This is the authors’ final peer reviewed (post print) version of the item published as:


Available from Deakin Research Online:

http://hdl.handle.net/10536/DRO/DU:30062276

Reproduced with the kind permission of the copyright owner

Copyright: 2010, ICMI
Meta-discursive rules and the introduction of new content in mathematics classrooms in Seoul, Shanghai and Tokyo
Lihua Xu, David Clarke, and May Ee Vivien Wan
University of Melbourne, Australia
xulh@unimelb.edu.au
d.clarke@unimelb.edu.au
vivienwme@yahoo.com.sg

Abstract
The importance of classroom dialogue in student learning of mathematics has been highlighted in several recent publications (e.g. Walshaw and Anthony, 2008) and the emphasis on talking is reflected in several official initiatives. Our studies on spoken mathematics (Clarke & Xu, 2008) revealed significant differences among those classrooms characterised as “East Asian”, in the opportunities that each classroom afforded for the students to employ relatively sophisticated mathematical terms in both public discussion and private student interactions. This study extends our previous studies and seeks to compare the ways in which classroom talk was conducted in classrooms from Seoul, Shanghai, and Tokyo, with a particular focus on meta-discursive rules (Sfard, 2001, 2008) that regulate the patterns of exchanges between the teacher and the students. The analysis reported in this paper centres on the events in which a new mathematical topic was introduced, in this case, the topic of “linear equations”. The similarities and differences will be illustrated through the data obtained from the Learner's Perspective Study (LPS) (Clarke, 2006).

Keywords: cross-cultural studies, classroom discourse, mathematics classroom, LPS, Confucius Heritage Cultures

Introduction
The benefits of engaging students in classroom dialogue have been highlighted in a number of publications (e.g. Alexander, 2008; Lampert, 1990; Mercer, 1996; Walshaw & Anthony, 2008). Recently, the emphasis on classroom talk has been entering the educational mainstream and is reflected in several official initiatives, such as the Principles and Standards for School Mathematics in the U.S. (NCTM, 2003). As Walshaw and Anthony (2008) observed in their review of recent research into mathematics classrooms, that “talking about mathematics becomes acceptable, indeed essential, in the classroom, and mathematical discussion, explanation, and defense of ideas become defining features of a quality mathematical experience” (p. 516).

The significance attached to talking is consistent with Western beliefs about the connection between talking and thinking, which assumes speaking is conducive to learning. In contrast, silence is usually perceived as a lack of engagement or at best passive participation because “silence is in conflict with the notion that knowing mathematics involves arguing, defending, challenging, and proving one’s own ideas and those of others” (Lampert, 1990, p. 56). Under such assumptions, the learning environment in East Asia is perceived to be non-conducive to learning, yet East Asian students outperformed their Western counterparts in various international mathematics achievement studies. This seemingly paradoxical phenomenon has attracted much discussion in recent years (e.g. Leung, 2001; Mok, 2006; Watkins & Biggs, 1996). Attention has been paid to those deep-rooted cultural values, typically associated with Confucius Heritage Cultures (CHC), in an attempt to explain the consistent success of East Asian students. For example, Leung (2001) identified six features that distinguish East Asian mathematics education from its Western counterparts. By relating these features to the cultural values derived from the Confucian tradition, Leung argued that these features should not be only considered as traditions of established practices, but that we should examine the deep-rooted cultural values for explanations.
When considering classroom instruction, the shared cultural values such as respect for authority and humility in social relationships, and an emphasis on listening over verbalization frame classroom practice of East Asian Classrooms in similar ways, such as a preference for teacher-dominant whole-class teaching (Leung, 2001). However, resorting only to the overarching values and beliefs of Confucius Heritage Cultures could not explain the differences in the patterns of spoken mathematics that we observed among those classrooms characterised as “East Asian”. Our study (Clarke and Xu, 2008) revealed considerable differences in the opportunities that each classroom afforded for the students to employ relatively sophisticated mathematical terms in both public discussion and private student interactions.

To what extent do these reported differences reflect something more fundamental and robust about the distinctive pedagogies employed in each classroom, and to what extent are these differences culturally specific? With these questions in mind, we want to go beyond simply considering culture as a set of values and beliefs functioning merely as background or external influences, but to see culture as an integrated part of how the work in a classroom is carried out and sustained. For the clarity of the paper, we define “culture” to be “any aspect of the ideas, communications, or behaviours of a group of people which give them a distinctive identity and which is used to organize their internal sense of cohesion and membership” (Scollon & Scollon, 1995, p. 127). We distinguish microculture from macroculture. We use the word “macroculture” to refer to a set of ideas, communications, or behaviours embraced by the majority of people in a particular society, whereas “microculture” defines regularities and patterns of interactions specific to mathematics classrooms.

In order to understand the microculture of mathematics classrooms, we need a tool that could help us to reveal those deep features of a mathematics classroom rather than simply a description of patterns of interaction. For this purpose, we employed the notion of “meta-discursive rules” (Sfard, 2001, 2008) to refer to those rules that regulate or govern discourse in classrooms. According to Sfard (2001), the meta-rules in mathematical discourse include those that underlie the uniquely mathematical ways of defining and proving; rules that regulate and guide interpersonal exchange and self-communication; the way symbolic tools should be used in a given type of communication; and those meta-rules involved in regulating interlocutors’ mutual positioning and shaping their identities. These meta-rules are the observer’s construct and mostly act “from behind the scene”.

The analysis reported in this paper centres on the events in which a new mathematical topic was introduced. We selected one classroom from each of the three cities: Shanghai, Seoul, and Tokyo, from the dataset of the Learner’s Perspective Study (LPS) because of their shared focus on the topic of “linear equation” or “linear function”. The LPS research design has been detailed elsewhere (Clarke, 2006) and will not be elaborated here. This paper reports the analysis of the first three lessons from each of the classrooms studied. The guiding question for the analysis is “what are the similarities and differences in the ways in which the topic ‘linear equations’ was introduced in each of the three classrooms”. To address this question, the data analysis was conducted in two phases. In the first phase, three lessons from each of the classrooms were analysed to reveal the forms and functions of the activities involved in introducing the new content. In the second phase, classroom dialogue was examined in detail to uncover not only the content of the exchanges between classroom participants, but more importantly, the meta-discursive rules governing those exchanges. This paper discusses the meta-discursive rules related to:

- Nature of mathematics: what is mathematics and who defines the rules and principles?
- Ways of learning mathematics: how is mathematics learned in the classroom?
Mathematical language: what is considered to be the appropriate use of mathematical language?
Mathematical explanation: what counts as a valid and acceptable explanation?
Mathematical solution method: what is regarded as an acceptable solution method?

We will discuss these meta-rules in relation to the beliefs, values and expectations from the broader macroculture and the traditions of a particular education system. Based on the comparison of the meta-discursive rules in the three classrooms, we conclude the paper by examining the affordances of these rules on student mathematics learning.

It should be emphasized that the selection of these three classrooms is not intended to signify any form of national typification. Instead, we want to illustrate the distinctive pedagogy that each classroom employs, and to show how the meta-discursive rules shape the forms of knowledge allowable in each classroom. Whether the set of meta-discursive rules identified for each classroom is culturally specific is open to further investigation.

Introducing Linear Equations in the three classrooms

Despite a common focus on linear equations, observation of the lessons showed different tasks and activities employed in each classroom. In the Shanghai classroom (SH1), the topic of the first lesson was on linear equations in two unknowns and solutions. Particular attention was paid to clarifying the meaning of 二元一次方程 (linear equations in two unknowns) and the concepts of a solution and a solution set. The second and the third lessons introduced the rectangular coordinate axes and coordinates as "a graphical method" for solving linear equations in two unknowns.

In the Seoul classroom (KR1), the emphasis of the first lesson was on the difference in the graphs of a linear equation in two unknowns, when the condition for variable X is a natural number as compared to the graph when X is a real number. Lesson Two focused on the notion of the intersection of the two straight lines as the solution of the simultaneous equations, and Lesson Three continued this focus and introduced the method of elimination by addition and subtraction.

The three lessons in the Tokyo classroom (JP1) were conducted around the same task: a stair problem (see Figure 1), which served as a context to introduce general forms of linear function. In the first lesson, the teacher invited the students to brainstorm about the variables that can be examined in the stair problem, and the class explored the relationship between the number of steps and the perimeter of the stairs in three forms of representation: a table, a formula, and a series of figures. In the second lesson, the class was asked to relate the mathematical relationship between the number of steps and the perimeter to the changes displayed in the figures. The students were also asked to formulate relationships between two variables of their choice. The definition of a linear function was introduced in Lesson Three.

In addition to the different tasks and activities, the way in which the lessons were organized in each classroom also differed. In the Shanghai classroom, the lessons were conducted in a very structured fashion, started with reviewing student prior knowledge relevant to the new content to be introduced, followed by several cycles of definition, practice, and note taking. The lessons usually ended with a summary of the key points in the form of teacher-led discussion with student input. Similar to the
Meta-discursive rules in the three classrooms

Doing mathematics as a collective activity

Our analyses of spoken mathematics in LPS classrooms revealed both similarities and differences in the way classroom dialogue was orchestrated in each classroom. Figure 2 shows the number of teacher utterances, student utterances, and choral utterances in each lesson analysed in this paper. The figure demonstrates that while teacher talk was the most dominant form of talk in all three classrooms, there are significant differences in the amount of choral and individual student utterances in each classroom. While very few choral utterances were found in the Tokyo classroom, this form of utterance was the most important means through which the students were given voice in the classroom discourse in the other two classrooms.

Further analyses of the classroom data revealed differences in the extent to which student contribution was valued in each classroom. In the Shanghai classroom, although the classroom discussion can be regarded as heavily guided by the teacher, the students were given many opportunities to contribute to the public classroom discourse, usually through teacher invitation. A distinctive feature of this classroom is that in many cases, the teacher would invite (either explicitly or implicitly) collective action (e.g. recite definition or answering questions together). One discursive indicator for such collective action is the frequent use of the words “we” or “classmates”, without differentiating individuals for a particular action. By using the word “we”, the students were regarded as part of the mathematics community that defines those mathematical rules (“we make a rule”, L02). Classroom activities were conducted in such a way that the conclusion could be seen to be the result of the collective contribution of the whole class. This approach of building on student contribution was expressed in his interview:

SH1-T: One characteristic (of a typical lesson) is that the teacher is the facilitator of learning. This lesson shows that students are the active agent in learning, from the beginning till the end. That is...(I raised) questions that let them to answer, and towards the end, students generate their conclusions. Even when we talk about the sample problems, the teacher does not tell them the conclusion directly. It is the students who have to think and talk about the problems by themselves. The role of the teacher is only to guide them. In other words, students are the active agent.

While the Shanghai teacher would weave student input into a coherent ongoing classroom discourse, student contribution to the public discussion in the Seoul classroom was minimal, with most of student responses reporting the result of a simple mental calculation or agreement with a statement made by the teacher. The teacher’s reluctance to the “new” way of teaching was clearly expressed in the interview:
KR1-T: These days there are many open classes in which students actively discuss in the class, I think the way of teaching is changing. But I think the teacher should teach. I think it is better. In the beginning, I teach and in the last part of the class I make students discuss what they learned. It is a good way to teach math, I don’t oppose to the open class. But I think teacher’s explanation is more important in teaching math.

Compared with the emphasis on uniform and collective action in both the Shanghai and the Seoul classrooms, the students in the Tokyo classroom were given autonomy to generate their own formulations of equations and come up with their own method of solving the problems. In the interviews, the teacher stated the importance of students having their own opinions and of raising these opinions in the public discussion. For example, in one of the interviews, she said:

JP1-T: Um, it went totally different from what I have planned, so I wouldn’t be able to evaluate this class right. But I had another thing I wanted to do in class if it had gone as I planned. That plan was to begin talking about a graph of a linear equation in general. So I had two plans for this lesson. But it was not important to do as planned. Students discuss with each other, and have their own opinions is the most important. And I think it is what was good about this lesson. Students didn’t only think about the problem for a while, but went further with the class. I think it was a very important part of the lesson.

The teacher valued the opportunity for the students to share their opinions with their peers, which was considered more important than having the lesson as planned. The observation of the Japanese lessons also showed that student expression of lack of understanding was acceptable and adequately resolved by the teacher. Arguably, this classroom is a different place from the one in which students are rarely given the chance to voice their own opinions.

In summary, despite the fact that all the three classrooms can be regarded as belonging to a collectivist culture, it can be argued that the form of collectivism was differently performed in each classroom. While in the Shanghai and Seoul classrooms, the students were given opportunities to verbally participate in the classroom discourse as a collective, the teacher in the Tokyo classroom respected the different opinions of individual students, and orchestrated the classroom discussion so that these student opinions were voiced and shared within the classroom as a community.

The use of mathematical language in the classroom
The significance attached to the use of standard mathematical language also differs from one classroom to another. In comparison with the other two classrooms, the Shanghai classroom showed a distinctive emphasis on the accuracy of mathematical language (see Figure 3). Through the classroom discursive interactions, the students were assimilated and institutionalised into a discourse of school mathematics that encourages the accurate use of standard mathematical terms. The modelling of mathematical language use by the teacher was a deliberate strategy, and the students were expected to follow such a model. Such expectations were never explicitly voiced in the classroom but were enacted in teacher-student interactions. The use of standard mathematical language can be regarded as a normative aspect of this particular classroom. The value attached to the use of accurate mathematical language and the completeness of student response was expressed in the teacher interviews, in which he explicitly evaluated student use of language in the classroom. This finding is
consistent with Leung’s study of Beijing classrooms (Leung, 1995), in which he reported that 15 out of the 36 lessons observed demonstrated the stress placed on the use of accurate and rigorous mathematical language.

**Mathematical explanations**
In many mathematics classrooms, it is not sufficient for students to simply provide an answer to a problem. Providing explanations is considered to be an essential component of mathematics discourse. What is regarded as an acceptable mathematical explanation might vary from one classroom to another. A mathematical explanation may involve students articulating their solution methods or students recalling a rule learned previously. An explanation may also involve students providing the meanings of words in relation to the task at hand or describing relationships between different types of representations.

In the Shanghai classroom, the students were frequently asked to provide explanations for their answers. Many of these explanations required the students to employ mathematical concepts or rules to justify their responses. The systemic way of defining and applying mathematical concepts (mediated by specifically designed tasks) could be seen as a reflection of the beliefs about the nature of mathematics and of mathematics learning. This is well grounded in a tradition of school mathematics in China that emphasizes basic knowledge and basic skills. As Li (2006) observes, under this tradition, the teaching process is usually deliberately organized to ensure that teachers and students concentrate on concepts, theories, rules, skills and techniques.

Compared with the Shanghai classroom, the rules or principles of solving linear equations or simultaneous equations in the Seoul classroom were given by the teacher with little explanation of their underlying meanings. The main aim of the lesson sequence was to help the students to understand the procedures of solving particular groups of equations rather than an explicit focus on conceptual meanings. This could be regarded as reflective of a belief that mathematics is composed of a given body of knowledge and truth, and the task of teaching is to pass this body of knowledge to the students. Such a “transmissive” way of teaching may also be influenced by the male dominant culture in Korea since this class was in a girls’ school with a male teacher.

In the Tokyo classroom, a student’s contribution was accepted and acknowledged no matter whether or not it was “mathematical” in a strict sense. In this classroom, mathematics was about formulating relationships and expressing them in different representational forms such as a table, a formula or figures. The students were asked to explain their understandings of the underlying relationships between variables and between representations of different forms.

The rules governing the legitimacy of mathematical explanations reflect the different priorities of each teacher in developing their students’ mathematical understanding. But we would also argue that these rules are likely to reveal the different beliefs about the nature of mathematics and of mathematics learning shared among particular school communities.

**Diversity and Simplicity of Solution methods**
Rather than restricting the class to a particular way of solving mathematical problems as demonstrated by the teacher in the Seoul classroom, different methods or solutions were encouraged by the Tokyo teacher. The encouragement of diverse ideas was demonstrated in two interrelated aspects: firstly, the students in this Tokyo classroom were given autonomy to generate their own formula about the variables of their choice; secondly, the students were encouraged to consider the relationships displayed in different representational forms from various perspectives. The Shanghai classroom also provided the students with opportunities to display various solution methods, but the purpose of displaying different solution methods was to examine which method was
better and simpler in solving particular types of problems. The emphasis on diversity and on simplicity represent two different meta-discursive rules, each having consequences for student learning. The respect for diversity of solution methods without evaluation of their superiority in the Tokyo classroom could foster student creativity, but it might overlook the consideration of the relative validity of those methods. On the contrary, the public evaluation of different solution methods may help students to see the merits of certain methods in terms of their simplicity and efficiency, but it might encourage rigid approaches to problem solving by fostering a belief in a single “best method”. Indeed, as Sekiguchi (2006) argued, maintaining the productivity of mathematical activity requires a delicate balance between the three components of a value system: validity, efficiency, and creativity.

**Concluding Remarks**

This paper compares the form, the content and the meta-rules of classroom talk on the topic of “linear equations” in three classrooms located in Seoul, Shanghai, and Tokyo. From the outset, there are similarities among the three classrooms studied, such as teacher-dominated whole-class teaching as the predominant mode of instruction in all three classrooms. One can attribute such a shared preference for instructional mode to large class sizes and a group-orientation derived from the Confucian tradition. However, this superficial similarity undermines the different functions of whole class discussion and the distinctive characteristics of such discussion displayed in each setting. As we have demonstrated in the above comparisons, the balance between uniformity and individualization was differently maintained in each classroom. While the Shanghai teacher expected the conclusions to be built upon student input, the Seoul teacher conceived that the role of the students was to follow the examples set by the teacher. Moreover, both the Shanghai and the Seoul classrooms encouraged uniform and collective action by the students. In comparison, the students in the Tokyo classroom had opportunities to raise their individual opinions. The whole-class discussion in the Japanese classroom was sustained by the belief that every child can and should expect moral support for her serious attempt to learn because helping individual students to learn is the ultimate goal of lessons in a classroom community (cf. Hatano & Inagaki, 1998).

The comparison of meta-discursive rules also reveals some fundamental differences in terms of what was considered as “mathematical” and “mathematically capable” in each classroom. In the Shanghai classroom, the students were required to use standard mathematical language as modelled by the teacher. In addition, to be considered as mathematically capable, the students should not only be able to articulate their understanding of the mathematical concepts or principles in standard mathematical language, but also be able to apply their understanding in solving mathematical problems. In the Korean classroom, to be regarded as mathematically capable, the students were required to understand the conditions of X and the consequence of these conditions on the solutions and the graphs of an equation. In this classroom, understanding means to know and to be able to apply those “established” mathematical routines and principles in solving equations of various sorts. In contrast, the students in the Tokyo classroom were interrogated by the teacher regarding their understanding of the relationships between different representations. In this classroom, understanding meant to be able to see the underlying relationships between the variables as expressed in different representational forms and the connections between those representations. The differences of the three classrooms, as discussed in this paper, suggest that while the shared macrocultural values and beliefs frame the social activity in each classroom in certain ways, the meta-discursive rules that constitute the classroom microculture determine the opportunities for student learning in mathematics.
Reference


Copyright (c) 2010 Lihua Xu, David Clarke, & May Ee Vivien Wan. The authors grant a non-exclusive license to the organisers of the EARCOME5, Japan Society of Mathematical Education, to publish this document in the Conference Proceedings. Any other usage is prohibited without the consent or permission of the author(s).