An Integrated Approach to
Shape and Topology Optimisation
of Mechanical Structures

by

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Submitted in fulfilment of the requirements for the degree of

Doctor of Philosophy

Deakin University

April 2014
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An Integrated Approach to
Shape and Topology Optimisation
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Acknowledgements

I would like to thank SFE GmbH, Berlin, for providing the software package SFE CONCEPT, without which this research would not have been possible.

A big Thank You also goes to the Australian Automotive Technology Cooperative Research Centre (AutoCRC), Australia, for their financial support in the form of a three year scholarship.

Finally, I want to thank my supervisors Bernard Rolfe and Tim deSouza. Your continuous commitment, encouragement and support was invaluable, and without your advice, creative thoughts and challenging questions I could not have accomplished this work. You dedicated a huge amount of time and thoughts into this project, so thank you very much.
Publications

A Systems Approach to Shape and Topology Optimisation of Mechanical Structures

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Conference paper presented at

OPTI 2012
12th International Conference on Optimum Design of Structures and Materials in Engineering
New Forrest, UK
2012

Published in

Computer Aided Optimum Design in Engineering XII
DOI: 10.2495/OP120131
2012
Integrated Shape and Topology Optimisation — Applications in automotive design and manufacturing

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Submitted to

IJVD — International Journal of Vehicle Design, Elsevier
Under review
2013
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Abstract

In the last ten years, topology optimisation methods for the design of structural components have become an indispensable tool for structural engineers. At the same time, shape optimisation methods have made further progress. However, the SIMP topology optimisation method (Solid Isotropic Material with Penalisation) still has limitations. They are generally applied on a fixed design space, and deliver results which require manual interpretation to ensure they are manufacturable.

Shape methods on the other hand are still hard to perform, mainly due to the difficulty of translating shape design parameters into meaningful analysis models. Also, shape methods usually cannot suggest new load path configurations.

When it comes to combining shape- and topology optimisation methods, most approaches put forward in the literature employ some parametrisation of the topology optimisation result in order to assist the transition from density distribution to a manufacturable design. In contrast to these ideas, this thesis proposes an integrated shape- and topology optimisation method, where large scale shape modification is done in conjunction with topology optimisation. The presented method allows two major new approaches: First, the initial design space that goes into the topology optimisation algorithm can be subject to large scale parametric shape variation. Second, the topology design volume may be part of a complex system, which itself is subject to parametric optimisation. This approach (named IST — Integrated Shape- and Topology Optimisation method) adds a geometric dimension to the design space for topology optimisation and to the system it is embedded in, and largely increases the size of the optimisation space open for exploration.

This thesis investigates whether the introduction of large scale parametric shape modification to the topology optimisation process can find better (i.e. stiffer and more mass efficient) structural solutions than traditional consecutive shape and topology optimisation methods.
In order to answer this question, a number of case studies involving different classes of structural applications have been examined, such as shell structures with lightning holes, multi-piece solid structures, and complex combined shell/solid structures. In all cases, the design space for topology optimisation and its surrounding system are coupled and subject to large scale parametric geometry variation. It is clearly shown that the added geometric dimension opens up the design space and finds light weight solutions that a sequential approach could only have found by chance.

A new type of problem then arises with the simultaneous modification of shape and topology: Variable geometry leads to structures with different mass and different performance, which creates a ranking problem as to which design is to be called better. A number of approaches to address this issue are described.

The optimisation loop governing IST is best approached using genetic algorithms with discrete design variables. The main reason for bionic algorithms is that topology optimisation results for varying input geometries can be discontinuous, which is why gradient based method are not suitable. Discrete design variables are preferable over continuous variables, because the small changes in the geometry input parameters do not necessarily translate monotonically to mass or performance figures for the topology optimisation.

When looking at limitations, certain types of structures do not lend themselves to the application of IST, namely when joint design is performed independently from structural beam members, for example when the structural layout is driven by buckling loads. Also, IST comes at an added computational cost, since a topology optimisation is performed for every geometric characteristic analysed.

To summarise, this thesis shows how IST extends the current topology optimisation approaches in a geometric dimension. This additional design freedom allows to find structural solutions that are lighter or perform better than traditional methods can deliver.
Nomenclature

\( \mathbf{x} \)  
Vector of geometry design variables, \( \mathbf{x} \in \mathbb{R}^n \)

\( \delta \)  
Density function on topology design space: \( \delta : \Omega \to [0, 1] \)

\( \bar{\delta} \)  
Prescribed mass/volume fraction for SIMP algorithm, or resulting mass/volume fraction in case the SIMP algorithm runs with performance constraint

\( \Gamma \)  
Combined SIMP design space and surrounding structure. \( \Gamma = \Lambda \cup \Omega \)

\( \Lambda \)  
Non-designable parts of a finite element structure

\( \Omega \)  
Design space for SIMP topology optimisation algorithm

\( \overline{\Omega} \)  
Topology optimisation result on \( \Omega \)

\( f \)  
Outer objective function \( f : \mathbb{R}^n \to \mathbb{R} \). In most cases, \( f \) measures the mass.

BIW  
Body in White. Automobile body structure.

CAD  
Computer Aided Design

CAE  
Computer Aided Engineering

DOE  
Design of Experiments

FE  
Finite elements

FEM  
Finite element method

IST  
Integrated Shape– and Topology optimisation method

SIMP  
Chapter 1

Introduction

1.1 Shape- and Topology Optimisation Methods

In structural mechanics, the requirements for structural elements and systems are highly complex. And yet, in a very competitive environment, engineers are expected to shorten development and test time, while also cutting material and production costs. Moreover, energy efficiency plays an increasingly important role in the life cycle of any product. One way to achieve this is through light-weighting of structures, where engineers aim to minimise material usage while maintaining or improving performance.

When assessing today’s body structures of automobiles, the mass market is still dominated by thin walled pressed and welded steel structures. This approach has great advantages since steel is an inexpensive and very versatile material. The forming processes and joining technologies (mostly spot welding) are well understood, and the employment of high strength steels has facilitated a significant improvement in crash safety performance while limiting the mass penalty.

Nevertheless, the introduction of non-conventional materials such as aluminium, magnesium and fibre reinforced composites provides a further potential for weight reduction (Cole and Sherman [25], Schindler [95]). These materials are lighter and offer new design possibilities. Examples are highly complex shapes of castings, or fibre reinforced composite materials that are tailored to meet specific loading conditions. On the other hand, for large volume production, there are a variety of issues to overcome, such as higher material cost, longer production cycle time (e.g. compared to sheet metal stamping), and the development of appropriate joining technologies.

Moreover, in the quest for reducing fuel consumption and CO₂ emissions,
light-weighting of structures is a key enabler for more energy efficient transportation systems (Saito et al. [92], Roth et al. [89], Carle and Blount [17]). In this context, topological layout and complex shape design play a crucial role, and here simulation and optimisation methods are very important.

With the possibility to analyse virtual geometry, comes the potential to optimise such structures, typically with the aim to find a set of design parameters among a myriad of possible combinations that best meet prescribed requirements and objectives.

In structural mechanics, often virtual geometry and derived Finite Element (FE) models serve as a basis for simulation. Furthermore, optimisation of these structures requires a parametric description, enabling the automated modification within a given mathematical parameter space.

In this thesis I focus on two important aspects of structural optimisation, namely:

- Shape optimisation methods and
- Topology optimisation methods.

Topology optimisation in this thesis mostly refers to the application of the Solid Isotropic Material with Penalty (SIMP) method, which determines the optimal material distribution within a given fixed geometrical design space. Shape optimisation, however, is understood to be based on parametric geometry variation of structures.

Shape optimisation methods are generally complex and difficult to perform. This is mainly due to problems encountered during the translation of geometry design parameters into analysis models. In addition, shape methods are typically unsuitable for developing topological layout design.

On the other hand, topology methods are designed to find general load path configurations, and are not well suited to shape design. Also, topology methods cannot easily account for specific characteristics of parts, such as thin walled structures or complex relationships between members.

For the design of complex assemblies such as automobile body structures, topology optimisation methods can generate load path ideas, but cannot provide shape characteristics such as details of a cross-section, or the intricate structure of a multi-layered panel with specific fibre orientations. Results usually require a substantial amount of human interpretation. Topology methods are therefore best applied to single parts.

The literature reveals two main approaches used to combine these two methods. First, extensions to shape methods which include topological layout design by including or excluding structural members. Second, topology methods that apply shape design after a topology layout has been derived,
mainly to fine-tune the geometry with respect to local stress concentrations and to ensure manufacturability. The limitations of current topology and shape optimisation methods raise the question, whether it is possible to devise an approach that overcomes some of these issues.

The aim of this thesis is to investigate whether parametric shape design can be done prior to and in conjunction with topology optimisation. This will enable the exploration of complex systems and extend the design space for a classical topology optimisation by a geometric dimension.

1.2 Research approach

The fundamental idea that I want to explore is depicted in Figure 1.1, where a traditional topology optimisation is compared with a proposed integrated process that incorporates large scale shape variation into the topology optimisation process.

![Figure 1.1: Schematic comparison between traditional topology optimisation process (a) and topology optimisation with integrated large scale shape variation (b)](image)

Here, shape optimisation should not be thought of as an additional step after topology optimisation (which is a field of research in its own right — Grujicic et al. [43], Hsu and Hsu [53], Schumacher [97]), but rather, can shape design in conjunction with topology optimisation add a geometrical dimension to the topology method?
1.3 Research question

The specific question this thesis seeks to answer is whether a parametric design domain based topology optimisation can provide improved results over standard topology optimisation with a fixed design space.

In particular, one needs to ask the following sub-questions:

1. Which classes of structural problems lend themselves to the application of such an integrated approach?

2. What new issues and types of problems arise from this new approach?

3. What are the specific benefits, and how does the integrated approach compare to traditional methods?

4. What are the disadvantages and the inherent limitations?

The research approach combines parametric geometry modelling with topology optimisation, in the following sense:

1. The design domain (denoted Ω), which is subject to a topology optimisation algorithm, can vary parametrically, subject to geometry design parameters;

2. The design space Ω can be part of a larger complex system, that itself is subject to parametric geometry variation.

This opens topology optimisation to systems analysis, which is significant and novel. When discussing this idea in the early stages of this research, I have often come across the argument that there should be no benefit in varying initial package space for the topology optimisation. The reason being, that the algorithms are designed to take away any surplus material, so increasing package space should not make a difference to the final result. Nevertheless, this thesis will examine a number of classes of applications where parametric package space is believed to be beneficial. For example, when a solid design volume is integrated into a complex structure, it is no longer clear that direct traditional topology optimisation will realise the best result.

Several chapters of this thesis present examples of applications that investigate the integrated approach on various scenarios. Case studies include:

- Multi–piece structures, where different components are subject to topology optimisation; and where a compromise between the different package spaces needs to be found.
Structures made from extruded sections. We attempt to find an optimal initial design space that differs from a conventional fixed design volume;

• Thin walled three dimensional shell structures, combined with lightening holes/load path layout, which traditionally are difficult to handle for topology optimisation;

• Integrated structures where shell structures are combined with solid structures. Here the trade-off between beam size and shape on the one hand and joint topology on the other hand is explored, which is currently impossible for traditional topology optimisation.

The current research has a particular focus on thin walled beam structures in combination with cast joining structures, which will become more and more important in the automotive industry. The results are transferable to other industries, where the main objective of the optimisation is light weighting. For example similar requirements can be found in the aerospace industry, as well as applications for concrete structures such as buildings or bridges.

The proposed integrated method is named Integrated Shape- and Topology Approach (IST).

1.4 Thesis outline

I conclude this introductory chapter with a summary of the layout and structure of the thesis. Chapter 2 on page 22 is a literature review that examines the current research in structural optimisation methods. The discussion focuses on shape- and topology methods, in particular parametric geometry tools for shape optimisation, the Solid Isotropic Material (SIMP) approach for topology optimisation, as well as research into the combination of both.

The insights lead to the research question introduced earlier: Is it possible to overcome some of these limitations by integrally linking large scale shape variation with topology optimisation methods?

Chapter 3 on page 52 explains the theoretical background and the methodology used to tackle the research question, and explains why I chose a particular approach. Requirements for the necessary tools are explored and a collection of classes of applications for the proposed method are examined. Chapter 3 also highlights new types of problems that appear with this methodology, such as the ranking problem that arises when mass and performance of a structure vary simultaneously. Possible solutions are highlighted, and then discussed in more detail in the following chapters.
Chapters 4 and 5 present case studies that investigate structural problems involving both large scale shape variation and load path layout design.

In Chapter 4 I apply our integrated method to derive a lightweight solution for both shell and solid based structures. This shows the application of varying the design domain on not only solid structures, but also thin walled structures.

Chapter 5 on page 93 examines more complex examples where intricate thin walled beam structures are connected with cast solid joints. This shows how this integrated approach extends the traditional design space of topology optimisation into a new class of problems combining complex thin walled and solid structures.

Chapter 6 on page 112 discusses the results of these studies. I identify those cases where the methodology can be applied successfully, and the advantages over traditional methods. The limitations of the method are also discussed, both in terms of computational cost as well as intrinsic limitations. This chapter concludes with a discussion about the range of applications the method could be applied across.

The concluding chapter summarises the thesis findings and alludes to future work that could help to further refine the field of integrated shape—and topology optimisation.
Chapter 2

Literature review

2.1 Introduction

Chapter 1 introduced the idea of combining and integrating the two main optimisation methods in structural mechanics, namely topology optimisation and shape optimisation.

After a brief introduction to what I mean by structural optimisation, I look closely into topology optimisation methods (Section 2.3 on page 24). In the last 15 years this field has made tremendous advances in theory and practical application. Particularly, I will look at homogenisation methods, which is an active field of research and development.

More recently, Level Set Methods have generated much interest within the research community, and I give brief account of this macro-structure approach (Section 2.3.2 on page 25). However, homogenisation methods are most relevant to the engineering industry, which is why I examine this approach in depth, especially the SIMP method and its numerous extensions that capture important features and demands for industry.

I then discuss shape optimisation methods, a field which is not as easily defined and characterized as topology optimisation (Section 2.4 on page 34). I describe traditional ideas, such as finite element mesh morphing methods, that range from simple grid coordinates as design variables to more complex parametric shape vectors. A section is dedicated to parametric geometry tools that can handle large scale shape variation better than finite element mesh based methods.

Finally, I discuss the combination of shape and topology methods, which is the central question under investigation in this thesis.
2.2 Structural Optimisation

Computational methods are readily available to engineers, and simulation plays an ever increasing role in the design of load bearing parts. As analytical resources become cheaper, finding the optimal topological layout and shape is a key to designing mechanical structures that meet complex performance specifications. The list of requirements is often long and may include several of the following targets:

- Stiffness (static and dynamic)
- Durability, longevity
- Crash safety
- Robustness
- Fluid-dynamical performance
- Resistance to temperature and temperature gradients

At the same time the engineer aims to find solutions that best satisfy a number of constraints, for example:

- Weight
- Cost
- Manufacturability
- Material use, availability of materials
- Life cycle cost
- Environmental impact (Energy use, use of natural resources, recyclability, emissions)

Often individual objectives and constraints conflict with each other, and it is a challenging task to derive solutions that find the best compromise when multiple targets are taken into account.
2.3 Topology Optimisation

Topology optimisation is now widely employed in the engineering world, and its application allows mechanical engineers to design structural components with, for example, minimum mass and maximum performance by determining the optimal material distribution layouts (Bendsoe and Sigmund [10], Chen and Usman [22], Soto [107]). Using topology optimisation, a design can be optimised at a very early stage of the design process. A requirement that has become more and more important in today’s quest for lightweight, strong and durable structures, while at the same time being cost efficient and easy to manufacture.

The first applications of topology optimisation were problems in two dimensions, which are easier to set up and solve than structures in three-dimensional space due to reduced complexity and computational cost (Figure 2.1 shows an example of topology optimisation in two dimensions.) Indeed, topology optimisation for 3D structure is fairly recent development, with first applications being published in the early to mid 1990s, see for example Díaz and Lipton [29] or Olhoff et al. [76]. In this thesis I am mostly concerned with structures in a three-dimensional space.

Today, there are two main approaches to topology optimisation, namely the micro-structure and the macro-structure approaches, often referred to as “material” versus “geometry” techniques. The following section introduces these methods. However, the main focus lies on the material approach, since this technique is used in the course of this research.

Figure 2.1: Topology optimisation of cantilever beam using OptiStruct
2.3.1 Macro-structure approach

Macro structure approaches to topology optimisation generally start with a design domain made from solid material. The optimisation process generates structures by removing material by one of the following means:

1. Changing the boundary of the design region;
2. Introducing voids into the structure / eliminating voids.

Changing the design boundary requires a parametric description of that border; and for every modification in the geometry of the new shape the border description has to be translated into an analysis model. In the context of finite elements, this means, modifications to the existing mesh or complete re-meshing is necessary (Papadrakakis et al. [80], Canales et al. [15]).

An early attempt is the **Variable thickness sheets** method which works for 2D-structures with in-plane loads and is very similar to the (micro-structure) SIMP method (described in more detail in Section 2.3.3 on page 28) in that it divides the design domain into a large number of sub-domains and allows the thickness of each sub-region to vary continuously (Rossow and Taylor [88]).

Another interesting method is the so called bubble method. Here the boundary of design region is propagated in the design space with the advantage that voids can be introduced or merged during iteration steps. This enables topological changes, see for example Luo et al. [68] or Eschenauer et al. [38]. Also very promising in the macro structure area is the level set method to which I dedicate the next section.

2.3.2 Level Set Method

In 1988 Osher and Sethian [78] describe the application of an implicit function method to track the movement of a surface boundary. The boundary is represented as the constant value set of some function $\varphi : \mathbb{R}^3 \to \mathbb{R}$, and the variation of the surface is traced by additionally introducing a time-dependence for $\varphi$, subject to the curvature of the surface at any point in time. The authors term the method **PSC algorithm** and apply it to solve Hamilton-Jacobi-type equations. This may be viewed as an early description of what today is known as the level set method for design optimisation in structural mechanics (Zhu et al. [123]).

One of the first applications of the level set method in structural mechanics is found in Sethian and Wiegmann [101] as recently as 2000, where the authors calculate the stresses inside a two-dimensional design domain under
given loading conditions. The design boundary — described as a constant value set — is then perturbed and propagated towards a design with smaller deformation energy, leading to a structure minimising compliance. Since then, the level set method has made great progress as can be seen from the large number of publications in the field; Osher and Fedkiw [77], Wang et al. [116], Yulin and Xiaoming [120], Challis et al. [19], to name a few.

The basic idea is to describe a domain $\Omega \subset \mathbb{R}^n$ by its boundary $\gamma$. To do so, $\Omega$ is embedded into a larger domain $\Omega \subset \tilde{\Omega}$ encompassing the total admissible design space. A Lipschitz continuous function $\varphi$ is introduced that lives on $\tilde{\Omega}$. Then $\gamma$ is defined as the zero level set (or iso-surface) of $\varphi$:

$$
\gamma = \{x|\varphi(x) = 0\}
$$

(2.1)

and basically is the boundary of the wanted design part $\Omega$. Introducing a “time”-dependence to $\varphi$, equation (2.1) becomes

$$
\gamma(t) = \{x|\varphi(x(t), t) = 0\}
$$

When differentiating equation $\varphi(x, t) = 0$ with respect to $t$, one obtains

$$
\frac{\partial\varphi(x, t)}{\partial t} + \frac{dx}{dt} \cdot \nabla \varphi = 0.
$$

(2.2)

Equation (2.2) is the so called Jacobi-Hamilton PDE and can be solved numerically on a fixed grid embedded in the support of $\varphi$. This allows tracking of the boundary $\partial \Omega$, and if a suitable function on $\Omega$ is defined, leads to the minimisation of that function by varying the boundary using the normal velocity field $v$.

It is worth noting that the computational cost of re-evaluating (2.2) during iteration steps scales with the surface area $\partial \Omega$ of $\Omega$, rather than with the volume $|\Omega|$, as pointed out by various authors (see for example Osher and Fedkiw [77]) and is comparable to boundary integral methods as far as computational complexity goes.

The level set method lends itself to applications in fluid dynamics and has been successfully applied to the simulation of compressible and incompressible fluid flow, wave propagation and related physical problems, and even in fields such as image processing. For structural mechanics, the method can optimise design boundaries for linear elastic problems, and has recently been extended to nonlinear elastic problems (Allaire et al. [5]).

For two-dimensional problems, the level set method — until extended methods were introduced — did not allow for the introduction of voids into the design domain. So it was essential to choose an initial design with a large number of holes, as illustrated in Figure 2.2 on the following page. In the
same paper, the authors propose an extended level set method that allows
the introduction of voids.

As far as structural mechanics is concerned, until this century the litera-
ture almost exclusively showed examples in two dimensions, such as in Luo
et al. [68]. In more recent publications three-dimensional structures have
been tackled, see for example Allaire et al. [5].

The level set method may be viewed as a combined shape and topology
development tool in that it is able to find load paths as well as clean bound-
daries of the structure.

A drawback remains the limitation to single parts. While
their topology and shape may be elaborate, complex in-
teraction between multiple parts cannot be described.
This means that a level set method can be well suited to develop a single cast component, but cannot account for structures consisting of multiple parts with intricate joining.

### 2.3.3 Micro-structure approach

In the micro-structure approach one starts with an initially solid volume describing the permissible design region — the “package space” — divided into a discrete mesh of finite elements. After applying loads and constraints the optimisation algorithm then determines whether a region in the design space should contain material or not. In practice, most algorithms also assign intermediate states, such as 'a design region is present to a level of 50%'. In mathematical terms, the topology optimisation problem can be formulated as follows.

Let $\Omega \subset \mathbb{R}^3$ be the permissible design region and $\delta : \Omega \rightarrow [0, 1]$ a density function.\(^1\) Then

$$V := \int_{\Omega} \delta$$

is the volume of the design with respect to the density $\delta$, and the aim of the optimisation is to find a suitable density distribution $\delta$ with some minimising property. In a computational context the design region is divided into $N$ finite elements $e_i$, so the volume defined above becomes

$$V = \sum_{i=1}^{N} \delta_i v_i.$$

Now there are two main approaches as to what the optimality criterion is. The first one aims to minimise a certain function $f : \Omega \rightarrow \mathbb{R}$ under a volume constraint $V^*$, i.e.

$$\min_{\delta} f(\delta) \quad \text{such that} \quad V(\delta) \leq V^*$$

This is sometimes referred to as the “volume fraction” approach, and the optimisation leads to a design with fixed volume and mass $V^*$. The function $f$ is in many cases the global compliance. In terms of finite elements this means to minimise

$$C := \frac{1}{2} u^T K u$$

\(^1\)It is assumed that $\Omega$ is a three-dimensional domain. In earlier years, topology optimisation was only done in two dimensions.

\(^2\)More precisely I should say $V := \int_{\Omega} \rho$, where $\rho$ is the specific density of the material.
where $K$ is the stiffness matrix, and $u$ the deformation under the given set of loads. In other words, one tries to find a material distribution $\delta$ that leads to a structure with minimal deformation energy.

The second approach is to minimise the volume under a number of performance constraints:

$$\min_{\delta} \int_{\Omega} \delta \quad \text{subject to} \quad g_k(\delta) \leq 0$$

In either of these approaches, topology optimisation can be understood as a material distribution problem. The topology optimisation algorithms assume that the stiffness $K$ of each element varies monotonically (e.g. linear or according to a power law) with its density $\delta$. If — as is often the case — the material is assumed to be isotropic (i.e. has identical mechanical properties in all directions), this method is referred to as the Solid Isotropic Material method, or short SIM. Here, the term solid, is to be understood as “filled” as opposed to “porous”, rather than solid versus fluid.

**Black-and-white topology optimisation**

Early approaches in topology optimisation aimed at finding structures where each element $e_i$ is either kept or taken away, i.e.

$$\delta_i \in \{0, 1\}$$

This led to a “black-and-white” structure and is — from an engineer’s point of view — a desirable outcome, but it has been shown that there are a number of problems associated with these approaches that in essence made them impractical for real world applications. The publication by Sigmund and Petersson [105] gives an overview of the shortcomings:

- Non-convergence of topology optimisation algorithm, which tends to alternate between different spacial mass distributions;
- Non-unique solution (Figure 2.3 on the next page);
- Checkerboard patterns: Formation of alternating solid and void elements.
2.3.4 Homogenisation Techniques, SIMP

To overcome the non-convergence problem, the most important advance was the introduction of intermediate densities $\delta \in [0, 1]$, called Relaxation [76] or Homogenisation [39]. Here, the topology optimisation algorithm assigns intermediate densities to the finite elements. $\delta$ is assumed to be constant within one element, and the stiffness of each element varies monotonically with the density of that element. Moreover, $\delta$ is chosen to be above a certain threshold, say $\delta \geq 0.01$, to avoid a singular stiffness matrix.

The main disadvantage with intermediate densities is that the proposed structures cannot always be manufactured and require interpretation by an experienced engineer. Therefore, it then remains a manual task to interpret the density distribution in terms of its practical implementation.

To mitigate this issue, SIM algorithms implement what is referred to as “penalisation” of intermediate densities. This is the ”P” in what is called the SIMP-method: Solid Isotropic Material with Penalisation. It works as follows: Let $\delta$ be the density distribution at any one iteration step. Now instead of assuming a linear relationship between density and stiffness, let the nominal density be $\delta^* = \delta^{1+p}$, and the resulting stiffness for each element

$$K^* = \delta^{1+p} K.$$ 

Here $p$ is called the penalisation factor. For values of $p > 1$ the penalisation drives densities smaller than 1.0 towards zero and thus leads to designs with a more distinct/discrete material distribution.

A number of questions have been raised with respect to the numerical stability and effectiveness of topology optimisation using the SIMP method. The introduction of relaxation — or intermediate densities — were able to solve the numerical instabilities with respect to alternating designs. A series of approaches have been proposed as solutions to checkerboard-patterns and mesh dependence problems. The most important ones are detailed by Eschenauer and Olhoff [39] and Rozvany [90]:

![Figure 2.3: Black-and-white topology optimisation has non-unique result](image)
• Local gradient method: Ensures continuous/smooth density transition between neighbouring elements. Disadvantage: introduces many additional constraints (Niordson [74], Sigmund and Petersson [105]).

• Perimeter method, limits surface area of design in every iteration step. Disadvantage: Limit for perimeter has to be chosen heuristically (Haber et al. [45], Eschenauer and Olhoff [39]).

• Mesh independence filter method, based on image processing algorithms, considers weighted averages of design sensitivities in each iteration step (Hsu and Hsu [52]).

It has been a matter of some debate as to whether the SIMP method actually is justifiable in a physical sense, i.e. whether it is possible to find a material micro-structure that has a stiffness proportional to $\delta^{1+p}$, which has been shown to be the case (Bendsøe and Sigmund [9]). For all practical purposes it has turned out that penalisation leads to far superior results, and the question of physical relevance is of little importance.

It may be worthwhile for the reader to experiment with basic features of SIMP topology optimisation, for example using MATLAB [142], or free software such as FreeFEM [143] and BESO3D [136]. The latter also allows self weight of structures to be integrated with Rhinoceros [146].

2.3.5 Extension of Homogenisation Method

When first developed, topology optimisation algorithms for structural mechanics were limited to problems with static loads in two dimensions, the objective being to minimise total deformation energy. In its famous 99 line Matlab implementation by Sigmund [103], the topology optimisation algorithm can handle one static load case.

In recent years there are a number of publications that show the progress that has been made in extending the capabilities of the homogenisation method to incorporate more requirements of real world applications, most importantly the step into three dimensions. Cherkaev and Palais [24] (at first for structures with cylindrical symmetry only), and Allaire et al. [3] were among the first to develop this important advancement.

The move into three dimensions entails the crucial demand for manufacturability control, predominantly the capability to avoid cavities and undercuts. This may not be an issue for certain applications, say for example when investigating a truss structure using a rather small mass fraction. Here — similar to a result in two dimensions — the engineer may be content with a material distribution showing voids, since it provides information how
to arrange structural members. But for actual cast components, topology optimisation results with voids may be hard to interpret and virtually impossible to manufacture. Interestingly, in his fairly comprehensive overview, Eschenauer and Olhoff [39] do not mention advances in the incorporation of manufacturing requirements, and indeed, the literature around process constraints is sparse. Very few publications talk about incorporating manufacturing constraints, and they do not lay a theoretical foundation to the extension of the homogenisation method, but rather employ a more or less manual process for specific design tasks. See for example Chang and Tang [20] and Del Prete et al. [27].

Nevertheless, industry applications have tackled the manufacturability issues of three-dimensional topology optimisation problems (See for example Vanderplaats [156] and Altair [132]), and have managed to find acceptable numerical solutions asserting

- Extrusion constraints, restricting the design to a constant cross section perpendicular to a specified direction;
- Draw constraints, also along curved paths;
- (Multi-piece) die cast constraints;
- Member size control: asserting minimum or maximum size of structural members;
- Sheet metal constraint: Evolve a design with constant thickness, (The design space needs to be very fine grained, and is often not very practical);
- Symmetry constraints and pattern repetition.

Advances in other directions are better represented in the literature. Important to mention is the incorporation of stress constraints. Duysinx and Bendsoe [37] introduce an extended SIMP procedure to penalise local stresses which introduces a large number of additional local constraints, while Yang and Chen [119] propose a global stress measure to approximate the local stresses. See also Navarrina et al. [73] for a minimum weight formulation with stress constraints. A number of other additional constraints have found their way into the the homogenisation method. The following is a brief overview:

- Stress constraints (Duysinx and Bendsoe [37], Yang and Chen [119], Paris et al. [81, 82], Bendsøe and Sigmund [10]);
• Periodic structures (Zuo et al. [127]);
• Draw constraints (Schramm and Zhou [96], IUTAM [141]);
• Member size constraints, minimum and maximum size (also Schramm and Zhou [96]);
• Frequency constraints (Zuo et al. [128], Allaire and Henrot [4]);
• Eigenvalues (Ma et al. [69], Pedersen [85]);
• Design dependent loads (Chen and Kikuchi [21]);
• Self weight (Bruyneel and Duysinx [13], Huang and Xie [57], Bendsøe and Sigmund [10]);
• Multiple displacements (Zuo et al. [128], Huang and Xie [56]);
• Non linear Materials (Sigmund [104], Schwarz et al. [99], Xia and Wang [118]);
• Large displacements, non linear loads (Sigmund [104], Jog [60], Pedersen [84]);
• Elastoplastic responses, variable strain rates (Schwarz et al. [99]).

However, the density distribution across the volume continues to be a clear disadvantage for homogenisation methods, as the final result still needs to be interpreted. On the other hand, many advanced methodologies for the homogenisation method have been developed with respect to applicable loads, constraints and manufacturability considerations, that make them very useful in the engineering industry. At any rate, in the industry SIMP software still prevails over other topology optimisation methods, including the level set method.

An important point to note here is that while homogenisation methods attempt to find a low mass or maximum stiffness solution, the actual mass figure returned by the topology optimisation algorithm is not predominantly important. More relevant is the general load path layout that can be interpreted and translated into a manufacturable design.

A critical question is, how much information the mass of a structure as returned by a topology optimisation algorithm actually provides.

Later I will investigate parametric modification of the design space for topology optimisation, and discuss the implications with respect to the somewhat fuzzy mass information that a density distribution delivers (Section 6.2.7).
2.4 Shape Optimisation

Shape optimisation is often described in the literature as hard to tackle. In 1994 Hsu [54] writes, there are no examples of three dimensional shape optimisation out there. Of course this has since changed, but as late as 2010 it was reported that

\begin{quote}
In shape optimisation, cumbersome parametrisation of design domain is required and time consuming re-meshing task is also necessary. (Seo et al. [100])
\end{quote}

I will now take a closer look at the current state of the art in structural shape optimisation.

2.4.1 Sizing design parameters

Early approaches to optimisation of load carrying structures considered predominantly scalar entities such as gauge and material property variables [46]. These include:

- Shell thickness parameters;
- Cross sectional area of bars;
- Inertia values of beams;
- Young’s modulus and other materials properties.

The advantage of these types of design variables is that they are easy to implement and integrate into an automated optimisation analysis loop since they involve solely replacing a scalar value in an ASCII file. Re-meshing of a finite element mesh is not necessary. However, their application is limited to structures with a fixed topological layout and shape.

Some authors argue that allowing members to become non-existent by down-sizing them to zero should be called topological layout design. For example, Torstenfelt and Klarbring [111] describe the optimisation of an automobile body structure, where they allow structural members to virtually disappear in that manner. Figure 2.4 on the following page shows a truss structure where the sizing of individual structural members effectively finds the best load-paths, though limited by the initial choice of available beams Bendsøe et al. [8]. Techniques around truss structure optimisation are well developed. Reddy et al. [87] for example employ simulated annealing methods that can incorporate stress and buckling constraints. See also Zhou and Rozvany [122], Rozvany [91], Miguel et al. [71], Deb and Gulati [26].
While sizing alone can alter the topological layout of a structure, it is limited to predefined potential load paths, the number of which can be very large, and comes at high computational cost. The benefit is that the resulting members do not need re-interpretation as is often the case for a load path structure generated by homogenisation. Cross section inertia values can be directly integrated into the optimisation.

![Truss structure optimisation using only sizing design variables](image)

**Figure 2.4:** Truss structure optimisation using only sizing design variables Bendsøe et al. [8]

### 2.4.2 Finite Element node coordinates as design variables

When it comes to true shape modification, the main problem is to translate changes in design variables to changes in some analysis model. A finite element mesh model is assumed to be the basis for a structural analysis, so the task is to update the finite element model after every modification of the design variables.

One very basic approach is to define the coordinates of individual finite element grid positions as design parameters. The disadvantage is that the number of design variables is high, and there is no intuitive connection between the local variation of the finite element nodes and the intended shape modification. This can be mitigated by a filtering scheme to provide length scale control (Le [63]). Also problematic is that the finite elements can easily become distorted and may not represent the intended structure anymore. For example, the analytical stress values may become unrealistic for a distorted
mesh (see for example Haftka and Grandhi [46]). Therefore, this method is limited to relatively small changes in the design. For example, to fine tune the surface of a three dimensional part. Zhang and Chen [121] describe an example where this is applied to refine the surface of a cast automotive suspension part for durability purposes.

### 2.4.3 Mesh morphing

A better approach is to use mesh morphing techniques where finite element grid coordinates are controlled by a relatively small number of parameters. Typically, a number of finite element grids are collected into a control volume, the size and shape of which can be adjusted by a few parameters. Let, for example, the control volume be a cuboid that can stretch and bend: All grids inside the volume then change their position with respect to the bounding box by some relative value, linearly being the most obvious relation. Other methods collect grids on the surface of a three dimensional part and move their coordinates in the normal direction with respect to the surface. This allows one to stretch or bend a finite element mesh in a defined way. Advantages over single grid positions as design variables are:

- The number of design variables is much smaller;
- Design parameters can have intuitive geometrical meaning;
- Bounds and interrelationship between design variables can be easily expressed.

Morphing techniques have great benefits in many areas outside of engineering; examples that come to mind are computer graphics or product design (Alexa [2]). In the engineering world, morphing has great potential, for example, in fluid dynamics when used to describe containers, ducts or aerodynamic structures. Here the mesh quality is not a huge issue, since only two-dimensional surfaces — used as contact areas — are involved. Moreover, the analysis task focuses on the fluid, and the requirements for the finite elements representing the containing structure are not very high.

For structural mechanics, mesh quality is more critical. In applications where the bandwidth of modification is relatively small, and simulation prediction accuracy needs to be high, mesh morphing is applied successfully. Important examples can be found in the medical field. Grassi et al. [42] apply morphing techniques to medical implants, Sigal et al. [102] show the benefits of finite element mesh morphing for human bone structures. Due to the comparatively small shape modifications in conjunction with a very fine initial finite element model, it is not necessary to re-mesh the structure.
Another interesting example is shown by Liu and Yang [66]: The authors describe an automated optimisation loop of a deep draw die shape design. An initial design is chosen after a number of simulations in AUTOFORM, and subsequent shape modifications are realised through mesh morphing. Again, the magnitude of adjustment were comparatively small with respect to the overall dimensions of the part.

For automobile applications, a number of commercial software packages are used in the industry e.g. Abaqus [150], ANSA [147], ANSYS [139], CATIA V5 [151], DEP Meshworks Morpher [144], Altair Hypermorph [130], or Pro/ENGINEER [145]. However, as described by various authors, there a some significant drawbacks with mesh morphing:

- [Morphing suitable] only for small geometrical changes (Duddeck [33]);
- [Mesh morphing] approach is highly labor-intensive and experience driven (Zhang and Chen [121]);
- The topology of the finite element mesh is fixed and large shape variations lead to bad finite elements (Schumacher et al. [98]).

While morphing tools are getting more sophisticated by incorporating automatic re-meshing, it can be concluded that in general, mesh morphing is better suited for single parts involving relatively small changes, and not very practical for complex applications such as body structures.

### 2.4.4 Mesh modification based on boundary nodes

Another idea involves procedures that allow the modification of finite element grids based solely on the variation of boundary nodes. The approach here is to exploit the fact that it is often easier to find a parametric description of moving boundary nodes, rather than having to trace every finite element grid inside the design domain. See, for example, Song and Baldwin [106], who outline a methodology in two dimensions where a finite element mesh is reconstructed based on the variation of outer grids only. A similar technique is presented by Bugeda et al. [14] where an adaptive meshing method with emphasis on fast re-meshing is presented, claiming to reduce computational cost for regeneration of finite elements in each iteration step.

### 2.4.5 Shape vectors

Describing variations of a finite element mesh allows one to generate shape vectors, which are sometimes called shape basis vectors. This technique
requires topologically identical meshes, and the ability to trace grid coordinates with changing geometry. In technical terms, a finite element mesh with \( k \) nodal coordinates corresponds to a vector \( \mathbf{v} \in \mathbb{R}^{3k} \). A small change in the geometry leads to a new mesh \( \tilde{\mathbf{v}} \), allowing one to define the differential \( \Delta \mathbf{v} := \tilde{\mathbf{v}} - \mathbf{v} \).

Given \( N \) design parameters \( \lambda_1, \ldots, \lambda_N \), \( N \) shape vectors \( \Delta \mathbf{v}_1, \ldots, \Delta \mathbf{v}_N \) have to be calculated. Performing \( N + 1 \) analyses, the finite element solver then calculates sensitivities for each design variable, and a gradient based optimisation using steepest descent on a linear response surface can lead to an optimal linear combination

\[
\tilde{\mathbf{v}} = \mathbf{v} + \sum_{k=1}^{N} \lambda_k \Delta \mathbf{v}_k
\]

(2.3)

of the design variables \( \lambda_k \) within the permissible design region, while observing predefined constraints \( g(\tilde{\mathbf{v}}) \leq 0 \).

While this approach sounds promising, it is hard to create shape vectors for complex models. Basic applications are discussed by Vanderplaats et al. [112]. Another problem is that for a given combination of design parameters the resulting deformed mesh \( \tilde{\mathbf{v}} \) (2.3) becomes easily distorted to a degree that the analysis results may become unreliable. For example, due to the deformed finite elements, the calculated stiffness generally over-estimates the stiffness compared to a mesh with better element quality.

Some progress has been made by the software SFE CONCEPT, which features the automatic generation of shape vectors based on small parametric modifications of the geometry model, and is capable of handling elaborate shapes (Kuhlmann et al. [61]). Also, re-meshing of the modified geometry can be triggered, in case the element quality of the distorted mesh \( \tilde{\mathbf{v}} \) does not meet the quality criteria of the solver.

### 2.4.6 Specific geometry descriptions

Often it is possible to directly link a design variable to a geometrically meaningful design parameter, which translates into an analytical concept in a straightforward way. In numerous studies this technique has been applied successfully (see for example Figure 2.5 on the next page) where 8 design variables track the x- and y-position of the four points \( P_1 - P_4 \). This leads to a cantilevered truss structure of minimal mass under the load \( F \), given certain compliance constraints (Vanderplaats et al. [112]).

Many other examples of the application of parametric geometry can be found in the literature; often B-splines are employed to describe varying
shape, the location of the spline control points being the design variables. An interesting example is the cross section of an airplane wing detailed by Brakhage and Lamby [11] where a small number of points vary cross sectional shapes. A subsequent discretisation and CFD-analysis leads to an optimisation of the shape with minimal aerodynamic drag. Derksen and Bender [28] optimise a nose cone for large fans by describing their shape with Bézier curves. See also Tecklenburg [110], or Pourazady and Fu [86] for further examples.

2.4.7 Parametric CAD tools

Since the early 1980s, CAD (Computer Aided Design) tools have been widely used in the engineering industry. Being at first merely two-dimensional drawing aids, they soon developed into highly complex three-dimensional design tools. The list of available software packages is long, and I mention only a few, such as CATIA, NX, AutoCAD, Pro/ENGINEER, Solid Works [151, 149, 134, 145, 153]. Two developments of these tools are important in the context of this work:

1. The integration of CAD geometry with CAE methods, namely the finite element method;

2. The ability to describe geometry parametrically.

The first point requires a finite element mesh generator, either internal or external. To name a few, ANSA, Hypermesh, ANSYS, are well established
meshing tools that import CAD data and generate shell or voxel based finite element models. Many CAD tools integrate a finite element mesher, for example CATIA (in combination with ANSYS), NX, AutoCAD, Solid Works, Rhinoceros, Abaqus, etc.

For the second point, many of the CAD tools have incorporated the idea of parametric geometry description, such as Pro/ENGINEER, CATIA V5, NX, SFE CONCEPT, solidThinking Inspire, Rhinoceros, and others [110, 48]. This is the ability to generate geometric descriptions that can easily be modified and regenerated by changing a small number of descriptive parameters. An example is an extruded part with a parametric description of the cross section. Whenever the designer modifies some of the values defining the section, one wants to automatically update the final extruded geometry without having to iterate each construction step. However, for complex parts, it becomes very challenging to define a useful parametric such that the subsequent design steps can be repeated without model update errors. This is significant especially when multiple parts have to interact. While such problems may be manageable for an interactive development, they often pose limits for automated optimisation loops.

In the remainder of this section I want to highlight two specific software packages that incorporate important features relevant for this thesis. The first one is Altair’s solidThinking Inspire [129], the second one SFE CONCEPT [154].

2.4.7.1 solidThinking Inspire

The software package solidThinking began as a 3D CAD design tool in the early 1990s. After being acquired by Altair in 2008, the modelling and rendering tool was renamed solidThinking Evolve, while the modelling engine was combined with both Altair meshing tools and topology optimisation algorithms into what is today called solidThinking Inspire. In the academic field, not many articles have been published involving Inspire (see [41] for a brief mention). In the engineering industry though, Inspire has great potential for conceptual design studies due to its capability to rapidly model a 3D design space, apply loading conditions, and perform a SIMP topology optimisation within one easy to use integrated environment. See Figure 2.6 for some examples of structural load path design using topology optimisation.

2.4.7.2 SFE CONCEPT

The parametric geometry tool SFE CONCEPT can handle large scale parametric shape variation better than other tools, especially when it comes to
thin walled sheet metal structures. The reason is that SFE CONCEPT is specifically designed for use in the concept development stage for welded thin walled structures. While the geometry representation is not as accurate as is typically the case for other CAD tools, the inherent parametric allows fast and automatic shape modifications while always maintaining connectivity between parts. While the literature on shape- and topology optimisation is vast, only a limited number of publications are available around shape design using SFE CONCEPT.

The development of the software package SFE CONCEPT began around the year 1990 by SFE GmbH in Berlin, Germany, as a package to describe topological layout and parametric geometry of structures. Unlike traditional CAD packages, SFE CONCEPT describes geometry in terms of topological connections between parts; the geometry is then generated automatically based on a relatively small number of geometric entities (Zimmer et al. [126]). Based on an implicit parametric, it can translate variations of shape design parameters into consistent geometry which can then be meshed and output as a finite element model (Zimmer et al. [125]).

Compared with more widely used CAD packages (e.g CATIA, Unigraphics, Solid Works, Pro/ENGINEER, Rhinoceros), SFE CONCEPT is by no means a mainstream application, but for many automobile engineers has become indispensable for finding optimal topological layouts and performing geometrical optimisation in an early stage of structural concept layout design. The main application lies in the development of automobile body structures at an early concept stage when not much detailed (CAD-) infor-
mation may yet be available ([Schumacher et al. [98], Schelkle and Klamser [94], Zimmer and Prabhuwaiagankar [124]]). It allows large scale shape modifications in a short time and has been described as the “state of the art in shape optimisation” ([Duddeck et al. [36]]).

Figure 2.7 shows a typical SFE CONCEPT automotive body in white geometry model in various stages of development (the first three pictures), and a resulting finite element mesh derived from the geometry.

![Figure 2.7: SFE CONCEPT automobile body in white geometry model at various stages, and internally generated finite element model (right) [124].](image)

One advantage over homogenisation methods is SFE CONCEPT’s potential to include crash load cases into the optimisation process. This is highlighted by various authors, see for example Hilmann et al. [51] or Hunkeker et al. [58]. Duddeck et al. [35] write:

A new approach for realising automatic shape alterations was developed based on the software SFE CONCEPT provided by SFE GmbH in Berlin, which is based on an implicit parametrisation technique. Here true shape variables can be defined for the description of the geometry. Then shape modifications are realised either on a small scale via morphing or on a large scale via re-meshing of the FEM mesh. This is a unique feature enabling the shape optimisation of structures for crash-worthiness or similar functionality even if larger changes in shape are required during the optimisation process (which is the usual case). The implicit parametric approach of SFE CONCEPT assures the connectivity of the different components in full car models if the geometry is changed.

Hilmann et al. [50] discuss a “fully automated method of structural optimisation for the body in white structure” using SFE CONCEPT, Hypermesh, Perl, MATLAB, and RADIOSS. SFE CONCEPT is used to create a parametric geometry model with shape design variables to optimise the shape and bead-structure of a frontal crash-box. The other tools mentioned are
then applied to chain together a closed process loop for multidisciplinary structural optimisation including crash load cases.

To conclude, I note that shape optimisation is an active field of research. It has been seen that in many cases, the prerequisite of automated shape optimisation — namely the transition from shape design parameters into analysis models — remains a difficult task. Finite element mesh morphing approaches have limitations in geometrical expression, while specific parametric CAD tools have their intrinsic limitations.

2.5 Combined shape and topology methods

In this section my aim is to identify structural optimisation methods that combine topological layout design with shape optimisation approaches.

The separation between sizing, shape and topology optimisation is not always clear cut. A simple sizing optimisation of a truss structure (Figure 2.4 on page 35) may well be called a topological layout optimisation, since members are allowed to be deleted by downsizing their cross section size to zero. Similarly, structures developed using some homogenisation or level set method may be called shape- and topology optimised, since the resulting design not only describes the structural members, but also the shape of the part.

Nevertheless, combined methods have specific limitations, which I aim to discuss. This then leads to the research question investigated in this thesis.

2.5.1 Shape optimisation after Topology optimisation

When it comes to combined shape and topology methods, very often one refers to the shape fine tuning performed after a SIMP topology layout has been devised. This is a very natural approach, since the homogenisation method returns a density distribution on a fixed design space which needs to be interpreted by a human operator. The engineer looks at a density distribution and make decisions as to where to place structural members and how to form them. This step may be called the translation from a grey-level design to a black-and-white structure. In a number of publications, this is done in a manual interpretation step, as for example in Müller et al. [72], Grujicic et al. [43], Bakhtiary et al. [7], Cappello and Mancuso [16], Schwarz et al. [99], Spath et al. [108]. Thereafter the derived shape is parametrised and fine tuned, often to meet stress constraints, as in Jang et al. [59]. In a more complex example, Volz et al. [114] describe the shape optimisation
of an automobile body structure using a parametric geometry description. Here too, the basic load path layout is determined by topology optimisation, however, has been derived independently beforehand, meaning before the application of shape optimisation.

Automated interpretation of density results

A number of methods have been proposed to automate this interpretation step. Of interest is the idea shown in Hsu and Hsu [53] and [55]: After a filtering step, the density contours are cut in a sweeping process, and the resulting sections are parametrised using B-splines and assembled into a three dimensional CAD model. Lin and Lin [65] and Bremicker et al. [12] use computer vision techniques to smooth density contours. A subsequent parametrisation allows shape optimisation. See also Cappello and Mancuso [16], Schumacher [97]. In all these publications, shape optimisation always refers to the modification of the outer boundary of a design and is limited to single components.

This begs the question whether the limitation to single components can be overcome by performing parametric geometry modification on complex assemblies beforehand topology algorithms are exercised.

Shape and topology design using Level Set Methods

While homogenisation methods generally require an interpretation step for the density result, level set methods avoid intermediate densities by describing the design boundaries explicitly. In addition to the stiffness requirements, shape control features can influence the design boundaries. This approach is referred to as simultaneous shape- and topology optimisation in many examples in the literature. For example, Chen et al. [23] (shape feature control favours beam structures), or Takezawa et al. [109] (phase field method and sensitivity analysis is used to described the design boundaries). Many other publications describe the application of level set methods as simultaneous design of shape and topology, such as Eschenauer et al. [38], Duan et al. [31], Luo et al. [67], Luo et al. [68], Victoria Nicolas [113], Wang et al. [117], Wakao et al. [115]. As for the homogenisation based approaches, here shape optimisation refers to the modification of the outer boundary of a design, and is limited to single components. This limitation applies to shape optimisation methods in general. See, for example, Hari Gopalakrishnan and Suresh [49].
CHAPTER 2. LITERATURE REVIEW

Here it is interesting to ask whether topology and shape optimisation can be combined in a way that overcomes this limitation, namely the restriction to modify the outer boundary of a single part.

2.5.2 Shape and topology optimisation of beam structures

Truss structures are still an active field of research, as can be seen from the great number of publications (See for example Reddy et al. [87], Zhou and Rozvany [122], Rozvany [91], Guo et al. [44], Miguel et al. [71], Deb and Gulati [26], Noilublao and Bureerat [75]). Various algorithms are employed to optimise for section properties and size of beam members. Here, the variation of cross section values is referred to as shape optimisation. As to the topology optimisation, members are swapped in and out of the design space using either discrete 0/1 variables, or by allowing the beam size to drop to virtually zero.

In contrast to the SIMP topology method, this approach cannot find topological load path concepts other than predefined members. On the other hand, it avoids the interpretation required for a homogenisation approach, and complex geometry models can be used. This is shown for example in Torstenfelt and Klarbring [111] for an automobile BIW structure, represented by beams with rectangular cross sections. Similarly, Donders et al. [30] apply a reduced beam and joint concept to optimise beam dimensions for an automotive BIW. A more complex geometry description is used by Duddeck [34] for a BIW structure. A number of predefined beam members are swapped in or out of the model, and the geometry description is detailed enough to allow crash load cases to be analysed. See also Schumacher et al. [98], and Schelkle and Elsenhans [93], Hänschke et al. [47] who show similar examples of body structure optimisation, where topology layout design is done independently from a subsequent shape optimisation.

2.6 Discussion

2.6.1 Limitations of current methods

Here I summarise the limitations of current topology and shape optimisation methods. Homogenisation methods and boundary descriptions, such as the level set method:

- applicable to single parts only;
• often require considerable interpretation and post processing to lead to manufacturable design;
• have limited ability to handle complex loading conditions such as non-linear deformation in a crash event;
• cannot handle thin walled three-dimensional structures well;

On the other hand there are tools that describe geometry parametrically, enabling automated shape design variation. Limitations are that they:
• require the creation of complex models;
• cannot derive topological load path layouts;
• have limitations in geometrical expression, for example;
  – mesh morphing cannot alter basic topology, and easily runs into finite element quality problems;
  – parametric CAD tools have certain limitation, such as being able to describe only thin walled structures;
  – Geometry tools usually do not capture complex interrelationship between parts.

When reviewing methods that attempt to combine shape and topology optimisation, there are two main approaches:

1. Topology methods:
   • which perform shape modification after a topology layout has been derived, or
   • which simultaneously modify the boundary of a single part design to capture certain physical properties (such as avoiding stress concentration)

2. Methods based on parametric geometry realise topological variation using predefined parts:
   • by either swapping them in and out, or
   • by downsizing them to (almost) zero.
As a general rule, the engineer first generates a coarse topological load path layout, and then refine the resulting preliminary design into a manufactureable structure using shape tools. Topology methods have difficulties capturing complex geometry such as compound and thin walled structures, whereas parametric geometry approaches cannot derive a fundamental load path topology. Moreover, the design space for topology optimisation using homogenisation or level set methods is fixed, and reveals single continuous parts.

2.6.2 Topology methods vs. parametric geometry methods

Two general types of structures are distinguished. The first class is represented by single parts that can be built in one piece, often made of one homogeneous, isotropic material. The second class comprises complex compound structures, made of multiple parts, possibly from mixed materials. With this distinction, a tension cable falls into the first category, an automobile body structure into the second. A suspension arm of that car — made from a single piece of cast aluminium — is simple, a crane cantilever is not. In this sense, an aeroplane wing spar falls into the first class, if it is made out of a single cast part, even though its loading conditions and structural requirements may be highly complex.

However, the separation between these two classes is not absolutely clear cut, and it is not always possible to unambiguously classify a structure as belonging to either one or the other. Nevertheless, when looking into structural optimisation methods, the distinction made here helps in understanding the scope and viability of a specific approach versus its limitations and constraints.

A main focus in structural optimisation methods is research around topology optimisation, with a lot of activity in the field of homogenisation techniques on the one hand and level set methods on the other. Here the design space is always fixed, as stated by numerous authors (see for example Eschenauer and Olhoff [39], Lin and Lin [65], Grujicic et al. [43], Paris et al. [83]). In all the examples described, the topology optimisation starts with a given fixed design volume, as well as a fixed set of loading conditions.

An important question to ask here is whether it is possible to extend current topology methods by allowing the design space to change parametrically.

When investigating shape optimisation, level set methods are a very important field of research. Generally speaking, these types of methods —
homogenisation, level sets, and related techniques — arrive at shapes and
topologies of parts that fall into the first of the above classes, in the sense that
they deliver a connected, single, three-dimensional structure. If an isolated
part is what the designing engineer is expecting, then the result returned by
topology optimisation methods may be perfectly acceptable, even though a
number of manual subsequent steps have to be performed to arrive at a final
manufacturable design.

On the other hand, when the optimisation procedure targets a complex
structure — such as an aeroplane hull structure or an automobile body —
the results obtained by homogenisation or level sets may be a long way
from an actual structural solution that can be implemented in production.
Often the solution returned by the optimisation algorithm may only give a
general indication as to where structural members are likely to perform best,
and it remains the responsibility of an experienced engineer to translate the
proposed load paths into a real world structure. This is especially true for
structures that incorporate sheet metal parts. Here, homogenisation or level
set based optimisation methods are not well suited.

Other techniques deal with compound and thin walled structures better.
The main approaches are finite element mesh morphing methods, and para-
metric geometry tools, combined with a finite element mesh generator.

Given a parametric geometry description, the engineer can — within the
limits of the actual tool — deduce an analysis model, and thus conduct an
optimisation for the geometry parameters covering the design space.

As to mesh morphing, this technique requires a finite element model to
start with. A limitation is that the topology of the structure is already put
into the optimisation process, so mesh morphing is inherently incapable of
modifying topological layout (Zhang and Chen [121], Zimmer et al. [125]),
and is limited to relatively small-scale modifications (Duddeck [33]).

Another approach to shape variation is what is generally referred to as
parametric geometry. CAD packages with a parametric geometry description
combined with a meshing tool provide the engineer with the potential to
describe design variation with intuitively meaningful parameters that can be
translated into analysis models. Often, the parametric geometry description
of CAD tools refer to the modification of the boundary, and cannot handle
the complex relationship between multiple parts well (Pantz and Trabelsi
[79], Papadrakakis et al. [80]).

A big advantage of shape methods such as morphing or parametric CAD
models is that they are capable of delivering a geometry that is representative
of the production design. This is generally not the case for homogenisation
approaches, where the optimisation algorithm output may indeed have to undergo a major redesign to reflect the actual structural intent. One may think of an automobile body structure, where the result of a topology optimisation delivered by a homogenisation algorithm is not representative of a manufacturing implementation at all. The topology algorithm is based on solid elements, whereas the production design relies on thin walled, stamped and welded sheet metal (See Figure 2.8).

![Figure 2.8: Topology optimisation result for an automobile body in white. From SAE website [148].](image)

It is obvious that a significant amount of interpretation is necessary in order to translate the proposed load paths into an actual production design, and it is indeed questionable whether this approach delivers significant value.

Another important point here is that the topology algorithm cannot take into account crash load cases, since the voxel-based homogenisation method only allows linear load cases, and cannot account for crush. Rather, these loads have to be approximated by linear static equivalent loads.

Once more, the design approach here is to first look at general load path layout, which has to be translated into a more realistic thin walled geometry model. The latter can then be used for further shape optimisation (c.f. Figure 2.7 on page 42). Thus, topology optimisation is applied first, followed by shape optimisation on the interpreted topology optimised structure. The interpreting process is non-trivial as it needs to convert a 3D density description into a thin walled assembly.

The question then arises as to whether a complex system consisting of thin walled and solid structures can be optimised for mass and/or stiffness by applying both

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large scale shape variation and topology optimisation. Thus, can a method be devised that takes into account the interaction between thin walled and solid component assemblies.

2.7 Conclusion

Optimisation methods using parametric geometry description as well as topology approaches have become indispensable tools for the engineer. But then, there are inherent limitations of current topology or shape optimisation tools in use today. Is it possible to find new ways to bridge some of the gaps?

Due to the characteristics and restrictions of current methods, a general trend in the process of structural design can be observed. The review indicated that the design space for topology optimisation using homogenisation or level set methods is fixed, and topology design is performed independently from shape optimisation, in the sense that the latter is conducted after the former has been established. In turn, shape methods are often hard to implement and have difficulties in developing topological load path layouts.

In order to get the best performance out of a complex structural system, the main question I want to answer in this thesis is whether it is feasible to integrate large scale parametric shape variation with a homogenisation based topology optimisation methods. This leads to the central research questions we want to investigate in this thesis:

By including system information, can an extended topology optimisation method find better structural results than stand alone topology optimisation? What are the advantages of parametric shape design performed in conjunction with topology optimisation? Which benefits can be gained by linking shape design variables to both the initial design volume for the topology optimisation and to the surrounding system?

In particular, I want to investigate whether the following objectives are achievable:

- Overcome the idea of a fixed package volume by allowing parametric variation of the design space.
- Overcome the limitation of shape optimisation to the outer boundary of single connected parts.
• Integration of large scale parametric geometry variation with topology optimisation methods for complex compound structures.

To achieve these outcomes the following questions need to be answered:

1. How can the integration of large scale shape variation and topology optimisation methods be realised?

2. Which classes of structural problems lend themselves to the application of such an integrated approach?

3. How does one compare mass and performance when they can vary across the solutions?

4. How can boundary conditions be created for dynamically generated analysis models?

5. How does the integrated approach compare to standard topology and shape optimisation methods?

6. What are the advantages and benefits, and which new issues arise? What are the inherent limitations of this integrated approach?

In the next chapter, I discuss the logic underpinning the integrated shape and topology optimisation approach.
Chapter 3

IST — An Integrated Approach to Shape and Topology Optimisation

3.1 Introduction

In this chapter I introduce the concept of a combined shape and topology method, in which large scale shape variation is performed prior to and in conjunction with topology optimisation (Section 3.2). I begin by describing the general idea and theory of what has been named IST (Integrated Shape and Topology approach), and show how this approach extends existing methods.

In Section 3.3, the building blocks required to realise the proposed approach are discussed, followed by a discussion regarding the classes of structural problems that the IST approach is able to tackle. Initially, the research and subsequent development of IST was motivated by looking at thin walled sheet metal structures, combined with cast joints (for example a complex aluminium joint used in automobile body structure that connects extruded profiles). However, it is also interesting to see how other types of structural problems can benefit from the application of the developed method (Section 3.4).

Using adjustable package space for the topology optimisation, as well as varying its surrounding structure and boundary conditions, new problems arise, such as the comparability of multiple topology optimisation results. I investigate these issues and propose solutions in Section 3.5.

Having described in detail the theory, I then look into practical examples and applications in the two subsequent chapters.
3.2 Integrating optimisation methods

In this section I explain the basis underpinning the integration of shape variation and topology optimisation methods.

Topology optimisation, as applied in the current thesis, is defined as the problem of finding a density function $\delta : \Omega \rightarrow [\delta_{\text{min}}, 1]$ on a design domain $\Omega$. This domain typically is a finite element mesh in three (or less often in two) dimensions, that may consist of either shell elements, solid elements or both. The finite element mesh is part of an analysis model including loads, constraints, and material definitions. The density function $\delta$ is to be chosen such that some performance figure, for example the overall mass $M$, is minimised:

$$M(\delta) = \int_{\Omega} \delta \, d\Omega \rightarrow \min,$$

subject to constraints $g_j(\Omega, \delta) \leq 0$. Alternatively, $\delta$ is chosen to minimise the total deformation energy in $\Omega$ for a predefined mass or volume fraction value $\bar{\delta} = \int_{\Omega} \delta \, d\Omega / \int_{\Omega} 1 \, d\Omega$. While these two objectives (minimise mass or maximise stiffness) are the most commonly used approaches, other optimisation targets are possible, such as finding a maximal first Eigenfrequencies (c.f. Chapter 2 on page 22). The stiffness of one finite element — assumed to be constant within that element — varies monotonically with the density $\delta$, as described in Section 2.3.4 on page 30.

Traditionally the design domain $\Omega$ is assumed to be fixed. The engineer would perform a load path optimisation on $\Omega$, and interpret the results. Now, the significant novel ideas of the integrated optimisation approach explored in this thesis are twofold:

1. Allow the design domain $\Omega$ to vary, subject to geometry design parameters.

2. Allow $\Omega$ to be part of a complex system, that itself can be subject to parametric geometry variation.

This is where shape design parameters come into play. $\Omega$ itself, as well as its surrounding structure $\Lambda$ is subject to variation. This geometry variation is described by shape design variables $x \in \mathbb{R}^n$: For each set of design variables $x = (x_1, \ldots, x_n)$, a new geometry model

$$x \rightarrow \Omega_x, \Lambda_x$$

needs to be created. A necessary next step is to integrate $\Omega_x$ and $\Lambda_x$ into an analysis model, denoted by $\Gamma(\Omega_x, \Lambda_x)$. Then $\Omega_x$ is subject to a SIMP
Parametric geometry model $\Omega_x, \Lambda_x$

Analysis deck $\Gamma(\Omega_x, \Lambda_x)$

Topology optimisation $\int_{\Omega_x} \delta_d x \Omega_x \rightarrow \min$

New design variables $\mathbf{x} = (x_1, \ldots, x_n)$

Figure 3.1: Topology optimisation embedded into large scale shape variation and optimisation process

algorithm, while $\Lambda_x$ is fixed with respect to the topology optimisation for this specific shape design $\mathbf{x}$. This IST approach is a significant extension of the traditional topology optimisation method. See Figure 3.1 for a generic process flow, and Figure 3.2 on page 58 for a more detailed explanation. I will come back to the latter in the next section.

To realise the approach depicted in Figure 3.1, an automated process loop needs to be established and implemented.

### 3.3 Requirements for the implementation of an automated IST process loop

In this section, I explore the building blocks and tools required to realise and implement what has been called IST. It will be seen that it is possible to use off-the-shelf tools for most aspects of the implementation, while some process steps turn out not to be readily available, and therefore have been developed by the author.

In order to perform an optimisation, it is necessary to fully automate all the process steps depicted in Figure 3.1. Without referring to actual implementation details or concrete software tools, I list the generic elements that are indispensable to realise the intended automation.

1. A CAD (Computer Aided Design) program that can create parametric 3D geometry. For complex geometry, this requires a sophisticated approach that goes beyond the ability of mesh morphing tools. It is necessary to incorporate shape design variables that describe geometry in an intuitive way.

2. A meshing tool that generates finite element models out of the CAD geometry. In the context of analysis of structures, I always mean
numerical analysis based on the finite element method (FEM). This means, one needs reliable and robust tools that can translate the generated geometry into a finite element mesh.

3. A preprocessing tool that assembles the finite element data into an analysis model. A finite element input deck suitable for a specific solver needs to be compiled. In particular, connection elements between disjoint parts of the model have to be generated, and loads and boundary conditions (constraints) need to be applied. This step needs to take the specific geometry into account.

4. A finite element solver incorporating a SIMP topology optimisation algorithm. This solver should incorporate relevant manufacturing restrictions such as extrusions and symmetries.

5. An optimisation tool to handle the shape design variables and outer optimisation loop. This optimiser should provide a number of optimisation methods, including bionic optimisation algorithms.

6. A mechanism that chains all of the above together. One of the ideas behind IST is to provide an easy to use user interface that is flexible and powerful enough to quickly set up an optimisation loop that links all the above steps together. This process handling tool must incorporate the possibility to pass data between different units of the analysis chain, extract solver results and handle errors appropriately. Moreover, the interface needs to be flexible enough to incorporate problem-specific user defined plug-in functions.

It makes sense to use available state of art technology wherever possible and practical. For the work at hand, this is the case for steps 1 on the preceding page, 2, 4, 5: Geometry creation tools are readily available, and solutions for meshing, suitable for a variety of requirements, are readily available. There are a number of analysis solvers to choose from, and optimisation algorithms don’t have to be reinvented. I describe the choice of concrete tools for these steps in Appendix A on page 131.

With respect to Point 3, a mechanism is required that is capable of automatically generating analysis models for varying geometries. Necessary features include the ability to

- Import finite element data;
- Translate, rotate, replace, copy, mirror entities;
- Define sets, based on part names or geometry (boxes, radii);
• Create connections between components using various element types (rigid body connections, beams, bars), based on part names, sets, distance, etc.;

• Renumber entities, merge and replace grids, assign specific grid id’s for handling of loads and for performance tracking;

• Apply single point constraints and loads, based on geometry, part names, sets, id range, etc.;

• Export solver specific analysis decks.

There are software tools out there that are suited to handle all of these tasks, and also offer the ability to automate particular steps via scripts, such as Hypermesh, ANSA, LS-PREPOST ([137]). But these tools need to be programmed for individual problems, and are difficult to integrate into a generic approach used for the current research. This is why I decided to develop proprietary software that realises the necessary process steps in a flexible and extensible way. This allows one to control process steps by a simple script language.

All these elements need to be chained together (Step 6 on the preceding page). This integration is realised by implementing a simple parser for a batch script language similar to the interface for Step 3 on the previous page. This allows one to define the optimisation loop within one scripting environment. Additionally, it is possible to add problem specific functions to the process chain. The details of the implementation are outlined in Appendix B on page 136.

With this tool set, it becomes possible to quickly set up an analysis that integrates large scale shape variation and topology optimisation. Figure 3.2 on page 58 shows a generic example of one such an optimisation loop: Starting with an initial package space, a parametric geometry model is created as a one-off manual step. Typically, this geometry model consists of a portion which are subject to topology optimisation (Ω), as well as components that are not (Λ). Both Ω and Λ can depend on geometry design variables \( x \), and may consist of any combination of thin walled structures as well as solid volume elements. Once the model is created, and a design space for \( x \) is defined, the process of compiling the analysis model needs to be described. This is also done in a one-off manual step, using connection elements provided by IST. For the topology algorithm, constraints and an objective (which in many cases is the mass or some structural performance metric) have to be defined within a solver specific input deck.
After these manual steps, the optimisation or DOE can be started, and the following steps are performed in an automated loop: The outer optimiser chooses design parameters $\mathbf{x}$, the geometry $\Omega_\mathbf{x} \cup \Lambda_\mathbf{x}$ is updated, a finite element mesh is generated, the analysis mesh compiled, and a topology optimisation and potentially additional analysis steps are performed. This results in performance figures passed back to the optimiser. Contingent on the global objective function, new design variable values are chosen, and the loop is closed. In Chapters 4 on page 71 and 5 on page 93, a number of case studies are presented, and it can be seen how these steps are performed in practice. Before that, I explore which classes of structural problems can be tackled using IST, and highlight issues arising with this approach.
Figure 3.2: Typical process flow for an integral shape/topology Optimisation (IST). The application sequence is flexible and can be managed via description files and expanded by plug-in functions.
3.4 Structural problems that can be targeted using IST

The benefits of integrating large scale shape optimisation with topology optimisation approaches occurred to me when dealing with automobile body structures at an early concept design stage. On the one hand, one is concerned with global load path layout development, where one attempts to find best locations for main structural members. This task can be tackled using topology algorithms, and requires only a relatively coarse package space input, together with a set of static loading conditions. On the other hand, in order to take into account complex loads such as crash scenarios, sufficient geometry detail needs to be available.

Topology methods cannot account for nonlinear deformation, and crash loads can only be incorporated using equivalent static loads [64, 32]. Much better suited for the shape design are dedicated CAD tools that can model thin walled sheet metal structures, in conjunction with finite element mesh generators.

The first application in the context of IST arose when looking at recent advances in body structure design. Complex cast joints play an increasingly important role as they are used to join relatively simple beams with constant cross section that can be manufactured from extruded profiles. One potential advantage is a reduction in weight by using lightweight materials, another advantage comes through simplified manufacturing processes, namely the use of extruded beams, which avoids costly stamping tools for pressed sheet metal parts.

A disadvantage (with constant cross section beams) is that the joint design becomes more complex. One may think for example of an extruded rocker (or often called sill), which is a main structural member carrying bending and torsional loads in an automobile body. When made from a constant cross section, the joints that link the rocker with other structural members — such as the A-pillar and cross car beams — need to be more sophisticated. While for a conventional stamped sheet metal approach, the load transfer can be managed using smooth local transition geometry, all these tasks need to be taken over by complex casting design. (See for example Figure 3.3 on the following page).

While the design of the cast joining structures is more complex than more traditional thin walled sheet metal surfaces, it offers added potential. Structural elements can be located exactly where loads have to be transferred, and the complex geometry can incorporate more functions, such as the integration of local reinforcements and bulkheads, or for providing attachment
Put simply, the design task for the automobile body structure is to find the size, shape and position of relatively simple beam members, and to simultaneously find suitable connection joints that transfer all necessary loads.

For the design and optimisation of beam members, a number of tools may be applied. Analysis tools such as OptiStruct, NASTRAN, Abaqus, LS DYNA, and others provide beam elements that can be assigned section properties. This may be a good start for a first layout design of the main load carrying members (Torstenfelt and Klarbring [111], Donders et al. [30]). If more information is available, the geometry can be modeled more accurately using shell or solid elements, and in order to run an automated performance optimisation, a parametric geometry model including a finite element generator is necessary.

For layout design of cast members, topology optimisation methods play an important role to determine main load carrying elements. A necessary input is the surrounding structural elements and forces acting on them. Looking at these two design approaches (parametric beam geometry and topology optimisation), the advantages to integrating the two methods became apparent. In the following sections I highlight potential applications for such an approach. The list is certainly not exhaustive, and it will be interesting to see more applications in the future. This coupling between parametric design volumes and topology optimisation for different problem classes indicates
that the topology design volume is not independent of external influences (as was suggested to me at the beginning of this thesis).

### 3.4.1 Trade-off between beam size/shape and topology of cast joints

One of the main motivations for the development of IST is to simultaneously vary beam geometry and joint topology. By combining these two aspects into an integrated method, it should become possible to find optimal solutions (e.g. lowest mass) that trades off necessary stiffness and mass of extruded beams with joint topology, size and mass.

![Diagram](image)

Figure 3.4: Variable initial geometries and structure with minimum system mass

Preliminary studies indicated that in principle this is possible. A simple example is shown in Figure 3.4. Different combinations of length and radius of the tube, different topologies are created, and a compromise between size and mass of the beam versus size and mass of the joint is determined. This approach has the potential to be applied for more complex designs, such as automobile body structures, reinforcements for aircraft wings, wind turbine housing designs, and many others. Detailed studies are presented later.

While investigating this idea and implementing tools to realise it, a number of other classes of applications have been discovered. I introduce these
new applications in the following section, and show application examples in the two subsequent chapters.

3.4.2 Variable geometry for shell based applications

Thin walled structures may be subject to topology optimisation to find

- a load path distribution,
- optimal thickness distribution, or
- locations for lightening holes.

In most applications found in the literature, the input geometry is fixed. An exception is found in Ansola et al. [6], where the finite element nodes of a simple flat sheet are allowed to vary in the vertical direction. While this approach has very limited application, it shows that shape design and topology algorithms can be combined to find better structural solutions than a sequential method (Figure 3.5). The method explored in this thesis goes beyond this approach because it allows arbitrary shape modifications. Figure 3.6 shows an example that is explored in more detail in Chapter 4 on page 71.

Figure 3.5: Simultaneous shape and topology optimisation, where finite element nodes can move in vertical direction [6].

Figure 3.6: Large scale shape variation in conjunction with topology optimisation of a thin walled structure.
3.4.3 Variable package space to accommodate for extrusion constraints and multi-piece assemblies

The parametric variation of the design space Ω for topology optimisation can also be beneficial for relatively simple solid parts. One application of IST is the design of parts made from extruded profiles. Constant cross section design can be a low cost alternative to more complex castings, and still offers a wide range of design freedom. When topology optimisation is involved, the task is to find the best cross section of such an extrusion. Other design parameters could and should be considered, such as the part length (i.e. the cut length of the extruded profile). In the easiest case this can be an angled cut, but can also include more complex cutting operations, as shown in Figure 3.7, (For a detailed discussion of this example refer to Fiedler et al. [40]).

![Figure 3.7: Brake pedal, to be manufactured from an extruded aluminium profile: different initial geometries and lightest topology optimisation result (Fiedler et al. [40]).](image)

In Chapter 4 a more complex example of a two-piece assembly made from an extruded section is discussed (page 76).

3.4.4 Variable boundary conditions

Another way to employ IST is to parametrically vary boundary conditions. An example is shown in Section 4.4.1 on page 81, where I look at the topology optimisation of a cast attachment bracket. Varying boundary conditions in this case means that the location of the attachment bolts is subject to parametric variation and optimisation.
3.5 Challenges arising with varying initial geometry

While using an IST approach, a problem associated with varying initial shapes needs to be addressed: When a number of different geometries $\Lambda_x \cup \Omega_x$ are generated, and subsequently material from $\Omega_x$ is removed by applying a topology optimisation, the resulting structures in general have different masses $m_x$, different performance results, different mass fraction $\bar{\delta}_x$, or any combination thereof, depending on the geometry $x$ (Figure 3.8). Here $\bar{\delta}$ is defined as the effective mass fraction value returned by the topology optimisation algorithm:

$$\bar{\delta} = \frac{\text{mass}(\Omega)}{\text{mass}(\Omega)}$$

where $\Omega$ is the remaining structure that is not subtracted from $\Omega$.

This creates a ranking problem. How does one determine which is the better design in terms of a global real valued objective function that is to be optimised in the outer geometry loop. In this section I describe the nature of the problems arising for different approaches to the topology part of IST, and explore solutions.

![Figure 3.8: Mass vs. performance vs. mass fraction for different initial geometries $i \neq j$. In general, at least two of these metrics are unequal, to the effect that different geometries are not directly comparable.](image)

Assume a global objective function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ that evaluates the mass of the designs $\Gamma(x) = \Lambda(x) \cup \Omega(x)$ for different geometry design parameters.
\( x \in \mathbb{R}^n \) (\( \Omega \) denotes the topology design space \( \Omega \) after SIMP). The target is to minimise the overall mass of the design, which is to find
\[
x_0 \in \mathbb{R}^n, \quad x_{\text{min}} \leq x_0 \leq x_{\text{max}}, \quad \text{such that}
\]
\[
f(x_0) \leq f(x) \quad \text{for all} \quad x, \quad \text{subject to constraints} \quad g(x) \leq 0.
\]

Now, when applying a SIMP algorithm to \( \Omega_x \), which returns a density distribution \( \delta_x \), two main approaches to establish a (SIMP) objective function stand out:

1. Maximise the performance of the structure for a fixed volume or mass fraction \( \bar{\delta} \), allowing the SIMP algorithm to take away a pre-defined volume/mass of material equal to \( 1 - \bar{\delta} \) times the initial volume/mass. *Performance* in this context most often is the stiffness of the structure, measured as the inverse of global deformation energy.

2. Minimise the mass under performance constraints (typically displacements, stresses and/or Eigenfrequencies)

In the first case, the amount of material (mass or volume) to be used is pre-determined, and a load path distribution is generated such that the performance of the structure is maximised. In the second case, the algorithm determines a material distribution with minimum mass, whereby the final amount of remaining material is a priori unknown.

I will investigate the two cases separately, highlighting the potential issues of each, and explore a number of ways to solve them (subsequent section).

Compared to traditional topology optimisation, the addition of variable initial geometry opens up the design space for more design solutions. The described ranking problem is not relevant to traditional topology optimisation because:

- topology optimisation traditionally only runs once;
- the weight of the density distribution resulting from topology optimisation traditionally is of indicative nature only and is not used to compare two designs;
- with a traditional fixed design space, either performance or resulting mass is pre-determined.
CHAPTER 3. INTEGRATED SHAPE AND TOPOLOGY OPTIMISATION

3.5.1 Topology Optimisation maximising performance under volume fraction constraint

Consider a structure that consists of non-designable parts $\Lambda$ (i.e. regions that are not subject to SIMP), and a designable component $\Omega$. For the SIMP algorithm, the objective function is assumed to be some performance figure, while the mass fraction value is to be constrained to a fixed value, say $\bar{\delta} = 0.2$. Now, for this mass fraction the aim is to find a density distribution that optimises the performance, which could be to minimise the displacements at the load points, or to maximise the overall stiffness of the structure.

Since now also the initial shapes are varied, it becomes necessary to find topology optimisation solutions that are comparable between different geometries. Consider the naming conventions:

- $m_\Lambda$ = Mass of the non-designable region ($\Lambda$);
- $m_\Omega$ = Mass of the designable region ($\Omega$);
- $m_{\text{init}} = m_\Lambda + m_\Omega$ = Initial mass before topology optimisation;
- $\bar{\delta}$ = Mass fraction for the topology optimisation.

Now a topology optimisation is performed, the resulting mass being

$$m_{\text{res}} = m_\Lambda + \bar{\delta} m_\Omega.$$ 

For two geometrically different designs $i \neq j$, the masses $m_{\text{init},i}$ and $m_{\text{init},j}$ vary, such that

$$m_{\text{res},i} \neq m_{\text{res},j}.$$ 

Performance results for these two designs in general are also different, and the two designs are not directly comparable anymore (See Figure 3.9 on the next page).

There are three ways to tackle this issue, described in the following sections.

3.5.1.1 Multi-objective optimisation

The first approach is to do a multi-objective outer shape optimisation, where mass and performance are simultaneously assessed. This leads to a Pareto Frontier (Censor [18], Kung et al. [62]) of feasible designs, that need to be interpreted and evaluated by a human operator. In this work I prefer the unambiguous outcome of a single objective problem, and do not pursue this path further.
Figure 3.9: Comparability issue arises when SIMP algorithm is run on different geometries with fixed volume fraction \( \bar{\delta} \). For two designs, both mass and performance results (e.g., compliance) differ, which renders them incommensurable in terms of the outer geometry objective function, since the objective can only be to either to minimise mass (under a performance constraint), or to maximise performance (given a fixed amount of material).

### 3.5.1.2 Adjustment of mass fraction

The second approach is to make sure that the resulting masses for the different initial geometries remain constant by adjusting the volume fraction for the SIMP algorithm accordingly: For each geometry, choose \( \bar{\delta}_i \) such that

\[
m_{\text{res}, i} = m_{\Lambda, i} + \bar{\delta}_i m_{\Omega, i} = \text{constant}, \quad \text{equivalent to} \quad \bar{\delta}_i = \frac{m_{\text{res}, i} - m_{\Lambda, i}}{m_{\Omega, i}}. \tag{3.1}
\]

Thus, for a lighter initial geometry with mass \( m_{\text{init}, i} \) the volume fraction value \( \bar{\delta}_i \) is higher than for a geometry with more mass \( m_{\text{init}, j} \). There are a couple of disadvantages with this idea: First, the resulting mass \( m_{\text{res}} \) has to be fixed beforehand. Second, varying mass fractions can lead to qualitatively
different results, especially for shell structures. (This effect can be seen for example in Figure 3.10). For solid structures this is less of a problem, and I apply the approach in Chapter 5, where a cast joint undergoes topology optimisation under a volume fraction constraint. For different initial geometries, the compliance is minimised, and the volume fraction $\bar{\delta}_x$ is adjusted such that the resulting mass stays constant.

![Figure 3.10](image)

(a) $\delta_0 = 0.2$  
(b) $\delta_0 = 0.3$  
(c) $\delta_0 = 0.4$

**Figure 3.10**: Qualitatively varying results for topology optimisation with different mass fraction values $\delta_0$

### 3.5.1.3 Restriction of geometry parameters

Finally, one may be able to constrain the shape design parameters that ensures that the resulting masses are constant, while the volume fraction $\bar{\delta}$ stays fixed: Assuming $n$ geometry parameters, find a function $h(\cdot)$ with $(x_{k+1}, \ldots, x_n) = h(x_1, \ldots, x_k)$ such that

$$m_{\Lambda,i} + \bar{\delta} m_{\Omega,i} = m_{\Lambda,j} + \bar{\delta} m_{\Omega,j}$$

for all different designs $i, j$, resulting in $m_{\text{res},i} = m_{\text{res},j}$. This effectively reduces the number of shape parameters from $n$ to $k$, and requires that a suitable function $h(\cdot)$ can be found that interconnects the design variables in such a way as to ensure constant system mass before topology optimisation. $h(\cdot)$ needs to be determined individually for a specific problem, which may be hard to impossible for complex structures. A simple example is presented in Chapter 4.2 on page 72.

### 3.5.2 Topology Optimisation minimising mass under performance constraints

I now look into IST problems, where the mass is minimised under performance constraints (c.f. page 65). The comparability issues discussed in Sec-
tion 3.5.1 do not arise for this type of setup. Since the SIMP algorithm is constrained by the same performance characteristics for all geometries (one may think of a displacement constraint), different geometries can be assessed purely on their mass figure, so that topology optimisation results for different initial geometries are comparable. This leads to a desirable single objective outer optimisation.

On the other hand, it can be hard to define performance constraints that are valid for all individual shapes within the geometry design space.

Another problem can be that the effective mass fractions $\delta$ is a priori not known, and can take values of a large bandwidth. Due to manufacturing considerations, mass fractions outside a certain range may be infeasible. This makes it necessary to track the mass fraction $\delta_x$ for different designs $x$, and constrain it to a suitable interval $[\delta_{\min}, \delta_{\max}]$. Designs with mass fractions outside of this interval are considered infeasible.

The applications in Chapters 4 and 5 give examples of this approach.

### 3.5.3 Specific stiffness

A different approach to avoid the ranking problem could be to optimise for specific stiffness of a structure, i.e. to maximise the ratio between performance and mass. When for example the performance metrics of a structure are condensed into a single stiffness value $K$ (e.g. by assigning weight factors to the results of all load cases), one may look at the quotient $f$ between the stiffness and mass

$$f := \frac{K}{m}$$

as the objective function. It turned out that this approach is not suitable to the IST method. The reason is that when maximising the objective function $f$, the structures tend to adopt their largest possible geometric extend: Generally, when the mass is increased by a certain factor, say $\tilde{m} = \lambda m$, structures are obtained with stiffness $\tilde{K}$ greater than $\lambda K$. So $f$ is maximal when the geometry parameters propose the use of a maximal amount of mass.

Thus, one does not find low mass solutions, which is why I have not pursued the specific stiffness approach any further.

### 3.6 Conclusion

In this chapter, the concept of integrated shape and topology optimisation has been introduced.
I suggest that the design space — denoted $\Omega$ — that goes into a SIMP topology optimisation algorithm, as well as its surrounding structure — denoted $\Lambda$ — may be subject to large scale shape variation. Particularly, the geometry variation is applied on $\Omega \cup \Lambda$ before the topology optimisation algorithm is performed. This couples the design space to the performance of the interacting external system.

In order to realise this method, a number of tools need to be integrated, and I have explained which parts of the process can be covered by off-the-shelf software packages, and which parts need to be implemented from scratch.

I have described how this could be used to tackle a number of classes of structural optimisation problems, most importantly beam structures that connect in cast joints. I have also stated the potential benefits of this new approach over traditional topology optimisation.

This integral approach — named IST — leads to new types of problems, most importantly the issue that structures with unequal initial geometry (and therefore mass) can produce topology optimisation results with differing performance. This makes it impossible to compare and rank them in terms of an outer optimisation loop; a number of solutions are proposed.

Within the geometry optimisation loop, the respective topology optimisation can be performed both with performance constraints (minimising mass) or with mass constraint (maximising performance). One potential issue is the effective mass fraction value $\bar{\delta}$; designs with mass fraction outside of a suitable interval have to be discarded.

Equipped with this background, I now proceed to the application examples, representing the classes of problems described in Sections 3.4.1 to 3.4.4. The case studies examining these problems are presented in the forthcoming chapters. Structures consisting of either only thin walled or solid geometry are dealt with in Chapter 4, and more complex compound structures, integrating thin walled beam geometry with cast joining components are explored in Chapter 5. Each of these classes of applications show that IST provides better results than traditional topology optimisation.
4.1 Introduction

In this chapter I present application examples of the IST approach. This chapter focuses on structures that consist of either only thin walled elements or only solid elements. The three case studies represent different classes of applications that were outlined in Section 3.4. The examples are not trivial and show how the proposed IST approach can find system solutions with lower mass or higher stiffness than traditional stand alone topology optimisation, by taking into account large scale parametric variations of the initial design volume which is subsequently subjected to topology optimisation. The case studies are representative of structural problems that involve only thin walled elements or only solid elements.

The first example is a simple I-beam with a sinusoidal shaped web. Shape variation is applied in order to find an optimal ratio between height and width of the beam, while topology optimisation is used to find a load path layout. The second example is a clamp assembly made from two extruded parts. A SIMP topology optimisation is used to determine an optimal cross section. Parametric shape modification variation caters for variation of the split line between the two parts as well as for the part width. The third example is an optimisation problem for a cast bracket, designed using topology optimisation. At the same time, the location of the attachment bolts is varied parametrically by 11 design variables.
4.2 Light-weighting a shell structure

In this simple example, I investigate the integration of shape modification and topology optimisation for an I-beam with a sinusoidal shaped web [70]. This example is chosen to highlight the potential of IST when applied to pure shell based structures. It also addresses the important question of comparability between different designs when both mass and performance figures change.

The structure is made from 2 mm thick aluminium sheet (both top/bottom flange and web); it is rigidly attached at one end, while three static loads are applied to the other end, as shown in Figure 4.1.

\[ \begin{align*}
F_x &= 1000 \text{ N} \\
F_y &= -1000 \text{ N} \\
M_x &= 1000 \text{ Nm} \\
\end{align*} \]

Figure 4.1: I-beam with loads

The aim is to achieve two things:

1. Through topology optimisation find a material distribution that minimises the weighted compliance for the three load cases (Eq. (4.2)).

2. Determine the optimal height/width ratio for the cross section such that the global stiffness is maximised.

To assert a consistent qualitative behavior of the topology results, it is desirable to keep the mass fraction value \( \delta \) fixed. (C.f. Figure 3.10, where an example of inconsistent behavior arising from varying mass fractions is shown.) Doing so, one runs into the problem of ranking described in Section 3.5.1: Variable geometry leads to topology with different mass \textit{and} different performance. To overcome this issue, the total mass \( m_{\text{target}} \) is kept constant at every iteration step: for any given width \( w \) of the beam, the height \( h \) is chosen such that

\[ m_{\text{target}} = \delta_0 ( \text{mass}_{\text{top/bottom}} + h \cdot \text{unitmass}_{\text{web}} ) = \text{constant}. \]  

(4.1)
Figure 4.2 shows how this is achieved: A temporary finite element mesh is built for a given width $w$, from which the mass of the top and bottom surfaces ($\text{mass}_{\text{top/bottom}}$), as well as the unit mass per millimeter height for the curved web ($\text{unitmass}_{\text{web}}$) are determined. Now, using (4.1), the corresponding height can be calculated as shown in Figure 4.3.

Next, the geometry is updated, from which a FE mesh is generated. Boundary conditions are applied and an analysis deck compiled. A subsequent topology optimisation with a constant mass fraction of $\delta = 0.3$ determines the material distribution. The objective $f$ of the topology optimisation is to minimise the weighted compliance for the three load cases

$$f = \sum_{i=1}^{3} w_i C_i \rightarrow \min.$$ (4.2)

Since the loads are of a similar order of magnitude, the weights $w_i$ are made equal to 1. The specific loop that is necessary to determine the height for every given width is realised as a plug-in function. An extract of the IST script that controls the analysis loop from geometry generation to extraction of analysis results is shown in Figure 4.4.
Figure 4.3: Cross section of I-beam (a). For the design variable $x_1 = \text{width}$, the height is chosen such that the overall mass of the structure is constant (b).

A full factorial DOE is then conducted, where the width $w$ varies between 100 mm and 200 mm in steps of 5 millimeters. This is done using tools provided by the implementation of IST, compare also to Figure 4.4. This shows the full range of results within the design space as displayed in Figure 4.5(a). Here the resulting weighted compliance for the structure after topology optimisation is plotted against each of the initial beam geometries. From the curve in Figure 4.5(a) one can already determine the optimal width to be approximately $w = 125$ mm, corresponding to a height $h = 180$ mm and an objective function value of $f = 2.59 \cdot 10^{-6}\text{mm}$. Figure 4.6 shows four resulting structures for varying initial geometries with subsequent load path optimisation. For the given loads, a height to width ratio of 1.44 as shown in Figure 4.6(b) yields the stiffest design.

A comparison can now be made to a traditional approach where shape optimisation is performed separately and independently of the topology optimisation. That is, the Engineer would have to first choose a width/height (shape optimisation) without considering the effect of topology load path optimisation, and it is a priori unclear which height to width ratio is best. To show that the IST approach delivers a better outcome, I conducted a DOE to show compliance results for different ratios of the beam without topology optimisation (Figure 4.5(b)). (The compliance values shown in this graph are lower because no topology optimisation has been performed yet). Here, a width of $w = 154$ mm appears to be optimal. When a topology optimisation is then performed for this geometry, a corresponding objective function value of $f = 3.03 \cdot 10^{-6}\text{mm}$ is found. This compares to $f = 2.59 \cdot 10^{-6}\text{mm}$ for the
# IST script file example for I-beam
# The commands are executed in order each time new design variable values are assigned.

MODEL
NAME ibeam

# Copy data to local directory and update parametric geometry
COPY SFE_DIRECTORY TO CALLING_DIRECTORY/sfe

READ DESVARS # Read the designated geometry design variables

DEFAULT PENALTY VALUE displacement=2.0 # Default values in case of failed analysis

# Run SFE CONCEPT
RUN SFE_CONCEPT -batch sfe01.DV.mac

# Create dynamic connections and boundary conditions

DEFINE SET 1 BY BOX (-2,-500,-500), (2,500,500)
DEFINE SET 2 BY BOX (998,-500,-500), (1002,500,500)

CREATE SPC ID=100 DOF=123456 NODES OF SET 1 # Create RBE2 at lower end of the pedal for the Load F

CREATE RBE2 ID=200 MASTERNODE=200 NODES OF SET 2 # Create connections between pivot axis and pedal

WRITE NASTRAN CALLING_DIRECTORY/ana/include/mesh.fem # Write analysis deck to disk

# Do OptiStruct check run to determine current mass, then run plug-in to find height/width ratio

RUN OPTISTRUCT CALLING_DIRECTORY/ana/ibeam.fem -check

EXTRACT RESULT mass FILE=CALLING_DIRECTORY/ana/ibeam.out

RUN PLUGIN ibeam.pl ibeam height width # to find the height to width ratio

# Run SFE again, this time with correct height design variable value

RUN SFE_CONCEPT -batch sfe02.DV.mac

# Create dynamic connections and boundary conditions

# Run SFE again, this time with correct height design variable value

RUN SFE_CONCEPT -batch sfe03.DV.mac

# Create dynamic connections and boundary conditions

# Run topology optimisation algorithm and extract results

RUN OPTISTRUCT CALLING_DIRECTORY/ana/ibeam.fem # Start the topology optimisation

EXTRACT RESULT mass FILE=CALLING_DIRECTORY/ana/ibeam.out

EXTRACT RESULT disp FILE=CALLING_DIRECTORY/ana/ibeam.dips GRID=200 # Masses, compliance, and displacement at loading point

# Analysis loop completed, pass control back to main loop
EXIT

Figure 4.4: Excerpt from IST code script for I-beam example. Not all of the steps are shown (Comments in red).

Figure 4.5: Optimal width for I-beam using IST (left) and apparent optimum width without IST (right): The right curve shows the compliance of the structure before topology optimisation has been performed, the left curve after topology optimisation. The right curve suggest a different optimum design region, showing the benefit of the integrated IST approach.
optimal width of $w = 125$ mm, found by IST.

The shaded regions in the graphs highlight objective function values within 5% of the minimum, and it shows that the true (a) and apparent (b) optimum design regions differ significantly.

It can be concluded that applying IST not only finds a structure with lighter mass, but also a more realistic design region for feasible height/width ratios than the traditional approach would suggest.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.6.png}
\caption{Topology optimisation results for different height/width ratios. The stiffest design yields structure 4.6(b)}
\end{figure}

4.3 Integrated shape- and topology optimisation for an extruded solid two-piece assembly

In this example I show the integrated optimisation of both geometry and topology of an extruded assembly. Figure 4.7(a) depicts the package space for a clamp that is used to attach a steel canopy to the tray of a pick-up truck. It consists of two main parts that are held together by a bolt ($\varnothing 8$ mm), which also generates the necessary clamping force.

To keep cost low, the clamp is to be manufactured from extruded aluminium profiles, which are simply cut to the desired width. The only necessary additional process steps are to drill the bolt holes and to deburr and round off sharp edges.

4.3.1 Optimisation objective

Given the load $F$ representing the clamping force (Figure 4.7(b)), the objective here is to find

(a) the optimal cross-sections for each of the two parts and
Figure 4.7: Schematic of the two piece clamp design (a). In the analysis model, the bolt is represented by a rotational spring in point P (b).

(b) the optimal width (= cut length) of the extrusion,

such that the mass of the final design is minimised.

To that end, three geometric design parameters are introduced as shown in Figure 4.8 and Table 4.1. The first parameter $x_1$ describes the width of the parts as cut from the extruded profiles. Parameter $x_2$ varies the $y$–position of the split line between the two parts, while parameter $x_3$ modifies the location of the gap in the $z$-direction.

Figure 4.8: Three geometry design variables, defined in the parametric geometry model.

The setup now works as follows: As an initial manual step, an SFE CONCEPT model representing the geometry is created. The geometry design variables are defined within this parametric model. New design variable values can now be passed to SFE in batch mode, where the geometry is updated accordingly and a finite element mesh is created. The design variables $\mathbf{x} = (x_1, x_2, x_3)$ describe the variation in millimeters with respect to some (arbitrary) initial configuration, referred to as $\mathbf{x} = (0, 0, 0)$, as shown in Table 4.1.
Applying the IST method, the automated loop starts with a file containing values for the three design parameters. The values $x_1$, $x_2$, $x_3$ are substituted into a SFE batch file. Then SFE is run in batch mode, reading the design variables and updating the geometry. A SFE macro then creates and exports a 3D hexa-dominant finite element mesh. At a mesh size of 2 mm, the analysis model consists of around 26,000 elements.

Next, the IST tools create the necessary connections and boundary conditions. This is shown schematically in Figure 4.7(b), where the lower side of the two clamping faces is constrained with single point constraints (SPC) in the vertical direction. For the facing side, a force $F = 1000$N is applied, representing the clamping force when the bolt is tightened. The bolt itself is modelled as a pivot point $P$ using a Rigid Body Element (RBE2) allowing rotation around the $x$-axis only, the bending stiffness of the bolt is represented by a rotational spring. Finally, rigid body elements are created between the two sliding surfaces (RBE2’s that allow the relative movement of the two parts in the $z$-direction while restricting the movement in the $x$- and $y$-direction). This completes the build of the analysis model.

As a next step, a topology optimisation is performed by Altair OptiStruct. The objective is to minimise the mass. The displacement of the load point $F$ is limited to a maximum of 0.2 mm. At the same time, an extrusion constraint is in place to ensure that material is taken away homogeneously in the $x$-direction.

<table>
<thead>
<tr>
<th>Description</th>
<th>DV</th>
<th>Min</th>
<th>Max</th>
<th>Values for DOE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width of clamp</td>
<td>$x_1$</td>
<td>-6</td>
<td>18</td>
<td>-6, 10, 18</td>
</tr>
<tr>
<td>$y$-location of split line</td>
<td>$x_2$</td>
<td>-8</td>
<td>4</td>
<td>-8, 4, 4</td>
</tr>
<tr>
<td>$z$-location of split line</td>
<td>$x_3$</td>
<td>-10</td>
<td>30</td>
<td>-10, 2, 14, 20</td>
</tr>
</tbody>
</table>

Table 4.1: Geometry design variables for clamp optimisation (in millimeters). A value of 0 refers to the initial design.

### 4.3.2 Full factorial DOE

The computation time for one topology optimisation step including model update, mesh generation, and compilation of the analysis model is about 10 minutes on an Intel i7 1.6 GHz processor (Table 4.2). This is relatively inexpensive and allows a comprehensive scan of the geometry parameter design space. A full factorial DOE for the three design parameters with 96 different shapes is conducted (Table 4.1 shows the specific values).
<table>
<thead>
<tr>
<th>Action</th>
<th>Duration (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SFE CONCEPT model update</td>
<td>0.5</td>
</tr>
<tr>
<td>Finite element mesh generation</td>
<td>0.5</td>
</tr>
<tr>
<td>Generate boundary conditions and constraints</td>
<td>0.5</td>
</tr>
<tr>
<td>OptiStruct topology optimisation</td>
<td>9.0</td>
</tr>
</tbody>
</table>

**Table 4.2:** Approximate computing time for one geometry update/topology optimisation step

The topology optimisation performed on the different geometries leads to a distinct mass figure for each design. Figures 4.9 (a)-(c) show cuts through the three–dimensional response space for fixed coordinates $x_1, x_2, x_3$, respectively. The response surfaces show a convex characteristic, indicating that the problem could be well suited to a gradient based algorithm. I tried this, and the optimisation converged after 13 objective function evaluations, close to the optimum found by the full factorial DOE. On the other hand, one cannot rely on a gradient based method, because it is a priori not known whether the optimisation problem would lend itself to this method. This is why in the examples shown later generally genetic algorithms are preferred.

Looking at the DOE result, $x_1$ and $x_2$ have a big impact on the objective function, while the mass is relatively constant for changes in $x_3$. Lowest mass solutions are achieved for small $x_1$, and $x_2$ around zero. A number of selected designs are shown in Figure 4.10.

Compared to the initial arrangement ($x = (0, 0, 0)$), the mass is reduced by about 14% to 218 grams. Another advantage of this approach shown in this example is how IST is able to capture the extrusion constraint for the topology optimisation while at the same varying the length of the extruded part. In effect, the approach allows one to introduce additional manufacturing constraints that are not possible to consider with traditional topology optimisation. More general benefits are mentioned in the discussion in Chapter 6.
Figure 4.9: Resulting masses (after topology optimisation). In each graph, one of the three parameters is constant. (Values for $x_1, x_2, x_3$ in mm).

(a) $x_1 = \text{constant} = -4.0$ \hspace{1cm} (b) $x_2 = \text{constant} = 2.0$ \hspace{1cm} (c) $x_3 = \text{constant} = -10.0$

$m = 273 \text{ g}$ \hspace{1cm} $m = 258 \text{ g}$ \hspace{1cm} $m = 226 \text{ g}$

Figure 4.10: Topology optimisation results for some DOE designs. While the characteristic topology is similar, the difference in mass is significant.

4.4 Topology optimisation of a solid bracket with simultaneous optimisation of attachment point location

In this example, I want to determine the optimal load path design for a solid attachment bracket design. This is typically done using a topology optimisation approach, which I apply here also. But in addition to the conventional topology optimisation approach, the location of the attachment bolts is to be optimised as well, which is the shape component of the IST method. This application shows how the IST methods extends the design space significantly, resulting in a great weight reduction compared to a conventional approach with fixed boundary conditions for a solid only system.
4.4.1 Problem setup

Figure 4.11 shows the design space for an aluminium bracket (in light blue), attached to an engine block (dark blue) with four bolts (red). An air conditioner pump (yellow) is bolted to the bracket with three bolts (green).

It is standard practice to employ topology optimisation to determine a concept load path design for the bracket. The design target is to minimise the mass, while applying the critical constraint that the first Eigenfrequency of the system is not lower than 950 Hertz, so as to avoid possible resonant frequencies at low engine revolutions. A draw constraint is applied, to make sure that the part can be manufactured in a die cast process using a two piece die.

The exact positions of the bolts can make a significant difference to the static and dynamic performance of the design. In this study, the engineer has the design freedom to modify the attachment location of the bracket, as well as the bolt location for the AC pump. The location of the connection bolts is allowed to vary as shown in the schematic setup shown in Figure 4.12: Bolts 1, 2, 5 and 6 can move in the $x$- and in $z$-direction. Due to package constraints, the lower bolts 3, 4 and 7 can only move in the $x$-direction.

Applying the IST method, an analysis loop is set up that allows one to assess the performance of different design configurations. First, a finite element mesh of the individual components is created manually as a one-off step. In an automated loop, the position of the seven bolts are then varied, connections between the bolts and the structure are created, boundary conditions are applied, and a topology optimisation on the bracket is performed. Finally, the resulting mass is fed back to the optimiser, and a set of new design variable values is chosen.

It is well understood that any density distribution returned by the topology optimisation algorithm requires manual rework in order to reduce local
Engine block with bolted on bracket (blue); the air conditioner (AC) compressor bolts onto the bracket. The bolt positions (red and green) can vary as shown in Table 4.3. The bracket is subject to topology optimisation.

**Figure 4.12:** Engine block with bolted on bracket (blue); the air conditioner (AC) compressor bolts onto the bracket. The bolt positions (red and green) can vary as shown in Table 4.3. The bracket is subject to topology optimisation.

stress concentrations and to create a final design for a manufacturable part. Nevertheless, one can show that the mass obtained for different bolt locations is indicative of designs with high potential for a lowest mass solution. To demonstrate this, a full factorial DOE is conducted for one of the bolts (bolt 2 in Figure 4.12), varying within 20 mm in the $x$-direction, and 40 mm in the $z$-direction. The graph in Figure 4.13 shows the resulting bracket mass, which varies significantly for different positions of this one bolt; for the variation in vertical direction the masses after topology optimisation lie between approximately 350 and 550 gram. Figure 4.14 shows selected topology optimisation results obtained when moving bolt 2 (shown in red) in the $z$-direction. The differences in mass are very significant, and prove the benefit of the approach.

In the next section I show that this result can be improved by allowing all 7 bolts to move.

### 4.4.2 Genetic optimisation with eleven design parameters

In a more comprehensive optimisation study, 11 geometric design parameters are introduced. They describe the location of the seven bolts as shown in Figure 4.12.

The response surface shown in Figure 4.13 (only one bolt moves) suggests the existence of a unique global minimum for that simplified case. However,
for the optimisation task with 11 design parameters, the response surface cannot be expected to be convex. Due to alternating characteristic load path solutions, the objective may even be discontinuous, so a gradient based algorithm is not suited here. This is why a genetic algorithm is employed. Also, it should be noted that the objective function values generated by the topology optimisation algorithm are fuzzy to some degree in the sense that the density distribution returned by the SIMP algorithm does not represent a final black-and-white design. Since this density result requires interpretation before being translated into a manufacturable design, this means that a small change in the resulting mass does not necessarily translate into a different mass for the final design. Thus, the mass figures can only be indicative. This fuzziness is the reason why the design parameters are allowed only discrete values, in steps of 5 mm. The bolts can vary by 15 to 35 mm in the $x$-
and z-direction, within the limits shown in Table 4.3. Another advantage of discrete design parameter values is an increase in convergence speed, since previously calculated results can more readily be reused than for continuous design variables. The parameter limits are due to geometrical restrictions, and the values referred to as “0” correspond to some (arbitrary) initial design (see also Figure 4.17 for a schematic view of the parameter space).

As in the DOE analysis discussed in Section 4.4.1, the objective of the optimisation is to minimise the system mass. Constraints are the first Eigen-frequency, which is required to be above 950 Hz, and a draw constraint to ensure manufacturability. Two static load cases represent forces of the driving belt (\(F_y\) and \(F_z\) in Figure 4.12), with displacement constraints in place for both loads.

<table>
<thead>
<tr>
<th>Description</th>
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<th>Min</th>
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<tr>
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<td>(x_1)</td>
<td>-5</td>
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</tr>
<tr>
<td></td>
<td>(x_2)</td>
<td>-20</td>
<td>0</td>
</tr>
<tr>
<td>Bolt 2 x-direction</td>
<td>(x_3)</td>
<td>-15</td>
<td>5</td>
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<td></td>
<td>(x_4)</td>
<td>-10</td>
<td>30</td>
</tr>
<tr>
<td>Bolt 3 x-direction</td>
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</tr>
<tr>
<td>Bolt 4 x-direction</td>
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<td>5</td>
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<tr>
<td>Bolt 5 x-direction</td>
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<td>15</td>
</tr>
<tr>
<td></td>
<td>(x_8)</td>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>Bolt 6 x-direction</td>
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<td>-10</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>(x_{10})</td>
<td>-15</td>
<td>5</td>
</tr>
<tr>
<td>Bolt 7 x-direction</td>
<td>(x_{11})</td>
<td>-30</td>
<td>30</td>
</tr>
</tbody>
</table>

Table 4.3: Design Variables for optimisation using a genetic algorithm. The parameters are allowed discrete values in steps of 5 mm.

### 4.4.3 Results

The actual genetic algorithm is performed by Dakota. With a population size of 40 individuals per generation and a survival rate of 10% percent, the Dakota optimisation evaluated 290 individual designs within 200 hours on an Intel i7 1.6 GHz processor before converging due to small (less than 2%) changes in the minimum mass of the lightest 20 individuals.
After 100 iterations, the objective function declines — on average — steadily (Figure 4.15). Optimal location for the bolts can be seen in Figure 4.16 (design variable history for some of the parameters), and in Figure 4.17, where the bolt locations for the five lightest designs are shown.

The design with lowest mass show very similar load path configurations, all weighing around 380 grams (Figure 4.18). It can be concluded that the integrated geometry variation has significantly narrowed the design space for feasible low mass solutions. Even though the mass figures change once an actual manufacturing design has been created, the potential to make a meaningful reduction in mass by simultaneously varying geometry parameters in an automated fashion is promising.
Figure 4.16: Design variable history for 6 out of 11 design variables.
**Figure 4.17:** Schematic of the design space and optimal bolt positions. The outer blue rectangle corresponds to the topology design space of the bracket. The grey areas represent the geometrical parameter domains for each bolt. The bolt locations for the five lightest designs are displayed, showing distinct optimal positions for the 11 design parameters.
(a) Design 286, 
m = 377 g  
(b) Design 217, 
m = 378 g  
(c) Design 288, 
m = 378 g

**Figure 4.18:** Topology optimisation results for the three lightest designs. (Top row: view from the AC-pump side, bottom row: view from the engine side)
4.5 Conclusion

In this chapter, the focus lay on the integrated shape and topology optimisation of structures consisting of only thin walled shell based or volume based structures. In the following I summarise the findings and draw conclusions for each of the examples.

4.5.1 Simple beam structure — shape changes combined with load path layout study

Section 4.2 dealt with the parametric modification of the initial geometry prior to OptiStruct’s topology optimisation. For this simple I-beam structure with a sinusoidal shaped reinforcement web, a traditional approach would be to choose a height and width, that for the given load cases has maximal stiffness, and then to perform a SIMP algorithm to find a good load path layout. Alternatively, by integrating an outer geometry modification loop the algorithm finds a height/weight ratio with lower mass than a conventional choice of geometry layout would reveal. This example therefore shows that the integration of topology optimisation into the larger context of variable geometry design can provide a better solution than a traditional approach.

In this example a fixed mass fraction is used for the topology optimisation algorithm. For any given SIMP algorithm run, the target is to minimise the compliance of the structure. This approach comes at a cost, because it leads to the ranking problem described in Chapter 3: when comparing two designs, one is faced with the problem that both the initial geometry (and their mass), as well as the resulting performance (stiffness) are different. To solve this ranking problem, I introduced a restriction for the design variables, so that the height and width are not modified independently, but only the height-to-width ratio. Thus, the resulting mass is always constant, and the outer geometry loop could target the structure with the best performance.

4.5.2 Two-piece extruded assembly

In Chapter 3 I briefly mentioned the optimisation of a brake pedal made from an extruded section (Figure 3.7 on page 63). Here the modification of the initial volume establishes an optimal initial design to go into the SIMP algorithm. (In Fiedler et al. [40] it is shown how IST manages to find an optimal width and cut angle of the go-in volume for the SIMP algorithm.)

In the more complex example shown in Section 4.3, I examine an assembly consisting of two parts. Shape modification simultaneously adjusts the initial volume of both parts, then tools provided by IST execute the
task of assembling the parts into an analysis deck, before being passed to OptiStruct for topology optimisation. Here the mass is to be minimised, while the performance (displacement at the loading points) is limited to a maximum deflection. For the SIMP algorithm, I exploit OptiStruct’s extrusion constraint to ensure that the final part can be cut from an extruded aluminium profile.

The advantage of the parametric volume modification is due to varying two parts of a compound assembly simultaneously. The benefit is twofold:

1. An optimal cut-line between both parts can be found with respect to the lowest mass solution while performance is kept constant.

2. At the same time, the optimal cut length for the extrusion is determined. This effectively allows an additional manufacturing constraint to be defined, in this case the prescriptive extrusion, rather than being limited to a fixed length.

If we look back at the literature review in Chapter 2, we can now positively answer some of the questions posed. The presented case study shows how the parametric variation a multi-piece design space boundary combined with flexible boundary conditions connecting these parts allows to overcome the restriction of standard topology methods to single parts (pages 44 and 45).

### 4.5.3 Optimisation of solid cast bracket in conjunction with parametric boundary conditions

In the example in Section 4.4, the topology volume does not change at all. Rather, I showed how IST tools can be employed to vary boundary conditions by moving the FE entities that represent the attachment bolts, and regenerating connection elements in every iteration. In the application shown, 11 geometry design variables are introduced to vary the location of attachment bolts. A subsequent topology optimisation generated a load path layout for the bracket. A genetic optimisation then produced suitable locations for the attachment point while minimising system mass. The following points are worthwhile highlighting:

1. After an initial geometry model is created — the geometry variation is done by IST (no other geometry tool required). For each set of geometry parameters, the bolts are shifted, and then attached to the structure.

2. The topology obtained from the lowest mass solutions is consistent. This was not the case when the loading conditions and constraints
were not as well defined as is the example shown. In earlier tests, I performed the topology with only the frequency constraint (The first Eigenfrequency was to be above 950 Hz.) After introducing additional static loads to represent the forces of the driving belt, the topology optimisation results became more consistent, confirming the importance of the proper choice of loads. (The latter is neither specific to IST, nor a new finding. For topology optimisation problems, the better the definition of the loading conditions, the more realistic the topology result will be.)

3. The results show that the integrated geometric variation is able to find a good combination of bolt locations, which can serve as a starting point for a manufacturable design. This may be very hard to achieve without an automated geometry variation setup.

This links back to the questions posed earlier: Can we enhance topology optimisation by coupling the design space with the surrounding system? In this example we have taken into account specific system information in the form of parametric boundary conditions. For now one can note that by the parametric variation of the boundary conditions the design space is largely increased and enables a superior engineering solution compared to a standard approach (Refer to page 50). In the following chapter we will take the coupling to the surrounding system one step further.

4.5.4 Benefits of IST approach

In the examples provided, we have shown how integrated large scale shape variation helps in finding better solutions than a traditional topology approach is able to deliver. For the shell based beam example in Section 4.2 IST finds a optimum design region that differs from the apparent optimum found when looking at beam dimension first independently from topology optimisation. Section 4.3 introduced the application of IST on a multi-piece assembly. It was seen that IST allows to find a optimum cut line between the two components, as well as a optimum part width. Another interesting application of IST is shown in Section 4.4, where the attachment location for a cast bracket are varied on a large scale prior to topology optimisation. Here it can be seen how IST increases the design space by a geometrical dimension, and thus allows to find an optimum location for the seven bolts.

It is worthwhile to restate that the optimisation result achieved by IST does not represent a final design and that the minimum mass figures will change, once a production design has been developed. Of course, this is the case whenever a topology optimisation algorithm is utilised, and the IST
optimisation provides the engineer with a good starting point for a final design.

In the following chapter, I will look into more complex structures, where thin walled parametric beam geometry is combined with shape and topology optimisation of cast joints.
Chapter 5

Application Examples: Combined Shell and Solid Structures

5.1 Introduction

This chapter looks into application examples where parametric shell geometry is combined with variable solid joints. Here, both thin walled shell geometry and package space for topology optimisation is varied simultaneously, which cannot be currently performed with traditional shape or topology methods independently. In the case studies, both the topology design space as well as the shape of the surrounding beam structures will be subject to large scale parametric variation. The examples show the advantages of this integration, and address the main research question posed in this thesis, namely the potential of IST in terms of lighter structural solutions compared to standalone shape and topology approaches. Moreover, these examples show that the topology design space is coupled to its external environment. Thus, when traditional shape and topology methods are applied independently, they are most likely to deliver sub-optimal results.

The idea to combine large scale geometry variation with topology optimisation arose when I was involved in the early design and development stages of complex automobile body structures. On the one hand, the engineer’s aim is to find the best load path layout for their structures, which is commonly done using topology optimisation methods on solid design volumes. On the other hand, the representation of the structure needs to be detailed enough to predict complex behavior, which calls for the use of sophisticated shell based finite element models. This impasse — detailed models require accu-
rate geometry that at an early stage may not yet be available — is typically resolved by using topology optimisation to first develop a coarse load path layout. This leads to a picture similar to the topology result discussed in Figure 2.8 of the literature review. The outcome may then be interpreted and translated to a much more accurate model representing actual sheet metal, which can be used for a detailed shape and sizing optimisation.

In any event, the outcome of the topology algorithm, applied to a complete body structure, is fairly ambiguous, and the ability to combine parametric shell geometry with topology methods becomes desirable. The integration can be especially useful in cases where a structure combines thin walled sheet metal with cast solids. For the former, parametric geometry tools are most suitable, whereas for the latter a topology optimisation algorithm can be applied successfully. After analysing a number of complex automotive body structures, it becomes apparent that the thin walled sheet components are coupled or dependent on the solid components and vice versa.

I will show how the combination of these two approaches is applied for a body structure example and for a vehicle subframe.

5.2 Parametric shape variation of an automobile body front structure combined with topology optimisation of cast aluminium connection joints

With demands to reduce vehicle weight, new body structure design architectures are being investigated. A common theme is the application of lightweight alloy extrusions combined with cast joints to form the vehicle's space frame. Unlike the common steel monocoque, the space-frame architecture separates the structural requirements of the vehicle body from its styling form. This segregation in functional requirements can lead to the implementation of simpler structural geometry and the introduction of low formability materials such as Aluminium, Magnesium and Ultra High Strength Steels (UHSS). Low volume vehicle manufacturers such as Aston Martin and Lotus have employed these techniques and achieved significant weight savings, and recent trends appear to show more mainstream vehicle manufacturers adopting these strategies.

A recent research project funded by the Australian Automotive Technology Cooperative Research Centre (AutoCRC [133]) adopted this design philosophy to develop a high-level conceptual design for a Lightweight Mod-
ular Vehicle Platform (LMVP). The body structure is based on a modular design employing simple constant cross section structural members. A mix of materials and joining technologies are explored. The architecture is representative of a space-frame, as shown in Figure 5.1.

![Automobile body structure](image)

**Figure 5.1:** Automobile body structure, developed for a lightweight modular vehicle platform project. The arrows stand for three static load cases applied in the topology optimisation of the cast joints (grey). Not all boundary conditions shown.

### 5.2.1 Shape modification combined with topology optimisation

One of the research objectives was to investigate design strategies of cast joints that serve as complexly shaped components to connect extruded beam members. Here I show how IST accomplishes this task by simultaneously varying beam geometry and applying topology optimisation on a parametric initial design volume.

In the current design task, the objective is to replace traditional spot welded sheet metal parts by cast joint components. As a schematic example I focus on the study of the joints that connect the front rails with the first lower cross member as shown in Figure 5.2. The layout design for the castings employs topology optimisation methods. In addition, the geometry of the beams that connect into these joints is parametrically modified. Now, for any given geometry, a topology optimisation on the cast connection joint is performed. Here the shape of the design volume that goes into the topology optimisation algorithm changes with the geometry, and the aim is to find a combination of the optimal shape of the beams and the optimal joint
topologies such that the overall system has the highest stiffness, given a fixed amount of material.

To this end, a parametric SFE CONCEPT geometry model of the body structure is created. The geometry contains all relevant structural members; connections between parts include spot welds, laser welds, and adhesives (Figure 5.1). Two geometry design parameters are defined within the parametric concept model. The first parameter \( x_1 \) varies the width of the front rails on the inboard side, the second parameter \( x_2 \) varies the width of the first cross member on the rear facing side (Figure 5.2).

![Figure 5.2: Front rails and lower cross member with geometry design parameters \( x_1, x_2 \). (The rail width \( x_1 \) varies symmetrically on both rails.)](image)

### 5.2.2 Automated analysis loop

As in the examples in the previous chapter, the closed automated analysis loop is realised in a number of steps. An IST batch script controls the process:

1. Design parameter values are written to a SFE command file. After reading this file, SFE CONCEPT updates the geometry, and generates and exports a finite element mesh. One complication in this step is that the SFE CONCEPT generated parametric volume mesh cannot be exported in batch mode. (The export of the shell mesh is doable; the software vendor has announced that the solid mesh export feature will be added to the software’s batch capabilities in a coming release.) For now, this issue requires a workaround: The export of the solid mesh is to be done using a windows library interface. In this case, the Linux tool xdotools is applied. This temporary solution has been implemented, and works well for the examples shown here. A potential problem is
the portability, and the application on distributed computer systems without a windows user interface, which can only be solved with the release of a SFE software update.

2. The solid & shell based FE mesh is preprocessed before being passed to the solver. The most important step here is to create the connection elements between the solid mesh and the shell mesh. By specifying a single command in the IST process setup script, the parts which are to be connected, as well as the type of connections, are defined. In the example at hand, Rigid Body Elements (RBE2’s) with a single dependent node between the shell elements and the solids are used.

3. Loads and boundary conditions have to be applied and adapted to the changed geometry. This is done by IST methods.

4. The ranking problem of assessing mass versus performance for different initial geometries needs to be addressed. In Section 3.5 I have proposed a number of ways to tackle this comparability issue. For the problem at hand, I chose the following solution: I fixed the resulting mass $m_{res}$, and for every specific geometry model, $\bar{\delta}$ is adjusted such that

\[
m_{res} = m_\Lambda + \bar{\delta} m_\Omega = \text{constant}.
\]

(Here $m_\Lambda$ is the mass of the non-designable shell based beam structure, while $m_\Omega$ is the mass of the solid design volume prior to topology optimisation). This is realised by measuring $m_\Lambda + m_\Omega$ before the topology optimisation is performed, and then choosing $\bar{\delta} = \frac{m_{res} - m_\Lambda}{m_\Omega}$. The value for $\bar{\delta}$ is then substituted into the OptiStruct input deck.

Of course, this approach requires a predetermined mass $m_{res}$, which is defined at the start by setting the geometry design parameters to their medium values with a subsequent topology optimisation run. Essentially, the aim is to find the stiffest solution for a given amount of material.

Also, it must be noted that one needs to keep track of the volume fraction $\bar{\delta}$ to make sure that it stays within an acceptable range.

5. A SIMP topology optimisation using OptiStruct is performed. For the latter, the optimisation objective is to minimise the weighted compliance for all static load cases. In order to ensure manufacturability of the casting, a draw constraint in the $y$-direction for a two-piece die is applied.
6. Finally, the analysis results are extracted from the solver output files. With this approach, I am ready for the automated geometry loop with integrated topology optimisation.

5.2.3 Parameter study for two geometry design parameters

Figure 5.1 shows the three load cases used for this study. Besides bending (1) and torsional loads (2), longitudinal forces into the front rails are applied, simulating an equivalent static load for a full frontal impact (3).

Since commercial topology optimisation codes cannot yet cater for non-linear behavior, this approach is standard practice. From physical tests done on earlier structures, and from insight gained through simulation, the engineer can estimate the size of forces that the main members have to withstand for various impact scenarios. These forces can then be used as an equivalent static load for the body structure design, particularly for topology optimisation problems [64].

The torsional load case is unsymmetrical, so no symmetry conditions are applied. For any one OptiStruct run, both solid joints undergo topology optimisation.

With respect to the initial position of \((x_1,x_2) = (0,0)\), the shape parameters vary for the rail width \((x_1)\) between \(-30\) and 30 millimeters, and for the lower cross member \((x_2)\) between \(-20\) and 40 millimeters. In 60 hours, a parameter study is conducted using the same Intel i7 1.6 GHz processor as for the previous examples, scanning the design space at 35 points, which results in the interpolated response surface shown in Figure 5.3.

![Graph showing parameter study results.](image)

**Figure 5.3:** Weighted compliance for the parameter study with two geometry design parameters (left). On the right the same data as contour plot.
It can be seen that there is a trade-off between the geometry of the beam members and the resulting mass of the topology optimised joints. The response surface suggests that a rail width of 95 mm and a cross member width of 85 mm is the best starting point for a refined design (corresponding to $x_1 \approx -5$, $x_2 \approx -15$). The resulting topologies are consistent in their geometrical characteristics, i.e. different beam sizes result in similar load paths. Figure 5.4 shows some topology optimisation results for the left hand side joint (the results for the right hand side joint turn out to be symmetrical).

![Figure 5.4](image.png)

Figure 5.4: Some topology optimisation results for different combinations of rail and cross member widths. The characteristic load paths are very similar.

This study does not claim to be representative of a production design. Furthermore, it may be beneficial to incorporate more load cases and to increase the number of geometry design parameters. For instance, the length of the extruded beams, and the cut angles could be added for both beams. Also, it would be interesting to include simple sizing parameters varying the gauges of the beam members. Nevertheless, it highlights the potential of IST to simultaneously vary complex thin walled structures and cast connection joints.

5.3 Automobile front subframe, integrating simple beams with cast connection joints

In automobile body structure design, a subframe denotes a structural component that is used in most front and rear wheel drive vehicles, bolted to the main rails of the body structure from below. Its main function is to support the engine and to provide structural attachment points for suspension components. Other functions include support for overall body structure stiffness and front crash performance.
Today, subframes are usually made in a traditional way from thin walled stamped and welded sheet metal components. Advantages of stamped steel parts are that manufacturing and joining processes are well understood, the production cycle time is very short, and parts can have reasonably complex shapes (within limits, e.g. the restriction to constant sheet thickness and stamping limitations). A downside is that the tooling cost for the stamping machines are high and require a large production volume. Whenever possible, manufacturers search for part designs that are inexpensive to make, such as beams made from extrusions, or roll formed parts (both with constant cross section). On the other hand, using simple beam geometry limits the characteristic shape of the parts, and can be problematic for the connection design.

Here, cast parts can come into play. Even though the manufacturing cost is high — injection molding requires expensive dies, and the production cycle time is high — they can provide parts with very complex shape and function, offering mass efficient solutions.

Figure 5.5: Subframe components with mounting points and loads. For the topology optimisation, suitable symmetric boundary conditions are applied, reducing the analysis to a half model.
5.3.1 Problem setup

In this section I combine the simple beam parts with the cast joints for a subframe. Figure 5.5 shows the schematic layout of a simplified design for a front wheel drive vehicle with cross car oriented internal combustion engine. Most parts are assumed to be manufactured from extrusions or folded sheet metal. The main longitudinal members and the cross member are connected by two aluminium castings. The concept design of the joints is developed using topology optimisation, and at the same time I want to find the optimal dimensions for the beams.

5.3.2 Loading conditions and geometry design variables

To this end, I introduced 8 geometry design parameters, varying the cross section shape of the main members, the length of the cross member, as well as the dimensions of the initial volume that goes into the topology optimisation algorithm (Refer to Figure 5.6 for details).

The loading and boundary conditions are indicated in Figure 5.5. Forces in the $x$-, $y$- and $z$-direction act at each of the two suspension control arm mounting points, as well as on the engine mount (Please note that in a real world scenario the direction of the forces maybe opposite to what is shown, but since only linear analysis is performed, this is not relevant).

As to the boundary conditions, I introduced springs where the subframe mounts to the body rails, and constrain the other end of the springs by SPC’s (Single Point Constraints). This is to represent the stiffness of the body structure, rather than assuming a fully rigid connection. To simplify the analysis, symmetric boundary conditions are assumed, and only the left hand side of the structure is analysed.

For the topology optimisation, the objective is to minimise the mass fraction of the topology design volume (red component in Figures 5.5 and 5.6), while the movement of each of the load points is constrained by a suitable maximum displacement. As discussed in Section 3.5.2, this setup avoids the ranking problem with respect to performance versus mass for different geometries.

With this setup, IST is applied to perform a closed loop optimisation: Design parameter values are determined by Dakota, passed to SFE via a batch script, where the geometry is updated and a finite element mesh is generated. The mesh undergoes the IST connection generation tool, and boundary conditions are applied. Then, OptiStruct performs a topology optimisation, after which the results are scanned and stored. After applicable
error handling, the loop closes.
CHAPTER 5. APPLICATION EXAMPLES: COMBINED SHELL AND SOLID STRUCTURES

Initial shape of casting \((x_5, x_6, x_7)\)

Longitudinal section in \(y\)-direction \((x_8)\)

Cross member length in \(y\)-direction \((x_3)\)

(a) Design variables, overview

Cross member section \((x_1, x_2, x_4)\)

(b) Section changes of cross member

(c) Shape of Casting (top view)

(d) Shape of casting and length of cross member (top view)

<table>
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<tr>
<th>Description</th>
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<th>Max</th>
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</tr>
<tr>
<td></td>
<td>Section height</td>
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<td>8</td>
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<tr>
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</tr>
<tr>
<td></td>
<td>Trapeze</td>
<td>(x_4)</td>
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<td>0</td>
</tr>
<tr>
<td>Casting</td>
<td>Length front</td>
<td>(x_5)</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>Length rear</td>
<td>(x_6)</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>Width in the (y)-direction</td>
<td>(x_7)</td>
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<td>30</td>
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<tr>
<td>Longitudinal member</td>
<td>Width in the (y)-direction</td>
<td>(x_8)</td>
<td>0</td>
<td>30</td>
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</tbody>
</table>

(e) Parameter range for design variables

Figure 5.6: Schematic view of the design variables [(a)–(d)]. Table (e) lists the boundaries and number of steps used in a genetic algorithm. The parameters are allowed discrete values in steps of approx. 5 to 10 mm, the value ‘0’ refers to the initial value.
5.3.3 Optimisation setup

The 8 design parameters are shown in Figure 5.6. The bandwidth of the variable values is determined by geometric conditions, and varies between 16 to 30 millimeters (Figure 5.6(e)). As an example, the cross section width \( x_1 \) ranges from 70 to 100 millimeters, the cross section height \( x_2 \) from 22 to 38 millimeters, corresponding to a change of around 40% and 70% respectively.

The objective function to be minimised is the combined mass

\[
f(x_1, \ldots, x_8) = m_{\text{res}} = m_{\Lambda} + \bar{\delta} m_{\Omega},
\]

where \( \Lambda \) consist of all sheet metal parts that are not subject to topology optimisation, and \( \Omega \) is the solid cast component. \( \bar{\delta} \) is the mass fraction value determined by the SIMP topology optimisation on \( \Omega \) (as mentioned, the objective within the topology optimisation is to minimise \( \bar{\delta} \), given displacement constraints under all load cases. The topology optimisation is performed by OptiStruct).

With this setup, I use Dakota to provide a genetic optimisation algorithm. As for the optimisation of the bracket shown in Section 4.4, I do not rely on a gradient based algorithm to be effective, since the response surface cannot be expected to be locally convex. Indeed, it is likely to not even be continuous. For two designs generated by very similar design parameters, the resulting density distributions are generally expected to be similar, but in some cases can have very different characteristics. Also, a small change in mass in the input geometry may not necessarily translate monotonically to a small change in mass in the topology result. For these reasons, I allow only discrete design variable values, with a relatively big increment of around 4 millimeters. This gives a design space with 180,000 possible combinations of design variables.

For the Dakota genetic optimisation, the population size in any one generation consisted of 50 individuals. Standard parameters for the crossover rate (80%), mutation probability (10%), and survival rate (10%) are chosen. With this, the Dakota optimisation evaluated 280 individual designs before converging due to small (less than 2%) changes in the minimum mass of the lightest 20 individuals.
(a) $x_1$, cross member section width
(b) $x_2$, cross member section height
(c) $x_3$, cross member length in the $y$-direction
(d) $x_4$, cross member trapeze
(e) $x_5$, casting, length front
(f) $x_6$, casting, length rear
(g) $x_7$, casting, width in the $y$-direction
(h) $x_8$, longitudinal member, width in the $y$-direction

Figure 5.7: Design variable history for the 8 design variables.
5.3.4 Results

Figure 5.7 on the preceding page shows the history for the 8 design variables. Without looking at individual results, one can already see preferred values for the variables: The length of the cross member ($x_3$) tends to be at the upper limit of the permitted range, its width ($x_1$) varies around the middle point, and the height ($x_2$) tends towards smaller values. As to the trapeze shape ($x_4$) of the cross member, values close to zero indicate that a rectangular cross section is preferred, while small values are favoured for the width of the longitudinal member in the $y$-direction ($x_8$).

Three design parameters alter the initial volume of the topology design space: While the rear elongation of the casting ($x_6$) is at its maximum, the length in the front ($x_5$) seems to be in the middle of its range, and the same holds for the width of the casting in the $y$-direction ($x_7$).

One can see these trends confirmed when one goes through the lightest mass solutions, two of which are shown in the top row of Figure 5.9.

It is interesting to note that out of the 280 designs, the lightest 195 variants show the same basic layout pattern for the casting. For the 35 designs with lowest mass, the longitudinal member ($x_8$) has minimal width, and the cross member ($x_3$) has maximum length, while the cross member height ($x_2$) hovers around the middle of its allowed range. Similarly, the variables defining the initial shape of the casting ($x_5, x_6, x_7$) are within one delta step of their allowed range. The most variation lies within the cross member section width ($x_1$) and the trapeze shape of the cross member.

If one now looks at the objective function history in Figure 5.8, a steady (average) decline in the system mass (blue graph) can be seen after 2 generations, with the lightest designs weighing in at around 4.63 kg. Interestingly, the solid design volume increases in the course of the outer geometry loop (i.e. before topology optimisation), while the SIMP algorithm outcomes show a fairly constant mass after around 100 evaluations. This implies that the effective mass fraction decreases, and the initial design volume gets used more efficiently.

At the same time, the mass of the shell geometry declines after around 150 evaluations without penalty to the mass of the joint structure.
Figure 5.8: The blue graph shows the objective function value (total system mass after topology optimisation). The mass of non-designable parts ($\Lambda$) is marked orange. In dark green the initial mass of the solid design volume ($\Omega$), and in light green the mass of the design volume after topology optimisation. $\delta$ is the mass fraction value resulting from the OptiStruct SIMP run. The small red dots designate infeasible design, showing that only a few occasional or early designs do not meet the displacement constraints. The curves in red are moving averages, taken over 15 individual results.
Figure 5.9: Subframe topology optimisation results. The top row shows the two lightest designs (a,b). In the second row, two examples are shown for a small cross member height (c) and a narrow cross member. The mass figures refer to the combined mass after topology optimisation for the whole (symmetric) assembly.
5.4 Conclusion

The idea of integrating parametric shell geometry with load path optimisation for cast connecting joints was originally conceived when thinking of novel manufacturing methods for automobile body structures. This was the significant knowledge gap identified at the end of the literature review (2). IST is one approach in the quest of finding ways to simultaneously frame the shape and dimensions of beam geometry together with complex joining members. I presented two examples, where the structural development can benefit from the approach taken. Referring back to the research question, where we asked what the benefits of the integrated shape modification approach, it can now clearly be stated that IST manages to find a trade-off between shape/size of thin walled structural beam members and the size and topology of cast joining members. In the section below I summarise the findings.

5.4.1 Automobile body structure: Study of cast joints connecting parametric thin walled beams

In the body structure example in Section 5.2.3, I use only two geometry design parameters. They vary the width of main longitudinal front members and the width of the first main lower cross member, that connect in two cast joints. Their range is 60 millimeters each, and with a step size of 10 millimeters, this allows a full sweep of the parameter space. The resulting 3D response surface shows the compliance (i.e. the inverse of the stiffness) with a distinct minimum. While the three load cases are basic, they capture the main requirements for an automobile body structure, in an early stage of conceptual design, namely global torsional and bending stiffness. Additionally, I include a longitudinal force into the main chassis rails, representing the equivalent static load for a frontal impact.

Important observations are:

1. The resulting topology is consistent over the geometry design space, i.e. the main characteristics of the load paths are very similar.

2. In this example, the objective for the topology optimisation algorithm is not a minimum mass solution, but a material distribution with maximum stiffness. This means that for this type of ranking problem, the mass fraction value $\delta_x$ needs to be fixed for every topology optimisation run. The way the comparability issue between mass, performance and mass fraction is tackled is to adjust $\delta_x$ such that the combined resulting mass $m_{Ax} + \delta m_{Ax}$ after the topology optimisation is constant for all geometry variations.
3. A suitable value for the (constant) overall mass needed to be defined beforehand.

4. In order to minimise the compliance of the structure, suitable weighting factors for the individual load cases need to be defined.

5. The tracking of the resulting mass fraction $\bar{\delta}$ shows that all values are within an acceptable range of $0.1 \leq \bar{\delta} \leq 0.25$, so no designs are discarded.

6. In order to ensure manufacturability, the topology optimisation is performed with a draw constraint in cross-car direction. I had wanted to also investigate a joint design made from an extruded profile, with cut-outs to connect into the front rail. Unfortunately, OptiStruct does not support this. While an extrusion constraint can be requested, this setting only works when the initial volume has a constant cross section.

7. In effect, the aim is to find the stiffest solution for a given amount of material. Here a trade-off between the use of the stiffer (but heavier) steel and the lighter aluminium is determined.

5.4.2 Automobile engine cradle

In the subframe (also often called cradle) example in Section 5.3 there are 8 geometry design parameters, and I perform an automated genetic optimisation. The objective of the outer geometry optimisation is to minimise the overall system mass, i.e.

$$m_i = m_{\Lambda,i} + \bar{\delta}_i m_{\Omega,i}.$$  

(Compare to the body structure example, where the objective is to minimise of the weighted compliance). Thus, the optimisation target for the SIMP algorithm also needs to be the minimisation of mass. The main findings from this example are

1. Displacement constraints for all load cases need to be defined. Unlike the previous example (body structure with maximisation of stiffness), one can choose the constraints individually for each load-case.

2. The SIMP algorithm establishes the lightest solution for the solid design volume, while still honoring all displacement constraints. This effectively leads to mass fraction value, which needs to be tracked. This proves to be no issue; I had $0.19 \leq \bar{\delta}_i \leq 0.3$ for all feasible designs, which is well within an acceptable range.
3. Only a few designs early in the optimisation loop do not meet the performance constraints (1, 2, 5, 10, 12, 14, 15, 23, 51), as well as the two scattered designs 212 and 218. (They are marked by little red dots in Figure 5.8 on page 107.) This indicates that not only are the boundary conditions and parameter ranges adequate, but also that the optimisation algorithm managed to effectively avoid parameter combinations leading to infeasible designs.

4. Genetic optimisation shows clear optimal choices for all of the design parameters.

5. The resulting topology is very similar across most input geometries. The few exceptions prove to have a large mass, and are therefore discarded by the genetic optimisation process.

6. Analysing Figure 5.8 on page 107, one can see that most of the design improvement stems from the modification of the shell structure. While lighter shell geometry goes hand in hand with increasing initial solid volume, the mass of the resulting design volume remains fairly constant. This indicates that the geometry variation adds significantly to the overall reduction of the system mass, as could be expected, and it is reassuring to see this notion confirmed.
Chapter 6

Discussion

6.1 Introduction

In this chapter I first want to discuss the findings from the experiments described in the two preceding chapters, and explore how the results are able to answer the questions that arose from the literature review and its impact on the academic field. Of particular importance are the new types of issues that arise with the IST approach, and the underlying limitations. Also, I want to investigate how the findings could affect the engineering industry.

6.2 Research questions

At the end of the literature review in Chapter 2, I set out to find answers to a number of questions focusing on the integration of large scale shape modification with topology optimisation. With the results obtained and described in the previous chapters, I can now propose answers to these questions.

6.2.1 Coupling shape design parameters to initial topology design volumes

With respect to the first question,

In order to extend topology optimisation by including system information, can parametric shape design be done in conjunction with topology optimisation?

I have clearly demonstrated that the combination of shape modification with subsequent topology optimisation has benefits over a traditional sequential approach. The way I implement the integration of geometry modification
in an outer geometry loop proves to be beneficial for a number of classes of applications. I will look in detail into the findings and how they relate to the research question in the following sections.

6.2.2 Parametric design space

In the context of topology optimisation, one significant condition identified in the literature review was that in the great majority of cases the package space provided for a topology algorithm is fixed [39, 65, 43, 83]. This led to the question as to whether this idea can be overcome by allowing parametric variation of the design space, and, of course, whether it makes sense to do so.

In Chapter 4 I focussed on this question, and I discussed examples where the design space provided to the SIMP algorithm transformed according to a set of geometry parameters. The simple example of the I-beam (4.2) proves that the integrated approach found a better shape than a sequential approach would have delivered. The parameter study of the clamp (4.3) shows how the integration of shape changes allowed one to simultaneously modify both parts of a two-piece assembly, thus giving an optimal cut-line between the two parts. In this application one can also see how IST extends the extrusion constraints provided by OptiStruct by parametrically varying the cut length of the parts.

The more complex examples in Chapter 5 take the idea of a variable package space still further by integrating the topology design volume with parametric beam structures. Here, the variable package space is an essential prerequisite for the simultaneous optimisation of beam structure and joint structure, the benefits of which are demonstrated distinctively. In both applications, I show how the parametric variation of the beam structures entails the modification of the initial volume to go into SIMP for the joining structures. This allows one to attain a combination of geometry design parameters representing the best shape for a subsequent detailed manufacturing design.

In more general terms, IST increases the design space for the structural optimisation by a geometric dimension. The added shape design parameters allow for a much bigger solutions space than a sequential approach — definition of a fixed topology design space and subsequent topology optimisation — does. The increased design space then allows the possibility to find more mass efficient solutions. Moreover, because the parametric variation of the design space links the topology optimisation to the system being optimised, a global low mass solution for the system can be found (as will be discussed in the next section).
6.2.3 Integration of shape and topology methods

The observation I make during the literature review is that in the context of topology optimisation, shape optimisation almost always refers to the modification of the boundary of a single part. In most cases this means applying some parametric shape description to the density result of the topology optimisation after the latter has been performed, with the aim to transform the somewhat fuzzy and potentially frayed topology optimisation output into a smoother and more manufacturable shape.

Here one can ask whether one can

Overcome the restriction of the shape optimisation domain being limited to the outer boundary of single connected parts.

I have to clearly state that the proposed IST approach does not attempt to improve on shape optimisation. Notwithstanding these methods’ usefulness, I have taken a supplemental approach in that shape modification comes before topology optimisation. The central ideas are that:

• the package space provided to a SIMP algorithm is subject to large scale shape variation;

• the topology design space is integrated into a larger system consisting itself of parametric geometry with parameters that can be optimised with respect to whole system performance.

With this idea, I cannot solve the general problem that is always present around SIMP topology optimisation methods, specifically, the need to interpret the density results delivered by these algorithms.

Rather, the IST method allows one to augment the design space by an additional geometric dimension. With the integration of large scale parametric shape variation in conjunction with topology optimisation, I have shown that one can find combinations of shape and topology of structures that conventional topology approaches fail to find. Thus, IST delivers stiffer or more mass efficient structures.

6.2.4 Applicable structural problems

Which classes of structural problems lend themselves to the application of such an integrated approach?
In Chapter 5 two examples of complex structure designs are presented, where thin walled beams connect in cast joints. These examples demonstrate that there is a trade-off between the sizing and shape of the beams with the size and topology of the castings that join the beams together, with respect to the overall mass or overall stiffness of the system. This type of application is certainly one of the most important use-cases for the proposed integrated method. But then, in the course the research other applications came to mind.

In Chapter 4, I have explored a number of examples that consist of either only shells or only solids, where the best initial geometry for a subsequent topology optimisation is found. Also interesting is the example where a cast solid bracket is optimised for minimum mass by modifying the locations of its attachment bolts. Put more generally, the concept is to parametrically modify boundary conditions, and to use the IST tool set to set up and control an automated design space exploration or optimisation loop. To summarise, I have shown the application of IST on the following types of structural problems:

- Thin walled three dimensional shell structures, combined with lightening holes/load path layout;
- Multi-piece structures, where more than one component is subject to topology optimisation, and where a compromise between the different package spaces needs to be established;
- Topology optimisation problems with parametric variation of boundary conditions, such as loading points or attachment locations;
- Structures made from extruded sections. It may be possible to find an optimal initial design space for the subsequent topology optimisation, leading to a structure with minimum mass;
- Integrated structures where complex beams are combined with cast/solid joining structures. Here a trade-off between beam size and shape on the one hand, and on the other hand the joint topology can be found.

There are likely to be more classes of applications that the IST approach can be applied that have not been analysed in this thesis. For example, the problem of finding the best location for a complexly shaped joint that connects a number of beams, or the combined optimisation of the shape and the topology of an overcast structure, where an extruded or otherwise economically manufactured beam — say a roll formed high strength steel bar
— is inserted into a die and overcast by a lightweight complexly shaped joint that provides additional local stiffness and caters for attachment points.

6.2.5 Mass vs. performance vs. mass fraction

Optimisation can only work for a single, well defined, real valued objective function. A critical aspect of the simultaneous shape and topology modification as in the proposed IST approach, is that different initial geometries with subsequent topology optimisation lead to results that differ in mass, performance, mass fraction values, or any combination thereof. This led to the question

How does one compare mass and performance when they can both vary across the solutions?

In Section 3.5 on page 64 I have addressed this issue, and proposed a number of ways to overcome it. I now want to look back and see how the proposed solutions are applied and how they have worked in practice.

6.2.5.1 Constant mass fraction

I review the last proposal first (Section 3.5.1.3 on page 68): In this approach the aim is to keep the effective mass fraction $\bar{\delta}$ for the topology optimisation constant. This comes at a cost, namely that the geometry design parameters have to be limited in such a way that the initial mass (i.e. the mass before topology optimisation starts) is identical for all designs. Assuming $n$ geometry parameters $x_1, \ldots, x_n$, this can only be achieved by introducing a function $h(\cdot)$ with $(x_{k+1}, \ldots, x_n) = h(x_1, \ldots, x_k)$ that restricts some of the design variables.

In the example shown, I managed to express one of the two design parameters in terms of the other, so that I only looked at the ratio of $x_1$ and $x_2$ (height vs. width of the beam). While this is acceptable and useful in this simple case, there are disadvantages with this method, namely:

1. The geometric design space has to be narrowed;
2. For complex examples, it may be very hard to find a suitable restriction function $h(\cdot)$.

This is why I do not recommend this method. This leaves the only feasible alternative which is to allow the mass fraction value for the topology optimisation to vary (i.e. between different geometry designs) and will be discussed in the following section.
6.2.5.2 Variable mass fraction

As discussed in Section 3.5, the ranking problem requires the mass fraction value to vary from one geometry to the next. Two main approaches are distinguished and discussed:

1. Topology optimisation under performance constraints, optimise for minimum mass;


In the first case, the SIMP algorithm delivers a mass fraction value, in the second it is passed into the algorithm. The way the integration of the outer geometry modification loop manages to keep different designs comparable is easy in the first case, since all geometries — aside from infeasible designs — have the same performance (such as maximum displacements, minimum Eigenfrequency). The resulting mass then distinguishes them perfectly. In the second case, a target mass is defined beforehand, and the effective mass fraction is calculated for every geometry and handed to the topology optimisation algorithm. Please note that the topology optimisation step needs to have the same type of objective as the outer geometry optimisation, i.e. when the geometry optimisation aims at minimising the system mass, then the SIMP part also needs to minimise the mass of the design volume. In turn, when the outer optimisation targets a maximal performance, then the SIMP also has to maximise performance under a fixed mass fraction constraint.

In the applications discussed in the previous chapters, I have used the first approach for the clamp (4.3), the AC-bracket (4.4), and the subframe (5.3), while the body structure example (5.2) used the second approach. As it turned out, both methods work equally well, and the potential issue of mass fraction values to be out of bounds proves to be less of a problem than anticipated for both approaches (c.f. summary in Section 5.4).

One reason to favour the performance constraint over the fixed mass approach is simply that often engineering problems are posed in terms of performance requirements. The problem formulation is to find the lowest mass design that meets the performance requirements. In cases where performance targets are not yet defined, the fixed mass approach may be better suited. The problem formulation then is to find the design with maximal stiffness (or any other suitable performance target), given a fixed amount of material. In this case, a target mass figure has to be defined. One way to do this is to set all geometry design parameters to their respective medium value, and to measure the resulting mass for a suitable volume fraction value. Also, in
CHAPTER 6. DISCUSSION

<table>
<thead>
<tr>
<th>Constraint for SIMP algorithm</th>
<th>Performance constraint</th>
<th>Mass fraction constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimisation target for outer geometry loop</td>
<td>Mass $\rightarrow$ min</td>
<td>Performance $\rightarrow$ max</td>
</tr>
<tr>
<td>Optimisation target for SIMP algorithm</td>
<td>Mass $\rightarrow$ min</td>
<td>Performance $\rightarrow$ max</td>
</tr>
<tr>
<td>SIMP result</td>
<td>$\text{performance}_i = \text{performance}_j$, $m_i \neq m_j$</td>
<td>$\text{performance}_i \neq \text{performance}_j$, $m_i = m_j$</td>
</tr>
<tr>
<td>Comment</td>
<td>Performance targets need to be available for all load cases</td>
<td>Target mass needs to be defined beforehand. Weight factors for individual load cases have to be defined</td>
</tr>
<tr>
<td>Advantage</td>
<td>Performance targets need to be defined only once</td>
<td>Works when no performance targets are available</td>
</tr>
<tr>
<td>Disadvantage</td>
<td>Mass fraction $\bar{\delta}$ has to be recalculated in each step of the geometry loop</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.1: Summary of the two major approaches to overcome the design ranking problem arising from varying initial geometries. Please note that the optimisation targets for both the outer geometry loop and the topology optimisation need to be identical. The performance constraint approach solves the comparability problem quite naturally, whereas the constant mass fraction approach requires this value to be recalculated for every step in the geometry loop.

every iteration step, the resulting mass fraction value has to be recalculated and passed on to the SIMP algorithm. I have taken this approach for the body structure example (5.2), where concrete displacement targets were not available.

6.2.5.3 Summary

As discussed in Section 3.5.1, I do not consider multi-objective problems leading to Pareto Frontiers. This essentially leaves two approaches to solve the ranking problem of differing performance and mass between multiple initial geometries. As already discussed both approaches work well; for all the examples examined, the resulting load path topology returned by OptiStruct is very similar in their general characteristics across varying initial shapes,
provided the loads are defined properly. Also, the effective mass fraction values for both approaches prove to be well within acceptable limits. In either case, a well defined objective function that can be optimised in the outer geometry loop is generated.

For most applications, the performance constraint method — provided performance targets for the load cases are available — is likely to be a better option, since it avoids having to calculate a new mass fraction value for every geometry iteration. Refer also to Table 6.1 for a summary of the comparison between the two approaches.

### 6.2.6 Comparison between IST and standard shape/topology methods

In this section I look at the important question posed at the end of the literature review, namely how the proposed integrated optimisation approach compares to standard topology and shape methods. According to the literature, in most cases combined shape and topology methods do one of two things:

1. Topology optimisation refers to the technique of swapping beam members in and out of a structure, either using discrete design variables, or by allowing them to downsize to zero [87, 122, 91, 44, 71, 26, 75, 30, 34, 98].

2. Apply shape optimisation techniques to a SIMP topology optimisation output, either manually [72, 43, 7, 16, 99, 108], or in some automated way [53, 55, 65, 12, 16, 97].

The IST method has been shown to find lighter and/or stiffer solutions than traditional topology optimisation. However, some problems still remain, such as the necessary interpretation of the topology optimisation density result, i.e. its translation into a manufacturable black-and-white design. In this regard, one advantage offered by IST may be the following:

When surrounding geometry is varied, one can look at the resulting load path layouts returned by the SIMP algorithm. If the essential characteristics change, this indicates a non-robust design. In this sense, IST can be used to prove robustness of designs or find indications for unstable solutions. This happens in the example shown in Section 4.4 (AC bracket, optimised for Eigenfrequencies), where the load path characteristics changed significantly for different boundary conditions. A stable result is achieved after more relevant load cases are introduced. While this is not the main goal of the
method, IST may be applied to improve the robustness of a topology optimisation study. Another positive effect of applying the combined shell and solid approach can be achieved when parts of the structure can be modelled as realistic representations of the actual physical model. I talk more about this in Section 6.3, where I discuss a typical application in the automobile industry, namely body structure design, in this case particularly the front rails. Rather than representing them as solids to go into a topology optimisation, it would be beneficial to actually model them as thin walled structures, and connect them to the solid design volume. IST can assist greatly in the process setup.

Returning to the comparison question, rather than improving a specific topology optimisation technique, the IST approach parametrically couples the external system to the topology optimisation. This coupling can then open the design space of the topology optimisation to find better system solutions.

In a number of classes of structural problems, I have shown how the increase of the design space by a geometrical dimension allows one to find lighter and stiffer solutions on a systems level. Within the allowed geometry design parameter ranges, the resulting topology characteristics are very similar across input geometries. Exceptions prove to have a large mass or low stiffness, and are discarded by the optimisation process. This is a good outcome in that it increases the confidence in solutions found by the IST method.

6.2.7 Limitations and new types of issues

The extension of topology optimisation by a geometrical dimension comes at a cost, and I now look into the question posed earlier:

**Which new issues arise, and what are the inherent limitations?**

I have already discussed the ranking problem between mass and performance, so will not mention it in this section any further.

I have also mentioned that one general problem associated with homogenisation topology optimisation methods is that they deliver a density distribution on their design volume. This is in most cases not a clear cut black-and-white result, and requires some interpretation and additional process steps to translate it into a final manufacturable design. IST cannot solve this issue, but only mitigate it to the extent that portions of the design space can be represented by realistic geometry.
6.2.7.1 Fuzzy homogenisation result

Because any SIMP method returns a fuzzy result that is not a manufacturable black-and-white solution, one needs to be aware that the objective function values (such as mass or compliance) are also fuzzy to some degree. This limits the degree of accuracy of any global objective function, and thus prohibits the use of fast gradient based optimisation algorithms. To give an example: The two lightest designs found by the genetic optimisation in Section 5.3 weigh in at 4625 gram and 4629 gram respectively (Figure 5.9). The difference of 4 grams cannot be expected to be suitable for a gradient calculation, since the density results returned by OptiStruct are not precise mass figures for a final design.

One solution is to employ bionic algorithms, with the downside that their intrinsic random approach often requires a big number of designs to be evaluated. On the other hand, they have the advantage that they do not easily get caught in local minima. And from a computational point of view, they lend themselves naturally to parallelisation on distributed computing systems.

Also due to the imprecise nature of the SIMP results, it makes sense to use discrete geometry design variables. This avoids two very similar geometries to be evaluated, thus achieves a clear separation of results. This also provides scope for the reuse of results.

6.2.7.2 Computational cost

This leads to the question of computational cost. The outer geometry loop requires a topology optimisation run for every set of design parameters. In Table 6.2 I have summarised the times required for the individual structural problems discussed to be processed on a single machine with an Intel i7 1.6 GHz processor with 16 GiB of RAM.

For the clamp (Section 4.3 on page 76) and for the body structure (Section 5.2 on page 94) the number of design variables is low (3 and 2 respectively), which allowed a full sweep of the design space, resulting in response surfaces that can quickly be visually assessed. The genetic optimisation conducted for the AC mounting bracket (Section 4.4 on page 80) and for the subframe (Section 5.3 on page 99) contained 11 and 8 design variables, with a total run time of around 8 and 6 days respectively. This is considered to be a long wait for the designing engineer. But then, more powerful computers and parallelisation will be able to bring this number down by an estimated factor of 100, which would bring the computing time down to an acceptable range. On the other hand, the examples shown are relatively simple, and real world engineering problems may require more complex models. It remains
### Table 6.2: Computing time for the examples discussed in Chapters 4 and 5. The analysis time comprises geometry update, mesh generation, assembly of the analysis deck, and topology optimisation, where the latter takes the majority of the time for all four case studies. (All computations were processed on a single machine with an Intel i7 1.6 GHz processor with 16 GiB of RAM.) The element numbers vary for different analysis runs and are approximate only.

<table>
<thead>
<tr>
<th></th>
<th>Total Number of elements</th>
<th>Number of designable elements</th>
<th>Analysis time</th>
<th>Number of runs</th>
<th>Total time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two-piece C-clamp</td>
<td>19,000</td>
<td>18,500</td>
<td>10 min</td>
<td>96</td>
<td>16 h</td>
</tr>
<tr>
<td>AC mounting bracket</td>
<td>28,000</td>
<td>26,000</td>
<td>40 min</td>
<td>290</td>
<td>195 h</td>
</tr>
<tr>
<td>Body structure, front joint</td>
<td>140,000</td>
<td>30,000</td>
<td>100 min</td>
<td>35</td>
<td>60 h</td>
</tr>
<tr>
<td>Subframe</td>
<td>45,000</td>
<td>27,000</td>
<td>30 min</td>
<td>280</td>
<td>140 h</td>
</tr>
</tbody>
</table>

6.2.7.3 Limitations with respect to applicable structures

The original motivation to develop IST was the need to optimise thin walled sheet metal structures, combined with cast joints. I have demonstrated how this idea can simultaneously tackle the shape design of beams and the topology layout of joining structures, allowing to find an optimal balance between the necessary size of the beams and joints.

One example discussed in this context is the common bicycle frame, such as the example shown in Figure 6.1. The beam members are made from carbon fibre reinforced composite material, with cross sections varying along its lengths, and are joined using intricately shaped cast magnesium connection elements. This structure seems to be well suited to the application of IST with the aim to minimise its system mass. As it turned out though, this is not the case, because there is no trade-off between the size/weight of the tubes versus the size/weight of the connecting elements.

The reason for this is that the design of the frame members is predominantly driven by buckling loads, and the failure modes are independent of
the connection elements. Once the beam dimensions (varying cross section shapes along the length of the beams etc.) are determined, topology optimisation can help to develop the joint structures. When increasing the section size of the beams (and therefore adding mass), the joint elements cannot be reduced in size to make up for it. On the other hand, IST could be applied to help design the ends of the beams where they connect into the castings.

More generally speaking, the benefit of IST on a combined beam/joint optimisation is limited when the beams encompass the majority of the structure and its design is driven by loads that are not supported by the joints. This can be said of most truss structures, where the joint design is more or less independent of the dimensioning of the connected beam members.

### 6.3 Industrial context

I now want to briefly discuss how the proposed IST method can be relevant to the engineering industry. Today, topology optimisation methods play an increasingly important role in structural engineering. In the automotive and aerospace industry, software packages such as OptiStruct or Genesis are the most widely used tools.

While extremely useful for single part applications, the challenge that density results need to be interpreted by an experienced engineer remains, as has been highlighted a number of times. He or she looks at a density distribution returned by the homogenisation algorithm, and make decisions as to where to place structural members and how to actually form them.

For complex systems made from multiple parts, the topology method delivers coarse results at best. To illustrate this, let’s look at an automobile
body structure where potential load paths are to be developed. The engineer may set out with a package space and a set of loading conditions, including static equivalent loads representing impact forces. The result may look similar to what is shown in Figure 2.8 on page 49.

Taking one example, I focus on the front rails. The package space for these critically important beam members is very restricted from the start, and it is obvious that these load paths need to be preserved. Moreover, the engineer has the task to translate the topology result into a multi-piece stamped and welded steel structure. The density result is not of much assistance here, since the design requirements for a manufacturable structure involve highly intricate decisions about the exact shape, materials, gauges, tailor welded blanks, crush initiators, local reinforcements for engine attachments, and so on. Indeed, the engineer is likely to have some detailed knowledge about the front rail structure before starting the topology run, and it would be beneficial to be able to feed that know-how into the topology optimisation. Moreover, in the development process for the body structure, a lot of detailed work goes into the construction of the rail members. This is where IST comes into play, enabling parametric shape optimisation methods to be applied to both the shell portions of the model as well as to the package space going into the topology optimisation algorithm.

The studies and results discussed in this thesis indicate that the proposed integrated shape and topology approach should add value to the engineering community.

IST relies on the integration of state-of-the-art software packages. In the presented studies in this thesis, I have noted several limitations that I want to repeat here, because they may impact on the industry application of IST, and deserve further development by the software providers.

- The parametric geometry tool SFE CONCEPT proves very valuable for the generation of thin walled welded sheet metal structures. Its capability in the creation of volume meshes is limited though, and required some workarounds to facilitate the integration of HEXA-based meshes into an automated shape optimisation process.

- For topology optimisation, I have relied on OptiStruct with its many features to incorporate manufacturing constraints. One problem I came across is that the extrusion constraint does not always work if the provided package space does not have constant cross section. (For this reason I reverted to die cast joints in the example in Section 5.2.)

- For many cast structure applications, ribbed structures with fairly constant thickness are beneficial. One may think of an automobile shock
tower made from cast magnesium. Here the support from the industrial topology optimisation packages is not very good. OptiStruct, for example, allows one to define a minimum member size, or a constraint to produce thin walled structures, but cannot cater for ribs on a constant thickness component.
Chapter 7

Conclusion

7.1 Limitations of current shape and topology optimisation methods

Looking at current topology or shape optimisation tools in structural mechanics, inherent limitations can be seen. In particular, the design space for topology optimisation using homogenisation or level set methods is fixed, and topology design is performed more or less independently from shape optimisation, in the sense that the latter is conducted as a fine tuning step of the outer boundary. Also, topology optimisation methods cannot account well for thin walled structures and complex compound structures. In turn, shape methods are often hard to implement because the transition from design modification ideas to an analysis model can be very complex. Moreover, they have difficulties in deriving topological load path layouts.

7.2 IST — An integrated approach to shape and topology optimisation

This thesis proposes a new approach to integrate these two methods by performing large scale parametric shape modification prior to topology optimisation. This effectively extends the topology optimisation method by a geometric dimension by coupling design volume parameters and the external system to the topology optimisation. Moreover, the external system is subject to large scale parametric shape variation.

The method is named IST — Integrated Shape- and Topology approach — and is realised through a combination of commercial software tools and proprietary software. The implementation of the IST method provides a
batch script interface to manage the automated process integration of shape
and topology tools. One important step in this loop is the non-trivial task
of the assembly of analysis models. I have developed a set of tools to realise
this step.

The implementation of IST allows fast and flexible setup of structural
optimisation problems that involve both shape variation and topology load
path design. I have shown that the combined approach can lead to lighter or
stiffer structures than a traditional sequential approach would have delivered.

IST was first devised for structures where complexly shaped beam mem-
bers are connected with cast joints. This is of particular interest for new
manufacturing approaches in the automotive industry, where space frame
concepts with integrated complex cast components promise further poten-
tial for light weighting, but can be applied in other fields as well, such as
aeronautics, and civil engineering.

However, a number of other classes of applications are discussed, that
involve either solely shell based or solely volume based structures. For exam-
ple, IST allowed to derive an optimal combination of both the attachment
point locations and load path layout for a complex cast structure bracket
design.

7.3 Ranking problem of mass vs. performance
vs. mass fraction

IST wraps complex shape variation around a SIMP topology method. This
entails the problem of comparability: How can one rank two designs that dif-
fer in mass, performance, and volume fraction? I have proposed a number of
solutions, one of which proves most practical: For the topology optimisation,
define suitable performance constraints, such as displacement or Eigenfre-
quencies. The SIMP algorithm objective is then to minimise the mass, and
leads to structures with identical performance. For well posed problems the
differing mass fractions then proves to be not an issue, and designs can be
ranked according to their mass.

Another approach is to run the topology optimisation with a fixed mass
fraction while optimising for minimal compliance. This proves to work as
well, but has the disadvantage, that the effective mass fraction value has to
be determined in an additional process step for every new set of geometry
design variables.

In one example I showed a third idea that manages to keep both mass
and mass fraction constant across the geometry parameter space, by impos-
ing suitable restrictions on the geometry design variables. However, this approach is difficult to implement and the restrictions are not desirable, which is why this method is not recommended.

7.4 Limitations

7.4.1 Interpretation problem

The proposed approach has some intrinsic limitations. One general problem associated with topology optimisation methods (here I am speaking of the homogenisation approach, namely the SIMP method), is that it delivers a density distribution on the design region. This is in most cases not a clear cut black- and white result, and requires interpretation and additional process steps in order to devise a final manufacturable design. IST cannot solve this issue, but only mitigate it to the extent, in which portions of the design space can be represented by realistic geometry.

Related to the fact that any SIMP method returns a fuzzy answer (in the sense that the result is not a manufacturable black and white solution), one needs to be aware that objective function values (such as mass or compliance) are also fuzzy to some extent. This leads to noise, and limits the degree of accuracy of any objective function within IST. This issue can be mitigated by applying genetic algorithms in combination with a discrete geometric design space. Even if the parametric geometry approach, combined with “fuzzy” topology optimisation results do not deliver a sharp global minimum of the objective function, the method can nonetheless determine a trade-off between geometry and load path characteristics, which then serves as a basis for a refined production design.

7.4.2 Computational cost

Another potential limitation is computational cost. With the addition of a geometric design space, wrapped around topology optimisation, the required computing resources is substantially bigger than for a traditional topology optimisation approach. Moreover, IST does not lend itself to fast optimisation algorithms such as gradient based methods. Rather, in most cases bionic algorithms have to be adopted. While these types of algorithms have the advantage that they do not easily get caught in local minimal, their intrinsic random approach often require a large number of design evaluations. On the other hand, genetic algorithms are intrinsically well suited to run on parallel systems, and if the trend to ever more powerful computing resource
at reduced cost continues, structural problems with substantial complexity and a high number of geometry design variables may well be manageable.

### 7.4.3 Structures not suitable for IST

Finally, for some types of combined beam/joint structures, the IST approach is not well suited, even though at first sight it seemed to be a good fit. Where there is very little dependency of the external systems on the topology optimisation, the IST approach adds little value. For example, when beam design is predominantly driven by buckling loads, then the joint design is almost independent from the beam design, and there is potentially only a very little trade-off benefit between beam section shape and size on the one hand, and joint topology.

### 7.5 Future Work

I want to give an outlook, first, of where I see potential for further research, and, second, how I see IST applied in the engineering industry.

A lot of research focuses on the automated interpretation of topology optimisation result, or more generally, developing topology optimisation methods that derive black and white structures, rather than fuzzy density results. It would be interesting to see some of these methods incorporated into the IST optimisation loop.

Another important field of research is the integration of non-linear load cases into the topology optimisation algorithms, in particular crash simulations. While today this is possible to a very limited extent, IST offers the potential to simulate relevant components as realistic thin walled stamped and welded sheet metal structures. This enables the integration of crash simulations. It would be interesting to explore how these would be best integrated into the IST geometry optimisation loop.

Then, I have elaborated the fact that an IST type of optimisation is best done using genetic algorithms with discrete design variables. I have not focused on the best choice of parameters for the genetic algorithm, such as the number of individuals per generation, or the mutation and cross-over rate. It may be beneficial to pursue this further.

In Section 3.5 I have discussed the problem of ranking different design with respect to their mass and performance, when both mass and performance can vary across multiple geometries. It could be of interest to explore multi-objective optimisation and Pareto Frontiers.
Also, one could explore how to derive sensitivities for the geometry design parameters. Very likely it would make sense to identify design parameters with either low impact on the objective functions, or with a clear tendency towards a specific value. In a two-step process (or a multi-step process), one could then eliminate these parameters, and thus increase the overall efficiency of the optimisation process.

I have already mentioned a few items for the wish-list of features of homogenisation algorithms, such as the incorporation of non-linear load cases. Of great importance is also the ability to restrain the homogenisation algorithm to thin walled ribbed structures. This is currently not possible, and may require some theoretical work to explore its feasibility.

From an industry perspective, I believe that the proposed IST approach has a great potential for structural concept development, because it adds a geometrical dimension to the optimisation space. The approach will be particularly useful in areas where topology optimisation methods are applied as a standard process today, such as in the automotive or aeronautic industry. However, it would be interesting to see IST applied to fields other than the ones mentioned in this thesis.
Appendix A

Tools integrated into IST

A.1 Building blocks integrated into IST

In Chapter 3 I outlined the building blocks necessary to implement the proposed integrated parametric shape approach. Here I give a summary of the choice of actual tools, together with a rational.

The author has decided to use state of art available technology wherever possible and practical. For most aspects of the implementation, off-the-shelf tools are employed, while some process steps required proprietary implementation by the author. Figure A.1 gives an overview of the main elements of the IST process chain.

A.2 Parametric geometry tool and finite element mesher

For a parametric geometry tool, the choice fell upon the software SFE CONCEPT. A main advantage over other tools is that is comes with an implicit parametric, meaning that inter-dependencies between parts are automatically defined. One example is the “mapping” functionality, that allows to project points, lines, or surfaces onto target surfaces. The projected geometry is calculated automatically, and — more importantly — is kept up to date whenever the projection target is modified. Another feature that SFE handles better than many other tools are thin walled structures connected in flanges. The problem of penetrating surfaces joining in bonded flanges is solved very elegantly by topological description of the stack-up. This unique feature separates SFE CONCEPT from many other geometry and optimisation tools, including solidThinking Inspire, which is why Inspire was not
used is this research. With these characteristics, SFE allows complex large scale shape modifications in a short time, and the geometry parameters can be modified in batch mode.

Another major advantage of SFE is that it embodies a mesh generator, including the creation of welding information. This allows fast automated structural assessments, which is why SFE CONCEPT has become an important tool in the automotive industry for early concept design assessment of body structures, and in this context has been described as the “state of the art in shape optimisation” (Duddeck et al. [36]). A downside of SFE CONCEPT is that it cannot handle solid geometry very well. The capability of generating volume meshes are limited to relatively simple geometries, and the generated 3D elements are not connected to surrounding shell elements. Another disadvantage of the software is that it is not easy to operate because of an outdated and often erroneous user interface.
In the most of the examples shown in this thesis, SFE CONCEPT is employed to generate an initial geometry model, to modify geometry according to design variables, and to generate finite element meshes. An exception is the bracket optimisation (Section 4.4), where only the initial geometry is created with SFE. In this example, no re-meshing is necessary, and all subsequent parametric variation is done by IST tools.

A.3 Automated assembly of analysis decks

After the generation of a finite element mesh, the next important step in the process chain is to assemble an analysis model. Particularly, all parts of the model have to be connected, and boundary conditions need to be applied. Some of the concepts are implemented within SFE CONCEPT, but are not sufficient. As an example, one can define Rigid Body Elements at specific locations, that parametrically adapt after geometry updates. But is it not possible to connect shell geometry with solid geometry.

It turned out that this is a general task that can not be automated easily with existing tools. Analysis preprocessors (such as ANSA, Hypermesh, Primer, etc.) allow the generation of connection elements and boundary conditions, but are not easy to program to work in batch mode for general cases.

This is why I have decided to develop proprietary tools that realise the necessary tasks. The functionality for a general tool capable of setting up analysis solver decks need to include the capability to

- Import finite element data;
- Translate, rotate, replace, copy, and mirror finite element entities;
- Define sets, based on part names or geometry (boxes, radii);
- Create connections between components using various element types (rigid body connections, beams, bars, springs), based on part names, sets, distance, etc.;
- Renumber entities, merge and replace grids, assign specific grid id’s for handling of loads and for performance tracking;
- Apply single point constraints and loads, based on geometry, part names, sets, id range, etc.;
- Export solver specific analysis decks.
I have implemented these features, with the possibility to control them with an easy batch script language. This tool set can be used for a wide range of applications, and is a significant part of the development work that went into realising the IST approach as described and applied in this thesis (Refer to Appendix B for more detail regarding the implementation).

A.4 SIMP topology optimisation

Central to IST is a solver that implements a Solid Isotropic Material with Penalty topology optimisation algorithm (SIMP), that can handle both shell and solid elements. The most widely used tools in the engineering industry are GENESIS (Vanderplaat’s [156]) and OptiStruct (Altair [132]). In this research, I have used OptiStruct, simply because it is readily available through the IFM (Institute for Frontier Materials [155]) Research Institute at Deakin University. OptiStruct’s control over manufacturing parameters — such as extrusion and draw constraints — are very good, and the integration with the Altair HyperWorks Suite is beneficial for pre- and postprocessing of analysis models. It is possible to integrate any other solver into the process chain. In the examples provided in this thesis, only linear static analysis is performed, so the application of OptiStruct is sufficient for all structural problems considered.

A.5 Optimisation algorithm

Then, a tool to create matrices for DOE’s (Design of Experiment) and optimisation is required. Here I have considered a number of options, such as MATLAB [142], Altair’s Hyperstudy [131], custom made algorithms, and finally decided to use the open source software Dakota [1]. Dakota contains algorithms for optimisation with gradient and non-gradient-based methods, sensitivity/variance analysis with design of experiments and parameter study methods, among others (From the Dakota Website [140]). Main advantages of choosing Dakota are the smooth integration into the analysis process, the number of readily available algorithms for parameter space sampling, and a wide range of both gradient based and bionic optimisation algorithms. Moreover, Dakota is well documented and free of cost.
A.6 Process Loop

Finally, The IST process requires an integration tool that combines all of the above components. A fully automated process needs to be established that generates parametric geometry, assembles analysis decks dynamically, launches analysis software and runs a topology optimisation for every geometrical specification. Then, errors have to be captured, and solver results need to be evaluated and fed back into a global optimisation algorithm. Also, the process needs to be flexible enough as to allow the user to plug-in problem specific functions.

This integration is realised by implementing a simple parser for a batch script language similar to the interface for the above analysis deck generator. This allows to define the complete optimisation loop within one scripting environment. Even though commercial alternatives — such as Isight [152], or modeFRONTIER [138] — may be available for some of the process steps, the main tasks constitute of writing customised scripts that interact with the specific analysis tools SFE CONCEPT and OptiStruct, which is not readily available through commercial software.
Appendix B

Implementation of IST

This chapter gives a brief overview of the implementation of the software necessary to realise the IST method as detailed in this thesis. I will not go into coding details, but rather show the features and the user interaction by means of a typical application example.

The execution of an IST analysis consists of an outer optimisation loop that integrates and controls a number of iteration steps. Typically one such step starts with the generation of a geometry model, followed by the assembly of an analysis deck. Then, a topology optimisation is employed, results are extracted, and new geometry design variables are chosen (C.f. Figure A.1 on page 132).

In Section 3.3 on page 54 and in Appendix A I have already outlined the tools necessary to realise this optimisation loop, as well an overview of components that are not available from commercial vendors. To bridge these gaps, I have written software in Perl and C that consists of three major parts:

1. Integration of external tools and provision for user defined plug-in functions;
2. Automated assembly of analysis decks (preprocessing);
3. Extraction of analysis results and error handling (postprocessing).

In the following, I describe these components, and how the user interaction is structured, based on a typical example, without going into too much coding detail. In order to differentiate between the IST method and the software described here, I will refer to the program as ist.

The process sequence for all the examples shown in this thesis follow a similar basic pattern: a base directory contains the a number of sub-directories

- sfe
In cases where Dakota is used for the geometry optimisation loop, there is a directory

- dakota.

For DOE's (as opposed to genetic optimisation) I have used Hyperstudy, and the base directory also contained a sub-directory

- hyperstudy

Both Dakota and Hyperstudy generate a number of sub-directories where they store analysis data and result files for each individual analysis step. The process directory contains the main process definition script, a text file storing current design variables, plug-in functions, result files, and others. The sfe directory contains the SFE CONCEPT model, the SFE batch script, and the SFE generated finite element mesh files. The optistruct directory contains the analysis header deck, as well as all mesh include files.

The interaction between the engineer and ist is realised via a text file defining all necessary process steps, such as the definition of default values, individual analysis steps, external commands, etc. At the beginning of each iteration step, Dakota, OptiStruct (or some other tool) runs the command

```bash
> ist <path>/ist_process_script,
```

where ist_process_script is the user defined main process definition script file. ist will now scan the file ist_process_script and process individual commands line by line. In the following section, a generic example is presented that shows the typical process steps in an ist analysis loop.

### B.1 Generic example of IST process definition script

The following example is based on the subframe optimisation in section 5.3, which shows some of the major features implemented within ist. An outer control instance (e.g. Hyperstudy or Dakota) manages the geometry design variables. The command file controls the process flow for a single step within this outer optimisation loop, from geometry update to topology optimisation run through to the extraction of solver results.
The user first defines a name and an execution path for the current study thus:

```
# Model name and directory.
MODEL_NAME subframe_004
MODEL_DIR ~/ist/subfram_004
```

(ist user commands are typeset in blue, comments in red). Typically, a number of load cases will be considered. In case that any of the analysis steps fail, default values for all the responses can be defined. The response value names (e.g. \texttt{disp\_frt\_x}) correspond to the names used in the OptiStruct deck:

```
# Default values.
# Suspension front
DEFAULT VALUE disp\_frt\_x = 0.1
DEFAULT VALUE disp\_frt\_y = 0.5
...
# Suspension rear
DEFAULT VALUE disp\_rr\_x = 0.1
...
# Engine mount
DEFAULT VALUE disp\_engine\_x = 0.1
...
```

Next, the geometry data and the analysis header deck is copied to the current working directory. This step is done in order to store the results of all iteration steps for the current parameter study or optimisation:

```
# Copy SFE model and OptiStruct deck to working directory
COPY BASE\_DIR/sfe TO WORKING\_DIR/
COPY BASE\_DIR/ana/deck.fem TO WORKING\_DIR/ana/
```

Now, the geometry can be updated. In this case, SFE CONCEPT is employed: It runs in batch mode, where a command file \texttt{sfe\_con} is read. This file contains instructions to load the geometry model, read design parameter values, to update the geometry accordingly, and finally to generate and export a finite element mesh.

```
# Run SFE CONCEPT in batch mode.
RUN SFE\_CONCEPT -batch sfe\_con
```

The next step is to preprocess the SFE generated mesh into an analysis deck. This may involve a number of preprocessing steps such as symmetrizing the SFE mesh, assigning specific GRID ID's used for loads and measurements, modifying gauges and materials, generating rigid elements, and finally ap-
plying boundary conditions and loads. A typical sequence of actions could look as follows:

```
# Preprocess SFE output, generate analysis model
#
# Import SFE finite element mesh and SFE generated WELD file
IMPORT NASTRAN sfe/sfe_mesh.bdf
IMPORT WELD CDH sfe/sfe_mesh.bdf
# Define control volume used for Single Point Constraints in the symmetry plane
DEFINE SET 1 BY BOX (-2,-500,-500), (2,500,500)
CREATE SPC ID=100 DOF=135 FOR ALL NODES OF SET 1
# Rigid Body Elements for suspension attachment etc.
DEFINE SET 2 BY PART ID 1000
CREATE RBE2 FOR ALL NODES OF SET 2
# More RBE2 and RBE3 definitions
...
# Symmetrize the model
SYMMETRIZE ELEMENTS OFFSET=100000
# Convert SFE generated welds into OptiStruct CWELD format
GENERATE WELDS TYPE=CWELD
# Export finite element data to include directory
EXPORT NASTRAN WORKING_DIR/ana/include/mesh.fem
```

This concludes the built of the analysis deck, and the solver can be started. The main solver deck is named `subframe.fem`, which expects the bulk data in an include file that has been generated in a the previous step. In the example shown here, a check run is executed, from which the mass is extracted. Then a user defined plug-in function is called, where the mass figure is used to determine the correct volume fraction value that guarantees a constant overall mass for all different initial geometries. The calculated volume fraction value is automatically substituted into the OptiStruct input deck.

```
# Determine mass fraction value
#
RUN OPTISTRUCT WORKING_DIR/ana/subframe.fem -check
# Run plug-in to determine mass fraction value.
EXTRACT RESULT mass FILE=WORKING_DIR/ana/subframe.out
RUN_PLUGIN subframe.pl determine_mass_fraction
```

Now, the SIMP topology optimisation run is performed:
# OptiStruct Topology Optimisation

RUN OPTISTRUCT WORKING_DIR/ana/subframe.fem

Finally, the status of the analysis is examined and the results are extracted. The default values defined earlier serve as penalty values in case parts of the analysis failed.

# Extract mass and displacements

EXTRACT RESULT mass FILE=WORKING_DIR/ana/subframe.out
EXTRACT RESULT disp FILE=WORKING_DIR/ana/subframe.dips GRID=210
EXTRACT RESULT disp FILE=WORKING_DIR/ana/subframe.dips GRID=220
EXTRACT RESULT disp FILE=WORKING_DIR/ana/subframe.dips GRID=230

# Analysis loop completed, pass control back to main loop
EXIT

With this, a geometry update/ topology optimisation step concludes, and the outer optimiser takes back the control and determines new design variable values.

## B.2 Preprocessing of analysis decks: Features of ist

An important aspect of ist is its ability to automatically generate analysis decks, based on finite element data encompassing a large bandwidth of variation. The variation stems from the fact that we allow large scale shape variation to take place in every iteration step of an IST optimisation.

I have outlined the main requirements for this automated preprocessing step in Section 3.3 on page 54. which include the ability to

- Import finite element data;
- Translate, rotate, replace, copy, mirror entities;
- Define sets, based on part names or geometry (boxes, radii);
- Handle weld connections in a number of formats;
- Create connections between components using various element types (rigid body connections, beams, bars), based on part names, sets, distance, etc.;
- Renumber entities, merge and replace grids, assign specific grid id’s for handling of loads and for performance tracking;
• Apply Single Point Constraints and loads, based on geometry, part names, sets, id ranges, etc.;

• Export solver specific analysis decks.

For any given structural problem, the required individual steps need to be specified in order. As said before, I will not go into the details of the code implementation, but only observe that ist provides an interface to all these preprocessing features based on simple command line instructions. I have attempted to keep the user interaction to the individual process steps consistent with the other commands provided, as shown in the example above.

B.3 User defined functions

In the optimisation examples presented in this thesis we have already seen a number of cases where the engineer needs to intervene in the standard process flow. In Section 5.3 (and in the example above), a user defined plug-in function was called to determine the effective mass fraction for the subsequent topology optimisation, and in Section 4.2, a specific function was used as an intermediate step to calculate the proper height to width ratio of a cantilever beam.

The way the interaction is realised is as follows: The user implements a function using Perl. The code resides inside a file in the process directory. The transfer of data between the user defined function and the rest of the ist code is managed via a class $GlobalVars that provides set/get accessor functions to all necessary values. For example, design variable values declared in the process script, such as

\[
\text{DEFAULT VALUE disp\_frt\_x = 0.1}
\]

can be accessed or redefined using the Perl code

\[
\text{my \$disp = \$GlobalVars->getDesVarValue("disp\_frt\_x" );}
\]

and

\[
\text{\$GlobalVars->setDesVarValue("disp\_frt\_x", 1.33 );}
\]

respectively. In an analogous way, the analysis responses can be accessed. As shown before, at any point in the process script, the user may request

\[
\text{# Run plug-in to determine mass fraction value.}
\]

\[
\text{EXTRACT RESULT mass FILE=WORKING\_DIR/ana/subframe.out}
\]

\[
\text{RUN\_PLUGIN subframe.pl determine\_mass\_fraction}
\]
which will run the subroutine `sub determine_mass_fraction{...}`, declared in the file named `subframe.pl`, and can be called during the IST process as shown above. Inside the subroutine, the extracted mass value is accessible for example like this:

```perl
my $m = $GlobalVars->getResponse( "mass" );
$m = ...
$GlobalVars->setResponse( "mass", $m );
```

Many other data fields are accessible, for example the directory paths for the current study, or default and penalty values.
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