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The Set-Theoretical Nature of Badiou’s Ontology and Lautman’s Dialectic of Problematic Ideas

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In the notes to Being and Event, Badiou declares: ‘[Albert] Lautman’s writings are nothing less than admirable and what I owe to them, even in the very foundational intuitions for this book, is immeasurable’ (BE 482). Nowhere in his published work, however, does Badiou explain in detail what he believes he owes to the philosophy of Lautman. In an interview with Tzuchien Tho, Badiou remarks that even though Lautman’s primary and secondary theses for his doctorat d’état were published in 1938, it was only well after the 1977 publication of Lautman’s collected works that he became familiar with them (CM 82).¹ This would certainly explain the lack of explicit engagement with Lautman’s writings prior to the publication of Being and Event. But even after 1988, Badiou primarily invokes the figure of Lautman for illustrative purposes, or to suggest a very general affinity. Thus, in Logics of Worlds, Badiou inscribes himself, but without indicating what such an inscription amounts to, in the French tradition of ‘mathematising idealism’ which includes Brunschvicg, Cavailles, Lautman, Desanti, Althusser and Lacan (LW 7). In Metapolitics, Badiou eulogises Lautman, both a philosopher of mathematics and active member of the French Resistance who was captured and shot by the Nazis in 1944, as one of the ‘exemplary resistant figures of French philosophy’ (M 4–5). Finally, in the first Manifesto for Philosophy, Badiou cites Lautman as an ally insofar as he, like Badiou, is an openly Platonic philosopher standing in opposition to the more general twentieth-century trend of Nietzschean anti-Platonism (MP 100–1). In each of these instances, it is clear that Badiou invokes the name of Lautman for purposes other than concrete conceptual or argumentative support.

Three exceptions to this tendency, however, can be found in Briefings on Existence: a note to the essay ‘Philosophy and
Mathematics’, originally published in *Conditions*, and the short interview with Tzuchien Tho mentioned above published alongside the English translation of *The Concept of Model*. In these three texts, Badiou indicates a more substantial philosophical affiliation with, and perhaps a philosophical debt to, Lautman.

In *Briefings on Existence*, Lautman’s work is cited in support of Badiou’s thesis, defended from the time of *Being and Event* onwards, that ‘mathematics thinks being qua being’ or that ‘set theory is ontology’. He writes: ‘As Albert Lautman’s works in particular demonstrated, as far back as the 1930s, every significant and innovative fragment of real mathematics can and ought to evoke, as a condition of existence, its ontological identification’ (TO 60 – translation modified). But how exactly does Badiou understand this apparently Lautmanian idea, which he believes himself to share, that mathematics evokes – and as a ‘condition of existence’ no less – its ontological identification?

We ought firstly to recall that, for Badiou, the proposition ‘set theory is ontology’ is a philosophical and not a mathematical proposition (TO 59). In *Manifesto for Philosophy*, Badiou argues that this particular proposition is grounded in his own philosophy insofar as this latter has successfully been able to ‘compossibilise’ a number of contemporary, extra-philosophical ‘conditions’ or ‘generic truth procedures’. Because the development of set theory from Cantor to Paul Cohen is one such truth procedure, within the conditioned philosophical space that Badiou constructs, the philosophical thinking of being qua being can be effectively handed over to set theory (MP 33–9, 79–81). Indeed, this metaontological justification of the ‘set theory is ontology’ thesis is the one normally discussed in the secondary literature.

In *Briefings on Existence*, however, Badiou adds a specifically Lautmanian element to his account of the justification of the philosophical proposition that set theory is ontology. He writes that while it is indeed a matter of philosophy ‘identifying what its own conditions are’, it is also necessary for ‘real mathematics to be crossed reflexively’ (TO, 59). In other words, the grounding of the proposition that mathematics thinks being qua being is not simply a matter of philosophy welcoming within its ‘space of composibility’ a novel, set-theoretical translation of well-established philosophical and ontological concepts. It is also a matter of philosophy fully entering into and traversing mathematical discourse (but without itself becoming mathematical discourse) in order
to discover how mathematics thinks being qua being in its *own* unique terms.

Badiou makes a similar point in a note to ‘Philosophy and Mathematics’ – the second text mentioned above. Here, Badiou makes it clear that his and Lautman’s (as well as Cavaillès’ and Desanti’s) philosophical relation to mathematics is a matter of treating mathematics ‘as a singular site of thinking, whose events and procedures must be retraced from *within* the philosophical act’ (TW 244). And indeed, this sounds very close to the way Lautman expresses his own position on the issue of mathematical philosophy:

Mathematical philosophy . . . consists not so much in retrieving a logical problem of classical metaphysics within a mathematical theory, than in grasping the structure of this theory globally in order to identify the logical problem that happens to be both defined and resolved by the very existence of this theory.⁴

Finally, in the interview with Tho, Badiou explicitly aligns himself with Lautman’s unique brand of ‘dialectical Platonism’ (CM 92–3).⁵ Badiou’s comments here, coupled with the remarks just examined, throw a little more light on his belief that he shares with Lautman the idea that mathematics elicits, as a condition of existence, its ontological identification, and hence also that he owes to Lautman’s writings some of the fundamental intuitions for his philosophical undertaking in *Being and Event*. In the interview with Tho, Badiou alludes to two aspects of Lautman’s work which will be examined further below. The first aspect is that Lautman understands there to be a ‘dialectic of Ideas’ which drives the historical development of mathematics (as its ‘condition of existence’), giving to mathematics its philosophical value and, as Badiou would say, its ‘ontological identification’. The second aspect of Lautman’s work to which Badiou alludes is that there is a dialectical interpenetration of, on the one hand, the abstract Ideas governing the development of mathematics and, on the other hand, the concrete theories in which such development is successively embodied. Indeed, as Badiou appears to read Lautman, it is because certain abstract ‘ontological’ Ideas are both transcendent and immanent with respect to concrete mathematical theory that philosophy must fully enter into and traverse mathematical discourse in order to discover how mathematics evokes, in its own terms, its ontological character.
In line with textual evidence such as this, this chapter proposes to explore in greater depth the precise nature of the debt that Badiou owes to Lautman in *Being and Event*. The chapter's thesis is that Badiou's assertion that set theory is ontology is grounded upon what Lautman calls a 'dialectic of problematic Ideas'. More specifically, it will be argued here that the proposition that set theory is ontology is grounded insofar as set theory simultaneously defines and resolves, in its own terms, a connected series of philosophical – or, more precisely, ontological – problems. As will be seen, following Lautman's work, these problems can be expressed in the form of yet-to-be-determined relationships between a number of opposed notions belonging to the history of philosophy and ontology: the one and the multiple, Nature as poem and Nature as Idea, the finite and the infinite, and the continuous and the discrete. In what follows, I shall demonstrate exactly how the Zermelo-Fraenkel (ZF) axioms which found Badiou's set-theory ontology together resolve this series of dialectical couples. In concluding, I will also raise a problem which ensues from Badiou's Lautmanian heritage. But let us first of all examine the major lines of Albert Lautman's only recently translated work.

**Albert Lautman and the Dialectic of Problematic Ideas**

Lautman distinguished several layers of mathematical reality. Apart from mathematical facts, objects and theories, Lautman also argued for the existence of a 'dialectic of Ideas' which governs the development of mathematical theories and provides them with their unity, meaning and philosophical value. This dialectic is constituted by pairs of opposites (same and other, whole and part, continuous and discrete, essence and existence, etc.), and the Ideas of this dialectic present themselves as the problem of establishing relationships between these opposed notions. As prior 'questions' or 'logical concerns' relative to mathematical discourse, these problematic Ideas are transcendent with respect to mathematics and can be posed outside of mathematics. Indeed, many of the pairs of opposites analysed by Lautman can be found in the history of philosophy. However, since the Ideas, in order to be thought concretely, require an appropriate 'matter' in which they can be thought, any effort to respond to the problems that they pose is to effectively constitute mathematical theory. In this sense, therefore, the dialectic must equally be said to be immanent
to mathematics.\(^8\) As Lautman puts it, an ‘intimate link thus exists between the transcendence of Ideas and the immanence of the logical structure of the solution to a dialectical problem within mathematics’.\(^9\)

In a letter to the mathematician Maurice Fréchet, Lautman provides the following example in order to clarify his thought on this point:

One can envisage abstractly the Idea of knowing whether relations between abstract notions exist, for example the container and the content, but it happens that any effort whatsoever to outline a response to this problem is ipso facto the fashioning of mathematical theories. The question of knowing whether forms of solidarity between space and matter exist is in itself a philosophical problem, which is at the centre of Cartesian metaphysics. But any effort to resolve this problem leads the mind necessarily to construct an analytic mechanics in which a connection between the geometric and dynamic can in fact be asserted.\(^10\)

In order to avoid the charge of naive idealism, Lautman is careful to qualify the transcendence of Ideas as simply the ‘possibility’ of experiencing concern for a mode of connection between two ideas.\(^11\) The anteriority of Ideas is here rational or logical as opposed to psychological or historical.\(^12\) And this is precisely why Lautman argues that mathematics not only incarnates traditional metaphysical problems, it can also give birth to problems that could not have been previously posed. As we mentioned above, Lautman’s particular approach to the philosophy of mathematics does not so much consist in finding a well-established metaphysical problem within a mathematical theory. It is rather a matter of grasping the overall structure of a given theory in order to extract the problem that is at once defined and resolved in it.\(^13\)

Nevertheless, as Lautman goes on to argue, just as, in the very meaning of these terms, ‘intention’ precedes ‘design’ and the ‘question’ the ‘response’, the existence of established mathematical relations necessarily refers to the prior, positive Idea of the search for such relations.\(^14\) Or to put it another way, because the ‘sufficient reason’ for the diversity and development of mathematical theories, along with their progressive integrations and interferences, cannot be found within mathematics itself,\(^15\) one is obliged to affirm the prior existence of something like the dialectic of Ideas.
In short, to conceive of the historical development of diverse mathematical theories and their 'mixes' as responses or solutions to problematic Ideas is to give unity and meaning to these theories.\textsuperscript{16}

Now, it is of course clear that Badiou's concern is not that of the unity and meaning of mathematics in Lautman's sense. However, it will be the contention of this chapter that there is something like a dialectic of Ideas that traverses Being and Event, and that it is precisely this dialectic that allows Badiou to make the claim that 'ontology is set theory'. Or to put it another way, it is only because Badiou shows set theory to be capable of providing a systematic response to a series of dialectically opposed notions which can be found in the history of ontology (and philosophy more generally) that set theory can be said to be ontology. These dialectical couples include: the one and the multiple, Nature as poem and Nature as Idea, the finite and the infinite, and the continuous and the discrete. This chapter shall demonstrate how the Zermelo-Fraenkel axiom system which 'founds' Badiou's set-theory ontology in Being and Event resolves these dialectically opposed notions.

The One and the Multiple

Being and Event begins by outlining and then advancing a solution to the problem of 'the one and the multiple'. This is a problem, Badiou argues, with which any possible ontology will have to deal. It can be unpacked in the following way. Firstly, any presented concrete thing must be \textit{one}. A thing is, after all, \textit{this} thing. Secondly, however, presentation itself is \textit{multiple}, which is to say that what can be presented is presentable in multiple and variable ways. When it is asked whether being is one or multiple, therefore, one comes to an impasse. For, on the one hand, if being is one, then the multiple cannot be. On the other hand, if presentation is multiple and there cannot be an access to being outside of all presentation, then the multiple must be. But if the multiple is, then being is not equivalent to the one. And yet there is a presentation of \textit{this} multiple only if what is presented is one. Badiou then says that this deadlock can only be broken by declaring that the one, strictly speaking, is \textit{not}: oneness is rather only a 'result', a multiplicity which has been 'counted for one'. Badiou calls such a multiplicity a 'situation', and every situation must have a 'structure' which is the operator of its count-as-one (BE 23–4).

For Badiou, then, every identifiable being is in situation. In other
words, every being is a consistent multiplicity, counted-for-one. What is not in situation, what is not counted-for-one as this or that thing, could only be qualified as ‘no-thing’. There is, then, no-thing apart from situations, that is apart from consistent one-multiples, and these situations must all be posterior or subsequent to a structuring or count-as-one operation. However, at the same time, to say as I did above that the one is a result must mean that anterior to any possible count-as-one or consistent multiple there must be, and could only be, inconsistent multiplicity. In the final analysis, therefore, if the one is only ever a result, then inconsistent multiplicity – this no-thing which is outside of any situation – must ultimately be presupposed as the one-less ‘stuff’ or pure unqualified being of any possible being: that which is included in what any presentation presents (BE 24–5).

But now, what can be said about this pure, unqualified being? More specifically, how can it be presented to thought, not as some specific ‘thing’ or consistent one-multiple, but qua being? In other words, what could ontology be – the science of being qua being? Ontology, for Badiou, must be a situation, but it clearly cannot present inconsistent multiplicity as a one-multiple. Ontology must rather be a situation capable of presenting inconsistent multiplicity as that from which every presented or in-situation ‘thing’ is composed, but without thereby giving inconsistent multiplicity any other predicate other than its multiplicity. In other words, ontology will be the situation which ‘presents presentation’ in general (BE 27–8). And the only way it can do this, following Badiou, is by somehow showing in its very structure that this no-thing or inconsistent multiplicity exists, and that everything in the ontological situation is composed out of it, but without thereby counting this inconsistent multiplicity for one. And for Badiou, it is the axioms of set theory which fulfil this prior structural necessity, since they only give an implicit definition of what it operates on: the pure multiple (BE 28–30, 52–9). In short, then, for Badiou, insofar as set theory alone can respond to the above analysed ontological problematic, it is the only possible ontology.

So how exactly do the ZF axioms fulfil ontology’s a priori requirements, the requirements which, it is evident, correspond to nothing internal to set theory? First of all, it reduces the one to the status of a relationship, that of simple belonging, written ∈. In other words, everything will be presented, not according to the one of a concept, but only according to its relation of belonging
or counting-for-one: ‘something = α’ will thus only be presented according to a multiple β, written α ∈ β or ‘α is an element of β’. Secondly, the theory has only one type of variable and hence does not distinguish between ‘objects’ and ‘groups of objects’, or between ‘elements’ and ‘sets’. In other words, to be an element is not an intrinsic quality in ZF. It is a simple relation: to-be-an-element-of. Thus, by the uniformity of its variables, the theory can indicate without definition that it does not speak of the one, and that all that it presents in the implicitness of its rules are multiples of multiples: multiples belonging to or presented by other multiples. Indeed, and thirdly, via the ‘axiom of separation’, the system affirms that a property or formula of language does not directly present an existing multiple. Rather, such a presentation could only ever be a ‘separation’ or subset of an already presented multiplicity. A property only determines a multiple under the supposition that there is already a presented multiple (BE 43–8). Everything thus hinges on the determination of the initial pure multiple. But as was seen above, as a necessary consequence of the decision that the one results – called for by the problematic relationship between the one and the multiple – there must be, anterior to any count, inconsistent multiplicity, and it is this which is ultimately counted. It appears, then, that this inconsistent multiple – the void, the unpresentable of presentative consistency – is the absolutely initial multiple.

How, then, can the void have its existence assured, and in such a way that ontology can weave all of its compositions from it alone? As Badiou says, it is by making this nothing ‘be’ through the assumption of a pure proper name: 0 (BE 66–7). That the void is named is not to say, of course, that the void is thereby one. What is named is not the one of the void, but rather its uniqueness or ‘unicity’. In what sense is the void unique? Another axiom of ZF tells us this. This is the ‘axiom of extensionality’ which will fix the rule of difference or sameness for any two multiples whatsoever, that is according to the elements which belong to each. The void set, then, having no elements – being the multiple of nothing – can have no conceivable differentiating mark according to this axiom. But then, if no difference can be attested, this means that there is a unicity of the unpresentable within presentation. There cannot be ‘several’ voids: the void is unique and this is what is signalled by the proper name, 0 (BE 67–9).

So how does set-theory ontology weave its compositions out of
this proper name? What is crucial to this operation is the ‘power-set axiom’ or ‘axiom of subsets’. This axiom guarantees that if a set exists, another set also exists that counts as one all the subsets of this first set, thereby regulating or counting as one the internal compositions of a given being or situation. It has been seen what belonging means: an element (a multiple) belongs to a situation (a set) if it is directly presented and counted for one by this situation. *Inclusion*, on the other hand, concerns subsets or parts of a situation rather than directly presented elements. In other words, elements directly presented by a set can be re-presented, that is grouped into subsets that are said to be included in the initial set. Inclusion is written $\subset$: $\alpha \subset \beta$ or $\alpha$ is a subset (a part) of $\beta$.\(^{17}\) The power-set axiom gathers together or counts as one all such inclusions, all of the sub-compositions of internal multiples. It says that if a set $\alpha$ exists, there also exists the set of all its subsets: its power set $p(\alpha)$ (BE 81–4).

What, then, can be said of the void from the point of view of the difference between belonging and inclusion?

It has already been seen that the void is never presented: it never belongs to another multiple. What is more, since the void is the multiple of nothing, nothing belongs to the void. However, it can be shown both that the void is a subset of any set – it is universally included – and that the void possesses a subset, which is the void itself (BE 86). Indeed, it is impossible for the empty set not to be universally included. For, following the axiom of extensionality, since the set $\emptyset$ has no elements, nothing is marked which could deny its inclusion in any multiple (see also NN 64). Furthermore, then, since the set $\emptyset$ is itself an existent-multiple, $\emptyset$ must be a subset of itself (BE 86–7).

One can now begin to see how the axioms of set theory – which are, for Badiou, the ‘laws of being’ – weave compositions out of the void. The argument is as follows: since the void admits at least one subset, itself, the power-set axiom can be applied. The set of subsets of the void, $p(\emptyset)$, is the set to which everything included in the void belongs. Thus, since $\emptyset$ is included in $\emptyset$, $\emptyset$ belongs to $p(\emptyset)$. This new set, $p(\emptyset)$, is thus ‘our second existent-multiple in the “genealogical” framework of the set-theory axiomatic. It is written $\{\emptyset\}$ and $\emptyset$ is its sole element’: $\emptyset \in \{\emptyset\}$ (BE 89). Now, let us consider the set of subsets of $\{\emptyset\}$, that is $p(\{\emptyset\})$. This set exists, since $\{\emptyset\}$ exists. What, then, are the parts of $\{\emptyset\}$? There is $\{\emptyset\}$ itself, which is the total part, and there is $\emptyset$, since the void is universally included in any multiple. The multiple $p(\{\emptyset\})$ is thus a
multiple with two elements, \( \emptyset \) and \( \{\emptyset\} \). Woven from the void, this is, as Badiou puts it, 'the ontological schema of the Two', which can be written \( \{\emptyset,\{\emptyset\}\} \) (BE 92, 131–2). It becomes clear that this is where the unlimited production of new multiples begins, woven from the void in accordance with the laws of being (and particularly the power-set axiom). For, since this set, \( \{\emptyset,\{\emptyset\}\} \), exists, one can consider its power set \( p(\{\emptyset,\{\emptyset\}\}) \), etc. . . . This process can obviously be repeated indefinitely and it is in fact in this way that one can generate our counting numbers, our 'natural' or 'ordinal' numbers (also called Von Neumann ordinals):

\[
\begin{align*}
0 & = \emptyset \\
1 & = \{\emptyset\} = \{0\} \\
2 & = \{\emptyset,\{\emptyset\}\} = \{0,1\} \\
3 & = \{\emptyset,\{\emptyset\},\{\emptyset,\{\emptyset\}\}\} = \{0,1,2\} \ldots
\end{align*}
\]

**Nature as Poem and Nature as Idea**

Indeed, it is from this generation of 'natural' numbers, all woven from the void in accordance with the axioms of being, that Badiou will establish his ontological concept of 'Nature'. Or more precisely, that Badiou understands Nature in this way is the result of the way in which set-theory ontology provides a resolution of the tension, highlighted since the work of Heidegger, between Nature understood poetically as appearance or the poetic coming-to-presence of Being (the pre-Platonic poem), and Nature interpreted as Idea, subtracted from all appearance (in the manner of Plato) (BE 123–9; on Plato see also BE 31–7). In other words, within the perspective of a set-theoretical ontology, Badiou will be able to find another arrangement of these two opposed orientations. In short, following Heidegger, he will maintain that Nature is 'the stability of maintaining-itself-there' within the opening forth of its immanent coming-to-presence. On the other hand, he will mathematise the Platonic subtraction of being from appearance. Or again, he will develop a concept of Nature as a network of multiples which are interlocking and exhaustive without remainder, but which are also woven entirely from what is subtracted from all presence: the void. The point is, of course, that without reference to the opposing conceptions of Nature belonging to Heidegger and Plato, the assertion that natural or ordinal numbers formalise the being of natural things would appear somewhat arbitrary or as a play on
words. Certainly, nothing within set theory itself authorises such an ontological appropriation of the generation of ordinals.

Let us follow Badiou as he formulates his concept of Nature in the wake of this dialectical couple. On the one hand, conceding the stability of Nature to Heidegger, a multiple $\alpha$ will be said to be ‘natural’ (also called normal, ordinal or transitive) if every element $\beta$ of this set is also a subset or part (that is, if $\beta \subseteq \alpha$, and if every element $\gamma$ of $\alpha$ is itself natural in this way (that is, if $\gamma \subseteq \beta$). This doubling of belonging and inclusion guarantees that there is nothing uncounted or unsecured in natural multiples which might contradict their internal consistency and concatenation. Just as Nature can never contradict itself, natural multiples remain homogeneous in dissemination. Every natural multiple is here obviously a ‘piece’ of another, for, by the definition of inclusion, if $\beta$ is included in the natural multiple $\alpha$, every element $\gamma$ that belongs to $\beta$ must also belong to $\alpha$, and so on (BE 123–9).

On the other hand, mathematising Platonic subtraction, it can be said that the name of the void founds the series of natural multiples, conceived of in the way that has just been seen, in the double sense of formalising its concept and acting as its indivisible limit or atom. As examined above, an unlimited series of natural multiples can be generated from the void and the laws of ontology. For not only does the element $\{0\}$ have $0$ as its unique element, since the void is a universal part, this element $0$ is also a part. Furthermore, since the element $0$ does not present any element, nothing belongs to it that is not a part. There is thus no obstacle to declaring it to be natural. As such, the power set of $\{0\} = p(\{0\})$ or the Two: $\{0,\{0\}\}$ – is natural, and all of its elements are natural, and so on. Ordinal numbers thus both formalise the concept of natural multiples within set theory and are themselves existing natural multiples. And what is more, the name of the void is the ultimate natural element or atom which founds the entire series, in the sense in which the void is the ‘smallest’ natural multiple. In other words, if every natural multiple is a ‘piece’ of every other, the void is the only natural multiple to which no further element belongs (BE 130–40).

Needless to say, however, in Badiou’s set-theoretical concept of Nature, there can be no possible formulation of Nature-in-itself. For Nature-in-itself would have to be a multiple which makes a one out of all the ordinals. But since this multiple would itself have to be an ordinal to make a one out of all the ordinals that
belong to it, it would have to belong to itself. However, since no set can belong to itself, Nature in itself can have no sayable being (BE 140–1). Indeed, that no founded or consistent set can belong to itself is a fundamental presupposition of set theory. The ZF axiom system can even be said to have arisen in response to the paradoxes induced by self-belonging, such as those demonstrated by Russell (BE 40–3). In fact, the ZF ‘axiom of foundation’ was formulated in order to exclude the introduction of sets which belong to themselves. This axiom says that a set is founded if it has at least one element whose elements are not themselves elements of the initial set, that is, if it contains an element which has no members in common with the initial set. It is thus obvious that no set founded in this way can belong to itself (BE 185–7).

The Finite and the Infinite

This last point leads to a further problem, even if Badiou does not pose it in quite this way. What is crucial here is that this problem corresponds to that of the ontological problem of ‘being-in-totality’. It has been seen that there cannot be a set of all sets which would govern the total count. But this does not in any way dispense with the task of examining the operation of the count. For precisely, when one turns to examine it, one notices something strange: because the one is not, because the count-as-one is only an operation, something always escapes the count-as-one and threatens thereby to ruin consistency. This ‘something’ is nothing other than the count itself, and this is true of natural as much as non-natural situations (BE 93–4). In other words, because the ‘one’ is only an operational result, if the count-as-one is not itself counted for one, it is impossible to verify that ‘there is Oneness’ is also valid for the counting operation. ‘The consistency of presentation thus requires that all structure be doubled by a metastructure which secures the former against any fixation of the void’, that is against any inconsistency (BE 93–4). This metastructure of a structured set – what Badiou also calls the ‘state of the situation’ (BE 95) – is precisely the power set which counts as one all of the initial set’s parts. That is to say that it counts all of the possible internal compositions of the elements of the initial set up to and including the ‘total part’: the composition of elements that is the initial set. ‘The completeness of the initial one-effect is thus definitely, in turn, counted as one by the state in the form of its effective whole’ (BE 98).
Be that as it may, one cannot dispense in this way with the problem of the completion of the count of one-results without also dealing with a second historico-philosophical problem, a problem which can be phrased as: what is the relationship between being-in-totality and the finite/infinite couple? Or again: in the shadow of the problem of being-in-totality, what does it mean to say with the moderns that Nature is essentially infinite (BE 143)? Following Badiou’s reconstruction of the history of the relationship between being-in-totality and the finite/infinite couple, one observes first of all that Aristotle’s ontology was a finite ontology, since he refused to accept the existence of anything actually infinite or ‘non-traversable’ in nature. Indeed, for Aristotle, infinity could only be ‘potential’. Medieval ontology, for its part, kept the finite Aristotelian ontology and supplemented it with an infinite being: God. Being-in-totality was thus here distributed into finite and infinite beings, God representing the ‘punctual limit’ of what finite beings cannot know (BE 142-3). Now, however, with the moderns, the concept of infinity shifts from God to Nature. But this does not mean that Nature is likened to a de-punctualised God. Indeed, as shown in Kant’s antinomies, the one of Nature is illusory. Thus, following Badiou, since the one is not, that Nature is infinite must necessarily mean that presentation itself is infinite, and indeed infinitely infinite. If the one is not, there cannot be any one-infinite-being but only, as will be seen, numerous infinite multiples. The recognition of the infinity of Nature, the infinity of being, is the recognition of the infinity of situations: the count-as-one, even of a finite natural multiple, concerns an infinity of infinite multiples (BE 143-6).

What does it mean exactly when Badiou says that Nature or the count-as-one concerns infinite multiples? To say that situations are essentially infinite must mean that the finite is itself derived from the infinite. For, precisely, would not the succession of finite natural multiples or ordinals have need of the infinite in order to qualify it as the one-multiple that it is, that is in order to form-one out of all of its terms? This is what the ‘axiom of infinity’ declares: there exists an infinite limit ordinal, \( \omega_0 \), and for all \( \alpha \), if \( \alpha \) belongs to this limit ordinal, and if \( \alpha \) is not void, then \( \alpha \) is a finite, natural successor ordinal (\( \emptyset \) of course is the initial existent multiple, not a successor). One can thus see that infinity counts-as-one all of the successor ordinals insofar as it is the ‘support-multiple in which all the ordinals passed through mark themselves, step by step’ (BE 155-6).
Strictly speaking, however, infinity is not simply equivalent to the limit ordinal $\omega_0$, for one can also generate infinite successor ordinals for it such that, precisely, $\omega_0 \in S(\omega_0)$ (also written $\omega_1$) (see BE 275-7). So, then, an ordinal is infinite if it is $\omega_0$ or if $\omega_0$ belongs to it. An ordinal is finite if it belongs to $\omega_0$ (BE 158-9).

It is thus in this way that Badiou can affirm, with the moderns but also within his set-theory ontology, that Nature is infinite. Or again, that being qua being is infinite. Or finally, that what can be said of being qua being – the presentation of inconsistent multiplicity or of what would be presentation in itself – essentially concerns infinite multiples and indeed, since one can always generate further infinite successor ordinals, an infinite number of infinite multiples (see BE 275-7). Yet this is not the end of the problem of the distribution of the finite and the infinite within being-in-totality. For it must now be asked: what here becomes of the necessary re-securing relationship between presentation and re-presentation – between the count and the count of the count – with respect to this understanding of the essential infinity of natural presentation? For a finite set of $n$ elements, the power set is obviously equivalent to $2^n$, but what could the power set of an infinite set possibly amount to?

**The Continuous and the Discrete**

In fact, the more precise question that Badiou asks is the following: is the power set $p(\omega_0)$ – that is to say, the count-as-one of all possible subsets of the complete series of finite natural numbers, sufficient for a complete numerical description of the geometrical continuum – equivalent to $\omega_1$, the smallest infinite natural multiple which directly succeeds and counts-as-one $\omega_0$? This is Cantor’s famous ‘continuum hypothesis’ (see BE 295). The importance of this hypothesis is that, if it were true, we would have a ‘natural measure’ for the geometrical or physical continuum. Or in other words, we would have a quantitative knowledge of being qua being. For, if the continuum could be numerically measured, every discrete multiple could be quantitatively secured therein. The ‘great question’ of Badiou’s set-theory ontology, translating the problematic couple continuous/discrete, is thus: is there an essential ‘numerosity’ of being (BE 265)? The answer is: we possess a natural measuring scale (the succession of ordinals), but it is impossible to determine where, on this scale, the set of parts of $\omega_0$
is situated (BE 277–8). Or more precisely, following the work of Cohen and Easton, it appears that

\[ p(\omega_0) \text{ is equal to } \omega_{347} \]

or \( \omega(\omega_1)_{18} \), or whatever other cardinal as immense as you like . . .

Easton's theorem establishes the quasi-total errancy of the excess of the state over the situation. It is as though, between the structure in which the immediacy of belonging is delivered, and the metastructure which counts as one the parts and regulates the inclusions, a chasm opens. (BE 280)

To recap: on the one hand, the One is not and being qua being essentially concerns an infinite number of rigorously defined, infinite, natural multiples, all woven from the void (BE 269: 'being is universally deployed as nature'). On the other hand, the 'there is Oneness' of the presentation of such multiples – the count of the count – must be completely secured in order to render these discrete 'one'-beings consistent (BE 93–4). But now this means that, if we had a measure for this void-less continuum we would also have a quantitative knowledge of being qua being. This measurement cannot, however, be fixed. This 'un-measure', that is to say this variant on the enduring metaphysical problem of the relationship between the discreet and the continuous – itself the more general expression of the question of the distribution of the finite and the infinite within being-in-totality – is what Badiou calls the 'impasse of ontology' (BE 279). To resolve it, Badiou will be led to a consideration of what, with Cohen's 'ontological' technique of 'forcing', corresponds to the philosophical notions of the event, the subject and truth.

We cannot examine in detail these further developments. Suffice it to say that what Badiou calls an 'event' will be an unfounded multiple (inscribed in ontology by the supplementary signifier \( \square \)) which supplements the situation for which it is an event. It will be a self-founding 'supernumerary' something – named or posited as existent – whose place cannot be recognised in the situation as given, even though it can come to belong to or be counted within that situation, giving thereby the general 'one-truth' of said situation. This supplementation by the event will call for a 'subject' who asserts and then verifies – by examining one by one the connection of the infinite number of in-situation multiples to the event – the existence of the supernumerary event in the situation. This
subject ‘is’ here nothing other than a finite multiple or ‘fragment’ of an infinite procedure of verification, a finite fragment which maintains a law-like relation to the aforementioned ‘one-truth’ which can be articulated in ontology (forcing). Finally, the ‘truth’ of the situation will be the ‘indiscernible’ or ‘generic’ multiplicity which will have resulted from the necessarily infinite procedure of verification which groups as ‘one’ all of the terms of the situation that are positively connected to the name of the self-founding event.\textsuperscript{21}

Or again, to put it more ‘ontologically’, Cohen’s technique shows that sets of conditions of a generic subset $\mathcal{Q}$ can be constructed which force, in a generic extension, the number of parts of $\omega_0$ to surpass an absolutely indeterminate cardinal $\delta$ given in advance (see BE 420-6). This is the effective ‘ontological proof’ of the ‘un-measure’ of the continuum. But at the same time, as Badiou argues, this proof produces within ontology a ‘one’ account of inconsistent being qua being. How? In short, it constructs an infinite generic multiple by collecting, starting from the void, series of multiples attached to a supplementary, evental signifier $\mathcal{S}_2$. But because it is not itself ‘discerned’, this generic multiple sets no limits to what it can rigorously collect as one and is thus, in the final analysis,

composed of terms which have nothing in common that could be remarked, save belonging to this situation; which, strictly speaking, is its being, qua being . . . It is rightfully declared generic, because, if one wishes to qualify it, all one can say is that its elements are . . . [This is] the truth of the entire situation, insofar as the sense of the indiscernible is that of exhibiting as one-multiple the very being of what belongs insofar as it belongs. (BE 338–9)

\section*{Conclusion}

Ultimately, then, it appears that the ontology outlined in \textit{Being and Event} is grounded insofar as it resolves, in specifically set-theoretical terms, a series of dialectical couples: the one and the multiple, Nature as poem and Nature as Idea, the finite and the infinite, and the discrete and the continuous. This way of establishing set theory’s ‘ontological identification’ constitutes Badiou’s debt to Lautman and is fully in line with the former’s scattered remarks about the latter examined above. But it also follows from
this that while Badiou's ontology fulfils its aim of 'presenting presentation' in set-theoretical terms, it only does so in relation to another, prior and very different presentation of the ontological situation itself: that is, insofar as it must be thought as a response to a series of dialectical couples. But then this is also to say that being is equivocal in Badiou's system. Being is said once for the ontological situation insofar as it is grounded in a Lautmanian dialectic of problematic Ideas, and it is said once again for what the ontological situation, so determined, can say of being in general in a set-theoretical vocabulary.

Of course, Badiou would reply to this that if having an equivocal conception of being is what is required in order to think the particular, generic truth-procedures which collectively condition his philosophy, then he is happy to bear the criticism. As he writes in *Deleuze: The Clamor of Being* (although in relation to a reading of his system that differs from the one presented here, except in relation to the charge of equivocity):

> Deleuze always maintained that ... I fall back into transcendence and into the equivocity of analogy. But, all in all, if the only way to think a political revolution, an amorous encounter, an invention of the sciences, or a creative work of art as distinct infinities - having as their condition incommensurable events - is by sacrificing immanence (which I do not actually believe is the case, but that is not what matters here) and the univocity of Being, then I would sacrifice them. (D 91–2)

But what disadvantages does such a conception in fact present, if any? The first disadvantage, of course, is that Badiou does not have a single or unified concept of being. It is true that, in the history of ontology, being has often been said in different senses: in Aristotle, for example, but also in the work of various medieval philosophers, for whom God 'is' in a different way from the way in which his creatures 'are' (Duns Scotus here being the notable exception). Nevertheless, Ockham's Razor could apply here, leading one to prefer an ontology in which being is said in a single sense of all there is.

A second disadvantage would be that, because Badiou's ontology presupposes a prior dialectic of problematic Ideas but does not itself think the nature of this dialectics, it cannot think its relation to another philosophical system presenting a different but equally systematic solution to the same problems resolved by Badiou's
ontology, except as irreducible subjective conflict pure and simple. Again, this would not concern Badiou, who has a militant conception of the subject. But perhaps it would be of concern for those seeking a more supple approach to thinking the relations between the antagonistic subjectivities – political, scientific and so on – which can fall under different philosophical world views.

Taking these two points together, then, one can ask oneself the following critical question: can we conceive of a univocal conception of being which would be grounded solely on a Lautmanian-style dialectic of problematic Ideas, wherein beings in general could be considered to be 'solutions' to this dialectic, and in such a way that the various antagonistic subjectivities characterising our contemporary world can be thought together without irreducible conflict? I believe that Gilles Deleuze, another follower of Lautman, has developed such an ontology, particularly in his *Difference and Repetition*. As Deleuze writes:

The problem is at once transcendent and immanent in relation to its solutions. Transcendent because it consists in a system of ideal liaisons or differential relations between genetic elements. Immanent, because these liaisons or relations are incarnated in the actual relations which do not resemble them and are defined by the field of solution. Nowhere better than in the admirable work of Albert Lautman has it been shown how problems are first Platonic Ideas or ideal liaisons between dialectical notions, relative to 'eventual situations of the existent'; but also how they are realized within the real relations constitutive of the desired solution with a mathematical, physical or other field.22

Indeed, in *Difference and Repetition*, Deleuze shows how entities in diverse domains – physical, biological, psychological, social, linguistic, mathematical – can all be considered to emerge as solutions to Lautmanian 'problematic Ideas'. Moreover, Deleuze shows how the subject who thinks and 'actualises' problematic Ideas is itself merely a provisional 'solution' to these latter, which means that subjective conflict is really only an illusory 'freezing' of the underlying and ever-shifting differential relations constituting the dialectic of Ideas.23 Such a Lautman-inspired ontology, then, it would appear, might offer a way around some of the difficulties associated with Badiou's fascinating project. Nevertheless, the full justification of this claim cannot be dealt with here.
Notes


2. The French reads: ‘Comme l’ont montré en particulier, dès les années trente, les travaux d’Albert Lautman, tout fragment significatif et novateur de la mathématique réelle peut et doit susciter, en tant que condition vivante, son identification ontologique.’

3. Emblematic in this regard is Oliver Feltham’s excellent ‘Translators Preface’ to *Being and Event*. See in particular BE (xvii–xviii).


5. Badiou here distinguishes quite sharply between his and Lautman’s Platonism on the one hand, and Anglo-analytic mathematical Platonism on the other. Whereas the former can be characterised as a ‘dialectical Platonism’ which focuses on the question of participation or the intersection between the sensible and the intelligible, the latter is characterised by its ontological commitment to the independent existence of abstract mathematical objects.


8. Ibid., pp. 28, 189, 223.


10. Ibid., p. 223 – original emphasis removed.

11. Ibid., p. 189.

12. Ibid., pp. 221–2.

13. Ibid., p. 189.

14. Ibid., p. 204.


16. The term ‘mixes’ is Lautman’s. See *Mathematics*, pp. 157–70.

17. It should here be noted that inclusion is not really another primitive relation, to be added to that of belonging. Rather, inclusion can be defined on the basis of belonging, for $\beta \subset \alpha$ is equivalent to saying $(\forall \gamma)(\gamma \in \beta \rightarrow (\gamma \in \alpha)]$, or again, for all $\gamma$, if $\gamma$ belongs to $\beta$ then $\gamma$ belongs to $\alpha$ (BE 82).


19. And not only infinite successor ordinals, but also infinite limit ordinals. Consider the series: $\omega_0,\omega_1,\omega_2 \ldots \omega_n,\omega_{n+1} \ldots \omega_{(\omega_0)},\omega_{(\omega_0)+1} \ldots \omega_0$ (BE 275–7).

20. On the admitted importance of ‘the famous “problem of the continuum”’ for Badiou, see BE (5 and 281).

21. The reference here to events in their opposition to structured situations, as well as the reference to processes of transformation of situations, makes it clear that what is in play here is not only the couples continuous-discrete and finite-infinite, but also the couple fixity-change.
