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Learning objectives

1. To create interest in and passion for learning and teaching primary and junior secondary school mathematics.
2. To develop a commitment to and persistence in deepening mathematical and mathematical pedagogical knowledge.
3. To look for and recognise student engagement in mathematics.
4. To be able to discuss attributes of a caring student-centred approach to teaching mathematics.
5. To plan lessons and teaching experiences using a student-centred approach.
Chapter 5 Student-centred teaching of mathematics

Introduction

A sense of service to community and a desire to make a difference in young people’s lives typically characterise people’s reasons for deciding to become a teacher. These objectives signify that prospective and practising teachers care about their students and are focused on enabling their students to learn and achieve their aspirations. Prospective teachers acknowledge that in the primary years of schooling lie the foundational knowledge, skills and learning behaviours necessary for lifelong learning, with each primary teacher working with their students across the broad range of disciplinary, social and cultural fields of knowledge. The student-focused assumption of primary teachers’ practice is revealed in the common saying that ‘primary teachers teach students while secondary teachers teach subjects’. But what does this actually mean? And is it true? How do primary teachers maintain a focus on the student while engaging him or her in particular fields of learning such as mathematics? How do secondary teachers of mathematics focus their attention on the students and their learning in mathematics classrooms? In this chapter original research and practice conducted by the authors are examined in tandem with the work of other researchers and teachers to examine and explain what it means to engage students in mathematical thinking and to teach mathematics using a student-centred approach.

Many pre-service primary teachers and practising teachers display a lack of confidence and even a degree of anxiety at the prospect of teaching mathematics. Such apprehensiveness is often a result of their own disengaged learning experiences in mathematics classrooms and pre-service teachers can usually describe specific experiences in classrooms in terms of a particular teacher who shaped their attitude to mathematics (Uusimaki & Nason 2004). Even those pre-service teachers who are confident in their knowledge of mathematics can feel anxious about the flexibility required to respond to the diversity of students’ understanding and attitudes to mathematics (Williams 2011). Pre-service teachers who lack confidence and have low or negative attitudes to mathematics normally take one of two paths during their pre-service education: some try to avoid mathematics and put off confronting their knowledge and beliefs by seeking teaching practice experiences only in the early years of primary school; others accept the challenge and take up opportunities to learn and teach mathematics whenever and wherever possible, by, for example, enrolling in appropriate mathematics units of study, choosing the upper primary grades for their teaching practice and volunteering to tutor students in mathematics (Livy 2012). It is hoped that by engaging with the ideas presented in this chapter that both pre-service and practising teachers will reflect on their own experiences of learning and teaching mathematics to develop an understanding of engagement in mathematics.

Evidence of widespread disengagement in mathematics is not hard to find. It is revealed in numerous studies of mathematics achievement, participation in senior secondary mathematics, and attitudes to mathematics, especially those studies that probe these outcomes for low socioeconomic students, Indigenous students and girls
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(Atweh, Vale & Walshaw 2012). Pre-service teachers typically want their students to be positive about mathematics learning; they want them to have fun and often choose tasks, such as games, which they hope their students will enjoy. Fun is an admirable and desirable objective, but some fun activities, including some games, result in superficial or disconnected learning. Games, and other learning tasks, need to be selected and structured so as to engage students with mathematical ideas and in mathematical thinking and to enable them to build with and on their mathematical knowledge (Bragg 2003, 2006). The bringing of joy and success to mathematics learning is a complex endeavour. In this chapter this complexity is explored and untangled and the elements and examples of practice that serve to engage students in mathematics learning that is both joyous and successful are described and explained.

Engagement in mathematics

When students are engaged in mathematics they are in a positive state of mind. They are motivated and optimistic, expect to experience success and are resilient in the face of challenges that they encounter or that they set themselves when solving problems (Bobis et al. 2011; Williams 2009). They display adaptive behaviours of persistence, planning and task management (Martin 2007). Persistent students are prepared to put effort into completing tasks and continue to work on mathematics tasks even after they have encountered blockages, errors and challenges. They break tasks down into manageable chunks and, in so doing, take control of their learning. They revise tasks and information that they don’t fully understand. They enjoy learning.

Learning with and from other students by collaborating on group tasks facilitates engagement with mathematical ideas and problems (Boaler 2008; Kazemi, Lampert & Franke 2009; Williams 2009). Respectful relationships are necessary conditions for fostering constructive interactions between students and their teacher in the classroom; and for collaboration with peers as they work with mathematical ideas that are within reach or on the brink of their understanding (Boaler 2008; Kazemi, Lampert & Franke 2009; Williams 2009). Noddings (1992) argues that respectful relationships are based on trust and care; if children are to listen, participate and learn, they must not only feel safe but also know that their teacher is interested in them. Children feel safe, and respectful relationships develop, when teachers establish classroom norms that provide for equitable and respectful participation. An equitable classroom is one in which students can speak and be heard in whole-class and small-group discussion and where teachers model the way to ask good questions and to seek further explanations or justifications. Students are respected and treated fairly when the teacher and other students listen to their ideas.

There are usually a number of ways to solve a mathematics problem and some problems have multiple solutions. When the teacher (or another student) asks a student to explain or justify his or her thinking, then that student’s idea is being valued
and the request invites the student to consolidate and deepen his or her own understanding as well as provides other students with the opportunity to learn from a fellow classmate. The posing of questions that enable students to make connections between different ideas enhances respect and serves to deepens students’ mathematical thinking. Respectful relationships are also displayed when students show a commitment to the learning of other students in their group and classroom (Boaler 2008). For example, when working collaboratively, these students check that their group members understand the task; they ask questions to prompt others’ thinking and so enable them to reach an understanding of what needs to be done or how to begin solving the problem. Respectful students also invite their peers to share their ideas with others so as to compare and contrast ways of thinking and assess possible solutions.

Lev Vygotsky (1978) describes working at the edge of one’s current understanding with the support of significant others, be they teachers or peers, as being in the ‘zone of proximal development’. Successful learning in this zone instils optimism and encourages effort and the belief that failure is only temporary (Martin 2007; Williams 2009). Teachers, especially those teaching low-achieving students in ‘like achievement’ groups, often believe that setting tasks only within the students’ comfort zone will provide opportunities for experience of success. However, repeated success on mathematical tasks that lie well within a student’s range of understanding leads to boredom, to uncertainty when confronted with new contexts or content, to the reinforcement of low expectation and poor self-efficacy, and contributes to behaviours wherein students seek to avoid failure (Bobis et al. 2011) by non-participation. Holding high expectations of students is universally accepted as a necessary condition for enhancing students’ self-worth and achievement in mathematics (Anthony & Walshaw 2007; Boaler 2002, 2008; Hayes, Lingard & Mills 2000; Vale et al. 2010). This means that teachers expect each student to learn and to develop more complex ideas and use demanding skills in mathematics, while simultaneously acknowledging and appreciating their current knowledge and skills.

Teachers focus, or centre, their teaching on their students when they design and set mathematics tasks that enable their students to build their understanding ‘from within’ (Boaler 2008). This means selecting, or designing, a task so that it connects with students’ prior learning and lies within their zone of proximal development so that they can construct new understandings of mathematics and thereby demonstrate a higher level of understanding, complexity and competency (Black 2007; Vygotsky 1978; Williams 2009). Mathematics tasks that also connect with students’ cultural knowledge and experiences demonstrate an ethic of care by the teacher and enable students to build their understanding ‘from within’ in a different sense, one where mathematics is personalised and hence meaningful to them (Black 2007; Boaler 1998, 2005; Bobis et al. 2011; Noddings 1992; Ocean & Miller-Reilly 1997; Vale 1999).

Just as engagement in mathematics builds optimism for school students, so too engagement in mathematics builds optimism for pre-service teachers as they prepare and learn to teach mathematics (Lampert et al. 2010; Livy 2012; Williams 2011). Gaye
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Williams (2011) observed the way in which pre-service teachers’ collaboration on mathematics tasks, including on problems they did not recall having done, led to feelings of success and contributed to an optimistic orientation towards the teaching of mathematics. When working on these problems the pre-service teachers were able to behave as their prospective students may behave in the way that they approached and solved these problems. By focusing on finding and sharing a method of solution rather than on getting the correct answer, and by doing so together, the groups of pre-service teachers built and shared a repertoire of methods and solutions for these problems. Engaging with others on these tasks enabled the pre-service teachers to focus on building their mathematical knowledge without the requirement that they simultaneously focus on pedagogical issues. Pedagogical understandings were developed subsequently, after the mathematical task was completed or almost completed, when the pre-service teacher class reflected as a group on the mathematical thinking of all its participants and when they discussed what was happening pedagogically to enable this thinking to occur.

Joy and success in mathematics

The discussion turns now to examples of mathematics tasks and teachers’ actions to illustrate these key ideas of a caring student-centred approach. Readers are invited to participate in each mathematical task as described and to share their mathematical thinking with their pre-service teacher colleagues prior to discussing the pedagogical characteristics.

Personalising learning

In the two lessons described here, optimism is established by connecting mathematical ideas to the personal lives of children. In the first lesson the teacher calls on the children’s knowledge of his or her family and in the second to imagine a character.

Teenagers: a context for teen numbers

For children in their first or second year of schooling (Year F–1), the numbers from 11 to 19 are challenging and some children develop misconceptions about these numbers and the number system itself. It is desirable that children understand that our number system is based on 10 and includes 10 digits. For multi-digit numbers the place of each digit determines its numerical value. In some other languages, for example, Chinese, Greek and Vietnamese, the words for the numbers written as 14 and 24 translate literally as ‘ten and four’ and ‘two tens and four’, respectively; the English names for these numbers confuse the relationship between the place of digits and the order of the words in its name. The word ‘twenty-four’ does not indicate to young children that it means ‘two tens and four.’ They must learn that it does even though they may be able to count way beyond 24. The suffix ‘-ty’ does mean a multiple of 10 and the prefix ‘twen’ is derived from the Old English word for two. For the numbers 11 to 19 the connection between the symbols for the number and its name is back to front. The term for the number of tens
is at the end of the number’s name rather than at its beginning, for example fourteen. The numbers 11 to 19 are the ‘teen’ numbers though 11 (eleven) and 12 (twelve) do not conform to the way of naming the other teen numbers. Two common misconceptions occur with teen numbers. First, some students will write 31 when they hear ‘thirteen’, others may read 13 as ‘thirty-one’. They confuse the sound and pronunciation of ‘thirty’ and ‘thirteen’ or they confuse the order of the number name with the order of the digits. Therefore it is important to teach the two-digit numbers up to 20 separately from other two-digit numbers during the early years of schooling (Booker et al. 2010).

Consider an actual mathematics lesson, observed by the authors, in which the teacher began a lesson about the ‘teen’ numbers with a whole-class discussion and followed this with a series of tasks about teen numbers. After the students gathered on the mat together the teacher introduced the lesson by asking the students their ages: ‘How old are you?’ She then asked: ‘Who has a brother or a sister or a cousin who is a teenager?’ Some students replied that they had: ‘I have a brother who is thirteen and a sister who is sixteen.’ The teacher made a list of all these ages on the board – that is, she wrote the ‘teen’ numbers on the board using symbols and words. In this way the teacher connected the mathematics lesson to the personal lives of the children. She then drew on this knowledge to probe further about the meaning of ‘-teen’ and of teenage: ‘Would I be a teenager?’ ‘Would the principal be a teenager?’ ‘Why or why not?’ The teacher then asked the students to look carefully at the numbers and their names and asked them if they noticed anything special about these numbers. One student explained, and others agreed, that they all had ‘teen’ in them. The teacher then underlined this part of the word under the students’ directions. Before explaining the tasks for the remainder of the lesson, one student suggested that 20 is a teen number. The teacher wrote the word twenty on the board and through a series of questions established, with agreement of the students, that it is not a teen number, though it did sound very similar. She told them to ‘watch out and not get tricked by twenty’.

This discussion was meaningful to the students because they were able to relate the ‘teen’ numbers to the various ages of people in their families rather than having them be a mere abstract list of symbols. The discussion also led to the understanding that ‘teen’ numbers represented ages greater than 10 (older than children) but less than 20 (younger than adults). The teacher listened and accepted all the students’ responses, asking questions in order to build their knowledge from within, to clarify misconceptions, to check the connections that they were making and to assess their understanding (Lampert et al. 2010; Parish 2010).

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they rolled out aloud and then locate it on their number track and place a counter on that number. The children took turns rolling the die to cover as many numbers as was possible in the time provided. Other tasks involved: pegging digit cards to order the numbers along a length of string; rolling the die to name and make the number on a ‘tens frame’; matching the written word to the digits and collection; and using the digit cards to order the numbers from zero to 20. The teacher set these tasks to emphasise that vocabulary is important and that oral language, saying things aloud, is important for recognition and making sense of symbols and words in mathematics.

The students enjoyed these tasks because they were worked with another student from whom they could seek help if needed. They checked each other’s spoken names for the numbers as they worked on the various tasks. Establishing expectations about the way in which these children were to work together and communicate with each other was an important action on the part of the teacher ‘launching’ these tasks. The teacher fostered respectful relations as an important feature of the lesson, which included purposeful tasks.

**Collaborative teachers in action 5.1**

**A shape story: contextualising mathematical reasoning**

For a lesson about the properties of shapes the teacher retold, from memory, the story of an adventurous Triangle from a picture storybook called *The Greedy Triangle* (1994) by Marilyn Burns:

Triangle loved being a triangle until one day when he decided he wanted to change. He went to the shape fix-it man and changed into a quadrilateral. The next day he went back to the shape fix-it man because he didn’t like being a quadrilateral. He got one more side and one more angle.
The story continued, with the character adding another side and another angle each day until the day it became a decagon. After sharing the story the teacher recorded the names of the various shapes – triangle, quadrilateral, pentagon, etc. – on the board and then asked the students to draw each of the different shape characters from the story and to name them. While the children worked on their drawings the teacher roamed the room, monitoring the students’ work on their drawings. She asked the children about their pictures and what they were doing. She asked such questions as: ‘How do you know this one is called hexagon?’; ‘What is the difference between a nonagon and an octagon?’; ‘Your pentagon has five sides and five angles. Is this the same for all your shapes?’ The teacher was assessing students’ attention to the number of sides and angles. At the end of the lesson, during share-time, the teacher followed up this line of questioning, inviting the children to discuss similarities and differences between the shapes. For example, she asked: ‘How is this octagon the same as or different from your octagon?’ The students provided original responses and the teacher probed further so that they focused on the properties of the shapes – that is, on the number of sides and angles in each shape and whether the sizes of the sides and angles were the same or whether they could be different. When she asked ‘Did you enjoy maths today?’; one student replied with her own question: ‘Is this maths?’

The use of a story and imaginary characters enabled the children to personify and personalise the mathematical concepts (Noddings 1992). For at least one child, this lesson on geometry that used a storybook and invited the children to create geometric characters was not what she had previously regarded as mathematics. The drawing in Figure 5.2 illustrates the ‘happy’ shapes with smiles and waving arms produced by one of the Year 3 students.

![Figure 5.2: A child’s drawing of the character in different shapes](image-url)
Mathematical inquiry

Investigations or inquiries in mathematics are tasks designed so that students discover for themselves mathematical concepts, properties or relationships. The process of inquiry can vary between a structured investigation in which the teacher breaks down the investigation into a series of tasks, and an unstructured investigation in which the students are expected to select an approach from a list of known problem-solving strategies and conduct the investigation independently. Usually students collaborate in groups to complete these kinds of mathematics tasks. The three examples below illustrate structured investigations, each involving all of the students in the class working together. Two lessons are concerned with measurement concepts and skills, with one of these being set in a primary classroom and the other in a junior secondary classroom. The third lesson uses a ‘choral counting’ task (Kazemi, Lampert & Franke 2009; Parish 2010) in which the children identify number patterns as they participate in a whole-class count and discussion.

Perimeter of the classroom

Figure 5.3 shows a class of Year 3–4 students measuring the perimeter of their classroom. In this lesson the teacher was focusing on the concept of a ‘perimeter’ as well as on the use of informal units to measure distances. The picture shows the students doing maths together.

The teacher posed the problem: ‘What is the perimeter of our classroom?’ and ‘If we use our hands and fingers to measure the perimeter how many fingers would we need?’ To assist the students in developing number and measurement sense, the
teacher invited the children to guess how many fingers it would take to measure one wall (see Figure 5.3) and received answers ranging from 20 to one million. The students also discussed methods for counting all the fingers and hands. They thought that counting by ones using all their fingers and thumbs would be very slow. To find the perimeter of the classroom all of the children lined up around the edge of the room and put their hands up on the wall. They counted by fives and then by tens to find and then check their solution. The children needed to listen to each other so that they could continue the count as it progressed around the room. In this way the teacher provided an opportunity for the students to also practise their oral language for numbers and counting.

This lesson also shows the teacher building positive relationships among the children and developing their sense of commitment to each other’s learning (Boaler 2008). An appreciation of care and collaboration is exemplified in one child’s statement: ‘I really liked it when we had to measure the perimeter of our classroom and everyone helped’ (Year 3 student).

**Choral count: investigating number patterns and mathematical reasoning**

A choral count (Kazemi, Lampert & Franke 2009; Lampert et al. 2010; Parish 2010) is a whole-class activity in which the group counts together and aloud to investigate and use number patterns to extend the count. As the class counts, the teacher records the count on the board. The teacher stops the count from time to time to ask the students questions about how they are calculating the next number and the patterns that they see and are using in their calculations.

In a class of Year 3–4 students in the United States, the teacher asked the students to count by three and recorded the responses on the board in three columns. When planning the lesson she had realised that recording the count using three columns would assist the students to reason and identify the patterns that were the focus of the lesson. Figure 5.4 shows a similar three-column number count.

The teacher stopped the count at 24 and asked, ‘Ana, how did you know that the next number was 24?’, to which Ana replied, ‘21 plus 3 is 24’. ‘Henry, how did you know it was 24?’ The teacher continued this line of questioning to gather the different ways in which students were thinking; some just ‘knew’ the answer and others counted on by ones.

The teacher restarted the count from three and continued to 45. Going beyond 36 means that students are less likely to be counting by rote or using known facts for the multiples of three and will instead be employing a different strategy to find the next number in the count. The teacher began the questioning by asking again how they had reached the next number, before asking, ‘What patterns do you see?’ The children identified a pattern in the digits in the ‘ones’ and another in the ‘tens’ place
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Figure 5.3: Finding the perimeter of the classroom
teacher invited the children to guess how many fingers it would take to measure one wall (see Figure 5.3) and received answers ranging from 20 to one million. The students also discussed methods for counting all the fingers and hands. They thought that counting by ones using all their fingers and thumbs would be very slow. To find the perimeter of the classroom all of the children lined up around the edge of the room and put their hands up on the wall. They counted by fives and then by tens to find and then check their solution. The children needed to listen to each other so that they could continue the count as it progressed around the room. In this way the teacher provided an opportunity for the students to also practise their oral language for numbers and counting.

This lesson also shows the teacher building positive relationships among the children and developing their sense of commitment to each other’s learning (Boaler 2008). An appreciation of care and collaboration is exemplified in one child’s statement: ‘I really liked it when we had to measure the perimeter of our classroom and everyone helped’ (Year 3 student).

**Choral count: investigating number patterns and mathematical reasoning**

A choral count (Kazemi, Lampert & Franke 2009; Lampert et al. 2010; Parish 2010) is a whole-class activity in which the group counts together and aloud to investigate and use number patterns to extend the count. As the class counts, the teacher records the count on the board. The teacher stops the count from time to time to ask the students questions about how they are calculating the next number and the patterns that they see and are using in their calculations.

In a class of Year 3–4 students in the United States, the teacher asked the students to count by three and recorded the responses on the board in three columns. When planning the lesson she had realised that recording the count using three columns would assist the students to reason and identify the patterns that were the focus of the lesson. Figure 5.4 shows a similar three-column number count.

The teacher stopped the count at 24 and asked, ‘Ana, how did you know that the next number was 24?’, to which Ana replied, ‘21 plus 3 is 24’. ‘Henry, how did you know it was 24?’ The teacher continued this line of questioning to gather the different ways in which students were thinking; some just ‘knew’ the answer and others counted on by ones.

The teacher restarted the count from three and continued to 45. Going beyond 36 means that students are less likely to be counting by rote or using known facts for the multiples of three and will instead be employing a different strategy to find the next number in the count. The teacher began the questioning by asking again how they had reached the next number, before asking, ‘What patterns do you see?’ The children identified a pattern in the digits in the ‘ones’ and another in the ‘tens’ place
value of the numbers. For example, they identified that in the third column the digit in the ones place ‘goes down’ or decreases by one: 9, 8, 7, 6, 5 … and the digit in the ‘tens goes up’ or increases by one: 1, 2, 3, 4 … The teacher brought other children into the discussion by asking whether this pattern occurred in each of the columns. Some children identified the counting by nines pattern in the column and so the teacher asked, ‘Does this happen for the other columns?’ These questions invited students to check the sums for 30+9 and 33+9 and asked students to predict what number would be in blank spaces in the columns that they had not yet reached. She asked the students, ‘How do you know?’ She then restarted the count at 30 to discover whether the students’ predictions had been correct. By using the information elicited from students and shared by the class, the students were able to choose which strategy to follow, including the use of the pattern to find the next number in the count.

This structured investigation could be extended by asking: ‘Do you notice anything about the digits that make up the numbers in this count?’ Students might notice that some numbers have the same digits as other numbers in the same column but in reverse order – for example 12 and 21 in the first column and 24 and 42 in the second column (see Figure 5.4). When the students identify a pattern in the sum of the two digits, invite them to pose a conjecture about the summing of digits when counting by three. Then pose the question: ‘If we were to keep counting would 104 be included? How do you know? What about 97?’ Invite the class to continue the count so as to find out if their conjectures and predictions are correct. Finally, pose an open-ended problem to find other numbers that would be included in the count-by-threes, that is, that are multiples of three.

Elizabeth Warren (see Siemon et al. 2011, p. 254) uses a similar task to investigate multiples of three and to find numbers that are divisible by three. Instead of
‘choral counting’ the children use a hundreds chart upon which they place counters. It is best to use transparent, coloured counters on the numbers in the count (or to block out the numbers not in the count using other counters). Try this way of recording the count by threes to find out what is the same and what is different when it is recorded on a hundreds chart. Questions posed should focus on divisibility, on proving the rule and on extending the students’ thinking to enable them to find a pattern for numbers that are divisible by nine.

In this task the children reason mathematically as they explain their thinking, describe patterns and formulate conjectures. It is a collaborative task. The children count together and, with the teacher’s careful attention to questioning, they listen to and learn from each other, building on each other’s explanations and reasoning (Kazemi, Lampert & Franke 2009).

Pi ($\pi$): investigating the relationship between circumference and diameter

Many students retain misconceptions about pi ($\pi$) throughout junior secondary school, some of which are humorous but oddly logical such as the belief that pi is associated with circles because pies (such as meat pies and apple pies) are circular. Teachers often set students the task of deriving the relationship between the diameter of a circle and its circumference by asking them to measure the circumference and diameter of one or more circles and then to calculate the quotient or ratio. However, when students work alone and don’t take care to measure these properties accurately, their solutions can be different to the known approximation for pi ($\pi \approx 3.14$). Designing a similar investigation in which students must work towards a group or whole-class solution can help to avoid this problem.

A teacher of Year 9 students decided to conduct a whole-class investigation using digital technology. For this investigation the students measured many different circular objects and entered measurements of their diameters and circumferences into a spreadsheet. The teacher connected her laptop to a data projector so that all the students could see the collaborative findings. The teacher used a spreadsheet software program and involved all of the class in the designing of the table into which students recorded their measurements. The table included a column for the ratio of circumference to diameter. As each student entered the measurements of their circular object, the ratio was displayed for all to see. A cumulative average ratio was also displayed as the results were tabulated as shown in Table 5.1, a sample spreadsheet with some data entered. Other teachers who employ a similar whole-class exercise include a scatterplot of the data – that is, the circumference and diameter of each circular object.

The teacher observed that there was much suspense and ‘the big drama was when you hit [the] Enter [key] and it calculated pi [the ratio]. There was lots of competition.
### Table 5.1: Table of data showing the measurements of different circular objects

<table>
<thead>
<tr>
<th>OBJECT</th>
<th>CIRCUMFERENCE (MM)</th>
<th>DIAMETER (MM)</th>
<th>RATIO C/D</th>
<th>CUMULATIVE AVERAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drink bottle</td>
<td>228</td>
<td>72</td>
<td>3.1667</td>
<td></td>
</tr>
<tr>
<td>Drink can</td>
<td>204</td>
<td>65</td>
<td>3.1385</td>
<td>3.1526</td>
</tr>
<tr>
<td>Cup</td>
<td>274</td>
<td>87</td>
<td>3.1494</td>
<td>3.1515</td>
</tr>
<tr>
<td>Wall clock</td>
<td>820</td>
<td>260</td>
<td>3.1538</td>
<td>3.1521</td>
</tr>
<tr>
<td>Sticky tape roll</td>
<td>325</td>
<td>104</td>
<td>3.1250</td>
<td>3.1467</td>
</tr>
<tr>
<td>Vase</td>
<td>390</td>
<td>108</td>
<td>3.6792</td>
<td>3.2354</td>
</tr>
</tbody>
</table>

Could they get to 3.14? How close?’ She noticed that the students appreciated the need for accurate measurements and went back to their object and measured it again. She noted that they would come back and say ‘Oh no, my diameter was out by about 2mm, will that change my answer?’ This investigation enabled the students to appreciate the need for accuracy when measuring, to see the dynamic calculations of the ratio of circumference and diameter and to observe the convergence of the average ratio to approximately 3.14.

In this lesson the embodiment of respectful relationships among students was an important element of the classroom (Boaler 2008; Noddings 1992). The teacher accepted all measurements since each student entered his or her results into the digital table. Students were held to account for any inaccuracies and invited to explain and review their individual results by the public display of their measurements. They displayed commitment to their own learning and to that of their peers by deciding to check their results and to redo their measurements when the ratio for their measurements differed from the ratios found for other circular objects.

### Collaborative problem solving

Non-routine problem-solving tasks are ideal for promoting collaboration and engagement in mathematics. Students are encouraged to collaborate and engage in mathematical thinking when a problem is rich with connection to real situations, when there is more than one answer, when they are challenged to find as many solutions as possible, and when they are invited to explain and justify patterns in the solutions found. The teacher’s role is the establishment of respectful relationships among the students and the teaching of respectful communication that students should use when working in groups on problems. The teacher must ‘scaffold’ the students’ learning by selecting problems within the zone of their proximal development and
guiding their use of various problem-solving strategies by the ongoing monitoring and questioning of students while they work on a problem and during the sharing of strategies and solutions. Teachers can also support engagement with a task by offering prompts in the form of materials or modifications to particular aspects of a task in order to enable some groups to get started or proceed (Sullivan, Mousley & Zevenbergen 2006).

**Crosses: finding all possible solutions**

A mathematical investigation called ‘Crosses’, suitable for students in Years 3 to 8, provides an opportunity for students to work in small groups to trial and test solutions, to find as many solutions as possible and to explain why they think they may have found all of the possible solutions. ‘Crosses’ is an open-ended problem from the maths300 online collection of mathematics tasks (maths300 2010, Lesson 112). For this problem students arrange the digits 1 to 9 in a cross so that each arm of the cross adds up to the same number. Figure 5.5 shows one possible solution in which the arms add up to 27.

To support some students in beginning the task the teacher might modify the problem for them by suggesting which number to try in the middle of the cross, or by suggesting that they arrange the numbers on the cross to add up to a particular sum such as 25. In this way students are given a helping hand, and from this beginning the teacher, or other students, may continue to scaffold their learning by monitoring their thinking and asking questions to assist them in experiencing success and so encourage them to persist with the task to find other solutions. The teacher might

![Figure 5.5: One solution to the ‘Crosses’ problem](image-url)
also question the group about how they are going about finding a solution and the way in which they are adding their numbers as a way of supporting students with the task.

In addition to monitoring each group of students as they work on the problem, the teacher must carefully plan the sharing of the various solutions and the strategies that each group used to find its solutions (Stein et al. 2008). It is advisable to stop the class midway through their inquiry to share strategies and so enable the groups to learn from each other and to support their persistence. The teacher should select and sequence the order in which students share their method for adding numbers so as to model methods from the least to the most efficient, taking time to ask other students to repeat or to add on an idea to the method or methods already presented by the students. Figure 5.6 shows some solutions with ‘7’ at the centre of the cross with a student using a projected calculator on an electronic whiteboard to check a solution.

Using the collection of solutions to the problem that have been recorded on the board, the teacher poses questions about whether all of the possible solutions have been found. ‘Are there other combinations of numbers that give two arms with a total of 26?’ By drawing students’ attention to the number at the centre of the crosses she can ask, ‘What do you notice about the centre number in all of the solutions we’ve collected so far?’ Then, by drawing a table on the board and recording in one column the total of the arm of a cross and the number at the centre of the cross in the other, she can ask: ‘Can we see a pattern with these numbers?’ ‘What other solutions might we need to test and discover?’ ‘What possible centre numbers are missing from our chart?’

These questions from the teacher and the responses provided by the students motivated them all to seek further solutions and to try other numbers at the centre

Figure 5.6: Solutions with ‘7’ at the centre
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to test the conjectures arising from the discussion. Asking questions that invited children to identify patterns in the different combinations of numbers that yielded the same total enabled the children to use this knowledge to build on and find multiple solutions for other totals. Placing the various solutions in an organised table and asking questions that invited the students to consider which numbers might be missing further encouraged the students to take a systematic approach to the investigation. The students returned to their group work to explore patterns, to discover the missing solutions and to seek an explanation for the patterns they discerned in the table of solutions.

Towards the end of the lesson, or when the teacher thinks that all the solutions have been found, the students are brought back together in order to present their conjectures about the numbers at the centre and the totals in each arm and discuss possible explanations. In this whole-class discussion, the teacher calls on students to explain and justify their set of solutions. ‘Why do you only get a total of 27 when “9” is at the centre?’ ‘Why are these numbers the only numbers that can be at the centre?’
The maths300 advice to teachers that accompanies this task provides exemplary questions and strategies for sharing solutions that teachers can follow.

All of these examples of exemplary teaching display the setting of tasks that enable students to build ‘from within’ (Boaler 2008; Vygotsky 1978; Williams 2009). Students are enabled to use their prior knowledge to find a solution and then to build on this knowledge to discover others. Through careful monitoring of group activity and attentive orchestration of the sharing of different solutions and strategies, the teacher can assist students in breaking down a problem down into manageable chunks. In turn the students will experience success and feel encouraged to persist with tasks (Bobis et al. 2011; Stein et al. 2008). These collaborative skills and thought processes will help them to achieve success and joy, not just in mathematics, but across the whole of their life experiences.

Reflective closure

What does this mean for you as a practitioner, researcher or scholarly teacher?

Student-centred learning is a gift for both student and teacher. It is always a privilege to engage with inquiring and eager minds and to assist in their development. The authors hope you have noticed something special about these lessons that you may be able to employ in your own classroom. It is never only the student who must continue to learn.

In reviewing this chapter, it is worth taking the time to respond to the questions posed by the teachers to their students in each of these lessons and discuss the mathematical concepts and possible student reasoning with colleagues. Trial one of
Curriculum-based engagement

these lessons during a teaching practice and then meet and share with colleagues what you learned about your students, what you learned about these tasks and what you learned about your actions as a teacher.

1. Which task did you choose? Why?

2. Did you adapt the lesson to meet the needs of your students? How? Why? If you chose to group students, what did this look like? Did you group together students with like or different mathematical prior knowledge?

3. Which mathematical language did you model when explaining the task?

4. At which point in the lesson were students learning the most? Why? Share the questions you believe probing the students’ understanding to bridge with their zones of proximal development.

5. What opportunities arose to extend children’s understanding? Did a particular student provide a comment or explanation that you were able to use to extend all or other students’ understandings or skills as they repeated or elaborated upon the initial student’s contribution?

Above all remember to aim to have your students become and remain engaged with mathematical learning that is both joyous and successful!

Further reading


Braher, D.J. and Speer, Mathematics. California: National Council for the

Carpenter, Arithmetic and


References

Anthony, G. and Effective Pedagogy in Mathematics/Pāngarau: Best Evidence Synthesis Iteration (BES).

Atweh, B. and Walshaw, M. (2012). ‘Equity, diversity, social justice and ethics: Common or agendas?’ B. T. and and J. Greenlees (eds), Research in Mathematics Education in (pp. 39–65). Rotterdam: Sense Publishers BV.


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Further reading


Carpenter, Thinking Mathematics. Portsmouth, NH: Heinemann


References

Anthony, G. and Effective Pedagogy in Mathematics/Pāngaroa: Best Evidence Synthesis Iteration (BES).

Atweh, B., and J. Greenlees (eds), Research in Mathematics Education in (pp. 39–65). Rotterdam: Sense Publishers BV.


Chapter 5 Student-centred teaching of mathematics

Approaches to


the Mathematics Education Research Group of University, Geelong: MERGA.

— (2006). ‘Students’ Impressions of the

Broadway, New


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Further reading


Brahier, D.J. and Speer, Mathematics. California: National Council for the

Carpenter, Arithmetic and


References

Anthony, G. and Evidence Synthesis Iteration (BES).

Atweh, B., Walshaw, M. (2012). ‘Equity, diversity, social justice and ethics: Common or agendas?’ B. T. and

and J. Greenlees (eds), Research in Mathematics Education in (pp. 39–65). Rotterdam: Sense Publishers BV.


ED501899).

Chapter 5 Student-centred teaching of mathematics


Curriculum-based engagement


Vale, C., and Hooley, N. (2010). ‘Student-Centred Teachers’ Learning and Practice’. In L. Sparrow, B. Kissane and C. Hurst (eds), Shaping the Future of Mathematics Education. Proceedings of the 33rd Mathematics Education Research Group of


(2011). Optimism Prospective Teachers: Characteristics Enabling Flexible Pedagogy’. In O. Zaslavsky and P. Sullivan (eds), Constructing Knowledge for Prospective and Practicing Tasks (pp. 307–23). New