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GROUPING: SUCCESSES, SURPRISES AND CATASTROPHES

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The Australian Mathematics Curriculum emphasises the need to develop mathematical creativity to foster deep learning. This paper extends our understandings of how progressive changes to group membership over time can increase student inclination to explore unfamiliar challenging problems. It reports on the successive changing of group membership in two different situations and uses this to identify key features in what occurred. Like in any classroom, there were successes, surprises, and catastrophes.

Promoting Creative Mathematical Thinking: A Need

The Melbourne Declaration of Educational Goals for Young Australians http://www.mceccdy.ac.edu.au/mceccdy/melbourne_declaration,25979.html and the Australian Mathematics Curriculum (see <http://www.australian_curriculum.edu.au/Mathematics/Rationale>) require the development of creative, innovative and resourceful problem solvers. Enabling such activity in mathematics classrooms includes providing opportunities for students to enter the Space to Think (Williams, 2005) which is a state of flow in which students lose all sense of time, self, and the world around because all their energies are focused on creatively engaging with the task at hand (Csikszentmihalyi, 1992).

My research shows that it is resilient (optimistic) students who enter that flow state. ‘Optimism’ is an orientation to ‘failures’ and ‘successes’ (Seligman, 1995). Optimistic children perceive failure as ‘temporary’ (able to be overcome), ‘specific’ (to the situation at hand), and ‘external’ (can be associated with factors beyond their control). They perceive successes as personal (achieved through their own effort), permanent (able to be achieved again), and permanent (normalized as characteristics of self: ‘I did this, I am good at this’).
Creating conditions for creative thinking includes providing accessible tasks that can be undertaken using various mathematical ideas, and representations. These tasks need to contain opportunities for students or groups to discover and want to explore mathematical complexities that were not apparent to them at the start of the task. They also include autonomous and spontaneous student action so the teacher does not hint or tell but rather asks questions to elicit further thinking. Conditions for flow include students or groups spontaneously asking a question that takes them outside their present understanding and into the Space to Think. ‘Same pace of thinking’ groups is one of the key aspects of My Engaged to Learn Approach to problem solving (Williams, Harrington, & Goldfinch, 2012) within which students work in small groups (3-4 students) with problem solving tasks. Each group reports to the class at regular intervals with a different reporter each time. Priming involves the groups brainstorming what to report, the reporter practicing this report, and the group refining it to fit what they want. This is a key element of the approach.

It is not always easy to engineer conditions for flow in lessons, and sustain it once it occurs. Crucial to optimising these condition can be changes purposefully made to the composition of groups in the class over time. This paper extends the thinking about groups beyond my previous ideas on how to compose groups (see for example: Williams, Harrington, & Goldfinch, 2012) to a focus on how to successively change group membership over a period of time to build the optimism of particular class members.

Change Sequences

In this paper, two different instances of this changing of group composition across a sequence of tasks are reported. These sequenced changes have been named: a) From Self-Focused to Task-Focused; and b) Prioritising the Majority. Each grouping sequence is discussed in terms of the dilemmas, reflecting on how to proceed, the result, the next step, and reflecting on the outcome.

From Self-Focused to Task-Focused

This three task sequence was intended to shift Tom (pseudonym) from a non-optimistic student who perceived success came from external judgments of his mathematical worth to an optimistic student who takes on mathematical successes as attributes of self: ‘I can do these, I am good at this’. The three tasks across which the composition of Tom’s group was changed are now briefly described.

Task 1

Using all 14 tiles (each time), make as many different flat 'filled' rectangles as you can.
Repeat using 12 tiles. Have you found all possibilities? Make an argument for how you know you have them all. Do more tiles make more rectangles? Why or why not? Select a number of tiles between 16 and 45 to make as many rectangles as possible using all each time. Explain your process of thinking.

**Task 2**

A large fish drawn on the board has an arrow pointing to the head stating 'the head is as heavy as four tails' and an arrow pointing to the body stating 'the body is as heavy as the head and the tail together'. Find all you can about the maths of such fish.

**Task 3**

Use four of the digit four, and any number of the following

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to make each of the whole numbers from 1-20. Then look for ways to find them all as fast as you can. Explain.

**During Task 1**

Tom was in a composite Grade 4-6 class. Tom (Grade 5) was grouped with Billy (Grade 4) and Sammy (Grade 5). As shown from the excerpt of group discussion before the last of four reporting sessions, Billy (Grade 5) has begun to develop an idea but Tom argued that it was his turn to report and he was going to report his own ideas not the ideas of the group. This was contrary to class 'rules' and a disagreement ensued. The majority of the group discussion time was taken up arguing and the group did not have time to refine Billy's idea.

Group opportunities to enter the Space to Think were inhibited by Tom's focus on self:

1. Billy  [trying to gain Tom's attention] Tom- Tom- Tom every time you get a number that can't be divided by three- two (pause) you are bankrupt you can't find any more
2. Tom  I am not going to say that
3. Billy  Yes you are
4. Tom  No I'm not
5. Billy  Tom (pause) you need to say it
6. Tom  I know you are priming me but I don't have to say it ... I am saying what I have been thinking.
7. Billy  We need no the group tells you what to say
8. Tom  Yeah but that doesn't mean I have to say it

Billy had found that if you continued dividing a number and using the answer for the
next step, you could not go any further once your answer would not divide by two. He thought the number of divides gave the number of rectangles. This is not yet correct but could have led to rich mathematical discussions of divisibility. Billy presented his idea to Sammy and Tom [Line 1] and began to summarise for Tom. Lines 2-4 show the types of disagreement between Tom and Billy that continued until almost the end of the reporting session. Both Tom and Billy were aware of the group rules [Lines 6, 7] but Tom disregarded them [Line 8]. He was too focused on reporting his own ideas so they could be valued by others including the teachers. He did not consider Billy’s idea and whether it always worked. Tom eventually stated: ‘It’s my time to report and I am going to say what I want to say’. In his post lesson interview, Tom showed he did not have anything new at that stage:

“Ah I was thinking—ah well (pause) I was thinking um—oh—well (pause) um I was thinking ah there must be other ways to do it than just following a pattern—and I know you should follow patterns but there has to be a different way…”

Tom wanted to report something original. He was focused on himself and not the task. It did not matter that Billy had presented the start of some idea that could turn out to be very useful, to Tom that idea was around patterns and patterns were something that many groups focused on. He wanted to be different rather than solve the task.

Preparation For and Undertaking Task 2

To increase Tom’s likelihood of gaining task focus, Tom was grouped with two Grade 6 students Natasha and Ken who had previously collaborated to build insights. They were both focused and calm and it was considered they would be able to stop Tom from exhibiting ‘take over’ activity and make him conform to group reporting rules. The group did calmly discuss ideas, and they quietly used body language to deter Tom’s attempts, initially, to ‘take over’. Tom listened and contributed by asking when he did not understand. The group primed Tom to present a clear report that contributed to class knowledge. Tom expressed his interest in this task after Task 3: “You can just draw something so small with so little detail and find out so much about it and end up [small laugh] (pause) just like you have known it forever.” This task contained an element of surprise that engaged Tom with the mathematics. His awareness had been raised that you could learn from these tasks and they may not be as simple as they first appeared. Tom was pleased with the result but as researcher and teacher, I wanted more. Although Natasha and Ken had entered the Space to Think and created new ideas as a result, Tom had not, he had listened and learnt from the other two. He had not contributed to the newly created ideas. My next question became, who could I group Tom with who was a creative thinker but thought at a similar pace to Tom, and would be able to control any ‘self-promoting’ activity? If I could place him in
such a group, would he now focus on the task and contribute new ideas?

Preparation for and Undertaking Task 3

Tom was grouped with Gabriele, a student who I considered would be able to 'control' any ‘take over’ activity and could think creatively but not at a fast pace. This was expected to give Tom more time to think. It was hoped that Tom and Gabriele would create new ideas together. The other two students in the group had not previously contributed creative ideas but had not disrupted their groups. In this group, they would hopefully have opportunity to learn and also experience the creative development of new ideas by others (as Tom had in Task 2). Tom was the final reporter for his group. In the previous reports, Alf from another group explained: ‘Well four over four is one whole so that’s just like saying one (pause) and four times four plus one you would get seventeen.’ Tom reported his excitement at hearing this in his interview: ‘When he said four over four and it is the same as one just that sentence just flung me [intense, twirls hand] like quickly in my mind abbb I could use that’.

Tom’s idea was to use two of the four fours to make a multiple of four then the other two fours as: ‘Four over four-... that’s one okay... so that means that using a whole you can go... minus 1 or plus one... we could get these numbers... eleven thirteen fifteen seventeen and nineteen and twenty’. Gabriele had either worked out that Tom would not be able to make each multiple of four, or wanted to know how he was doing this but Tom just kept repeating his idea about plus or minus four over four. Gabriele insistently requested elaboration as she tapped the table in front of Tom to gain his attention and asked with intensity: “how how how are you going to get seventeen and nineteen twenty?” When Tom again focused on the end not the stem of the calculation, Gabriele did not give up, she picked up and handed the pencil and paper to Tom stating: “So if you were going- if somebody asked you to make- to get answers for every single number [ruling up sheet] ...”. Gabriele’s request for elaboration led to Tom’s revised ideas which limited the stem to 8, (4+4) and 16, (4x4). He elaborated on the reason for his changed position: “something like 12 (pause) you won’t be able to do it because four plus four plus four is one of the only ways to get to twelve and... [there] will be five fours”. Tom’s ideas were refined through Gabriele’s insistence for justification and elaboration. He had located some correct possibilities but did not see new possibilities for making the stem using two fours and other operations.

The Outcomes

In his final interview after Task 3, Tom showed he now considered himself as a problem solver after ‘seeing’ his activity over time. He described how his Mum had told him he was good at maths but he had not believed her: “but when I have gone through with
these [tasks] (pause) I have believed that I am a bit smart”. Tom has begun to shift from perceiving success as external—others acclaiming his ideas—to making his own assessment about his maths ability by reflecting on his task activity. Tom had begun to perceive Success as Personal. Tom changed from self-focused (Task 1), to group focused (Task 2), and task focused (Task 3). He experienced surprise as complexities became apparent in what had appeared to be simple (Task 2), and displayed positive affect during his creative thinking leading to insight (Task 3). Gabrielle’s insistent ‘push’ for elaboration was crucial to Tom extending his understanding of the detail of the ideas he was putting forward. Without this, Tom would not have refined his initial ideas.

In Tom’s case, creative mathematical activity accompanied by high positive affect was associated with increased indicators of optimism as expected (Seligman, 1995). Indicators were tentative not strong. This grouping sequence highlights the important role group composition played in changing Tom’s orientation to problem solving. This student who had required external affirmation to perceive himself as successful inhibited opportunities for his group to think creatively in Task 1. The non-optimistic indicator he possessed (perceiving Success as judged Externally) created an unproductive interaction because Tom was not focused on the task but on ‘showcasing’ himself. By successively grouping Tom to inhibit this non-productive activity, then giving him an opportunity for undertaking creative thinking himself, there is now another optimistic student in the class to promote rather than inhibit creative thinking.

**Prioritising the Majority**

One of the most surprising and devastating findings in my study of the role of optimism in collaborative problem solving has been the inhibiting of creative activity by high performing confident students who are not willing to ‘step outside’ what they know mathematically to explore new mathematical ideas (Williams, 2012). These are students who perform near the top of the class on class tests and are perceived by the class and the teacher to be ‘good at maths’. They differ to other high performing students in the way they learn, and in the way they perceive their mathematical ability. Where optimistic high performing students describe how they learn using active words like ‘thinking things out’ these high performing non-optimistic students describe learning as ‘listening to the teacher, reading books, and using the internet’. They judge their mathematical ability using external judgments made by others whereas optimistic high performing students primarily judge their mathematical ability through self assessments of their ability to work things out. Now here is the dilemma! In one class there were four of these high performing confident
students who were unwilling to explore new ideas. In the first task, they were in four of the seven different groups. In each of these groups, other students did not have opportunity to enter the Space to Think because these non-optimistic students took up all the talk time with mathematics that was already known and cut off the talk of any student who raised a question about a mathematical complexity that was new to all group members.

What to do? These students were stopping opportunities for others to think! What would happen if we grouped them all together? This would give other groups an opportunity to explore new ideas that arose but what would happen to this group of high-performing non-optimistic students? These students are not optimistic because they perceive success as external (they learn only through external sources not the reorganisation of ideas, and they draw on external sources as evidence of their mathematical ability). We grouped these students together and guess what happened!

Many other groups did begin to think creatively, and the group of non-optimistic high performing students did not ‘step outside’ their present understandings. They were unable to progress with the problem solving task. What to do now though? It is the obligation of the teacher to develop the potential of all students. This had been addressed in other groups, but what about this group? We decided to leave them together and see what we could do by asking them more questions and keeping a watch so we could accent anything useful that came into their conversation. There had been no progress with this group one task later and the following year three of them were in the same Grade 5-6 class. We decided to continue to group them together and see if we could find a person who had demonstrated they thought creatively who was strong enough to continue to do so ‘out loud’ despite the inhibiting activity of these three students. We were surprised and delighted at what occurred in the next task.

One of the three non-optimistic high performing boys was absent. The other boy we decided to add to this group was intensely interested in mathematics and willing to explore new ideas and sufficiently ‘strong’ to make sure that his ideas were heard by and used by groups he had participated in. Adding this student to make a group of three worked extremely well. The group listened to the new ideas presented because this boy was persistent in presenting them, he was willing to explain what he was thinking to the others, and he was willing to answer their questions as they developed an understanding. Through his passion, and determination, it appears that this creative thinker was able to present himself as a mathematical authority ‘super alpha’ (Vygotsky, 1978). This lifted how these high performing confident non-optimistic students learnt taking in information from an authority figure. In their report to the class, each group member was excited about...
what the group had found, and able to communicate it clearly to the class.

What we learnt was: it was possible for two non-optimistic high performing students grouped together with a forceful, creative student (willing to take time explaining his ideas) to come to the types of realisations Tom had developed in Task 2—that there was more to these tasks than initially apparent, and that presenting ideas that interested the class was accompanied by positive experiences. Further research is needed to find whether these students do become optimistic over time when placed in such groupings. In addition, we need to know: was the *sequence* of groupings necessary or would this final grouping have worked straight away? In other words, did these high performing non-optimistic students need to be *surprised* that other groups (of students they perceived to be not as 'good at maths' as they were) developed more creative ideas (than they did) before they would be responsive to new ideas from another student? Was this sequencing needed for this responsiveness to occur?

**What Have We Learnt?**

What can we take out of these experiences as teachers? New ways to begin to think about building the creativity of members of your classes. Not only are the criteria for the composition of groups important but there is also further thinking to do. By attending over time to the personal characteristics of group members, and selecting those who might best be able to productively channel the activity of others, there is opportunity to raise the awareness of students about the complexities that lie hidden within what appear at first to be simple tasks. There is also opportunity for students to vicariously share in the creative successes of other group members by becoming sufficiently familiar with the new ideas to be able to communicate them clearly to the class, and experiencing the pleasure associated with class exclamations about what has been found. This is optimism building.

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References


