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Exemplifying a Model-Eliciting Task for Primary School Pupils

Chan Chun Ming Eric
eric.chan@nie.edu.sg

Wanty Widjaja
wanty.widjaja@nie.edu.sg

Ng Kit Ee Dawn
dawn.ng@nie.edu.sg
National Institute of Education, Nanyang Technological University, Singapore

Abstract

Mathematical modelling is a field that is gaining prominence recently in mathematics education research and has generated interests in schools as well. In Singapore, modelling and applications are included as process components in revised 2007 curriculum document (MOE, 2007) as keeping to reform efforts. In Indonesia, efforts to place stronger emphasis on connecting school mathematics with real-world contexts and applications have started in Indonesian primary schools with the Pendidikan Realistik Matematik Indonesia (PMRI) movement a decade ago (Sembrining, Hoogland, Dolk, 2010). Amidst others, modelling activities are gradually introduced in Singapore and Indonesian schools to demonstrate the relevance of school mathematics with real-world problems. However, in order for it to find a place in the mathematics classroom, there is a need for teacher-practitioners to know what mathematical modelling and what a modelling task is. This paper sets out to exemplify a model-elicitng task that has been designed and used in both a Singapore and Indonesian mathematics classroom. Mathematical modelling, the features of a model-eliciting task, and its potential and advice on implementation are discussed.

Key words: Mathematical Modelling, Singapore, Indonesia, Model-Eliciting Task, Primary Mathematics, Task Scaffoldine

Introduction

Research have found that modelling activities help children in the promotion of important mathematical reasoning processes such as constructing, explaining, justifying, predicting, conjecturing, and representing (Chan, 2008; English & Watters, 2005; Lehrer & Schauble, 2000); reasoning aspects that are valued as a powerful way to accomplishing learning with understanding. Furthermore, studies have shown that children have been able to manage complex mathematical constructs irrespective of their academic mathematics achievement (Lesh & Doerr, 2003). What is it about mathematical modelling that is asserted to be a promising reform effort in the mathematics classroom? This paper introduces mathematical modelling from a Models-and-Modelling Perspective (MMP) and shares the features of a modelling task (both from a Singapore and Indonesian context) towards understanding the potential of using such tasks. It will also discuss the potentials of the task and suggest teacher scaffolding strategies for task implementation.

Mathematical Modelling

There are various conceptions as to what mathematical modelling is just as there are various dynamic representations of the modelling process found in
design of the modelling task needs to ensure that the mathematics embedded can be surfaced when students are deeply engaged in it. Lesh, Cramer, Doerr, Post, and Zawojewski (2003) asserted a model-eliciting task is designed based on six instructional design principles: (1) the reality principle (i.e., elicits sense-making and extension of prior knowledge); (2) the model construction principle (i.e., warrants the need to develop a mathematically significant construct); (3) the self-evaluation principle (i.e., requires self-assessment); (4) the construct documentation principle (i.e., requires students to make visible their thinking); (5) the construct generalization principle (i.e., sharable, adaptable, reusable in other similar situations); and (6) the simplicity principle (i.e., the simplicities of the problem-solving situation). Thus the construct ‘model-eliciting’ circumscribes a problem-solving situation, its mathematical structure, the problem-solving processes and the mathematical models generated that are invoked by the problematic situation. The features of a model-eliciting task are what are the principles aim to promote.

**An Example of a Model-Eliciting Task**

Figures 2 and 3 show the “Bus Route Task” for grade 5 (aged 10-11) students implemented in Singapore and Indonesian classrooms respectively between August to early November 2011. The two versions are deliberately designed to be parallel tasks so as to investigate the nature of student mathematical thinking and teacher pedagogical decision-making in the two educational systems. The two tasks share the same goal of determining the most efficient bus route for travelling from point A to point B.

**Determining the most efficient bus route**

Ms Chang recently moved to Block 297C Punggol Road. She is going to start teaching at Punggol Primary school next week and needs to know how to travel to the school. However, the MRT is always too crowded for her to take and it also requires her to take a feeder bus which results in inconvenience. Ms Chang realizes that there are three bus services that ply different routes to her school. Help her to find the most efficient route to travel by bus from her home to the school. The location of her home is marked in the diagram. Currently the three bus services that are available for Ms Chang to choose are Service 124, Service 62 and Service 89. The routes for Service 124, Service 62 and Service 89 are marked as blue, yellow, and pink lines respectively on the map. The bus stops along each bus route are marked with stickers with corresponding colours.

Your task is to give Ms Chang a proposal consisting of the following:

1. How your group determines what is meant by the ‘most efficient’ bus route
2. Assumptions about the problem your group made in order to help Ms Chang
3. The mathematics used to decide which route is the most efficient
4. How your group justifies that the selected route is the most efficient
5. The final recommended route for Ms Chang

For us to better understand your work, you can attach the following to your proposal:

(a) A map containing the chosen bus route
(b) The information you found useful for this task
Menentukan jalur bus paling efisien


Tugas kalian adalah membantu Bu Mustari untuk memberi rekomendasi dengan menjawab pertanyaan berikut:

1. Bagaimana kelompok kalian menentukan jalur bus yang "paling efisien"?
2. Asumsi apa yang harus kalian buat untuk dapat membantu Bu Mustari?
3. Bagaimana kalian menggunakan matematika untuk membantu Bu Mustari memilih jalur bus yang paling efisien?
4. Bagaimana kelompok kalian menyikahkan kelompok lain lewat penjelasan matematika bahwa jalur bus yang kalian pilih adalah yang paling efisien?
5. Rekomendasi akhir jalur bus yang paling efisien untuk Bu Mustari

Untuk membantu kelompok lain memahami penjelasan kelompok kalian, lampirkan dalam poster kalian:
(a) Peta yang memuat jalur bus yang kalian pilih
(b) Informasi penting yang kalian gunakan untuk menyelesaikan masalah

Potentials of the Task

The tasks incorporated features of MMP and were presented to the students together with detailed maps of the respective areas mentioned in the task sheets. In particular, the information contained in these maps served as important data for use during the modelling process. For example, Figure 2 shows the map of Punggol area, as mentioned in the Singapore version of the Bus Route Task. Students participating in this task were to assist Ms Chang, a new teacher in their school, to select a most efficient bus route among three given colour-coded routes for travelling from Ms Chang's residence to the school. The starting point and ending point of travel were circled for ease of recognition. In addition, the positions of bus-stops along the three bus routes were marked by coloured stickers. The chosen map was deliberately a comprehensive one which also included information on the housing density along the routes, the scale measurement, and various land mark places within the area.

Through participation in the tasks, students engaged in the negotiation of meaning within the context of the task, interpret the problem situation and the data given in the map using their everyday experience, and use their chosen relevant
mathematical concepts and skills to represent a solution to the problem in a coherent mathematical model. The tasks present platforms for students to organise their approaches based on their interpretations, explain and describe their mathematical reasoning, make conjectures and justify them, as well as compare among various solutions of mathematical models.

On the mathematical front, the two tasks carried rich potentials to bring forth relationships among pertinent variables such distance, time, and costs interpreted in real-world contexts. As the tasks were open-ended in nature, it allowed for a variety of mathematical approaches. There were multiple pathways to derive at the mathematical solution depending on interpretations, assumptions and choice of variables, as postulated by proponents of the MMP perspective (e.g., English, 2010a; 2010b). Students’ knowledge of pertinent mathematics (i.e., length, time, distance, angle) and their flexibility of use for determining a valid mathematical model in getting a solution to the problem would be important during mathematization of even real-world situation.

The modelling tasks were set as a group work for students to complete over 3 hours on three consecutive mathematics lessons. This offered a good platform for students to develop their mathematical communications skills during group and class discussion (English, 2010b). Students were required to justify their ideas and check the reasonableness of their solution. It was also ideal that students share their assumptions and the conditions set in the task in order to check if the assumptions needed further refinement. However, numerous studies on mathematical modelling have identified that validating results and re-interpreting the results in the real world context is challenging (Galbraith & Stillman, 2006). The teachers involved in the implementation of the two tasks in the Singapore and Indonesian classrooms did their best in helping their students refine the models within the constraints.

Suggestions to Task Scaffolding

This section will propose some teacher scaffolding suggestions for the completion of the Bus Route Task which may apply across the Singapore and Indonesian classroom contexts. Firstly, it is important that the teacher sets aside time for students to discuss their interpretations of the context provided in the problem. For instance, students should be encouraged to brainstorm on what they mean within the given context and how they would determine “efficiency” of their models. At this stage, it is plausible that students may be able to suggest efficiency based on time, speed, and cost of travelling, well within their mathematical pre-requisites of the task. The scaffolding strategies of the teacher would be concentrating on questioning for ideas, exploration of possible choice of mathematical concepts and skills useful for the task, and helping students determine their next stage in the problem solving process. Students need to be reassured that multiple interpretations of “efficiency” are recognised and differing solution pathways for mathematical diversity are thus encouraged. They should also be allowed to put forth questions about task for clarifications.

Secondly, it is also important to help students understand the role of assumptions, conditions, and variables within the task. Assumptions are made about the context of the task (e.g., the bus travels at a constant speed, regular traffic conditions) so as to narrow down the focus for choice of appropriate mathematical approach. For example, it may be logical for students to assume regular traffic conditions (i.e., no vehicle break-downs, traffic lights are working) because data on traffic conditions vary day-to-day and it may not be readily available for the feasibility of classroom implementation. Conditions of the task need to be articulated and recorded because it would be necessary for students to decide on the parameters in which they would like to work within in order to develop a “mathematically valid” model as a proposed solution to the problem. For instance, one of the conditions in the Bus Route Task (Singapore version) where students can work with was to discuss the problem situation with the peak hour (i.e., 06 30 – 07 30) of travelling stipulated, based on the starting time of their school. This helps in decision making about the frequency of bus services, along with other real-world considerations of day-to-day bus journeys. Variables of the tasks include the ways in which efficiency is measured (e.g., time, speed, cost). Students can be prompted to explore how these could be determined from the given information (i.e., map) and whether more information needs to be collected for more accurate calculations. Furthermore, these variables are also related to each other. Students can work on how to develop a more sophisticated mathematical model holistically connecting the variables to help determine the most efficient route from the given three routes.

Last and not least, model refinement requires continue evaluation of the mathematization process and critique of initial models based on the validity and applicability to the given situation. It is crucial for students to be encouraged to reflect and review their initial models in several ways. For one, the appropriateness of assumptions made, conditions set, variables chosen for exploration. These form the foundation of the mathematical model or representation of the students’ arguments and reasoning for their choice of the most efficient bus route. Students have to decide if the final decision made is a logical one, based on the above. Another way where students could review their initial model is to check their mathematical calculations
accuracy and reasonableness, whether these calculations present substantial
port for their decision. Finally, students can reflect on their models to determine if
meets the requirements of the task. They need to recognise the possible limitations
their models for applications across other similar contexts (e.g., the most efficient
route) based on the parameters they had initially chosen to work with.

Conclusion

This paper serves three purposes to: (a) introduce the Model-Modelling
perspective (MMP) in mathematical modelling from a practice-oriented point of view;
exemplify a model-eliciting task (The Bus Route Task); and (c) discuss the
potential of the task and teacher scaffolding strategies. However, the implementation
modelling activities in classrooms can be challenging (Ng, 2011). Teachers who
use to prescriptive teaching approaches would need to explore other scaffolding
strategies to more student-centred focus, encouraging multiple interpretations
and on pathways. Teacher beliefs about mathematics and how mathematics should
ought and applied may be challenged as mathematical models can take many
; tables, graphs, and drawings, amidst abstract algebraic representations and
dictions. In view of bridging the gap between the potentials of mathematical
ling and the recognition of these potentials by teachers, Ng (2011) recommends
ng teacher repertoire in two specific areas such as teacher questioning
ques during scaffolding and facilitating a conducive modelling climate in the
oom. The former recommendation has been elaborated in detail above based on
Bus Route Task. The latter recommendation is just as crucial for both teachers and
its engaged in mathematical modelling as a positive climate which encourages
, discussion, and fruitful mathematical arguments enhances sophisticated
mathematical thinking.

Acknowledgements

The authors would like to thank the teachers (Mdm. Thomas Mary Magdalene
Mustari Admiru) and schools (Punggol Primary School and SD Bopkri 3
Baru Yogyakarta) involved in the implementation of the Bus Route Task
apane and Indonesia. In addition, we are also grateful to Mrs Cynthia Seeto,
Teacher, Singapore, Ministry of Education, for her assistance in the research
based on the Bus Route Task.

References


mathematical reasoning about data. Journal for Research in Mathematics


L. Galbraith, C. R. Haines & A. Hurford (Eds.), Modeling students' mathematical
modeling competencies (pp. 287-299). New York: Springer.

Faragher & M. McLean (Eds.), Mathematics education for the third millennium:
Towards 2010 (pp. 207-214): MERGA.

classrooms: Some implications for teaching applications and modelling. In Qi-
Xiao Ye, W. Blum, K. Houston, & Qi-Yuan, Jiang (Eds.), Mathematical
modelling in education and culture: ICTMA 10 (pp. 111—125). Chichester, UK:
Horwood.

blockages during transitions in the modelling process. ZDM, 38(2), 143-162.

Lehrer, R., & Schauble, L. (2000). Inventing data structures for representational
purposes: Elementary grade students' classification models. Mathematical
Thinking and Learning, 2(1&2), 49-72.

development sequences. In R. Lesh & H. Doerr, (Eds.), Beyond constructivism:
Models and modeling perspectives on mathematics problem solving, learning
and teaching (pp. 35-58). Mahwah, NJ: Lawrence Erlbaum Associates.

in mathematics teaching and learning. In R.A. Lesh & H. M. Doerr (Eds.),
Beyond constructivism: Models and modeling perspectives on mathematics
problem solving, learning and teaching (pp. 3-34). Mahwah, NJ: Lawrence
Erlbaum Associates.

(Ed.), Second handbook of research on mathematics teaching and learning: A
project of the National Council of Teachers of Mathematics (pp. 763-803).
Charlotte, NC: Image Age Publishing.

MOE. (2007). Ministry of Education Mathematics Syllabus – Primary, Singapore:
Curriculum Planning and Developmental Division.

Ng, K. E. D. (2011). Facilitation and scaffolding: Symposium on Teacher
Professional Development on Mathematical Modelling - Initial perspectives from
Singapore. Paper presented at the Connecting to practice - Teaching practice and
the practice of applied mathematicians: The 15th International Conference on the
Towards Mathematical Literacy in the 21st Century: Perspectives from Indonesia

Wanty Widjaja  
<wanty.widjaja@nie.edu.sg>  
National Institute of Education, Nanyang Technological University, Singapore

Abstract

The notion of mathematical literacy advocated by PISA (OECD, 2006) offers a broader conception for assessing mathematical competences and processes with the main focus on the relevant use of mathematics in life. This notion of mathematical literacy is closely connected to the notion of mathematical modelling whereby mathematics is put to solving real-world problems. Indonesia has participated as a partner country in PISA since 2000. The PISA trends in mathematics from 2003 to 2009 revealed unsatisfactory mathematical literacy among 15-year-old students from Indonesia who lagged behind the average of OECD countries. In this paper, exemplary cases will be discussed to examine and promote mathematical literacy at teacher education level. Lesson ideas and instruments were adapted from PISA released items 2006. The potential of such tasks will be discussed based on case studies of implementing these instruments with samples of pre-service teachers in Yogyakarta.

Key words: Mathematical Modelling, Mathematical Literacy, Indonesia, Mathematical Tasks

Introduction

The notion of mathematical literacy advocated by the Programme for International Student Assessment (PISA) has gained wide acceptance globally. Mathematical literacy goes beyond curricular mathematics and covers a broader conception of what constitutes mathematics. The main focus of PISA assessment is on measuring the potential of 15-year-old students in activating their mathematical knowledge and competencies to solve problems set in real-world situations. PISA’s (OECD, 2006) definition of mathematical literacy captures this:

Mathematical literacy is an individual’s capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements, and to engage in mathematics in ways that meet the needs of that individual’s current and future life as a constructive, concerned and reflective citizen. (OECD, 2006, p. 72).

Indonesia has participated as a partner country in PISA since the start in 2000. The trend from PISA results in mathematics from 2000 to 2009 consistently revealed poor performance. Indonesia is ranked among the lowest performing countries that performed below the OECD average (Table 1). Figure 1 presents changes in some of countries performance from PISA 2003 to 2009. The comparison between 2003 and 2009 results showed that Indonesian 15-year-olds improved their performance by 11 score points. However, a worrying note from the 2009 PISA results was that almost 80% of the Indonesian sample performed below the baseline of level 2 of