here the product of $\kappa_{\text{rot}}$ and $\kappa_{\text{trans}}$ in the center of the workspace. If parameters that fulfill the constraints can not be determined it is immediately obvious that the Octahedral Hexarot is not useful for the selected application. The proposed optimization strategy is computationally expensive but it is possible to reduce the calculations by noticing that the plots in Fig. 5 are symmetrical on both sides of $z = 1$ m. Assuming that the positions closer to the center have a larger range of achievable rotations it is also possible to only evaluate positions on the edge of the selected workspace area and only for the minimum required platform rotations. This assumption can then be verified after the optimization is finished. To exemplify this procedure one optimization has been performed where:

- Constraint 1: Minimum cross-sectional area is a circle with radius 0.30 m.
- Constraint 2: Rotation angle ($\xi_{\text{all}}$) $> 30^\circ$.
- Criteria: Minimize $\kappa_{\text{rot}} \times \kappa_{\text{trans}}$ in the center of the workspace.

### 3.4. Resulting manipulator

The task based parameter optimization in the previous section leads to

$$P_{\text{taskopt}} = \{a_\lambda, h_\lambda, r_\lambda, s_\lambda, t_\lambda\}_{\text{taskopt}} = \{0.60, 0.46, 0.07 m, 0.35, 0.06 m\}, \quad (32)$$

$$[x_c, y_c, z_c]_{\text{taskopt}} = [0.97 m, 0 m, 1 m], \quad (33)$$

$$[\phi_c, \theta_c, \psi_c]_{\text{taskopt}} = [-2^\circ, 90^\circ, 0^\circ], \quad (34)$$

$$\kappa_{\text{rot}} = 1.15, \quad (35)$$

$$\kappa_{\text{trans}} = 3.57. \quad (36)$$

The workspace using the optimized parameters is shown in Fig. 5(c). However, this plot only illustrates how much rotation that is possible in the worst case direction for each position. For increased understanding of the possibilities and limitations of the manipulator, the range of achievable rotations in the roll, pitch and yaw directions are studied separately here. Figure 6 shows an Octahedral Hexarot with the optimized parameters (32). All plots are for the central workspace position (33). Starting from the central platform orientation (34) only one platform rotation (roll, pitch or yaw) has been changed in each plot. The last valid configuration before any limitation was reached has been plotted. The limitations are given by the solution to the inverse kinematics, joint limitations, Type 2 singularities, and collisions.

As shown in Fig. 6(a), a maximum positive roll of $80^\circ$ is possible before additional roll is limited by a collision between link $L_5$ and $L_6$. Due to symmetry, the same range of negative roll can be achieved in this position. Figure 6(b) demonstrates the maximum achievable positive pitch. Starting from the platform orientation (34) $56^\circ$ of positive pitch is possible until limited by the working range of the platform joint $P_1$. Due to symmetry, the same range of negative pitch can be achieved in this position. Figures 6(c) and 6(d) show the maximum negative and positive yaw for the manipulator. Starting from the central configuration, an additional $55^\circ$ of negative yaw is possible until limited by the working range of the platform joints $P_3$ and $P_4$. The achievable positive yaw from the start configuration is $54^\circ$ until limited by the working range of the platform joints $P_1$ and $P_6$. 

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Figure 6: Four configurations of the Octahedral Hexarot using the parameter set (32). In all four plots the center of the manipulated platform is in the same position in the middle of the radial workspace (33). In each plot one platform rotation has been changed, starting from central platform orientation (34), until some limitation has been reached. Only positive roll and positive pitch have been plotted, since in this position positive and negative roll and pitch are symmetric. Both the cases with negative and positive yaw are plotted since yaw is not symmetric.

Using the same set of optimal parameters (32) and repeating the calculations done for Fig. 6, for all reachable positions in the xz-plane, leads to the plots in Fig. 7. These plots show all reachable radial positions $(x, 0, z)$. Since these plots are approximately circular, the total positional workspace will be toroidal in shape. Only one rotation angle is varied in each plot, starting from
the central platform orientation (34). Each tested position has been colored according to maximum achievable platform rotation angle in one direction. Different markers have been utilized to show the limiting factor for further rotation in each position. Achievable roll and pitch are mirrored on both sides of $z = 1$ m (maximum achievable positive roll and pitch in $(1 + \Delta z)$ m is equal to maximum achievable negative roll and pitch in $(1 - \Delta z)$ m) so the achievable negative angles have not been plotted for these two cases.

The main limitation on maximum positive roll, displayed in Fig. 7(a) is due to collisions and reach of the manipulator, while in a smaller section of the workspace roll is limited by Type 2 singularities. The maximum positive pitch is displayed in Fig. 7(b). The main limitation on further pitch is the working range of the universal joints on the platform, but sections of the workspace are also limited by collisions and reach. Due to the joint limitations, the range of positive pitch is strongly limited for $z > 1$ m while negative pitch is strongly limited for $z > 1$ m. To increase the range of achievable platform pitch the three pairs of upper arms must be closer to each other (reduced $h_{1}$).

The maximum negative and positive yaw is displayed in Fig. 7(c) and Fig. 7(d) respectively. The range of positive yaw is mainly limited by the working range of the platform joints while the main limitations for further negative yaw also include collisions and joint limitations.

4. Conclusion and future work

Fully parallel manipulators with a large range of platform rotations are unusual, even more so if combined with a large positional workspace. In this paper we propose a novel parallel manipulator, the Octahedral Hexarot, which has the potential of a large workspace. The Octahedral Hexarot has six actuated upper arms that can rotate indefinitely around a cylindrical base column. An octahedral-like link arrangement is used to achieve high isotropy and to limit the risk of arm collisions. The properties of the Octahedral Hexarot, including the size of its workspace and its isotropic properties, are strongly dependent on the choice of structural parameters. The isotropic properties of the manipulator have been analyzed and an optimization process for task dependent parameters proposed. The workspace of the resulting manipulator has been studied extensively.

The simulations indicate that there are no Type 2 singularities in the central volume of the workspace. However, future work should include an analytical study of the singularities for this manipulator. One limiting factor for a larger range of platform rotations are collisions between the bodies of the manipulator. The workspace limitations due to collisions between the upper arms and the lower arm links could be reduced by modifying the shape of the upper arms in order to separate the main sections of two upper arms in a pair. One possible approach is to use upper arms that are L-shaped when viewed from the front or L-shaped or U-shaped when viewed from above.

The promising results for the Octahedral Hexarot encourages further study of manipulators using six actuated rotating upper arms with coinciding axes of rotation. It is possible that other useful members of this class of manipulators could be found. Most of the tools developed in this paper, including the solutions to the inverse kinematics and the algorithm for collision detection, have been derived in such a way that they can be applied to arbitrary manipulators of this type.

References

Figure 7: The plots show all reachable positions in the xz-plane (x > 0, y = 0) for an Octahedral Hexarot with the parameter set (32). Only one platform rotation angle is varied in each figure, starting from the central platform orientation (34). If the maximum achievable rotation angle is less than $30^\circ$, the platform position is colored red, if it is between $30^\circ$ and $45^\circ$ it is colored blue, while positions where a larger angle than $45^\circ$ can be achieved are colored green. Different markers are used to show the limiting factor for further rotations in each position. The use of a triangle signifies a singularity, a ‘+’ signifies joint limitations, a square signifies collision and a filled circle that the inverse kinematics lacks a solution due to reach.

Appendix A. Nomenclature

Table A.1 lists the used nomenclature.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>Manipulator base column</td>
</tr>
<tr>
<td>P</td>
<td>Manipulated platform</td>
</tr>
<tr>
<td>A_i</td>
<td>Upper arm i</td>
</tr>
<tr>
<td>L_i</td>
<td>Lower arm link i (5-DOF)</td>
</tr>
<tr>
<td>R_i</td>
<td>Rod in lower arm link i</td>
</tr>
<tr>
<td>U_i</td>
<td>Upper arm joint i (1-DOF)</td>
</tr>
<tr>
<td>P_i</td>
<td>Platform joint i (2-DOF)</td>
</tr>
<tr>
<td>h_i</td>
<td>Height of U_i in F</td>
</tr>
<tr>
<td>a_i</td>
<td>Kinematic length of A_i</td>
</tr>
<tr>
<td>l_i</td>
<td>Kinematic length of L_i</td>
</tr>
<tr>
<td>s_i</td>
<td>Side length of P</td>
</tr>
<tr>
<td>s_2</td>
<td>Joint distance on platform</td>
</tr>
<tr>
<td>F</td>
<td>Fixed base coordinate system</td>
</tr>
<tr>
<td>M</td>
<td>Tool coordinate system</td>
</tr>
<tr>
<td>q_j</td>
<td>Joint angle of upper arm A_j</td>
</tr>
<tr>
<td>n_p_j</td>
<td>Position of P_j in M</td>
</tr>
<tr>
<td>p_{xyz}</td>
<td>Elements of M_p at xyz</td>
</tr>
<tr>
<td>u_i</td>
<td>Position of U_i in F</td>
</tr>
<tr>
<td>q_L</td>
<td>Larger joint solution to I.K.</td>
</tr>
<tr>
<td>q_S</td>
<td>Smaller joint solution to I.K.</td>
</tr>
<tr>
<td>x, y, z</td>
<td>Platform translations in F</td>
</tr>
<tr>
<td>θ, θ, ψ</td>
<td>Platform rotations in F</td>
</tr>
<tr>
<td>R_{θ_1}, R_{θ_2}, R_{θ_3}</td>
<td>Euler rotation matrices</td>
</tr>
<tr>
<td>R_{Ω_1}</td>
<td>Origin of M in F</td>
</tr>
<tr>
<td>R_{Ω_M}</td>
<td>Orientation of M in F</td>
</tr>
<tr>
<td>n_{u_1}</td>
<td>Angle of joint u_1</td>
</tr>
<tr>
<td>n_{p_1}</td>
<td>Vector U_1 to P_1</td>
</tr>
<tr>
<td>n_{u_1}</td>
<td>Mounting direction of U_1</td>
</tr>
<tr>
<td>n_{p_1}</td>
<td>Vector P_1 to U_1</td>
</tr>
<tr>
<td>n_p</td>
<td>Platform normal vector</td>
</tr>
<tr>
<td>B_i</td>
<td>Body i</td>
</tr>
<tr>
<td>w_{B_1}</td>
<td>Vector to one endpoint of B_i</td>
</tr>
<tr>
<td>v_{B_2}</td>
<td>Vector to other endpoint of B_i</td>
</tr>
<tr>
<td>s_{B_1}</td>
<td>Distance between two bodies</td>
</tr>
<tr>
<td>w_{B_1,B_2}</td>
<td>Minimum distance of w_{B_1,B_2}</td>
</tr>
<tr>
<td>r_i</td>
<td>Radius of B_i</td>
</tr>
<tr>
<td>h_B</td>
<td>Height of the base column B</td>
</tr>
<tr>
<td>P_c</td>
<td>Complete set of structural par.</td>
</tr>
<tr>
<td>P_R</td>
<td>Reduced set of structural par.</td>
</tr>
<tr>
<td>λ</td>
<td>All lengths l_i are set to λ</td>
</tr>
<tr>
<td>a_3</td>
<td>All lengths a_i are set to a_3 λ</td>
</tr>
<tr>
<td>b_i</td>
<td>Dist. betw. A_i, A_j in a pair is 2h_i</td>
</tr>
<tr>
<td>r_B</td>
<td>Radius of the base column B</td>
</tr>
<tr>
<td>r_A</td>
<td>Radius of all upper arms A_i</td>
</tr>
<tr>
<td>r_A</td>
<td>Radius of all upper arms A_i</td>
</tr>
<tr>
<td>η</td>
<td>The joint speed vector</td>
</tr>
<tr>
<td>k</td>
<td>The Cartesian speed vector</td>
</tr>
<tr>
<td>J</td>
<td>The Jacobian</td>
</tr>
</tbody>
</table>

Table A.1: The used nomenclature.