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INTRADAY DYNAMIC HEDGING
AND FUTURES MARKET VOLATILITY

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In this paper, we study the risk minimizing behavior of traders in financial markets dealing with intraday information. We focus on the optimal dynamic rebalancing of hedge positions. We extend the existing BEKK-GARCH models to power versions to generate dynamic hedge ratios. These are then compared with performance of Dynamic Conditional Correlation (DCC) models of Engle (2002) and Tse and Tsui (2002). We employ intraday 15 minute sampled data from the Commodity Futures Trading Commission (CFTC) trade databases for the S&P500 index futures contracts supplied by the CFTC. The futures price series is matched with underlying cash market prices. We have conducted both in sample and out of sample studies for comparisons. Our results recommend that the management of the risk of physical- and derivative-based portfolios can be substantially reduced by employing active hedging strategies.

Since the introduction of the exchange traded futures market, many studies have investigated the minimum variance or Optimal Hedge ratio (OHR) using futures contracts. (see Baillie and Myers 1991; Cecchetti, Cumby, and Figlewski 1988 among others). An OHR is defined as the proportion of a cash position that should be covered with an opposite position in the futures market. The hedge ratio can be calculated as the ratio of the covariance between cash and futures prices to the variance of the futures price (see Anderson and Danthine 1981; Baillie and Myers 1991). An alternative OHR estimator is defined as the ratio of the standard deviation of the cash returns over the standard deviation of the futures returns multiplied by the correlation coefficient between the cash and futures returns (static or time varying). OHRs are now well established instruments in the field of risk management. It is well known in the finance literature that OHRs need to be determined in order to adequately hedge against exposure to risk in underlying assets.

The traditional approach to hedging assumed a constant hedge ratio over time (see Figlewski 1984; Ghosh 1993). However, extensive empirical research over the years has supported the view that financial time series data displays time-varying volatility, so estimating a time-invariant OHR may not be appropriate (Baillie

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and Myers 1991). As such, current studies tend to adopt time-varying conditional variance models, such as the autoregressive conditional heteroskedasticity (ARCH) framework of Engle (1982) or the generalized ARCH (GARCH) approach of Bollerslev (1986), to estimate optimal or minimum risk hedges. The ARCH and GARCH models have become popular as their variance specifications can capture commonly observed features of financial time series. In particular, these models recognize that volatility is not constant but a time-varying process. They are also particularly useful for modelling the volatility “clustering” typically associated with financial time series. By explaining volatility as a function of the lagged errors, large changes in past error terms are fed into further large changes in volatility, and small changes in past error terms are fed into further small changes in volatility.¹

Pennings and Meulenberg (1997) provide a comprehensive review of hedging performance measures in the literature. The evaluation generally contrasts between the combined cash-futures position versus the cash position alone. Hedging performance is measured by the reduction in the variance of the combined position in Ederington (1979), the ratio of Sharpe ratios (cash-futures position divided by cash position only) in Howard and D’Antonio (1994) and expected utility maximization in Kroner and Sultan (1993). Hedging applications based on a BGARCH framework include Baillie and Myers (1991) for US commodity futures, Kroner and Sultan (1993) for foreign currency futures, and Park and Switzer (1995) for US stock index futures. In these studies, hedging performance is based on minimizing the variance of the overall position.

There is also an extensive literature (see Lien and Tse 2002 for a comprehensive survey on futures hedging) comparing the relative performance of static and time-varying hedge ratios. However, many of these studies provide conflicting results. For example, Baillie and Myers (1991) and Park and Switzer (1995) find that the time-varying hedge ratios computed from Bivariate GARCH (BGARCH) models showed better out-of-sample performance in terms of variance reduction than the ordinary least squares (OLS) hedge ratio. On the other hand, Lien, Tse, and Tsui (2002) found that the Constant Conditional Correlation GARCH (CCC-GARCH) hedge strategy provided no benefits in variance reduction over the OLS hedge. So far, there is no definite conclusion concerning the benefits of using BGARCH models in optimal hedging performance.

This study extends the current literature: first by using an intra-day data set, as opposed to the more commonly used daily or weekly data. When working with higher frequency data sets, many empirical studies have found significant evidence of long memory in the conditional variance process of many financial and economic time series (see deLima, Bredt and Crato 1994; Ding, Granger, and Engle 1993; and Harvey 1993). The persistence of these lagged shocks will not be as apparent when working with lower frequency data sets. Dunis and Lequeux (2000) compared

¹ A new class of Simultaneous Volatility Models (SVL) was introduced in a paper by Gannon (1994) to account for contemporaneous futures volume of trade effects in a system of volatility equations. This class of models can also be employed to minimize cash index risk reduction within and out of sample and also in a trading strategies framework.
the hedging performance of hedge ratios constructed using a daily data set to those
using an intraday data set. They found that the hedge ratios calculated from an
intraday data set exhibit a substantially lower variance than those calculated using
daily data. By using an intraday data set, the effects of trading throughout the day
are captured, allowing for a more accurate estimation of volatility.

Secondly, the study will utilize the unity constraint bivariate Power BEKK-
GARCH model developed by Bhattacharya, Singh, and Gannon (2007). Since
the introduction of the ARCH and GARCH models, many studies have developed
variations or extensions of the original models (see Lien and Tse 2002). The unity
constraint BEKK-GARCH model is an extension of the classic BEKK-GARCH
on the parameter matrices of the conditional covariance matrix to ensure positive
definiteness. By relaxing this constraint, and employing an alternative unity constraint,
this model allows for mean reversion in volatility; that is, the GARCH parameters
do not need to be strictly positive. Freeing up this restriction also allows for more
flexible volatility estimates with the potential to impose greater weights on the most
recent shocks. The study also compares the performance of the above models with
the Dynamic Conditional Correlation (DCC-E) model of Engle (2002) and (DCC-
T&T) model of Tse and Tsui (2002).3

All models are benchmarked against the (0,1) Beta constant hedge ratio model.
This study restricts attention to within sample comparison of residual model risk
relative to cash index market risk. What is of real interest here is relative performance
of the dynamic GARCH based models on minimizing residual risk.

The analysis is performed on the most heavily traded stock index worldwide,
the Standard and Poor's 500 (S&P 500) index. The S&P 500 is a free-float
capitalization-weighted index comprising 500 large-cap common stocks actively
traded in the United States. The stocks included in the S&P 500 are those of large
publicly held companies that trade on either of the two largest American stock
market exchanges, the NYSE Euronext and the NASDAQ OMX. It is considered
a benchmark of the US economy and is widely regarded as the best single gauge of
the large cap US equities market. With over US$ 3.5 trillion benchmarked, of which
approximately US$ 915 billion comprises index assets, the S&P 500 index captures
approximately 75% of the value of the US equities market.

In the derivatives market, the Chicago Mercantile Exchange (CME) offers
futures contracts that track the index. The S&P 500 index futures contract is the
exchange's most popular product. This study uses trade, resampled to coincide

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2. They constructed three hedge ratios (two calculated from daily data and one calculated using intra-
day data), using two government bond futures contracts and their respective 3-month interest rate
futures contracts as traded on London International Financial Futures and Options Exchange (LIFFE).
3. The two step SVL methodology, reported in Gannon (1994), is extended in Chng and Gannon
(2003) and includes a three equation model of the variance/covariance matrix and a four equation
system to allow for contemporaneous implied volatility from both the cash index and index futures
volatility to enter the systems. The latter also allowed for volume of trade effects from the cash index
and index futures markets to enter the systems. This class of models is considered as part of further
model evaluation and comparison in terms of cash index risk reduction in another paper.
with the 15-minute reporting intervals of the Commodity Futures Trading Commission (CFTC) trade databases. The sample analyzed in this study comprises the S&P 500 index futures contract from January 2000 to June 2004. We restrict attention to synchronously sampled data when floor trading is conducted on the futures exchange.

In this study we find the less restrictive BEKK-GARCH model, with the unity constraint applied, provides a greater variance reduction than the classic BEKK-GARCH model with the square-root constraint. However, the DCC-Tse and Tsui (2002) performs overall best within the GARCH class of models while the DCC-Engle (2002) the worst.

The remainder of this paper is structured as follows. Section I provides a brief review of the relevant literature. Section II explains the methodology and estimation techniques used in this study. Section III contains details about the data sourced and the descriptive statistics of the data used in this study. Section IV discusses the results following estimation of the hedge ratios and their respective performance. Finally, Section V concludes and summarizes the main findings of the study, as well as providing motivations for future research.

I. LITERATURE REVIEW

There is an extensive literature on the use of futures contracts for the purpose of hedging. Hoos (1942) investigated how futures contracts, written on perishable commodities, could be used to hedge traders against losses resulting from the revaluation of stocks on hand due to variations in market prices. Working (1953) examined the benefits of futures trading, and saw the advantages to be more utilized by merchants and processors for the purpose of hedging rather than those desiring to speculate. Bailey (1968) investigated how the choice of steer futures contracts can be used by cattle feeders in order to take the price risk out of the traditionally risky business of cattle feeding.

It is also well known in the finance literature that OHRs need to be determined for risk management, and more so in cases where futures and cash price movements are not highly correlated, generating considerable basis risks. Among these studies, Ward and Fletcher (1972) use live beef futures to calculate optimal hedging ratios. Heifer (1972) and Rolfo (1980) compute utility maximizing hedge ratios for cattle feeding and cocoa futures. Stultz (1984) examines hedging strategies and the optimal futures hedge when applied to foreign currency risk. Witt, Schroeder, and Hayenga (1987) apply OLS methodology to estimate OHRs in sorghum and barley spot markets cross hedged with corn futures.

The above studies, however, make no allowance for variation in the distribution of cash and futures prices over time. To overcome this problem, Ghosh (1993), adopted an Error Correction Model (ECM) to estimate OHRs for the S&P 500 index, the Dow Jones Industrial Average (DJIA), and the New York Stock Exchange (NYSE) composite index. The ECM model recognizes the long-run equilibrium relationship between cash and futures as well as the short-run dynamics. The results showed an improved hedging performance over the traditional static OLS model.
Ghosh and Clayton (1996) later expanded on this study to consider other major international stock indices such as CAC 40 (France), FTSE (UK), DAX (Germany), and the Nikkei (Japan). The ECM was again successful in outperforming the time-invariant OLS model. More recently, Kenourgios, Samitas, and Drosos (2008) investigated the hedging effectiveness of the S&P 500 stock index futures contract using weekly settlement prices from July 1992 to June 2002. Testing a variety of econometric methods, including an ECM, their results were consistent with earlier studies with the ECM proving superior to others in terms of risk reduction. They also find the OHR derived from the ECM remains stable over time.

The criticism of the ECM is that the problem of heteroskedasticity still remains. Moreover, it cannot be used for a stationary time series despite the ability to capture both long- and short-term dynamics in a single statistical model (De Bof and Keele 2008). As such, the current literature tends to employ extensions of either the ARCH framework of Engle (1982) or the GARCH approach of Bollerslev (1986) to estimate hedge ratios in markets where underlying cash and futures prices show time-varying, persistent volatility.

Cecchetti et al. (1988) allowed for time variation in the distribution of cash and futures price changes by employing a bivariate ARCH model to generate OHRs involving T-bond futures. Baillie and Myers (1991) and Myers (1991) use BGARCH models of cash and futures prices for six agricultural commodities (beef, coffee, corn, cotton, gold and soybeans) to calculate OHRs in commodity futures. Their results show that the standard assumption of a time-invariant OHR is inappropriate. Bera et. al. (1997) use a diagonal vech (DVEC) representation of a BGARCH model to estimate time-varying hedge ratios for corn and soybeans. Moschini and Myers (2002), using weekly corn prices, show that a new parameterization of BGARCH processes establishes statistical superiority of time-varying hedge ratios over constant hedges. Recently, Yang and Lai (2009) examine the hedging performance of the major international stock index futures based on the bivariate asymmetric GJR-ECM. For conservative investors, both the GJR-ECM-GARCH and ECM-OLS models perform very well.

In other studies, Yeh and Gannon (2000) consider the Australian Share Price Index Futures (SPI) and Lee, Gannon, and Yeh (2002) analyze the S&P 500, the Nikkei 225, and the SPI Index Futures data. In all of these cases the static models provide a Beta of around 0.7 (substantial excess volatility in the futures relative to the cash index), and the Constant Conditional Correlation (CCC) GARCH model dominates in terms of out of sample re-balancing of hedge positions. The hedge ratios in Lee, Gannon, and Yeh report substantial variation around the average of 0.7 for the S&P 500 as well as the SPI; and for the Nikkei, the variation ranged from 0.3 to 2.0.

Au-Yeung and Gannon (2004, 2005) employ an MGARCH (BEKK) model modified to allow for multiple structural breaks and volatility spillovers between the Hang Seng Index Futures cash and index futures markets and also overnight spillovers.

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4. Abbreviated from the original authors' names Baba, Engle, Kraft and Kroner.
Review of Futures Markets

from the S&P500 index futures. Furthermore, they also report statistically significant structural breaks in either or both of their cash and/or futures data, which correspond to the timing of regulatory interventions. Lee, Wang, and Chen (2009) utilize four static and a dynamic BGARCH models to find the OHR for the S&P500 and five major Asian market index futures.

Bhattacharya et al. (2007) employed the Au-Yeung and Gannon (2004) BEKK model to test volatility spillovers between Indian stock and share futures. By relaxing the square-root constraint on the classic BEKK model of Engle and Kroner (1995), and employing an alternative unity constraint, they were able to achieve considerable variance reduction for six out of seven highly traded Indian stock futures. The unity constraint model outperformed the square-root constraint model in the following aspects: (i) it showed that all firms' data satisfy the conditional stationarity condition for the conditional covariance matrix; (ii) it generated hedge ratios consistent with the commodities and index futures related findings; and (iii) it led to variance reduction in all cases in their stock futures dataset. The study, therefore, shows promise in terms of generating better time-varying hedge ratios after taking care of mean reversion in volatility.

Giovannini et al. (2006), apply the Dynamic Conditional Correlation (DCC) GARCH model of Engle (2002), which allows for time-varying volatilities and correlations to construct OHRs for the stocks of various oil companies from several different countries. They find the DCC model provides more accurate hedging strategies than the CCC model. Hsuku (2008) employs the DCC model, along with an ECM and an M-GARCH model, to investigate the optimal hedge for British and Japanese currency futures markets. The DCC model yielded the best hedging performance in both markets, while the traditional MGARCH model provided the worst hedging effectiveness being out performed by both the time-invariant OLS and ECM hedging models.

In all of these studies involving financial futures and BGARCH models, the spot and futures are cointegrated in the variance, but there is substantial variation in

5. Gannon (2005) repeated this analysis but employed a Full Information Maximum Likelihood (FIML) set of simultaneous volatility equations.
8. Alternative OHR estimators specified include Bhaduri and Durai (2007), with daily US index futures data from the DVEC representation had a higher average variance reduction and also find that the simple OLS strategy performed well at shorter time horizons. Lee and Yoder (2007), develop a Markov regime-switching time-varying correlation (RS-TVC) GARCH model. This model is a generalization of time-varying correlation model proposed by Tse and Tsui (2002), a generalization of the CCC-GARCH model. Sultan and Hansan (2008), estimate time-varying OHRs for European stock indices using a BGARCH-ECM and a GARCH-X. Hsu et. al. (2008) present a class of new copula-based GARCH models for the estimation of the OHR. Lai, Chen, and Gerlach (2009) extended the copula methodology to construct a bivariate copula-threshold-GARCH (copula-TGARCH) model to construct OHRs in spot and futures markets. Complicated models such DCC and copula did not prove effective risk reducers when compared with the traditional OLS model.
Intraday Dynamic Hedging

the basis risk, which allowed these time-varying hedge ratio estimators to sometimes perform well. If the spot and futures are not cointegrated in the variance, then these GARCH type estimators do provide substantial variation in hedge ratio estimates and substantially reduce spot market risk.

All of the above mentioned studies have used a data set of no higher frequency than daily to construct OHRs. As such, volatility throughout the day is ignored. The use of high frequency intraday data in the estimation of realized volatility acknowledges that information arriving during the trading day contributes to the daily realized volatility. In this study we adopt an intraday data set to capture the effects of day trading and compare cash index variance reduction using competing multivariate GARCH models.\textsuperscript{9}

For the SVL model of Gannon (1994) intraday data was synchronously at 15 minute intervals for the Australian share price index, index futures, and futures volume of trade. Employing the same models and an extended dataset, Gannon (2010)\textsuperscript{10} considered effects on SVL model estimates from (i) misspecifying the returns generating equations and (ii) sequentially reducing the intraday sampling interval from 30 minutes down to 5 minutes. Chng and Gannon (2003) employ intraday data sampled at 30 minute intervals for the S&P500 cash index, index futures, and options on both the cash index and futures. We will also extend the SVL methodology and compare the reduction in cash index risk using hedge ratios generated from this class of SVL models.

II. EMPIRICAL METHODOLOGY AND ESTIMATION OF OHRs

A. The Optimal Futures Hedge

An OHR is defined as the proportion of a cash position that should be covered with an opposite position in the futures market (see Anderson and Danthine 1981). Consider an investor with a fixed long cash position who wishes to hedge some proportion of this cash position in the futures market. The expected return on the hedged position is given as:

\textsuperscript{9} Other related multivariate applications include Lafuente and Novales (2003) find multivariate models lead to a lower number of daily futures contracts being rebalanced than under a systematic unit ratio using two intraday data sets for the Spanish stock market index. Lai and Sheu (2009) propose a new class of multivariate volatility models encompassing realized volatility (RV) estimates. They find that in an out-of-sample context, with a daily rebalancing approach, substantial improvements in the performance of RV-based GARCH models can be made when switching from a daily to intraday data set to estimate the risk-minimizing hedge ratio compared to hedging performance to those generated by return-based models. Harris et. al. (2010) attributed the poor performance of conditional hedging models in portfolio variance reduction to be more likely due to the integrated hedge ratio using intra-daily data for three different currency exchange rates against the USD.

\textsuperscript{10} Gannon (2010) revisited the simultaneous volatility class of models to test transmission and spillover effects when the intraday sampling interval reduces. In that paper further theoretical conditions for this class of models and systems Error Correction Terms (ECT) are defined. In all of the above estimators, hedge ratios can be extracted, but no out of sample hedging performance was undertaken.
\[ R_h = R_c - hR_f \]  

where, \( R_h \) is the return on the hedged position, \( R_c \) is the return on the cash position, \( R_f \) is the return on the futures position, and \( h \) is the hedge ratio.

The return on a hedged position will normally be exposed to risk caused by unanticipated changes between the position being hedged and the futures contract (Cecchetti et al. 1988). This "basis risk" ensures that no hedge ratio completely eliminates risk. The OHR is the value of \( h \) which minimizes the variance of the return on the hedged position. It can be represented as:

\[ \text{OHR} = \frac{\sigma_{cf}}{\sigma_{\varepsilon_f}} \text{or } p_{cf} \ast (\sigma_{\varepsilon_f}/\sigma_{\varepsilon_f}) \]  

where, \( \sigma_{cf} \) is the covariance of the cash and futures returns, \( \sigma_{\varepsilon_f} \) is the variance of the futures returns \( \varepsilon_f \) is the correlation coefficient between the returns and \( (\sigma_{\varepsilon_f}/\sigma_{\varepsilon_f}) \) is the ratio of standard deviation of the cash over that of the futures.

After estimation of the different models (two variants of dynamic BEKK-GARCH models and two variants of the DCC models), we obtain the variance (denoted by \( \sigma^2 \)) ratio in the following way:

\[ \frac{\sigma^{2}_{\text{model}}}{\sigma^{2}_{\text{benchmark}}} \]  

where "benchmark" stands for the variance from the cash returns series. The term "model" in the above expression will represent the residual risk, calculated as the variance of \( R_h \) from equation (1) for the four different models we are employing in this study. These variance ratios are benchmarked against a \((0,1)\) Beta model where \( h \) in equation (1) is set to unity.\(^{11}\)

B. Time Varying Hedge Ratios

Numerous studies on MGARCH have been documented. We are concerned with the specification of the matrix process \( H_t \):

\[ H_t = \text{Var}(r_t \mid I_{t-1}) \]  

Various parametric formulations have been developed. These models can be divided into three categories. In the first one, the conditional covariance matrix \( H_t \) is modelled directly (see Engle and Kroner 1995; Au-Yeung and Gannon 2004, 2005; and Bhattacharya et al. 2007). This class includes, in particular, the VEC and BEKK models. The models in the second class, the factor models, are motivated by parsimony. The Data Generating Process is assumed to be generated by a (small)
number of unobserved heteroskedastic factors; Models in the third class are built on the idea of modelling the conditional variances and correlations instead of straightforward modelling of the conditional covariance matrix. Members of this class include the CCC model and DCC models and extensions. The latter class of DCC multivariate GARCH models allow the conditional covariance matrix of the dependent variables to follow a flexible dynamic structure, that is, DCC (Tse and Tsui 2002) and DCC (Engle 2002). The appeal of this class lies in the intuitive interpretation of the correlations, and models belonging to it have received plenty of attention in the recent literature. In our study, we focus on the first and third classes of models.

C. Time-Varying Hedge Ratios: BEKK

To calculate the first two of the time-varying OHRs we use two different versions of the bivariate BEKK-GARCH model. The first is the generic bivariate BEKK-GARCH model of Engle and Kroner (1995) with the square-root constraint imposed on the parameter matrices. The second is the bivariate BEKK-GARCH model with the nonrestrictive unity constraint applied.

The continuous returns are generated using the formula below:

\[ R_{1,t} = \ln \left( \frac{P_{1,t}}{P_{1,t-1}} \right) \]

\[ R_{2,t} = \ln \left( \frac{P_{2,t}}{P_{2,t-1}} \right) \]  

where, \( R_{1,t} \) and \( P_{1,t} \) represent the continuous return and price of futures at time \( t \), respectively, and \( P_{1,t-1} \) is the price of futures at time \( t-1 \). Similarly, \( R_{2,t} \) and \( P_{2,t} \) represent the continuous return and price of the spot at time \( t \), and \( P_{2,t-1} \) is the price of the spot one period prior. Preliminary estimation is done allowing the returns to follow an autoregressive process of order one and restricting the first lag parameter to zero in the returns equation. The parameter estimates obtained are then compared to those obtained when a GARCH procedure is followed.

To capture the second-order time dependence of cash and futures returns, a bivariate BEKK-GARCH (1,1) model proposed by Engle and Kroner (1995) is utilized. The model that governs the joint process is shown below:

\[ R_t = \alpha + u_t \]

\[ u_t | \Omega_{t-1} \sim N(0, H_t) \]

where the return vector for cash and futures series is given by \( R_t = [R_{1,t}, R_{2,t}] \); the vector of the constant is defined by \( \alpha = [a_1, a_2] \); the residual vector \( u_t = [\epsilon_{1,t}, \epsilon_{2,t}] \) is bivariate and conditionally normally distributed; and the conditional covariance matrix is represented by \( H_t \), where \( \{H_t\} = h_{ij,t} \) for \( i, j = 1, 2 \). \( \Omega_{t-1} \) is the information set representing an array of information available at time \( t-1 \). Given the above expression, the conditional covariance matrix can be stated as follows:
\( H_t = C_0C_0 + A_{11}^t \varepsilon_{t-1}^t \varepsilon_{t-1}^t A_{11} + G_{11}^t H_{t-1} G_{11} \) (7)

where the parameter matrices for the variance equation are defined as \( C_o \), which is restricted to be lower triangular. \( A_{11} \) and \( G_{11} \) are two unrestricted matrices. The conditional second moment can be represented by:

\[
H_t = C_0C_0 + \begin{bmatrix}
  a_{11} & a_{12} \\
  a_{21} & a_{22}
\end{bmatrix}
\begin{bmatrix}
  \varepsilon_{1,t-1}^2 & \varepsilon_{1,t-1}^t \varepsilon_{2,t-1}^t \\
  \varepsilon_{2,t-1}^t & \varepsilon_{2,t-1}^2
\end{bmatrix}
\begin{bmatrix}
  a_{11} & a_{12} \\
  a_{21} & a_{22}
\end{bmatrix}
+ \begin{bmatrix}
  g_{11} & g_{12} \\
  g_{21} & g_{22}
\end{bmatrix}
H_{t-1}
\begin{bmatrix}
  g_{11} & g_{12} \\
  g_{21} & g_{22}
\end{bmatrix}
\]

(8)

When expanded using matrix multiplication, the equation takes the following form:

\[
h_{11,t} = c_{11}^2 + a_{11}^2 \varepsilon_{1,t-1}^2 + 2a_{11}a_{12} \varepsilon_{1,t-1}^t \varepsilon_{2,t-1}^t + a_{12}^2 \varepsilon_{2,t-1}^2 + g_{11}^2 h_{11,t-1} + 2g_{11}g_{12} h_{12,t-1} + g_{12}^2 h_{22,t-1}
\]

\[
h_{12,t} = c_{12} c_{21} + a_{11}a_{12} \varepsilon_{1,t-1}^2 + (a_{12}a_{12} + a_{11}a_{22}) \varepsilon_{1,t-1}^t \varepsilon_{2,t-1}^t + a_{21}a_{22} \varepsilon_{2,t-1}^2 + g_{11}g_{12} h_{11,t-1} + (g_{21}g_{12} + g_{11}g_{22}) h_{12,t-1} + g_{21}g_{22} h_{22,t-1}
\]

\[
h_{22,t} = c_{21}^2 + c_{22}^2 + a_{12}^2 \varepsilon_{1,t-1}^2 + 2a_{12}a_{22} \varepsilon_{1,t-1}^t \varepsilon_{2,t-1}^t + a_{22}^2 \varepsilon_{2,t-1}^2 + g_{12}^2 h_{11,t-1} + 2g_{12}g_{22} h_{12,t-1} + g_{22}^2 h_{22,t-1}
\]

(9)

Although the current version of the BEKK-GARCH model guarantees that the covariance matrices will be positive definite, the same result will be expected for the unity version even though it is not being strictly imposed on the estimation procedure.

As previously mentioned, the majority of existing literature imposes simplifying but restrictive constraints on the parameters of the conditional covariance matrix. This involved putting a square-root constraint on the parameter matrices to make the conditional covariance matrix positive definite. Engle and Kroner (1995) call this a sufficient condition, and show that this implies \( \alpha_{11} \) and \( \beta_{11} \) to be strictly positive. Also, to ensure covariance stationarity of the conditional covariance matrix, Engle and Kroner (1995) prove that these necessary and sufficient conditions need to be satisfied: (i) \( \alpha_{11}^2 + \beta_{11}^2 < 1 \) and (ii) \( \alpha_{22}^2 + \beta_{22}^2 < 1 \).

In this study, we deviate from the approach above, and instead also adopt the framework developed by Bhattacharya et al. (2007). This involves relaxing the square-root constraint on the parameter matrices and imposing an alternative unity constraint to leave the power parameter as one. As a result, the interpretation of the parameter estimates obtained will be different from the current literature.

When the Beta GARCH parameters are allowed to be negative, the underlying interpretations change and indicate mean reversion in volatility in the system. Allowing negative Betas also enables the potential to impose greater weights on the most recent shocks. When calculating the OHR with the unity constraint applied, the estimated covariance and variance terms still need to be positive as the square root
of the ratio of the conditional variance between the spot and the futures over the variance of the futures needs to be calculated. In this case, the parameter estimates can be viewed as similar to the square of those estimated from the former specification, so that the time-varying variance covariance matrix from this new specification can be viewed as similar to the square of that obtained using the former model specification. Therefore, the estimated hedge ratio will be the square root of the projected term from the covariance equation between the spot and the futures divided by the projected term from the variance of the futures. This works well so long as none of the projected covariance or variance terms are negative.\footnote{One approach is to set these values to zero or remove returns from the database when this occurs. There were 119 cases where this occurred and the latter approach was adopted.}

D. Time-Varying Hedge Ratios: DCC

The assumption that the conditional correlations are constant may seem unrealistic in many empirical applications. Tse and Tsui (2002) and Engle (2002) propose a generalization of the CCC model by making the conditional correlation matrix time dependent. The model is then called a dynamic conditional correlation (DCC) model.\footnote{For some background on CCC models, see the Appendix.} An additional difficulty is that the time dependent conditional correlation matrix has to be positive definite $\forall t$. The DCC models guarantee this under simple conditions on the parameters. These models are estimated in two steps; first, a set of univariate GARCH equations are estimated, and, second, based on the conditional variances, these procedures then model the conditional correlation matrix (imposing positive definiteness $\forall t$).

The DCC model of Tse and Tsui (2002) is defined as in equation (A1) in the Appendix but with $R_t$ replacing $R$:

$$H_t = D_t H_{t-1} D_t,$$

(10)

where $D_t$ is defined in (A2), $h_{it}$ can be defined as any univariate GARCH model, and:

$$R_t = (1 - \theta_1 - \theta_2) \ R + \theta_1 \ \psi_{t-1} + \theta_2 \ R_{t-1}$$

(11)

In (11) $\theta_1$ and $\theta_2$ are non-negative parameters satisfying $\theta_1 + \theta_2 < 1$, $R$ is a symmetric $N \times N$ positive definite parameter matrix with $\rho_{ii} = 1$, and $\psi_{t-1}$ is the $N \times N$ correlation matrix of $\epsilon_i$ for $t = t - M, t - M + 1, \ldots, t - 1$. Its $i, j$-th element is given by:

$$\psi_{ij, t-1} = \frac{\sum_{m=1}^{M} u_{i,t-m} u_{j,t-m}}{\sqrt{(\sum_{m=1}^{M} u_{i,t-m}^2)(\sum_{h=1}^{M} u_{j,t-m}^2)}}$$

(12)
This is a special formulation of a constant correlation coefficient, but the weights $\theta_1$ and $\theta_2$ determine the loadings on past standardized shocks. This has an appearance of an Autoregressive Moving Average (ARMA) structure.

The standardized residual is $u_{it} = \epsilon_{it}/\sqrt{h_{iiit}}$. The matrix $\psi_{t-1}$ can be expressed as:

$$
\psi_{t-1} = B_{t-1}^{-1}L_{t-1}L'_{t-1}B_{t-1}^{-1}
$$

and $B_{t-1}$ is a $N \times N$ diagonal matrix with $i$-th diagonal element given by $\left(\sum_{h=1}^{M} u_{i,t-h}^2\right)^{1/2}$ and $L_{t-1} = (u_{t-1}, \ldots, u_{t-M})$ is a $N \times M$ matrix, with $u_t = (u_{1t}, u_{2t}, \ldots, u_{Mt})^T$.

Alternatively, Engle (2002) proposes a different DCC model which again is defined as in equation (A1) and with $R_i$ replacing $R$. The DCC model of Engle (2002) is formulated with $R_i$ defined as follows:

$$
R_i = \text{diag} \left( q_{i1,1}^{-1/2}, \ldots, q_{iN,N}^{-1/2} \right) Q_i \text{diag} \left( q_{i1,1}^{-1/2}, \ldots, q_{iN,N}^{-1/2} \right)
$$

where the $N \times N$ symmetric positive definite matrix $Q_i = (q_{ij,i})$ is given by:

$$
Q_i = (1 - \alpha - \beta)\bar{Q} + \alpha u_{t-1}u'_{t-1} + \beta Q_{t-1},
$$

$\bar{Q}$ is the $N \times N$ unconditional variance matrix of $u_{it}$ and $\alpha$ and $\beta$ are non-negative scalar parameters satisfying $\alpha + \beta < 1$. Now the parameters look much more like an ARCH structure.

Since we are only interested the bivariate case in this paper, we write the expression of the correlation coefficient in the bivariate case; for the DCC of Tse and Tsui (2002):

$$
\rho_{12t} = (1 - \theta_1 - \theta_2)\rho_{12} + \theta_1 \frac{\sum_{m=1}^{M} u_{1,t-m}u_{2,t-m}}{\sqrt{(\sum_{m=1}^{M} u_{1,t-m}^2)(\sum_{m=1}^{M} u_{2,t-m}^2)}}
$$

and for the DCC of Engle (2002):

$$
\rho_{12t} = \frac{(1 - \alpha - \beta)\bar{q}_{12} + \alpha u_{1,t-1}u_{2,t-1} + \beta q_{12,t-1}}{\sqrt{((1 - \alpha - \beta)\bar{q}_{11} + \alpha u_{1,t-1}^2 + \beta q_{11,t-1})((1 - \alpha - \beta)\bar{q}_{22} + \alpha u_{2,t-1}^2 + \beta q_{22,t-1})}}
$$

Equations (15) and (16) present the difference between these two DCC models. Unlike Tse and Tsui (2002), Engle (2002) formulates the conditional correlation as

---

14. A necessary condition to ensure the positivity of $\psi_{11}$, and therefore also of $R_i$, is that $M \geq N$.

15. The elements of $\bar{Q}$ can be estimated or alternatively set to their empirical counterpart to render the estimation even simpler.
a weighted sum of past correlations. The matrix $Q$ is written like a GARCH equation, and then transformed to a correlation matrix.\textsuperscript{16,17}

The implications for estimating the hedge ratio: $\text{OH}_t = \frac{\sigma_{df}}{\sigma_{gf}}$ is in the numerator. In the first step of DCC models the conditional variance of the futures is obtained from the first step GARCH equations. These estimates are identical. It is the dynamic covariance estimates obtained via modifying the CCC structure of equation (A1) with a different structure for $R_t$ that differentiate estimates of the conditional covariances.

III. DATA AND DESCRIPTIVE STATISTICS

The S&P 500 futures and stock index data for this study was obtained from the Commodity Futures Trading Commission (CFTC) in Washington, DC. The window of observation is from January 2000 to June 2004.\textsuperscript{18} The original dataset contains records of every trade in the S&P500 futures, including time stamp, trade price, volume, and masked trader identifiers. Both sides of the trade are visible enabling identification of buy/sell transactions. As well, the CFTC has classified four types of trader into CFT1 market makers, CFT2 Institutions, CFT3 brokers and brokers trading for other floor traders, and CFT4 the general public. We have sampled this dataset into 15 minute files corresponding to the CFTC-required reporting interval for these futures contracts trade. The S&P500 cash index is synchronously sampled into 15 minute files from records refreshed every 15 seconds.

The data employed in this study consist of the S&P 500 stock index trade price (open, close, high, low) and the S&P 500 stock index futures trade price (first, last, high, low) and volume of trade for 15 minute sampled files. This gives us 28,782 data points. The index futures contracts are rolled over to the next month’s contract on the expiry date.

In Figure 1 plots of the S&P500 cash index levels are reported for the sub-period 2000 to end June 2004. During this sub-period there seems to be a bear run accompanied with substantial price volatility during 2000, 2001, and 2002 followed by an upward less volatile movement in the market during 2003 and 2004.

The higher order moments from the descriptive statistics indicate that all the return series are leptokurtic, a common feature of financial returns series. We also looked for trends and seasonal components in these return series but could not find

\textsuperscript{16} However, for both DCC models, one can test $\theta_1 = \theta_2 = 0$ or $\alpha = \beta$, respectively, to check whether imposing constant conditional correlations is empirically relevant.

\textsuperscript{17} Note that $\theta_1$, $\theta_2$ and $\alpha$, $\beta$ are scalars, so that all the conditional correlations obey the same dynamics. This is necessary to ensure that $R_t$ is positive definite $\forall t$, through sufficient conditions on the parameters. If the conditional variances are specified as GARCH(1,1) models then the DCC models contain $(N + 1)(N + 4)/2$ parameters, that is 9 parameters in the bivariate case. CCC and DCC models can be estimated consistently in two steps, which make this approach feasible when $N$ is high. Of course, when $N$ is large, the restriction of common dynamics gets tighter, but for large $N$ the problem of maintaining tractability also gets harder.

\textsuperscript{18} We have also available the transactions in the S&P500 futures for 1994–1999 and the cash index sampled into 15 minute files.
The daily index levels are sampled 15 minutes after opening of trade in the stocks making up the S&P500 index. The intention is to avoid excess volatility usually associated with the market opening and closing mechanisms.

any evidence of trend or seasonality. Note that all cash and futures returns series show a non-normal distribution. Unit root tests involving Augmented Dickey-Fuller and Phillips-Perron tests show the futures and cash returns series to be stationary in log-difference, and therefore integrated of order one in (log) levels (see panel B of Table 1). Tests for cointegration show that the futures and cash returns series are cointegrated, and therefore share a long-term equilibrium relationship (see panel C of Table 1).

IV. RESULTS AND DISCUSSION

A. Estimation and OHR Effects

The results from estimated unrestricted versions of bivariate BEKK-GARCH of Engle and Kroner (1995) are presented in Table 2. The first two rows on the table report the conditional means while rows 3 to 13 show the conditional covariance parameters from log-likelihood estimation. Skewness and kurtosis statistics are reported in rows 15 and 16 while 10th-order serial correlations from standardized residuals are reported in rows 17 to 22.

The conditional covariance between futures and cash prices for the square-root model (parameters $\hat{A}_{22}$ and $\hat{\sigma}_{22}$ in column 2), display strong evidence of interactions between futures and cash returns, as the parameter coefficients are
both positive and significant. The conditional covariance matrix is positive definite as it satisfies the sufficient condition for positive definiteness that $\hat{A}_{11}$ and $\hat{G}_{11}$ coefficients are strictly positive. However, the matrix is not covariance stationary as it violates the necessary and sufficient conditions for stationarity: (i) $\alpha_{11}^2 + g_{11}^2 < 1$ and (ii) $\alpha_{22}^2 + g_{22}^2 < 1$, which translates to (i) $\hat{A}_{11}^2 + \hat{G}_{11}^2 < 1$ and (ii) $\hat{A}_{22}^2 + \hat{G}_{22}^2 < 1$ after estimation.

Specification tests (see rows 15 to 22) indicate a high level of excess kurtosis for the residuals of both futures and cash returns series. Evidence of serial correlation in the standardized residuals is also detected. These diagnostic test results support the findings of past literature recognizing the long-memory property and persistence of shocks commonly associated with a higher frequency data set (see deLima et al. 1994; Ding et al. 1993; and Harvey 1993 for reference). Overall it appears that the bivariate BEKK-GARCH model of Engle and Kroner (1995) produces consistent results in terms of model estimation.

Results from the bivariate BEKK-GARCH model with the unity constraint on the parameter matrices also shows strong evidence of interactions between futures and cash returns as the conditional covariance parameters $\hat{A}_{22}$ and $\hat{G}_{22}$ are both positive and significant. The conditional covariance matrix is positive definite as the parameter coefficients $\hat{A}_{11}$ and $\hat{G}_{11}$ are strictly positive. However, the matrix is not covariance stationary as it violates the necessary and sufficient conditions for stationarity, that is, (i) $\hat{A}_{11}^2 + \hat{G}_{11}^2 < 1$ and (ii) $\hat{A}_{22}^2 + \hat{G}_{22}^2 < 1$.

Diagnostic tests from the standardized residuals (see rows 15 to 22) show evidence of a high level of leptokurtosis from both the futures and cash returns series. Box-Pierce test statistics also show serial correlations in the residuals as was the case for the square-root constraint BEKK-GARCH model. The bivariate BEKK-GARCH model with unity constraint on parameter matrices shows a higher degree of leptokurtosis than that of the square-root constraint model.

As noted in Section II, the DCC models involve a two step procedure where univariate GARCH equations are first estimated for the cash and futures returns. In the second step the DCC estimates are then computed using the formulations from Engle (2002) and Tse and Tsui (2002) as represented in equations (15) and (16). For both cash and futures returns GARCH equations both exhibit strong persistence in variance but still below the non-stationary boundary. Both second step DCC results display results on the non-stationary boundary but with subtle differences. There is a slight dampening of longer term memory in the Tse and Tsui Beta parameter relative to Engle Beta parameter and slightly more weight given to recent effects. The striking difference is between the Rho .21 terms, which are 0.288 in the Tse and Tsui estimates compared with 0.874 from the Engle model.

Considering the variance ratios presented in Panel A of Table 4 (calculated using the first formulation in equation 2), we find that the BEKK-GARCH model with unity constraint applied outperformed the generic square-root model in terms of variance reduction. However, despite outperforming the (0,1) Beta model, both are outperformed by the DCC Tse and Tsui model with the DCC Engle model the
Table 1. Statistical Features of the Returns Series.

| Panel A: Descriptive Statistics for Futures Returns, Cash Returns and Basis Series |
|---------------------------------|--------|--------|--------|
|                                  | Cash   | Futures| Basis  |
| Mean                            | 0.000  | 0.000  | -3.538 |
| Median                          | 0.000  | 0.000  | -0.580 |
| Maximum                         | 0.039  | 0.060  | 34.550 |
| Minimum                         | -0.041 | -0.060 | -96.280|
| Std. Dev.                       | 0.002  | 0.003  | 6.247  |
| Skewness                        | 0.202  | -0.055 | -1.776 |
| Kurtosis                        | 23.262 | 53.062 | 8.043  |
| Jarque-Bera                     | 492531.587 | 3005468.632 | 45638.116 |
| Prob.                           | 0.000  | 0.000  | 0.000  |
| Obs.                            | 28781  | 28781  | 28782  |

Notes: “Std. Dev.” denotes standard deviation. “Jarque-Bera” represents Jarque-Bera test statistic for normality and “Prob.” denotes the corresponding p-value for testing normality. “Obs.” stands for the number of sample observations.

| Panel B: Unit Root Tests for Returns and Basis Se |
|---------------------------------|--------|
| DF-FuPL                         | -1.736 |
| PP-FuPL                         | -1.760 |
| ADF-FuRD                        | -191.287 |
| PP-FuRD                         | -190.864 |
| ADF-CaPL                        | -1.722 |
| PP-CaPL                         | -1.764 |
| ADF-CaRD                        | -168.746 |
| PP-CaRD                         | -168.911 |
| ADF-BaL                         | -3.787 |
| PP-BaL                          | -44.834 |

Notes: “ADF-PL” denotes the value of Augmented Dickey-Fuller test statistic for price series in log-levels. "ADF-RD" stands for Augmented Dickey-Fuller test statistic for returns series (or price series in log-difference). Similarly, “PP-PL” and “PP-RD” represent Phillips-Perron test statistics for prices in log-levels and in return series, respectively. “Fu” stands for futures, “Ca” stands for cash and “Ba” represents basis price series in the entire above notation.
**Table 1, continued. Statistical Features of the Returns Series.**

Panel C: Cointegration Test between Futures and Cash Return Series

\[
\Delta \hat{e}_t = \delta + \gamma \hat{e}_{t-1} + \nu_t ; \quad \hat{e}_t = Fu_t - \hat{\beta}_1 - \hat{\beta}_2 Ca_t
\]

|  \hat{\delta}  | 0.000 (0.000) |
|  \hat{\gamma}  | -1.356 (0.006) |
|  \tau_{test}   | -246.305   |
|  \tau_{critical} | -3.900 |

Notes: Numbers in parentheses are p-values; \( \tau_{test} \) is the \( \tau \) (tau) test statistic for the estimated slope coefficient. \( \tau_{critical} \) shows the Davidson-MacKinnon (1993) critical value at 99% level of significance to test the null hypothesis that the least squares residuals are nonstationary. In the above table, we reject all the null hypotheses that least squares residuals are nonstationary, and conclude that futures and cash returns series are cointegrated.

poorest performer of this group of four. However, all four GARCH based models outperform the naive “hedge ratio set to one” approach.19

Following these results we have undertaken preliminary analysis by calculating the one step ahead risk reduction of the cash portfolio using both DCC models. We have used 2,000 initial data points for estimation, then set up a loop picking up one new and deleting the first to re-estimate the parameters. The next step is to replace lagged values in the models with current and thus calculate the time varying hedge ratio at time \( T+1 \). The estimations are then dynamically re-estimated generating a sequence of one step ahead forecasts. The portfolio risk reduction is calculated as in equation (3) but employing the cash return and OHR at time \( T+1 \) and futures return at time \( T \). As we can see from Panel B of Table 4, the DCC Tse and Tsui dominates the simple model and the risk reduction is very impressive. This is a very important result as the third level of this analysis will consider the profit/loss from rebalancing a hypothetical futures position in a dynamic setting.

**V. CONCLUSION**

This has been an interesting exercise in model comparison between two separate sets of competing models, the classic BEKK and Power BEKK models and two competing DCC models. As noted above, the ranking of these models is in terms of minimizing the risk of the underlying cash index returns. The rankings from first to last are DCC Tse and Tsui number one, Power BEKK number two, BEKK number 3, DCC Engle number 4 and (0,1) Beta model number five.

19 Note as mentioned earlier these results for the BEKK unity constraint model, although unbiased, the 119 observations were removed from both cash and residual returns when calculating the variance ratios, there could be minor variation comparing these with the other models. However, the proportion of observations are extremely small relative to the overall sample size.
Table 2. Estimation of BEKK Model with Square Root / Unity Constraint on Parameter Matrices.

\[ R_t = \alpha + u_t, \quad R_t = \left[ R_{1,t}, R_{2,t} \right], \quad u_t \mid \Omega_{t-1} \sim N(0, H_t) \]

\[ H_t = C_0'C_0 + A_{11}'e_{t-1}e_{t-1}'A_{11} + C_{11}'H_{t-1}G_{11} \]

<table>
<thead>
<tr>
<th></th>
<th>SQRT</th>
<th>UNITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\alpha}_1 )</td>
<td>0.006 (0.000)</td>
<td>0.011 (0.000)</td>
</tr>
<tr>
<td>( \hat{\alpha}_2 )</td>
<td>0.002 (0.001)</td>
<td>0.006 (0.000)</td>
</tr>
<tr>
<td>( \hat{\epsilon}_{011} )</td>
<td>0.119 (0.001)</td>
<td>0.030 (0.000)</td>
</tr>
<tr>
<td>( \hat{\epsilon}_{021} )</td>
<td>0.088 (0.001)</td>
<td>0.073 (0.000)</td>
</tr>
<tr>
<td>( \hat{\epsilon}_{022} )</td>
<td>0.001 (0.078)</td>
<td>0.009 (0.000)</td>
</tr>
<tr>
<td>( \hat{\epsilon}_{111} )</td>
<td>0.504 (0.005)</td>
<td>0.470 (0.001)</td>
</tr>
<tr>
<td>( \hat{\epsilon}_{121} )</td>
<td>0.046 (0.003)</td>
<td>0.030 (0.001)</td>
</tr>
<tr>
<td>( \hat{\epsilon}_{211} )</td>
<td>-0.059 (0.006)</td>
<td>0.755 (0.001)</td>
</tr>
<tr>
<td>( \hat{\epsilon}_{222} )</td>
<td><strong>0.484</strong> (0.004)</td>
<td><strong>1.023</strong> (0.002)</td>
</tr>
<tr>
<td>( \hat{\epsilon}_{112} )</td>
<td>0.866 (0.002)</td>
<td>0.555 (0.000)</td>
</tr>
<tr>
<td>( \hat{\epsilon}_{122} )</td>
<td>-0.089 (0.002)</td>
<td>-0.170 (0.001)</td>
</tr>
<tr>
<td>( \hat{\epsilon}_{212} )</td>
<td>-0.054 (0.002)</td>
<td>0.049 (0.000)</td>
</tr>
<tr>
<td>( \hat{\epsilon}_{222} )</td>
<td><strong>0.884</strong> (0.002)</td>
<td><strong>0.773</strong> (0.001)</td>
</tr>
<tr>
<td>Log-L</td>
<td>214.61946</td>
<td>10539.841</td>
</tr>
</tbody>
</table>

\[ S_{Fu} = 0.843 \quad -0.125 \]
\[ K_{Fu} = 50.212 \quad 63.930 \]
\[ Q_{Fu}^{(10)} = 50.025 \quad 43.727 \]
\[ Q_{Fu}^{2(10)} = 4.370 \quad 34.416 \]
\[ S_{Ca} = 0.296 \quad 0.259 \]
\[ K_{Ca} = 21.223 \quad 20.309 \]
\[ Q_{Ca}^{(10)} = 16.070 \quad 9.209 \]
\[ Q_{Ca}^{2(10)} = 17.190 \quad 48.406 \]

Notes: Numbers in parentheses are p-values; \( S \) represents skewness of standardized residuals and \( K \) represents kurtosis of standardized residuals with subscripts "Fu" and "Ca" denoting futures and cash equation respectively; \( Q(10) \) and \( Q^2(10) \) show the Box-Pierce statistics for tenth-order serial correlations in the residuals and squared normalized residuals, respectively.
Table 3. Estimation of DCC GARCH Models.

<table>
<thead>
<tr>
<th></th>
<th>CASH</th>
<th>FUTURES</th>
<th>DCC-E</th>
<th>DCC-T&amp;T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cst(M)</td>
<td>-0.002 (0.000)</td>
<td>-0.001 (0.271)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cst(V)</td>
<td>0.001 (0.000)</td>
<td>0.001 (0.000)</td>
<td>0.874 (0.000)</td>
<td>0.288 (0.800)</td>
</tr>
<tr>
<td>$\hat{\alpha}_1$</td>
<td>0.100 (0.000)</td>
<td>0.268 (0.000)</td>
<td>0.001 (0.000)</td>
<td>0.006 (0.001)</td>
</tr>
<tr>
<td>$B_1$</td>
<td>0.800 (0.000)</td>
<td>0.695 (0.000)</td>
<td>0.998 (0.000)</td>
<td>0.994 (0.000)</td>
</tr>
<tr>
<td>$S$</td>
<td>0.202</td>
<td>-0.055</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K$</td>
<td>23.26</td>
<td>53.06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log-L</td>
<td>25852</td>
<td>22917</td>
<td>70750</td>
<td>71697</td>
</tr>
</tbody>
</table>

Notes: Numbers in parentheses are p-values; $S$ represents skewness of standardized residuals and $K$ represents kurtosis of standardised residual. The univariate GARCH results are in columns 2 and 3 with relevant DCC estimates in columns 4 and 5.

Table 4. Comparison of Variance Ratios from Different Models.

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Within Sample</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0,1) Beta</td>
<td>0.439</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sqrt_paras</td>
<td>0.357</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unity_paras</td>
<td>0.331</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DCC T&amp;T</td>
<td>0.338</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DCC Engle</td>
<td>0.405</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>One Step Ahead</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DCC T&amp;T</td>
<td>0.147</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DCC Engle</td>
<td>0.205</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: "(0,1) Beta" stands for the ratio of variance from cash returns less futures returns. "Sqrt_paras" specifies the ratio of variance from a BGARCH model with square root constraint on parameter matrices and variance returns following Engle and Kroner (1995). "Unity_paras" refers to the ratio of variance from a BGARCH model with unity constraint on parameter matrices and cash returns following Bhattacharya et al. (2007). DCC T&T refers to the DCC model of Tse and Tsui (2002) and DCC Engle the DCC model of Engle (2002).
What was novel in this exercise was that the analysis was undertaken using a
dataset compiled by the official regulator of every trade in the S&P500 over an
extended time period. This database was validated and resampled to the regulators
required 15-minute reporting intervals by sorting via time stamps attached to each
trade.

What has not been considered in this study was the performance of competing
simultaneous volatility models of the time varying variance/covariance matrix by
employing the procedures employed in Chng and Gannon (2003) to calculate the
hedge ratios. As well, the database has been resampled to account for accumulated
volume of trade within each 15-minute reporting interval. So a further extension
would be to modify the simultaneous volatility models reported in Gannon (1994,
2005, 2010) to account for contemporaneous volume of trade effects in the index
futures when calculating the hedge ratios.

One restriction imposed in this study was to restrict attention to within-sample
model comparison for the BEKK models. However, it is straightforward to formulate
the one-step ahead out-of-sample hedge ratios so as to further evaluate the
performance of these competing estimators as the BEKK, DCC (and simultaneous
volatility classes of models) have been formulated for such a comparison. A further
analysis would be to rank the models in terms of out of sample trading performance.

In general, the number of futures contract to short \( N_i \) against a long position
in a widely held equity portfolio can be expressed as \( N_i = \beta_t \times (\Pi_i / \tau_i) \), where \( \Pi_i \)
is the value of the physical position, \( \tau_i \) is the value of each futures contract, and \( \beta_t \)
is the OHR. Employing the forecasted OHR, we can see if the optimal number of
contracts should be varied by either going long or short in additional contracts, then
the profit/loss analysis can be conducted truly out of sample.

**APPENDIX**

Let us start with the CCC model which is defined as:

\[
H_t = D_t RD_t = (\rho_{ij} \sqrt{h_{ii} h_{jj}})
\]

where

\[
D_t = \text{diag}(h_{11}^{1/2}, \ldots, h_{Nt}^{1/2})
\]

\( h_{it} \) can be defined as any univariate GARCH model, and

\[
R = (\rho_{ij})
\]

20. See Yeh and Gannon (2000) for an example of such an approach for a variant of the CCC model and competing estimators of the out of sample hedge ratios.

21. See Chng and Gannon (2003) for an example of such an approach for variants of simultaneous volatility, CCC and weighted GARCH models where contemporaneous volume of trade is employed in some of these competing classes of estimators of out of sample hedge ratios.
is a symmetric positive definite matrix with $\rho_{ii} = 1, \forall i$.

$R$ is the matrix containing the constant conditional correlations $\rho_{ij}$. The original CCC model has a GARCH $(1,1)$ specification for each conditional variance in $\mathbf{H}$:

$$h_{ii} = \omega + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i h_{ii,t-1} \quad i = 1, \ldots, N. \quad (A4)$$

This CCC model contains $N(N + 5)/2$ parameters. $H$ is positive definite if and only if all the $N$ conditional variances are positive and $R$ is positive definite. The unconditional variances are easily obtained, as in the univariate case, but the unconditional covariances are difficult to calculate because of nonlinearity.

References


