DEVELOPMENT AND CONTRIBUTING FACTORS IN PRIMARY PRE-SERVICE TEACHERS’ MATHEMATICAL CONTENT KNOWLEDGE

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Diploma of Teaching, Bachelor of Education and Master of Education

Submitted in fulfilment of the requirements for the degree of
Doctor of Philosophy

Deakin University
June, 2014
I am the author of the thesis entitled

Development and Contributing Factors in Primary Pre-service Teachers' Mathematical Content Knowledge

submitted for the degree of Doctor of Philosophy

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Abstract

The knowledge needed for effective mathematics teaching is specialised. It is reasonable to expect that effective mathematics teachers possess a sound understanding of the mathematics they teach, including specialised content knowledge. This study was informed by an historical overview of theoretical frameworks that included categories used to describe mathematical content knowledge (MCK). The purpose of this study was to extend understanding of the MCK that pre-service teachers enrolled in a primary to Year 12 program developed during their program experiences, identifying contributing factors that enhanced their MCK for teaching primary mathematics.

The research used a mixed-methods research design. The quantitative component was an analysis of a large cohort of first and second-year pre-service teachers’ responses to MCSK test items. The item responses were ranked and reported as least difficult, difficult and most difficult by mathematical content strands. Similarly, a second analysis reported items by cognitive sub-domain, including Knowing, Applying and Reasoning. Qualitative data were collected during a four-year longitudinal study of 17 pre-service teachers’ MCK during primary mathematics coursework and practicum experiences. Categories of the Knowledge Quartet: foundation knowledge, transformation, connection and contingency (Rowland, Turner, Thwaites, & Huckstep, 2009) and other MCK frameworks (e.g., Ball, Thames, & Phelps, 2008; Chick, Baker, Pham, & Cheng, 2006a; Ma, 2009) were used to code, interpret and discuss the findings.

Most pre-service teachers relied on procedural knowledge during fourth year; few had breadth and depth of MCK or demonstrated the ability to make connections. Contributing factors that assisted pre-service teachers to develop their MCK included an emphasis on passing a MCSK test during second year, a lack of sustained engagement in opportunities to extend MCK, absence of depth of practicum experiences across different year levels and few opportunities to extend MCK when reflecting on primary mathematics lesson experiences with mentor teachers. Teacher education program designs can be improved by focusing on these factors and lost learning opportunities to extend pre-service teachers’ MCK.
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<tbody>
<tr>
<td>ACARA</td>
<td>Australian Curriculum Assessment and Reporting Authority</td>
</tr>
<tr>
<td>ACER</td>
<td>Australian Council for Educational Research</td>
</tr>
<tr>
<td>ACM</td>
<td>Australian Curriculum Mathematics</td>
</tr>
<tr>
<td>AITSL</td>
<td>Australian Institute for Teaching and School Leadership</td>
</tr>
<tr>
<td>CCK</td>
<td>Common Content Knowledge</td>
</tr>
<tr>
<td>HCK</td>
<td>Horizon Content Knowledge</td>
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<tr>
<td>IAE</td>
<td>International Association for the Evaluation of Educational Achievement</td>
</tr>
<tr>
<td>MCK</td>
<td>Mathematical Content Knowledge</td>
</tr>
<tr>
<td>MCSK</td>
<td>Mathematical Competency Skills and Knowledge</td>
</tr>
<tr>
<td>MERGA</td>
<td>Mathematical Education Research Group of Australasia</td>
</tr>
<tr>
<td>MKT</td>
<td>Mathematical Knowledge for Teaching</td>
</tr>
<tr>
<td>NAPLAN</td>
<td>National Assessment Program – Literacy and Numeracy</td>
</tr>
<tr>
<td>PCK</td>
<td>Pedagogical Content Knowledge</td>
</tr>
<tr>
<td>Prep</td>
<td>Preparatory</td>
</tr>
<tr>
<td>PUFM</td>
<td>Profound Understanding of Fundamental Mathematics</td>
</tr>
<tr>
<td>RQ</td>
<td>Research question</td>
</tr>
<tr>
<td>SCK</td>
<td>Specialised Content Knowledge</td>
</tr>
<tr>
<td>TEDS-M</td>
<td>Teacher Education and Development Study in Mathematics</td>
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<tr>
<td>TIMSS</td>
<td>Trends in International Mathematics and Science Study</td>
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List of Publications related to this thesis

Peer-reviewed journal articles


Peer-reviewed conference papers


CHAPTER ONE

Are teacher educators preparing teachers who can extend students’ mathematical learning?

If Australia’s primary students are to maintain or improve the quality of science and mathematical research, we need to improve the standard of mathematics education and not limit students’ opportunities to learn or allow them to fall behind on international rankings. A recent media release stated that “presently, Australia is in critical need of graduates from the mathematical sciences… that the need for PhD graduates in the Mathematical Sciences will increase by 55.6 percent by the year 2020” (Pradier, 2013, p. 1).

National Assessment Program – Literacy and Numeracy (NAPLAN) assessments between 2008 and 2013 show no improvement in Victorian students’ numeracy (Australian Curriculum Assessment and Reporting Authority (ACARA), 2013b). This stagnation is also present in the results of the Trends in International Mathematics and Science Study (TIMSS) assessment, conducted every four years between 1995 and 2011. Thomson, Hillman, Wernet, Schmid et al. (2012) reported that “[t]he performance of Australian Year 4 students has not changed since TIMSS 2007” (p. 32), whilst other countries have significantly improved their students’ achievements in mathematics. Australian results might be due to a crowded primary curriculum and/or primary teachers’ lack of preparation to teach mathematics, consequently resulting in lower student achievement by the time they commence secondary education, hence also contributing to fewer students completing Year 12 mathematics (Chinnappan, Kinham, Herrington, & Scott, 2007).

1.1 Rationale

A study designed to identify the development and contributing factors that assist training teachers (pre-service teachers) to develop their own understanding of mathematics, or mathematical content knowledge (MCK) will assist the quality of future teacher education programs and graduate pre-service teachers. Therefore the
research questions were designed to extend our existing understanding of the MCK that primary pre-service teachers develop during their program experiences, and specifically to identify contributing factors that enhanced pre-service teachers’ mathematical understanding and the strategies they used when interpreting different mathematical problems.

This study was designed to generate new knowledge regarding primary pre-service teachers’ MCK that would provide guidance for future teacher education program design to maximise primary pre-service teachers’ opportunities to improve their MCK. In turn, assisting teachers to extend their skills and MCK for teaching allows their students to improve their mathematical understanding. The MCK needed by primary teachers is different to the mathematical knowledge of other adults (Ball, Thames, & Phelps, 2008). In this study, pre-service teachers’ MCK was defined as the knowledge of mathematics they relied upon during coursework and practicum experiences for primary mathematics teaching. Understanding how and when pre-service teachers develop MCK during their teacher training programs is important for program design and developing effective primary mathematics teachers.

Shulman (1987) characterised teachers’ content knowledge as concerned with expertise in the particular discipline being taught. Others have described MCK as important knowledge required when learning to teach mathematics (e.g., Carre & Ernest, 1993b; Reynolds, 1992). Research aimed at understanding and describing MCK and pedagogical content knowledge (PCK) of primary mathematics teachers (e.g., Chick, Baker, Pham, & Cheng, 2006a; Hill, Ball, & Schilling, 2008; Ma, 1999; Rowland, Turner, Thwaites, & Huckstep, 2009; Shulman, 1987) and examining the MCK pre-service teachers demonstrate during their teacher education (e.g., Callingham et al., 2011; Tatto et al., 2012) continues to refine our understanding.

Building on the seminal work of Shulman (1986; 1987) and other authors (e.g., Chick et al., 2006a; Hill et al., 2008; Rowland et al., 2009), researchers have constructed frameworks as a means for understanding the complex relationship between the types of knowledge required for mathematics teaching. The domains of Mathematical Knowledge for Teaching (MKT) includes three categories of subject matter knowledge:
common content knowledge (CCK), specialised content knowledge (SCK) and horizon content knowledge (HCK) (Ball et al., 2008); the Knowledge Quartet has four categories: foundation knowledge, transformation, connection and contingency (Rowland, Turner, Thwaites, & Huckstep, 2009); and Ma (1999) described accomplished teachers as demonstrating profound understanding of fundamental mathematics (PUFM) demonstrating breadth, depth and thoroughness of their MCK.

Furthermore, it is expected – and vital – that teachers know the content they teach (Australian Institute for Teaching and School Leadership (AITSL), 2011). However, Sullivan (2003) suggested it is unrealistic to expect a graduate teacher to know everything about mathematics, rather they should know how to learn the mathematics they will need for teaching primary students. Rowland et al., (2009) suggested that beginning teachers, including those learning to teach should focus on making sense of categories of foundation knowledge and make connections so that students can make sense of the mathematics. Nevertheless, ensuring pre-service teachers have the opportunity to extend their MCK during their program experiences is important for their development as effective primary teachers.

There is worldwide concern regarding pre-service teachers’ lack of understanding of mathematics (Cooney & Wiegel, 2003). Adler, Ball, Krainer, Lin and Novotna’s (2005) survey of international research found that teacher educators frequently conducted research on their own pre-service teachers’ mathematics education. Extensive research has been completed on pre-service teachers’ mathematical knowledge, including in Australia (Goos, Smith, & Thornton, 2008). Adler et al.’s (2005) review of mathematics teacher education also noted that few studies focused on “what helps some teachers to develop from their own teaching while others do not” (p. 376).

Theoretical frameworks of teacher knowledge can deepen our understanding of the different categories used to describe teacher knowledge (Bobis, Higgins, Cavanagh, & Roche, 2012). Less clear in the literature are the differences between the categories used to describe the MCK primary pre-service teachers demonstrate and how to define their MCK. This study was designed to identify the various categories of MCK that pre-service teachers acquire at different stages of their pre-service teacher education and
establish the extent to which the knowledge they demonstrate during the final year of their program is sufficient for teaching primary mathematics.

These findings will be important because researchers have agreed that teacher educators need to assist pre-service teachers to make the transition into the classroom as smooth as possible (Smith & Lowrie, 2001; Sparrow & Frid, 2001), and there are many challenges in assisting pre-service teachers and practicing teachers to develop the complex knowledge required for effective teaching of mathematics (Callingham, Chick, & Thornton, 2012; Frid, Goos, & Sparrow, 2009).

1.2 Context

The university where this study took place offers programs across technical and further education (TAFE) through to higher education, including teacher education programs. It has international campuses, thus providing education for international students as well as local students. However, the pre-service teacher education program is only delivered on its Australian campuses.

In Australia, students commence primary school between the ages of four and five. Students commence secondary school in Year 7 when aged 11 or 12, and complete their final year of education in Year 12 between the ages of 17 and 18. After completing Year 12 students may choose to commence further education in programs such as teaching at a university and are called pre-service teachers whilst completing their training.

Pre-service teachers study core teacher education units and units in a selected secondary discipline specialisation, and complete school-based teaching experiences (practicum experiences). School based experienced were designed to develop pre-service teachers’ understanding and practice of teaching by creating partnerships between schools and universities that enhanced learning for all: students (children), pre-service teachers, teachers and university lecturers. After graduation, pre-service teachers who complete this program qualify as teachers with the skills to teach primary mathematics. They also qualify to teach a chosen secondary specialisation, such as mathematics education, physical education, science or creativity and the arts.
The core units of study aim to extend pre-service teachers’ knowledge of emerging theories of teacher education, providing experiences in primary classrooms during first, second and fourth years and secondary classrooms during third year. Three units, one in first-year and two during second-year, are designed to extend pre-service teachers’ knowledge of teaching primary mathematics, including pedagogy, curriculum and MCK. An elective unit is provided for pre-service teachers who choose to revise their MCK for primary teaching or fail a hurdle task (the MCSK test, which assesses mathematical understanding of content taught from Year 5 to Year 8 (ACARA, 2013)).

Classroom experiences are intended to provide opportunities for pre-service teachers to observe the classroom teacher (referred to as a mentor teacher), teach, assist students during mathematics lessons and practise primary mathematics teaching under the supervision of their mentor teacher. These experiences usually occur once a week during each semester, commencing in the second semester of the first year of the program, in addition to full week practicum experiences at various times during each year of the program. The number of days spent in schools increases for each year of the program.

By the end of this program, pre-service teachers are expected to have experienced four different educational setting during their practicum experiences. Of the 144 days on which pre-service teachers undertake supervised practicum school experiences, 102 days are in primary school settings. This means pre-service teachers in this program substantially exceed the recommended minimum 45 days of supervised practise teaching required for teacher registration (Victorian Institute of Teaching, 2011), and receive significantly more practicum experience than their counterparts at other teacher education institutions (Parliament of Victoria Education and Training Committee, 2005).

1.3 Research questions

A major research question and three subsidiary research questions were designed to guide the identification of the factors contributing to the development of primary pre-service teachers’ MCK.
What MCK do primary pre-service teachers develop during their teacher education and how is this demonstrated and achieved?

What MCK is demonstrated at different stages of pre-service teacher education?

What opportunities and influences enhance pre-service teachers’ MCK during units in primary mathematics teacher education?

What opportunities and influences enhance pre-service teachers’ MCK throughout teaching practice in primary and secondary classrooms?

1.4 Organisation of the thesis

This chapter introduced the importance of this study, provided some context, and described the rationale for the research and the research questions.

Chapter 2 contains the literature review, including a historical review of theoretical frameworks, and a classification of the knowledge needed for teaching primary mathematics. This literature review also describe the different MCK frameworks, list the varied categories of MCK, and compare the terms used to describe teachers’ MCK.

Chapter 3 is the methodology chapter, and describes the quantitative and qualitative methods used in responding to the research questions. The quantitative methods section includes detail used for analysing pre-service teachers’ responses to MCSK test items. The qualitative methods section outlines the methods used in the longitudinal study.

Chapter 4 reports the results of quantitative analysis of first-year and second-year pre-service teachers’ responses to MCSK test items. The results report first-year then second-year pre-service teachers’ strengths and weaknesses in MCK, including ranking by difficulty for content strands and sub-strands of the Australian Curriculum Mathematics (ACM) (ACARA, 2013) as well as by cognitive sub-domains and sample behaviours (Tatto et al., 2012). Finally an in-depth analysis of a ‘most difficult’ first-year MCK ratio scale test item is reported.

Chapters 5 and 6 describe the results of the longitudinal qualitative study of pre-service teachers’ MCK. They include an in-depth analysis of experiences that provided pre-service teachers with opportunities to develop breadth and depth (Ma, 1999) of MCK, as well as the four categories of the Knowledge Quartet, foundation knowledge,
transformation, connections and contingency (Rowland et al., 2009). Chapter 5 presents findings regarding opportunities and influences that enhanced pre-service teachers’ MCK during coursework, including pre-service teachers’ prior mathematical learning and opportunities that began to shape their MCK during the first two years of their primary mathematics teacher education. Chapter 6 reports results with respect to opportunities and influences that enhanced pre-service teachers’ MCK during practical teaching experiences in primary schools. First, the distribution of the longitudinal study pre-service teachers’ practicum experiences included the year levels they taught and their opportunities to practice teaching mathematics across different year levels. Then, mathematics practicum experiences for each year of the program were compared to identify growth and change in foundation knowledge (Rowland et al., 2009) and development of SCK (Ball et al., 2008).

Chapter 7 reports a comparison of pre-service teachers’ MCK in their second and their fourth years to identify development of their MCK. The topics chosen for analysis were fractions, decimals, area and perimeter. This analysis provides insights into pre-service teachers’ strategies, including correct and incorrect solution methods, to identify evidence of procedural knowledge or mathematical structure and connections (Chick et al., 2006a).

Chapter 8 contains a summary and detailed discussion of findings and concludes the study. The discussion conceptualises the findings reported in Chapters 4 to 7 by categorising pre-service teachers’ MCK as not yet demonstrating foundation knowledge, demonstrating foundation knowledge (Rowland et al., 2009) or SCK (Ball et al., 2008). This discussion, elaborates upon the different categories of MCK acquired while learning to teach primary mathematics.

The discussion also covers factors that assisted pre-service teachers to learn and develop MCK, including their identity, attitudes, beliefs and perceptions (Lerman et al., 2009), before commencing the program; when learning to teach; and as they developed professional values as teachers of primary mathematics. The chapter concludes with a discussion of the implications of the findings and recommendations for future research and primary teacher education programs.
CHAPTER TWO

Literature review

It is well documented that teachers use and need different types of knowledge for teaching primary mathematics (Chick et al., 2006a; Hill et al., 2008; Ma, 1999; Rowland et al., 2009; Shulman, 1986). Knowledge for teaching mathematics is important as it underpins a teacher’s decisions about which examples or representations to use, what connections to make during a lesson as well as how to respond to students’ thinking (Rowland, Turner, Thwaites, & Huckstep, 2009). Primary teachers, including graduate teachers, are expected to “know the content and how to teach it” (AITSL, 2011, p. 3). Pre-service teachers develop their mathematical content knowledge (MCK) and knowledge of how to teach mathematics content during coursework experiences, when responding to assessment requirements, and when planning, teaching and reflecting on their lessons with primary students in schools.

For the past two decades, research on mathematics teaching has included a focus on the knowledge teachers use and need for their craft of teaching (Grossman & McDonald, 2008). Shulman’s (1987) theoretical framework includes seven categories which became the foundation for describing the knowledge base for teaching. In addition, Shulman’s two broad classifications of content knowledge (CK) and pedagogical content knowledge (PCK) have been further developed through other studies and theoretical frameworks (e.g., Ball et al., 2008; Chick et al., 2006a; Ma, 1999; Rowland et al., 2009; Tato et al., 2012), enabling a deeper understanding of what teachers should know and what they might demonstrate when teaching mathematics.

Content knowledge is a central feature of Schulman’s (1987) framework and is referred to as the, “…amount and organisation of knowledge in the mind of the teacher…” (p. 9).

This review of the literature begins with definitions of the terminology used to describe the mathematical knowledge that teachers use when teaching. Next is an historical overview of the theoretical frameworks used to categorise the knowledge needed for teaching, including mathematics teaching. This is followed by a summary of findings of
recent studies of Australian and international studies of pre-service teachers’ and teachers’ MCK, including what beginning teachers need to know and factors impacting on knowledge. Finally key issues and future directions for research are presented.

2.1 Teachers’ mathematical knowledge

Shulman’s (1987) seminal work highlighted the importance of considering the different types of knowledge required for teaching. Initially Shulman (1986) proposed a framework involving three characteristics of teacher knowledge: CK, PCK and curricular knowledge. These categories continue to be referred to when describing the knowledge needed for teaching mathematics. Later, Shulman (1987, p. 8) suggested that a teacher’s knowledge base was comprised of seven teacher behaviours and strategies for improving teaching:

- content knowledge;
- general pedagogical knowledge, with special reference to those broad principles and strategies of classroom management and organization that appear to transcend subject matter;
- curriculum knowledge, with particular grasp of the materials and programs that serve as “tools of the trade” for teachers;
- pedagogical content knowledge, that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding;
- knowledge of learners and the characteristics;
- knowledge of educational contexts, ranging from the working of the group or classroom, the governance and financing of school districts, to the character of communities and cultures; and
- knowledge of education ends, purposes and values, and their philosophical and historical grounds.

Schulman argued that teaching a subject requires more than simply knowing the subject (Siemon et al., 2011). Shulman (1987) explained CK as concerned with expertise in the particular discipline being taught. While it is generally accepted that teacher knowledge encompasses more than just content (e.g., Ball et al., 2008; Rowland et al., 2009), Shulman (1987) and other researchers identified it as subject matter knowledge that impacts upon other types of knowledge.

Grossman’s (1990) review of the literature on teacher knowledge noted that researchers differ on their definitions of components of teacher knowledge but the categories of teacher knowledge are similar and typically include CK, PCK and other areas. Subsequent reviews have reinforced the value of these categories. For example
Liljedahl, Durand-Guerrier, Winsløw, et al. (2009) noted that there are three strands of teacher knowledge: knowing the mathematics (similar to MCK), knowing teaching, and knowing how to teach mathematics.

### 2.2 Theoretical frameworks

“Learning to teach requires a balance between teachers’ theoretical and practical knowledge and skills including knowledge of mathematics” (Novotná, 2009, p. 13). It also seems reasonable to suggest that effective mathematics teachers possess a sound understanding of the mathematics they teach (e.g., Askew, Rhodes, Brown, William, & Johnson, 1997; Fennema & Franke, 1992). Teachers’ mathematical knowledge has continued to be a much-discussed issue in contemporary debates about improving mathematics teaching and learning (e.g., Grossman, 1990; Hill, Ball, & Schilling, 2004; Mewborn, 2001; Ponte & Chapman, 2006; Tatto et al., 2012).

Many different frameworks (e.g., Ball, Bass, & Hill, 2004; Ball et al., 2008; Chick et al., 2006a; Rowland et al., 2009) have emerged documenting the complex knowledge needed for teaching mathematics. The common elements in these categories, such as MCK, have become as important to teaching as the categories themselves (Bobis et al., 2012).

#### 2.2.1 Early models of teachers’ knowledge

Early studies focused on quantitative studies and compared teachers’ responses to written tests (e.g., Eisenbert, 1977). In contrast, Fennema and Franke (1992) in their review of research published in the 1980s, presented examples that identified a change in research methodology. The studies they reviewed were described as cognitive studies, and involved qualitative research methods and related to studying teachers’ CK.

To illustrate, Leinhardt and Smith (1985) designed semantic nets to describe the relationship between elementary (primary) teachers’ subject matter knowledge and lesson structure. They reported on core knowledge of the lesson and compared beginning (pre-service teachers) and experienced teachers. These nets were similar to flowcharts, summarising teachers’ knowledge of fractions. They concluded that
experienced teachers can rely on procedural rules and also understand the interrelationship of the procedures to a greater extent than beginning teachers.

Fennema and Franke (1992), in reviewing Leinhardt and Smith’s (1985) early model of teachers’ knowledge and other studies of teachers’ knowledge in the context of the classroom, proposed a Teachers’ Knowledge framework. They combined components of teachers’ knowledge and beliefs including cognitive and affective characteristics (Figure 2.1).

Figure 2.1 Teachers’ Knowledge: Developing in Context (Fennema & Franke, 1992, p. 162).

Fennema and Franke (1992) described teachers’ knowledge of mathematics as knowledge of concepts, procedures and problem solving. Knowledge of mathematics includes how knowledge is organised as well as the teachers’ knowledge of the relationship between mathematical ideas. They explained how mathematics educators would use teacher knowledge and transform it when using materials and during teaching, and how they would continue to rely on their knowledge as students’ understanding change. They also suggest that teacher knowledge and knowledge of mathematics can evolve, is continuous, and changes during teaching; if teachers have conceptual understanding, this will assist their explanations when teaching students.
Similarly, interpreting students’ strategies can provide opportunities that promote a teacher’s MCK (Carpenter, Fennema, & Franke, 1996).

### 2.2.2 Teachers’ mathematical beliefs

In their international review of past mathematics education, Ponte and Chapman (2006) found that since the 1990s there has been a significant increase in studies related to teachers’ beliefs. Some suggest that teachers’ beliefs about mathematics usually develop during teachers’ own school experiences of learning mathematics (e.g., Beswick, 2012; Handal, 2003) and impact on their teacher knowledge (e.g. Fennema & Franke, 1992). Similarly, a review of Australasian studies of pre-service teachers acknowledged that attitudes and beliefs can influence teaching (Goos et al., 2008).

The literature has provided a range of definitions to describe teachers’ mathematical beliefs (Grootenboer, 2008). In New Zealand and Australia (Schuck & Grootenboer, 2004), there has been disagreement over whether beliefs belong to the affective domain or the cognitive domain of knowledge (Schuck & Grootenboer, 2004). Beswick (2011) used the results of her study of Australian secondary mathematics teachers to argue that beliefs form a subset of teacher knowledge. Similarly, Beswick and Callingham (2011) categorised beliefs with MCK and PCK within their conceptual framework. In contrast, others have reported that beliefs fall within the affective domain and include what teachers might believe they can do: “For example, beliefs that mathematics is ‘difficult’, ‘useless’, ‘all about one answer’, or ‘all about memorizing formulas’ stem from experiences that introduced these ideas and then reinforced them” (Liljedahl, 2005, p. 1).

Handal’s (2003) review of teachers’ beliefs about mathematics identified three concepts included in most frameworks for teachers’ mathematical beliefs: belief about what mathematics is, how mathematics teaching and learning actually occurs, and how mathematics teaching and learning should occur. An, Kulm and Wu’s (2004) conceptual understanding of teachers’ beliefs suggested that a teacher who holds “the belief of learning as understanding realises that knowing is not sufficient and that understanding is achieved at the level of internalising knowledge by connecting prior knowledge
through a convergent process” (p. 149) and described two kinds of teaching beliefs: learning as knowing and learning as understanding.

Grootenboer (2008) described three categories of belief change of primary pre-service teachers’ perceptions of mathematics. The first category was non-engagement, for example, when the pre-service teacher lacked passion or did not attempt to reconsider their mathematical beliefs during the course. Others developed a new set of mathematical beliefs during their course experiences and the third developed existing views and attitudes of mathematics and mathematics education, reforming their existing beliefs.

Newton, Leonard, Evans and Eastburn (2012) suggested that high-quality mathematics instruction relies on teacher efficacy and is related to student achievement and motivation. Similarly, Pajares (1996) wrote that:

> Efficacy beliefs help determine how much effort people will expend on an activity, how long they will persevere when confronting obstacles, and how resilient they will prove in the face of adverse situations—the higher the sense of efficacy, the greater the effort, persistence, and resilience. (p. 544)

Liljehahl (2005) described effective mathematics teaching as complex, including CK, pedagogy and PCK, as well as efficacy, which relates to the practice of teaching and perceptions or beliefs.

This literature strongly supports the notion that teachers’ beliefs are complex and most likely play an important role in a teachers’ knowledge. However, not all studies choose to include beliefs within their frameworks, as shown later in this chapter.

### 2.2.3 Cognitive knowledge

Within the past 20 years, more detailed classifications of MCK have emerged that describe and identify the knowledge teachers require when teaching mathematics. Even and Tirosh (1995) proposed two levels for MCK, explaining the terms “knowing that” and “knowing why” after observing secondary teachers’ cognitive approaches and mathematical thinking.
Knowing that includes knowledge of rules, algorithms, procedures and concepts related to specific mathematical topics.

Knowing why refers to the meaning and understanding, this affects the way a teacher presents the subject matter being able to explain the rules and respond to students’ answers, providing meaningful learning (Even & Tirosh, 1995, p. 17).

Even and Tirosh (1995) also mentioned that it can be difficult to determine when a teacher knew that or knew why, because a teacher may become confused as the situation becomes challenging and it is not always clear what MCK the teacher knows or understands. For example, knowing why could cause difficulties when a teaching situation became more complicated.

Similarly, procedural knowledge and conceptual knowledge refer to different aspects when learning mathematical concepts. Procedural knowledge can be described as knowing the rules or a step-by-step process for solving a problem (Rittle-Johnson, Sielgler, & Alibali, 2001). In contrast, conceptual knowledge is defined as understanding the principles and the interrelations between units of knowledge within a domain (Rittle-Johnson et al., 2001), and is not just knowing isolated facts and methods but being able to represent mathematics problems in different ways, having learnt with understanding (Kilpatrick, Swafford, & Findell, 2001). Furthermore, a teacher’s conceptual understanding of mathematics is important as it can support student learning (Fennema & Franke, 1992). In addition, “procedural fluency”, the “skill in carrying out procedures flexibly, accurately, efficiently, and appropriately” (Kilpatrick et al., 2001, p. 116), relies on both procedural and conceptual knowledge.

Relying solely on procedures may result in misconceptions or errors as students do not understand the mathematical connections behind the procedure (Booker, Bond, Briggs, Sparrow, & Swan, 2010; Kilpatrick et al., 2001). Students can be taught rules and formulas by rote, but those who develop understanding by means other than rote tend to remember it for future use and can apply it to new situations (Booker et al., 2010). For example, when learning multiplication facts, students should develop strategies for meaningful learning (Siemon et al., 2011), and when being taught measurement concepts they should progress through stages of development for understanding before being introduced to meaningless formulas (Booker et al., 2010).
Australian researchers have asserted there is a place for procedural learning such as naming shapes or symbols and learning, but

[m]athematics needs to involve conceptual understanding; that is, it needs to build on meaningful ideas and multiple representations and be supported by collaborative discussion, rich and challenging tasks, and personal success. (Siemon et al., 2011, p. 21)

Rittle-Johnson et al. (2001) proposed in their study of Grade (Year) 5 and Grade 6 students that the procedural and conceptual knowledge could not always be separated, and that they developed concurrently and repeatedly: as one develops it assists the other. Likewise, pre-service teachers, as part of their teacher education program, should consider how they use and extend their conceptual and procedural knowledge for teaching. Teachers are expected to understand what they teach, and require a certain depth of understanding in order to provide sound explanations of mathematical ideas (Ma, 1999). Ma used the term profound understanding of fundamental mathematics (PUFM) to refer to MCK that has breadth, depth, connectedness and thoroughness. Schoenfeld and Kilpatrick (2008) also referred to the importance of teachers knowing school mathematics in breadth and depth, with the general consensus being that this knowledge impacts upon PCK and therefore upon the effectiveness of instruction. However, the literature reveals that many elementary teachers lack conceptual understanding of mathematics (e.g., Mewborn, 2001), and that both in-service and pre-service teachers’ limited MCK and confidence with doing mathematics is of particular concern (e.g., Ball, 1990; Lange & Meaney, 2011; Ryan & Williams, 2007).

Mewborn’s (2003) review of American teachers’ knowledge concluded that many teachers have a strong procedural understanding of mathematics yet lack conceptual understanding. Likewise, reviews of Australian studies have continued to identify weaknesses in pre-service teachers’ MCK, with many indicating that pre-service teachers rely on procedural methods (Goos et al., 2008). This reliance on procedural methods is arguably a result of teachers’ pre-program identity, developed when they were taught at school – before commencing teacher education (Ponte & Chapman, 2008).

How pre-service teachers learn when at school can be affected by different factors depending on when and where they completed their primary and secondary education.
Until the mid-1980s, learning mathematics in Australian primary and secondary schools focused on procedural techniques in which students were taught the rules and completed exercises from textbooks (Siemon et al., 2011). Now, each of the eight states and territories follows the Australian Curriculum Mathematics (ACM), and four proficiency strands describe how the mathematics content is explored and developed: Understanding, Fluency, Problem-solving and Reasoning (ACARA, 2013).

Kilpatrick, Swafford, and Findell (2001), in their research into cognitive psychology and mathematics education, suggested that to learn mathematics successfully required the following intertwining five strands of mathematics proficiency:

- **Conceptual understanding** – comprehension of mathematical concepts, operations and relations;
- **Procedural fluency** – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- **Strategic competence** – ability to formulate, represent and solve mathematical problems;
- **Adaptive reasoning** – capacity for logical thought, reflection, explanation, and justification; and
- **Productive disposition** – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy. (p.116)

Note that the first four of the five strands of proficiency are similar to the ACM’s four proficiencies: Understanding, Fluency, Problem Solving and Reasoning of the (ACARA, 2013). Kilpatrick et al.’s (2001) fifth strand – productive disposition – incorporates one’s own efficacy of mathematics, but this concept is not a proficiency strand of the ACM. Productive disposition is a domain of knowledge of mathematics that both the student and teacher can possess. However, it should be expected that primary teachers have the knowledge to demonstrate the four proficiencies as well as value mathematics with productive disposition if they are to teach the ACM.

### 2.2.4 Comparison of teachers’ MCK

During the 1990s researchers conducted projects that provided insights into models for improving teaching (Mewborn, 2003). Ma (1999) documented the differences between American and Chinese elementary teachers’ MCK by distinguishing the key elements of their knowledge of mathematics. For example Ma reported on teachers’ understanding related to teaching subtraction with regrouping, noting that even with a simple two-digit subtraction problem a wide range of MCK was demonstrated by the teachers in her
study. When subtracting with regrouping, many of the American teachers focused on “borrowing”; Ma described these teachers as the “procedurally directed group” (p. 5). In comparison, the Chinese teachers were able to explain that the algorithm was not the only way to explain subtraction and discussed various ways of decomposing a digit, for example, “regrouping” 53 as 40 and 13 or 40 and 10 and 3. In their explanations, the Chinese teachers described a step-by-step process, justifying their choices of methods and the connections within their knowledge package.

Ma (1999) represented the thinking that the Chinese teachers described as a “knowledge package”. Figure 2.2 depicts a knowledge package used to demonstrate the connections and relation of subtraction with regrouping. The shaded ellipses are the key ideas; the unshaded ellipses represent other related pieces of knowledge, and the arrows assist with knowing the order for developing mathematical connections and understanding both conceptually and procedurally when learning subtraction.

![Knowledge package, subtraction with regrouping](Ma, 1999, p. 19).

Ma (1999) reported that the American teachers followed direct procedures that lacked conceptual underpinnings. By comparison, the Chinese teachers discussed the problems with interconnections and demonstrated conceptual understanding, wanting to “know how and know why” (p. 108). The Chinese teachers’ knowledge was described as PUFM.
A profound understanding of mathematics has breadth, depth, and thoroughness. Breadth of understanding is the capacity to connect a topic with topics of similar or less conceptual power. Depth of understanding is the capacity to connect a topic with those of greater conceptual power. Thoroughness is the capacity to connect all topics. (Ma, 1999, p. 124)

Ma (1999) suggested that PUFM was attained during Chinese teachers’ teaching careers and built on what the teachers learnt during their own schooling. Chinese teachers learn from their colleagues, learn by doing mathematics and solving problems in several ways; they learn when teaching mathematics with their students and when studying teaching materials.

Mewborn (2003) noted that 80 percent of the Chinese teachers only taught elementary mathematics, whereas American elementary teachers were required to teach a range of subjects. In this regard, Australian teachers are similar to American. At the time of this study, Australian primary teachers were generally expected to teach in seven key learning areas: English, Mathematics, Science, Technology, Health and Physical Education, The Arts and Studies of Society and Environment (DEECD, 2009b).

Ma’s study was described as a new approach and perspective on what teachers knew and how they articulated their mathematical knowledge when teaching (Even & Ball, 2003). The findings were significant for the professional development of teachers as they identified criteria that promoted multiple approaches for improving the quality of teachers’ knowledge of the elementary mathematics curriculum and a much broader interest in the nature of teachers’ subject matter understanding (Ball et al., 2004).

Schoenfeld and Kilpatrick (2008), for example, in their provisional framework for proficiency in teaching mathematics, described one category a proficient teacher required as knowing school mathematics in breadth and depth relating to “broad and connected knowledge of the content at hand… an understanding of big ideas… knowledge of effective ways to introduce students to particular mathematical ideas, and ways to instil understanding” (p. 327).

2.2.5 Domains of mathematical knowledge for teaching

Ball’s (1993) study of dilemmas in teaching elementary school mathematics was a turning point that commenced her journey of investigating different domains of mathematical knowledge for teaching and learning during primary mathematics lessons.
When teaching she focused on significant mathematical content. She audiotaped and videotaped her mathematics classes and spent considerable time unpacking her teaching whilst analysing students’ learning.

Later Hill, Ball and Schilling’s (2004) review of literature confirmed that further studies were required to identify what MCK teachers must know for elementary teaching. Together they developed a set of multiple-choice questions designed to measure growth in American elementary teachers’ MCK consisting of two types of questions: those relating to common content knowledge (CCK) and specialised content knowledge (SCK). These items were designed to investigate how teachers hold their knowledge: for example, “whether they can use their mathematical knowledge to generate representations, interpret student work, or analyse student mistakes” (Hill et al., 2004, p. 28). They videotaped the teachers during their mathematics lessons and categorised the mathematical skills and knowledge teachers displayed when they posed questions and gave explanations, chose tasks, used representations, recorded mathematics on the board, sequenced examples, analysed students’ errors, appraised, mediated, and so on (Thames & Ball, 2010).

Building on the scholarly work of Shulman (1987; 1998) Ball, Hill and colleagues (Ball & Bass, 2003; Ball et al., 2004; Ball et al., 2008; Hill et al., 2004; Hill et al., 2008) developed the MKT framework (Figure 2.3). They extended the theories of the 1980s, and were described as broadly distinguishing the categories of teaching-specific content knowledge (Chick, 2011); however, it did not include affective characteristics, for example, beliefs about mathematics. Ball et al.’s (2008) MKT framework was designed for identifying mathematical knowledge when investigating teachers in practice. The framework included two domains: subject matter knowledge and PCK (Figure 2.3). The MKT framework built on the scholarly work of Shulman (1987; 1998) but was practice-based because it was used and tested in the field of teaching.
Within the MKT framework were three categories of Shulman’s (1986) initial categories of subject matter knowledge, PCK and curriculum knowledge. However, Ball et al. (2008) chose to include curriculum knowledge within PCK. The first part of the MKT framework classified subject matter knowledge, including CCK, SCK and horizon content knowledge (HCK); the second part related to PCK, including knowledge of content and students, teaching and curriculum.

Although Ball and Bass (2009) agreed with Shulman’s definition of PCK, they argued that a richer understanding of this commonly used term, including three sub-domains, knowledge of content and students (KCS), knowledge of content and curriculum and knowledge of content and teaching (KCT) were required. Similarly, Brophy (1991) explained PCK as the organisation of knowledge, of what and how to teach a topic in combination with knowledge of instruction, activities and evaluation tools. Others have suggested that if teachers lack MCK it may affect their PCK and the way they respond to student’s questions (Goulding, Rowland, & Barber, 2002), and that a teacher’s CCK may also shape the way they use their pedagogical judgements (Barnes, 1989; Bennett, 1991).
1993; Shulman, 1986). Therefore, subject matter knowledge and PCK overlap as one relies on the other.

Teachers rely on CCK for planning and teaching, enabling teachers to know the mathematics they teach, check a student’s answer, respond to the definitions of a concept or complete a mathematical procedure (Ball & Bass, 2003; Thames & Ball, 2010). However, CCK is not exclusive to teachers; any adult may have well-developed CCK but most likely will lack the knowledge required to teach it (Hill et al., 2004). In addition, Thames and Phelps (2008) compared CCK and SCK and concluded that an effective primary teacher requires more, not less, content knowledge than the average adult.

Effective teachers who have SCK demonstrate CCK which is more than knowing the content and is unique to teaching (Ball et al., 2008). The following is a list of tasks that teachers with well-developed SCK can perform:

- Selecting/designing instructional activities;
- identifying and working toward the mathematical goal of the lesson;
- listening to and interpreting students’ responses;
- analysing student work; teaching students what counts as “mathematics” and mathematical practice;
- making error a fruitful site for mathematical work;
- attending to ambiguity of specific words; and
- deciding what to clarify, make more precise, leave in student’s own language. (Ball & Bass, 2009, p. 4)

Morris, Hiebert, and Spitzer’s (2009) research showed that pre-service teachers could demonstrate SCK and learn to explain mathematical ideas in several ways and justify their responses when responding to tasks but had difficulty applying this knowledge to teaching and learning situations. Their participants were asked to provide an ideal student response to a task that demonstrated understanding of the concept – for example, “solve one quarter plus three eighths by drawing a diagram on graph paper” (p.500). In the context of this study, pre-service teachers were beginning to develop SCK as part of their teacher education program.

HCK is the third category of subject matter knowledge (Ball et al., 2008). HCK is a kind of peripheral vision that informs mathematics teaching practice; when a teacher demonstrates understanding of the complexities of mathematical topics, has advanced
knowledge, broad understanding of mathematical ideas and connections, and links their CK with curriculum that their students know and will know in future years (Ball & Bass, 2009; Ball et al., 2004; Ball et al., 2009; Ball et al., 2008). This is similar to breadth, depth and thoroughness, or PUFM (Ma, 1999).

HCK might be demonstrated when teachers know the questions to prompt student understanding of mathematical proofs or know when to assist learning as well as when to be patient, thereby allowing their students to work through problems independently, as well as noticing and evaluating mathematical opportunities (Ball & Bass, 2009; Ball et al., 2009; Ball et al., 2008). Australian researchers have suggested that primary teachers should develop their knowledge of content so that they possess HCK and are aware of the range of strategies students will bring to tasks (Sullivan, Clarke, & Clarke, 2009).

Ball et al. (2008) defined “knowledge of content and students” as a combination of a teacher’s knowledge about the content of mathematics and about the students they teach. For example, they must anticipate how a student might think or if the task will be easy or hard for a student as well as anticipating students’ misconceptions of a topic.

“Knowledge of content and teaching” is used when teachers choose tasks that assist students with how they think about and learn a particular mathematical concept (Hill et al., 2008). Ball et al. (2008) described knowledge of content and teaching as a combination of what teachers know about teaching and their MCK. For example, the teacher can choose a task which will assist students to deepen their knowledge of mathematics. This category is drawn from Shulman’s (1986) theoretical framework and the category of PCK and understanding learning (Hill et al., 2008). Finally, “knowledge of content and curriculum” (Ball et al., 2008) also stems from Shulman’s (1987) category of curricular knowledge, consisting of the scope and sequence of the teaching syllabus.

2.2.6 Teacher knowledge for pre-service teachers

Liljedah et al.’s (2009) knowledge for teaching diagram illustrates teacher knowledge as more united than Ball et al.’s (2008) MKT framework. A braid illustrates the
developing proficiency used to describe the knowledge pre-service teachers need for teaching, including “knowing the mathematics, knowing teaching, and knowing how to teach mathematics” (p. 25) (Figure 2.4).

![Figure 2.4. Teacher knowledge needed during teacher education (Liljedahl et al., 2009, p. 31).](image)

The Knowledge for Teaching framework represents the interconnections of CK to pedagogical knowledge and didactical knowledge. Didactics is another term used for describing aspects of knowledge of teaching. In Winsløw’s (2007) study of the didactics of mathematics education, didactics was defined as “the study of the teaching and learning of specific knowledge, usually within a disciplinary domain” (p. 532). Bennet and Turner-Bisset (1993), in their study of the relationship of pre-service teachers’ subject matter knowledge for teaching, used didactic to mean direct instruction during teaching.

Liljedah et al. (2009) suggested that CK, pedagogical knowledge and didactical knowledge should become the unified knowledge of pre-service teachers by course completion, winding tighter and tighter during their program. They start as discrete knowledge and form integrated knowledge before uniting as initial teacher education knowledge.
2.2.7 Content knowledge in a pedagogical context

Chick, Baker and Cheng (2006) combined PCK and MCK in their theoretical framework, which was used to define teachers’ and pre-service teachers’ knowledge when analysing their responses to interview items. This framework was particularly useful for identifying “subtle difference between teachers’ responses, which may be attributed to differences in knowledge” (Chick & Baker, 2005, p. 256).

Chick et al.’s (2006a) framework for analysing PCK has three sections: Clearly PCK, Content knowledge in a Pedagogical Context and Pedagogical Knowledge in a Content Context. The second section, Content Knowledge in a Pedagogical Context, was selected for this review of literature because it focuses on MCK and lists five categories for classifying different aspects of how a teacher may demonstrate their MCK (Table 2.1). Many of these categories combine or include categories from other frameworks (e.g., Ball, 2000; Ma, 1999; Shulman, 1986, 1987).

Table 2.1
Content Knowledge in a Pedagogical Context (Chick et al., 2006a, p. 299)

<table>
<thead>
<tr>
<th>PCK Category</th>
<th>Evident when the teacher…</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profound Understanding of Fundamental Mathematics</td>
<td>Exhibits deep and thorough conceptual understanding of identified aspects of mathematics</td>
</tr>
<tr>
<td>Deconstructing Content to Key Components</td>
<td>Identifies critical mathematical components within a concept that are fundamental for understanding and applying that concept</td>
</tr>
<tr>
<td>Mathematical Structure and Connections</td>
<td>Makes connections between concepts and topics, including interdependence of concepts</td>
</tr>
<tr>
<td>Procedural Knowledge</td>
<td>Displays skills for solving mathematical problems (conceptual understanding need not be evident)</td>
</tr>
<tr>
<td>Methods of Solution</td>
<td>Demonstrates a method for solving a mathematical problem</td>
</tr>
</tbody>
</table>

Deconstructing Content to Key Components is evident when a method or estimation is used to check an answer and a teacher can identify the critical elements of the concepts (Chick et al., 2006b). This category relates to Ball’s (2000) thinking that expert
knowledge of mathematics may not be sufficient as teachers are also required to unpack MCK when teaching.

Mathematical Structure and Connection relates to the teacher’s connection of concepts and topics when teaching (Chick et al., 2006b). This could be the depth and thoroughness of teachers’ mathematical knowledge (Ma, 1999; Schoenfeld & Kilpatrick, 2008).

Ma (1999) suggested that a teacher learns breadth and depth of MCK when teaching. These first three categories (Table 2.1) demonstrate more than CCK, as they are features a teacher requires for their teaching, referred to as SCK (Ball et al., 2008). All categories are relevant to PCK (Chick et al., 2006a). The last two categories of Content Knowledge in a Pedagogical Context (Chick et al., 2006a), procedural knowledge and methods of solutions, can be used by teachers and most adults in their work, therefore these could be described as CCK.

Procedural knowledge can be used for solving mathematical problems (Chick, et al., 2006b). When elaborating on the mathematics and comparing different student responses, the teacher with only procedural knowledge may not be able interpret a student’s solution insightfully. Such teachers lack the ability to deconstruct the method of solution, have difficulties identifying common misconceptions, and may not display conceptual understanding (Baker & Chick, 2006). Methods of solution are evident when the teacher displays one method to solve the problem (Chick et al., 2006a).

2.2.8 The Knowledge Quartet

The final framework presented in this review of literature focuses on identifying teachers’ MCK in action. The Knowledge Quartet framework has been used when observing beginning primary mathematics teachers (Rowland et al., 2009; Thwaites, Huckstep, & Rowland, 2005; Turner, 2008) and more recently secondary pre-service teachers (Thwaites, Jared, & Rowland, 2011) to help them to improve their mathematics teaching. Pre-service teachers and teachers are encouraged to use the framework as a tool for reflecting to enhance their development of MCK when teaching (Rowland et al., 2009).
The Knowledge Quartet framework focuses on MCK rather than organisation and classroom management (Rowland et al., 2009). Rowland and colleagues observed trainee teachers teaching, then identified 18 codes and grouped these into four categories – foundation knowledge, transformation, connection and contingency – to classify how they demonstrated MCK (Table 2.2).

Table 2.2  
*The Four Categories and 18 Codes of the Knowledge Quartet Framework (Rowland et al., 2009, p. 29)*

<table>
<thead>
<tr>
<th>Category</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foundation</td>
<td>Adheres to textbook</td>
</tr>
<tr>
<td></td>
<td>Awareness of purpose</td>
</tr>
<tr>
<td></td>
<td>Concentration on procedures</td>
</tr>
<tr>
<td></td>
<td>Identifying errors</td>
</tr>
<tr>
<td></td>
<td>Overt subject knowledge</td>
</tr>
<tr>
<td></td>
<td>Theoretical underpinning</td>
</tr>
<tr>
<td></td>
<td>Use of terminology</td>
</tr>
<tr>
<td>Transformation</td>
<td>Choice of examples</td>
</tr>
<tr>
<td></td>
<td>Choice of representation</td>
</tr>
<tr>
<td></td>
<td>Demonstration</td>
</tr>
<tr>
<td>Connection</td>
<td>Anticipation of complexity</td>
</tr>
<tr>
<td></td>
<td>Decisions about sequencing</td>
</tr>
<tr>
<td></td>
<td>Making connections between concepts</td>
</tr>
<tr>
<td></td>
<td>Making connections between procedures</td>
</tr>
<tr>
<td></td>
<td>Recognition of conceptual appropriateness</td>
</tr>
<tr>
<td>Contingency</td>
<td>Deviation from agenda</td>
</tr>
<tr>
<td></td>
<td>Responding to children’s ideas</td>
</tr>
<tr>
<td></td>
<td>Use of opportunities</td>
</tr>
</tbody>
</table>

Foundation knowledge relates to knowledge and procedure of content; the other three categories relate to a teachers’ use of knowledge in the classroom. The two categories of Shulman’s (1987) framework, subject matter knowledge and PCK, were considered when designing this framework (Rowland et al., 2009); therefore, some of the codes include evidence of MCK and/or PCK. Subsequent work showed that the Knowledge Quartet was an effective tool for developing teachers’ MCK as well as their PCK (Turner & Rowland, 2011).
Foundation knowledge assists teachers to make decisions for primary mathematics teaching and underpins the other three dimensions of the Knowledge Quartet framework (Rowland et al., 2009). This category includes codes concerning subject knowledge and is evident when planning and teaching. In addition, and unlike the MKT framework (Ball et al., 2008) (Figure 2.3), foundation knowledge also includes a teacher’s beliefs about mathematics, such as a clear awareness of the purpose of the mathematics education (Thwaites et al., 2005).

The knowledge a teacher brings to the classroom is fundamental as it underpins the decisions they make during teaching (Rowland et al., 2009). In particular, foundation knowledge concerns pre-service teachers’ readiness for their teaching role (Rowland, Huckstep, & Thwaites, 2005). Others agree a teacher needs to understand how to present facts and concepts, how to make representation of ideas comprehensible to support student learning (Shulman, 1987) as well as create learning opportunities for their students (Reynolds, 1995).

As MCK and understanding are key components of foundation knowledge (Rowland et al., 2005), three descriptors of Chick et al.’s (2006a) framework including deconstructing content to key components, procedural knowledge and methods of solution align with the Knowledge Quartet framework (2009). Deconstructing content to key components may be demonstrated when teachers understand how to identify and check errors. Procedural knowledge and methods of solution include an accurate understanding of mathematical ideas.

Thwaites et al. (2005) described transformation as the teacher’s ability to transform their MCK. This category identifies how the teacher is required to use what they know when presenting ideas to their students through examples, procedures or student activities. A teachers’ choice of representation will assist student learning (Rowland et al., 2009). Transformation relates to Ball’s (1988) category knowledge of content and teaching when teachers choose tasks that assist with learning. It also relates to Chick et al.’s (2006a) category of being able to deconstruct content to key components, as the pre-service teacher makes choices for representing or carrying out and checking a procedure.
Transformation would be evidence of Ball et al.’s (2008) SCK, as pre-service teachers develop this knowledge as part of their teacher training considering different ways to make mathematics accessible to students. Similarly, Fennema and Franke (1992) specified, “transform” as an important teaching action and suggested that this action “distinguishes a mathematics teacher from a mathematician” (p. 153).

Connection relates to the “coherence of the planning or teaching across an episode, lesson or series of lessons” (Rowland et al., 2009, p. 31). This category is demonstrated when a teacher sequences tasks within a lesson and across a series of lessons (Thwaites et al., 2005) so as to assist students to progress and make connections in their mathematical learning (Rowland et al., 2009). Making connections has been explained by other researchers as having breadth and depth of knowledge, connecting a topic within topics (Ma, 1999; Schoenfeld & Kilpatrick, 2008); mathematical structure and connection of concepts and topics when teaching (Chick et al., 2006b) and connected knowledge within the curriculum (Ball et al., 2009; Ball et al., 2008).

Contingency is when a teacher is presented with an unexpected teaching event during their lesson and has to decide how to respond (Rowland et al., 2009). A student might ask a question or provide a response which the teacher did not expect – a “teachable moment” (Clarke et al., 2002); in responding, the teacher will be required to draw on their MCK and PCK which will determine the quality of the response (Rowland et al., 2009). Other frameworks presented in this review of literature do not describe actions relating to contingency with reference to MCK.

2.2.9 Mathematics cognitive domains

An international study, Teacher Education Development Study (TEDS-M) implemented a Cognitive Domains framework to classify the MCK pre-service teachers demonstrated when responding to MCK and PCK test items (Tatto et al., 2008b). TEDS-M included data collected from 17 countries and reported pre-service teachers’ knowledge for teaching mathematics by cognitive domains; this coding is also used in Trends in International Mathematics and Science Study (TIMMS) when reporting students’ results.
The mathematics framework by cognitive domain has three sub-domains: Knowing, Applying and Reasoning. Each of these domains has sample behaviours used to identify the differences between the subdomains (International Association for the Evaluation of Educational Achievement (IEA), 2012) (Table 2.3).

**Table 2.3**


<table>
<thead>
<tr>
<th>Sub-domain</th>
<th>Sample Behaviour</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowing</td>
<td>Recall, recognize, compute, retrieve, measure, classify/order</td>
</tr>
<tr>
<td>Applying</td>
<td>Select, represent, model, implement, solve routine problems</td>
</tr>
<tr>
<td>Reasoning</td>
<td>Analyze, generalize, synthesize/integrate, justify, solve non-routine problems.</td>
</tr>
</tbody>
</table>

The three sub domains of Knowing, Applying and Reasoning make links to the categories of Rowland et al.’s (2009) Knowledge Quartet (Table 2.2). Knowing would be coded as foundation knowledge and evidence of concentrating on procedures. The sub-domain of Applying would be coded as transformation and making choices of representations. Reasoning would be coded as connection as decisions are made between concepts or procedures. Other researchers have discussed the cognitive demands, the act of knowing, which can be demonstrated by a teachers’ knowledge, reasoning, decision-making and reflecting (Ball, 2000), including an awareness of the cognitive demands of the topics and tasks they select (Rowland et al., 2009).

### 2.2.10 Summary of frameworks

From a research perspective, frameworks of teacher knowledge can be used to describe the knowledge used for effective teaching (Bobis et al., 2012). Shulman (1987) initially focused on the knowledge a teacher holds for teaching, his theory of teachers’ mathematical knowledge was recognised as a seminal theory of teacher knowledge. Subsequently, others added cognitive aspects such as beliefs (e.g., Fennema & Franke, 1992). Many of the other frameworks within this literature review (e.g., Ball et al.,
2008; Chick et al., 2006b; Ma, 1999; Rowland et al., 2009), deepen our understanding of the similarities and differences of the categories of MCK.

2.3 Learning to teach and MCK

Research on learning to teach and MCK is important for understanding how to help prepare pre-service teachers to teach mathematics effectively. Previous reviews of teachers’ MCK have reported on Australasian research (e.g., Clements, 2008) as well as international studies (e.g., Adler et al., 2005; Mewborn, 2001). Studies of pre-service teachers’ MCK tend to be small scale and often involve teacher educators reporting on their own pre-service teachers’ mathematics education (Adler et al., 2005; Anthony, Beswick, & Ell, 2012). Larger comparative studies of pre-service teachers’ MCK have recently been published. TEDS-M compared pre-service teachers’ MCK across 17 countries (Tatto et al., 2012) but did not include Australia. Callingham, Beswick, Chick et al. (2011) reported on Australian pre-service teachers’ MCK and PCK from seven universities (not including the university, in which this study took place). These and other studies provide evidence of strengths and weaknesses in pre-service teachers’ MCK.

2.3.1 Review of Australasian studies

Since 1984 the Mathematics Education Research Group of Australasia (MERGA) has published four-yearly reviews of research in mathematics education conducted in Australia, New Zealand and the Pacific Nations. Each of the past three issues has included a review of research into teacher education and/or pre-service teacher education (Anthony et al., 2012; Goos et al., 2008; Southwell, White, & Klein, 2004). The research reviewed, and summarised below concerned pre-service teachers’ knowledge and beliefs.

Southwell, White and Klien’s (2004) review of teacher education and learning to teach mathematics noted an increase in the number of studies relating to pre-service teachers’ MCK as well as teacher beliefs about mathematics teaching. Ambrose (2004) noted that changing pre-service teachers’ beliefs about mathematics could be difficult. Burges and
Bicknell (2003) raised concerns regarding the limited time devoted to learning to teach primary mathematics as part of the teacher education program.

The process of becoming a teacher can be shaped by prior beliefs about mathematics teaching (Anthony et al., 2012) and in turn can shape how pre-service teachers imagine mathematics is taught (Hodgen & Askew, 2007; Walshaw, 2008). Smith and Lowrie (2001) suggested that primary pre-service teachers require reflective learning experiences during teacher education so they develop a mathematics teaching persona and beliefs that reflect teaching mathematics for understanding. Dole and Beswick (2002) asserted that tutorials provided hands-on tasks that helped reduce anxiety and change of primary pre-service teachers’ beliefs. Zevenbergen (2004) identified the use of study groups as enhancing positive learning for pre-service teachers requiring support in developing their MCK. These studies identified how MCK changed over a short time frame such as after completing a single mathematics, teaching unit.

However, Southwell, White and Klien (2004) stated that it is not clear whether pre-service teachers could sustain change and what factors contribute to sustainability. Anthony et al. (2012) identified a need to further understand how pre-service teachers use knowledge in practice with students and how to assess pre-service teachers’ learning of knowledge.

There are many ways to examine pre-service teachers’ MCK (Callingham et al., 2012) however, many studies have included multiple choice or questionnaires rather than observation or interviews. Clement’s (2008) review between 2004 and 2007 identified many researchers used questionnaires to compare pre-service teachers’ MCK and suggested multiple data collection methods would have strengthened these studies. Chick (2003) commented that written tests are useful for larger studies but interviews would be more likely to provide high-quality information about teachers’ understanding of their MCK. Southwell and Penglase (2005) recommended that all pre-service programs should include a screening test to identify any weaknesses pre-service teachers have and then design appropriate programs to support them. In contrast, Linsell and Anakin (2012) suggested that tests might not provide evidence of pre-service
teachers’ understanding of MCK apart from procedural knowledge or methods of solutions.

Butterfield and Chinnappan (2010) concluded that although pre-service teachers can build on their MCK, they have difficulty in transforming this knowledge when designing tasks for students and providing rich learning experiences that demonstrate SCK. To support pre-service teachers’ development of the categories of MKT, Butterfield (2012) suggested that programs immerse pre-service teachers in practicum experiences in which they experience activities with students and can draw out mathematical concepts related to the curriculum. Partnerships between the university and schools are also important for maximising pre-service teachers’ capacity to learn (Arnold, Edwards, Hooley, & Williams, 2011; McDonough & Sexton, 2011). Experiences need to be shared, that is, between the pre-service teacher and university lecturers, and practising teachers sharing or learning together within the school setting (McDonough & Sexton, 2011; Walshaw, 2009). However, Zevenbergen (2005) expressed concern that pre-service teachers’ practicum experiences could override theory learnt at university, and recommended pre-service teachers undergo practicum experiences that align with effective teaching.

Researchers have explored pre-service teachers’ development of teacher identity as a means of fostering knowledge for teaching mathematics. Walshaw’s (2008) theory of learning to teach recognised the construction of identity as important, including three factors that contribute to and influence pre-service teachers’ identity: past educational experiences as a student, teacher education programs (including coursework) and practicum experiences.

Identity as a mathematics thinker can develop through self-directed learning using a combination of learning experiences, social interactions and technology supports (Owens, 2007/2008). Klien (2008) encouraged pre-service teachers to initiate new pathways as part of their developing teacher identity by experiencing tasks in tutorials that emphasised reasoning and making connections. Some pre-service teachers in Klein’s study indicated that tutorial experiences were different to how they had learnt mathematics at school.
Several studies have identified factors that contribute to pre-service teachers’ MCK. As noted earlier, many of these studies have been small in scale, often involving researchers investigating their own students (e.g., McDonough & Sexton, 2011; Walshaw, 2009). Few large, comparative studies of pre-service teachers’ MCK across universities exist (e.g., Callingham et al., 2011; Tobias & Itter, 2007).

Tobias and Itter’s (2007) study of rural Australian first-year pre-service teachers from diverse backgrounds found that more than half of a cohort of 397 pre-service teachers had difficulty when responding to Year 8 MCSK test items. They also relied on incorrect procedural knowledge; males scored better than females; mature-age pre-service teachers had slightly more difficulty responding to items than those who had recently completed secondary school studies. Meany and Lange (2010) reported that some primary pre-service teachers in their program attempted a similar test four times before passing and that this experience promoted emphasis on procedural rather than conceptual understanding. Others agree that pre-service teachers who continue to struggle with their MCK will have difficulty identifying student errors (Walshaw, 2012; Zevenbergen, 2005) and transforming their knowledge for lesson planning (Murphy, 2011).

Callingham et al. (2011) undertook a two-year project, Building the Culture of Evidence-based Practice in Teacher Preparation, across seven Australian universities representing all states and the Northern Territory. The purpose of this study was to identify (using online surveys) the MCK and PCK of pre-service teachers at the end of their program. No statistically significant differences in MCK or PCK existed between part-time and full-time pre-service teachers, but pre-service teachers had more difficulty with the PCK items than the MCK items. The MCK item with which pre-service teachers had most difficulty was knowledge of prime factors of 30; they also struggled with the MCK item related to knowledge of fractions, including placing a set of fractions in proportion on a number line. Teachers had most difficulty with the PCK items related to measurement and geometry.

Few Australian longitudinal studies of pre-service teachers’ MCK exist. Afamasaga-Fuata'i (2007) reported findings about first-year Samoan pre-service teachers’ MCK in
the first part of a study designed to assess their MCK again in fourth year. They demonstrated conceptual connections after creating concept maps of mathematics topics. Further findings from this longitudinal study have not been reported as yet.

Australasian reviewers of research in this field agree that further studies are needed that focus on how teachers learn the knowledge for teaching; there has been limited research on the development of pre-service teachers’ mathematical knowledge over time (Anthony et al., 2012; Goos et al., 2008). Southwell, White and Klien (2004) suggested that “longitudinal studies that track the mathematical conceptual development of outstanding teacher education students” (2004, p. 209) are needed.

2.3.2 Review of international studies

Difficulties in pre-service teachers’ MCK have not been confined to Australia; educators in other countries, such as the UK and USA, have also expressed concerns. Early in this chapter, different theoretical frameworks for teachers’ knowledge were described. Some international reviews of teachers’ MCK combined elementary (kindergarten to grade 8) pre-service teachers and teachers (Ball, Lubienski, & Mewborn, 2001; Mewborn, 2001). Ball et al. (2001) presented an early critique of American research focusing on the role of teachers’ MCK, noting “354 articles that dealt specifically with mathematics teaching and learning” (p. 434) including research on teacher knowledge and beliefs.

Mewborn (2001) reviewed the different phases and genres of research into teachers’ MCK. The first phase, 1960-1970, mainly included quantitative studies, followed by descriptive studies that attempted to identify the strengths and weakness of content areas. For example, Eisenbert (1977) compared results of written tests and found no correlation between junior high school teachers’ MCK and student performance in algebra. A “flurry of descriptive studies” followed, using both qualitative and quantitative research methods, focusing on strengths and weakness of teachers’ MCK with respect to different mathematical topics (Mewborn, 2001). During this period, Ma (1999) completed her comparative study of American and Chinese elementary teachers’ MCK. This study was discussed earlier in this chapter, and described Chinese teachers as having PUFM because of the breadth, depth and thoroughness of their MCK when
responding to interview items. In contrast, the American teachers had difficulty demonstrating the knowledge needed for teaching because of how they approached mathematical problems; they had limited conceptual understanding and demonstrated less breadth and depth of MCK when compared with their Chinese counterparts.

Goulding, Rowland and Barber’s (2002) qualitative study of four English and Welsh pre-service teachers found a link between difficulties in MCK and weaknesses in their planning and teaching of mathematics when in schools. Their results suggest some pre-service teachers are unable to respond to students’ questions and choose to teach in lower year levels because of difficulty in understanding or demonstrating procedural knowledge.

Mewborn (2001) noted that longitudinal studies of pre-service teachers’ development during coursework and teaching experiences were missing from the literature. In addition, she suggested the field needed studies of pre-service teachers who demonstrate conceptual knowledge of mathematics, stating that “it would be useful to know how, when, and where these teachers develop this conceptual understanding” (2001, p. 34). One such study, by Cady, Meir and Lubinskie (2006), followed two American pre-service teachers who had completed the same teacher education program but demonstrated differences in their instructional practice in their sixth year of teaching: one reverted to traditional beliefs when teaching mathematics, the other demonstrated cognitively based instruction. The findings suggested that professional learning, specific to mathematics education rather than general professional learning as a teacher caused the differences.

Adler et al. (2005) reviewed international studies from 1999 to 2003 across three major international journals from English-speaking countries (the Journal of Mathematics Teacher Education, Educational Studies in Mathematics, and the Journal for Research in Mathematics Education) as well as conference papers (e.g., the Psychology of Mathematics Education (PME) conference proceedings) and the Second International Handbook of Mathematics Education (2003). They classified pre-service teachers as teachers for the purposes of their survey. Adler et al. (2005) asserted that three types of studies of teachers’ MCK were missing from previous research: large-scale studies,
including across countries; cross-case analyses; and longitudinal studies. Ball commented that “cross-case analyses would provide opportunities to learn about different approaches, programs, and settings that affect the content knowledge teachers need to learn how to teach” and because content knowledge develops over time, longitudinal studies are important for “understanding how teachers learn” (p. 370). Lin added that replication of existing studies could provide a means to challenge and strengthen current theories.

Ponte and Chapman (2006), in a review of PME papers of mathematics teachers’ knowledge and practices, reported that many papers focus on deficiencies in teachers’ mathematics knowledge; they noted a variety of methodologies, but a lack of studies that explored effective teachers’ knowledge and what this could look like; and encouraged future researchers to reconsider theoretical and methodological orientations as well as develop innovative research designs.

Blömeke and Delaney (2012) agreed that most studies of pre-service teachers were small scale and local. They noted that most studies were of primary pre-service teachers and few of practising primary teachers, and a focus on secondary mathematics pre-service teachers and teachers, was growing. They suggested that further funding should be made available for larger studies of practising teachers’ MCK across countries.

Two larger studies have investigated pre-service teachers MCK and PCK (Beswick & Callingham, 2011; Tatto et al., 2012). The TEDS-M study across 17 countries assessed prospective (pre-service) teachers’ MCK and PCK using multiple-choice items (Blömeke & Delaney, 2012). Due to the diversity of program structures within and across countries, it was difficult to compare results across the 17 countries represented in the TEDS-M study (Tatto et al., 2012). Pre-service teachers were provided with different opportunities to learn that lead to a range of knowledge outcomes. However, the overview of results did suggest that by the end of their program pre-service teachers’ “knowledge varies considerably among individuals within every country and across countries” (p. 202). In addition, pre-service specialist mathematics teachers had higher MCK scores than pre-service teachers qualifying to teach lower-primary
generalists; it was also notable that primary level teachers were predominantly female. Other results are summarised in the next section.

The findings of both these studies were limited because none of the pre-service teachers was interviewed to extend understanding of their method of solutions used when responding to the multiple-choice items. Data collection occurred near the end of their program, included responses to multiple-choice items so could not identify factors influencing pre-service teachers’ MCK during different stages of the program or map development of MCK over time.

2.3.3 Reviews of strengths and weaknesses of teachers’ MCK

A range of studies have focused on the weaknesses and strengths of teachers’ MCK and highlighted the challenges for teacher education in assisting pre-service teachers to develop and extend their MCK. Mewborn’s (2001) review of literature concluded that previous studies mainly focused on number—addition and subtraction of whole numbers, patterns and counting, with few studies on knowledge of probability, data analysis, functions, transformation geometry and number theory.

A later review found that pre-service teachers have difficulties with many mathematical topics with concerns of how they rely on their MCK “in terms of conceptual and procedural knowledge” (Ponte & Chapman, 2006, p. 5). Teachers and pre-service teachers can rely on procedural methods having difficulties with their conceptual or connected MCK (Goos et al., 2008; Ma, 1999; Mewborn, 2003). Other studies suggested that many pre-service teachers begin their teacher training with only basic MCK (Carre & Ernest, 1993a; Grossman, Wilson, & Shulman, 1989; Zevenbergen, 2004).

Tatto et al. (2012) found that pre-service teachers’ MCK varied substantially: the “highest achieving countries had some future teachers achieving relatively low scores,” (p. 138) and lower achieving countries also had a range of scores. In brief, while pre-service teachers “could solve some problems involving proportional reasoning, they often had trouble reasoning about factors, multiples, and percentages” (p. 137). Other findings of TEDS-M were that opportunities to learn varied because of the different
structures of teacher education systems across countries, there was also a substantial
difference between the mean score of the lowest and highest ranking countries when
comparing pre-service teachers’ MCK (Tatto & Senk, 2011). Similarly, most programs
provided primary pre-service teachers with opportunities to learn to teach mathematics
during practicum experiences in schools; during coursework experiences, primary
programs provided more opportunities to learn the topics of number, measurement and
graphs, with less coverage of probability and statistics (Tatto et al., 2012). Few
studies of teachers’ MCK (e.g., Tatto et al., 2012) have chosen to report by cognitive
domain: Knowing, Applying and Reasoning. Tatto et al. (2012) suggested that although
pre-service teachers “could solve some problems involving proportional reasoning, they
often had trouble reasoning about factors, multiples, and percentages” (p. 137).

Other studies have recognised fractions and decimals as a particular significant source
of learning difficulties for middle-year students and primary pre-service teachers (Pearn
& Stephens, 2004; Stacey et al., 2001; Steinle & Stacey, 1998; Ubuz & Yayan, 2010;
Widjaja et al., 2011). Similarly, Newton’s (2008) review of the literature found that
many studies of elementary (primary) pre-service teachers showed they had limited
fraction knowledge. Other studies demonstrated that many primary pre-service teachers
had measurement difficulties associated with area and perimeter, including accuracy
with measuring shapes with diagonal sides and conversion between square units (Ryan
& Williams, 2007b), conservation of area and perimeter (Ma, 1999; Murphy, 2012) and
use of inappropriate units when calculating area and perimeter (Yeo, 2008). Ma (1999)
for example, found that 8% of Chinese teachers and 9% of American teachers accepted
the claim that “as the perimeter of a closed figure increases, the area also increases” (p.
84). Yeo (2008) also reported that teachers confuse area and perimeter, often assuming
a constant relationship between the two measures. Concerns within pre-service teacher
education are even more prevalent, with studies indicating that many pre-service
teachers have poor conceptual understanding of area, relying on rules and formulae, and
have difficulties in explaining why these formulae work (Baturo & Nason, 1996;
Berenson et al., 1997; Menon, 1998; Reinke, 1997).

In Australia, both in-service and pre-service teachers’ limited MCK and confidence
with doing mathematics is of particular concern (e.g., Ball, 1990; Lange & Meaney,
Many Australian studies have reported on weaknesses in pre-service teachers’ MCK (e.g., Goos et al., 2008). Studies of pre-service teachers do not often include ratio and have focused on other areas of number knowledge such as place value, operations with common fractions, multiplication of decimal fractions, percentages and measurement (Southwell & Penglase, 2005); the concept of decimals is recognised as a significant source of learning difficulties for students and pre-service teachers (Stacey et al., 2001; Steinle & Stacey, 1998; Ubuz & Yayan, 2010).

Ryan and McCrae’s (2005/2006) analysis of first-year pre-service teachers’ MCK identified a range of errors and misconceptions across different strands of the mathematics curriculum. Zevenberg (2005) reported that many pre-service teachers lack understanding of number sense, measurement sense and spatial sense and that pre-service teachers’ MCK is highly variable. Afamasaga-Fuatai, Meyer, and Falo (2008) also noted that pre-service teachers have most difficulties with problem solving, fractions and interpreting diagrams. These studies suggest that many pre-service teachers have difficulty with a range of mathematical topics and problem types that they would be expected to teach primary students.

Morris (2001) suggested that it was difficult to address some pre-service teachers’ weaknesses in MCK because they were unaware they lacked it, which could consequently affect their teaching. Similarly, Maher and Muir (2011) found that many of the pre-service teachers in their study were unaware that they had developed flawed understandings or misconceptions about decimals and were therefore unlikely to address this unless these misconceptions were explicitly uncovered and addressed. Widjaja, Stacey and Steinle (2008) also highlighted the danger of misconceptions being “covered over rather than overcome” (p. 1), while others have found that success with procedural fluency can sometimes hide underlying misconceptions (e.g., Ball et al., 2008; Ryan & Williams, 2007a). Providing opportunities for pre-service teachers to identify and reflect on their errors may assist pre-service teachers to extend their MCK (Ryan & Williams, 2007b).

Pre-service teachers require assistance to improve their MCK (Even & Tirosh, 1995). Groves, Moulsey, and Forgasz (2006), in their review of Australian studies, reported
that, “many pre-service teachers believe they are insufficiently prepared in terms of MCK” (p. 192). Insufficient MCK can impact negatively on student learning. To illustrate, Stacey et al. (2001) were concerned that pre-service teachers with difficulties in decimal fraction knowledge would pass their misconceptions onto their students. Similarly, Carre and Ernest (1993a) found that after completing a one year Postgraduate Certificate in primary teaching many United Kingdom pre-service teachers could understand students’ learning in routine tasks and identify their errors but had difficulty using their MCK when assisting students with non-routine tasks.

2.4 Summary and implications

This review of literature provided the theoretical background for this study and identified the similarities and differences of terms used to define MCK. It began with a discussion of the complexities of MCK revealed since the publication of seminal work by Shulman (1986). Shulman’s (1987) theoretical framework included seven categories used to describe the knowledge base of teachers, including CK. Thereafter, Ma (1999) incorporated content knowledge as part of her definition of PUFM extending understanding of the complexities of a teachers’ MCK. Ball’s et al.’s (2008) MKT used three categories when defining subject matter knowledge (CCK, SCK and HCK), whereas Rowland et al’s (2009) Knowledge Quartet included four categories – foundation knowledge, transformation, connections and contingency – that combined MCK, PCK and beliefs. Similarly, Chick et al. (2006a) combined MCK and PCK in their PCK category of content knowledge in a pedagogical context.

The review shows that there have been few longitudinal studies of how teachers’ (e.g., Copur-Gencturk & Lubienski, 2013; Turner & Rowland, 2008) or pre-service teachers’ (e.g., Afamasaga-Fuata’i et al., 2008; Cady et al., 2006) MCK changes over time (Goos et al., 2008; Mewborn, 2001). Larger studies have assessed pre-service teachers’ MCK near the end of the program (Callingham et al., 2011; Tato et al., 2012) rather than throughout different stages of their program. The review of literature also highlighted small-scale studies of primary pre-service teachers and their areas of mathematical difficulties mostly during coursework experiences rather than during practicum
experiences. McDonough and Sexton (2011) agree that development of pre-service teachers’ MCK during practicum experiences is less common.

What is missing are studies of how program structure impedes or assists pre-service teachers’ MCK, rather than reports of performance on a mastery test (Callingham et al., 2011; Meany & Lange, 2010; Stacey et al., 2001; Tobias & Itter, 2007) or in one primary mathematics unit (e.g. Maher & Muir, 2013; Ryan & McCrae, 2005; Ryan & Williams, 2007b). Therefore a longitudinal study is needed to identify how MCK develops during teacher education and if significant improvement in MCK can be sustained and demonstrated during the final year of study. An in-depth analysis of pre-service teachers’ MCK will assist with further understanding of why teachers and pre-service teachers rely on procedural methods (Goos et al., 2008; Ma, 1999; Mewborn, 2003) or what opportunities and influences enhance their MCK during their program experiences.

Continuing to refine ways to measure and analyse the knowledge teachers use in teaching mathematics is important in developing an understanding of how teachers’ MKT affects student learning (Ball et al., 2004). Longitudinal studies have the potential to describe the development of pre-service teachers’ identity as a factor contributing to opportunities to learn MCK (Ambrose, 2004) because beliefs and identity impact on teachers’ opportunities to learn how to teach and the aims of the programs they enter (Anthony et al., 2012).

In summary, the review showed that research designed to extend our understanding of pre-service teachers’ MCK was needed, particularly a longitudinal study of pre-service teachers’ experiences during coursework, designed to identify the MCK they developed and how this was demonstrated and achieved during their program. As suggested by Clements’ (2008) multiple and complementary data collection methods would strengthen findings and so assist with identifying the categories of MCK pre-service teachers develop throughout different situations of their program experiences. The next chapter presents the quantitative and qualitative methodology used in responding to the research questions.
CHAPTER THREE

Methodology

After reviewing the literature on primary pre-service teachers’ mathematical content knowledge (MCK), as described in the previous chapter, the research questions (first presented in Chapter 1) and the methodology required to answer them were designed.

Following the research questions is a description of the setting for the study – including the course from which primary pre-service teachers were recruited, the course entry requirements, program structure, description of mathematics education units and practicum experiences in school settings. Next the overall research design is presented, including the mixed-methods research approach and the quantitative and qualitative methods used. The quantitative methods includes a descriptive research design for analysing pre-service teachers’ responses to MCK test items; the qualitative methods and longitudinal study research design involves investigating opportunities and influences for each year of the program that enhance pre-service teachers’ MCK.

The findings of the literature review identified that few longitudinal studies have investigated primary pre-service teachers’ MCK (Anthony et al., 2012; Goos et al., 2008). It was evident that a mixed-methods longitudinal study designed to describe and interpret primary pre-service teachers’ development of MCK during their teacher education program was needed. The question of how pre-service teachers’ MCK develops during program experiences was not covered in the literature – a gap this study was designed to fill.

3.1 Research questions

The methodology was designed to address the following major research question and three subsidiary research questions:

3.1.1 Major research question

What MCK do primary pre-service teachers develop during their teacher education and how is this demonstrated and achieved? (RQ1)
3.1.2 Subsidiary research questions

What MCK is demonstrated at different stages of pre-service teacher education? (RQ2)

What opportunities and influences enhanced pre-service teachers’ MCK during units in primary mathematics teacher education? (RQ3)

What opportunities and influences enhanced pre-service teachers’ MCK throughout teaching practice in primary and secondary classrooms during the program? (RQ4)

3.2 Research design

A mixed-methods design, integrating qualitative and quantitative methods, was chosen to explore the development of pre-service teachers’ MCK and their opportunities to learn. Many approaches have been used in educational research; an advantage of a mixed method design is that quantitative methods and qualitative methods can be used across multiple data (McMillan & Schumacher, 2006), allowing the researcher to, use one approach as a starting point for the other and test the findings of one approach against those of the other (Hammersley, 1996).

Longitudinal studies usually collect data from the same group of participants over time (Kervin, Vialle, Herrington, & Okely, 2006). For this longitudinal study, the research questions were best answered by inviting pre-service teachers from one university, enrolled in their first year of study in the same program. An in-depth, comparison of pre-service teachers could also be made because they completed three core coursework units in primary mathematics during their program and the same number of days completing practicum teaching experiences in primary schools. A longitudinal study design enabled collection of multiple data for identifying change in pre-service teachers’ MCK over time, as well as the factors that contributed to change, such as where and when opportunities to learn occurred. Pre-service teachers’ program experiences related to their MCK were studied using observations of practice teaching experiences, interviews, analyses of test data and assignments completed as they undertook units in primary mathematics teacher education.

Table 3.1 shows how the major and subsidiary research questions relate to the research methods, data gathering techniques and methods of analysis of pre-service teachers’ MCK presented later within this chapter.
Mixed-methods designs are suitable when answering a range of research questions (Kervin et al., 2006; M. L. Smith, 2006). Quantitative research is typically specific or ‘closed’ to enable it to determine the statistical significance of relationships between variables, whereas qualitative research is open, enabling flexible exploration of the experiences, ideas or feelings of participants (Kervin et al., 2006; McMillan, 2004) and “seek[ing] to make sense of social phenomena as they occur in natural settings” (Klein, 2006, p. 37).

As Table 3.1 shows, the questions in this study were open-ended and addressed pre-service teachers’ experiences related to their MCK during teacher education; therefore, qualitative methods were required. However, the major research question and first subsidiary research question required both quantitative and qualitative methods. Qualitative methods were used to respond to the second and third subsidiary research questions. Mixed-method designs can vary in the order in which quantitative and qualitative data is collected, or both sets of data can be collected at the same time (McMillan & Schumacher, 2006). In this study, quantitative and qualitative data were simultaneously gathered so as to enable triangulation and provide the best possible evidence to support the interpretations.
Table 3.1
Mixed Method Design Chosen for Research Questions Including Research Method, Data Gathering and Analysis

<table>
<thead>
<tr>
<th>RQ</th>
<th>Method</th>
<th>Data gathering</th>
<th>Analysis of MCK</th>
</tr>
</thead>
<tbody>
<tr>
<td>RQ1</td>
<td>Qualitative</td>
<td>Longitudinal data of pre-service teachers’ MCK (see RQ2, 3 and 4)</td>
<td>Content analysis and triangulation of data identifying program experiences that influenced and enhanced pre-service teachers’ MCK; using categories of MCK frameworks (Chick et al., 2006a; Hill et al., 2008; Ma, 1999; Rowland et al., 2009) to code data</td>
</tr>
<tr>
<td>RQ1</td>
<td>Quantitative</td>
<td>All pre-service teachers’ MCSK test item responses (Yrs 1 and 2, 2008)</td>
<td>Descriptive research design, comparison of correct and incorrect responses of MCSK test items, rank MCSK test items by percentage of correct responses to indicate level of difficulty for content domains (ACARA, 2013) and cognitive categories (Tatto et al., 2012)</td>
</tr>
<tr>
<td>RQ1</td>
<td>Quantitative</td>
<td>Longitudinal study pre-service teachers Response to questionnaires (Yr 2)</td>
<td>Descriptive research design of demographic responses of longitudinal study pre-service teacher participants</td>
</tr>
<tr>
<td>RQ2</td>
<td>Quantitative</td>
<td>All pre-service teachers’ MCSK test short answer item responses (Yrs 1 and 2, 2008)</td>
<td>Ranking MCSK test items by percentage of correct responses to indicate level of difficulty for content domains (ACARA, 2013) and cognitive categories (Tatto et al., 2012)</td>
</tr>
<tr>
<td>RQ2</td>
<td>Qualitative</td>
<td>Longitudinal study pre-service teachers’ responses to MCSK test items (Yr 2) and MCK interview items (Yr 4)</td>
<td>Descriptive research design, comparison of correct and incorrect responses of MCSK test items using categories of MCK frameworks (Chick et al., 2006a; Hill et al., 2008; Ma, 1999; Rowland et al., 2009) to code data and identify different categories of pre-service teachers’ MCK</td>
</tr>
<tr>
<td>RQ2</td>
<td>Qualitative</td>
<td>MCSK test item (Yr 2) and interview responses (Yr 4)</td>
<td>Content analysis of pre-service teachers’ responses to short answer items to identify categories MCK using MCK frameworks (Chick et al., 2006a; Hill et al., 2008; Ma, 1999; Rowland et al., 2009)</td>
</tr>
<tr>
<td>RQ3</td>
<td>Qualitative</td>
<td>Longitudinal data of pre-service teachers’ MCK; assignments Unit 1A (Yr 1); assignments Units 2A and 2B (Yr 2); primary mathematics lesson observations and field notes (Yr 2 and 3); one-on-one interview (Yrs 2, 3 and 4); responses to MCK interview items (Yr 4); responses to MCSK tests Units 1B (various years) and 2B (Yr 2)</td>
<td>Content analysis and triangulation of data identifying program experiences that influenced and enhanced pre-service teachers’ MCK; using categories of MCK frameworks (Rowland et al., 2009) to code data</td>
</tr>
<tr>
<td>RQ4</td>
<td>Qualitative</td>
<td>Interview responses (Yrs 2, 3 and 4) and lesson observations (Yrs 2 and 4)</td>
<td>Same as RQ1 qualitative analysis of MCK</td>
</tr>
</tbody>
</table>
3.3 Study setting

A large pre-service teacher education program was identified for this study. In 2007, when this study commenced, 300 pre-service teachers were enrolled in the first year of the course. Since then numbers of pre-service teachers have increased, with 700 commencing their first year of the program across two campuses in 2014. At the beginning of the study (but not now), the same program was delivered across four campuses of the university.

The Bachelor of Education (Preparatory –Year 12) program combines learning at the university with partnership-based teaching experiences in schools in which the practise of learning to teach is combined with theory. At graduation, pre-service teachers have the qualifications to teach Preparatory (age 5) to Year 12 (age 17) students.

3.3.1 Course entry requirements

Before enrolling in teacher education, pre-service teachers are required to have passed the Victorian Certificate of Education (VCE) (Victorian Curriculum and Assessment Authority, 2011). This certificate usually takes two years to complete and students study a minimum of five subjects, including English (compulsory). VCE qualifications provide secondary school students with a pathway for further study and entry into various university degrees, including teacher education programs.

When this study commenced, VCE mathematics students could choose from Foundation Mathematics, General Mathematics, Mathematical Methods and Specialist Mathematics. Four units must be completed per subject: units 1 and 2 are normally completed during Year 11 and units 3 and 4 during Year 12. These programs are described in Appendix A.

Almost all of the pre-service teachers who gained a place at the university in 2007 had completed Year 12 education prior to enrolment. A small number were accepted into the program without Year 12 because of other qualifications or life experiences. In addition, an adult entry option was available to applicants who had commenced a different degree and wished to transfer to the Bachelor of Education program.
3.3.2 Program structure

For successful completion of the Bachelor of Education program, pre-service teachers had to complete 20 core education units, 11 units for their elective specialisation and additional units as needed (3.2).

Table 3.2
Number of Core and Specialisation Units and Practicum Days Completed by Pre-service Teachers During the Four Years of their Program (2007-2010)

<table>
<thead>
<tr>
<th>Yr of program</th>
<th>Mathematics Education Units</th>
<th>Other Education Units</th>
<th>Practicum Days</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Core</td>
<td>Specialisation</td>
</tr>
<tr>
<td>1</td>
<td>Unit 1A (core)</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>Unit 2A (core)</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Unit 2B (core)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>None</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>None</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Any</td>
<td>Unit 1B(^1) (elective)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Practicum teaching experiences occurred in each year of the program, with pre-service teachers completing experiences in primary schools during first, second and fourth years. During third year, pre-service teachers practised teaching in their secondary specialisation, usually in a secondary school.

The core units of study focus on praxis inquiry, a pedagogical approach that fuses practice with theory (Cherednichenko & Kruger, 2002). The students engaged in reflection on personal experiences of learning and considered what impact their inquiry and learning would have on their own practice as they prepared for their first practicum experience. By fourth-year the final praxis inquiry unit provided opportunities for pre-service teachers to prepare to enter the teaching profession. They were required to demonstrate their understanding of experiences in teaching with an emphasis on

\(^{1}\) Unit 1B becomes a core unit in year three and year four for those students yet to pass the MCSK test instrument offered during Unit 2B
reporting the standards for graduating teachers (Victorian Institute of Teaching, 2010). Two tutorials provided an opportunity for pre-service teachers to reflect on their literacy and numeracy teaching.

All pre-service teachers completed three core units (1A, 2A and 2B) in primary mathematics teacher education and some completed an elective unit (1B). Unit 1A was the first primary mathematics unit and was studied in first semester of first year for one semester. Each week pre-service teachers attended a two-hour tutorial and a one-hour lecture. Units 2A and 2B were studied in either first or second semester during second year. For unit 2A, pre-service teachers attended a weekly two-hour tutorial. For unit 2B, pre-service teachers attended a weekly one-hour lecture followed by a two-hour tutorial. Unit 1B was an elective unit designed to assist pre-service teachers who needed additional mathematics understanding for primary mathematics teaching. This unit involved a one-hour lecture and two-hour tutorial each week.

The specialisation units provided pre-service teachers with the knowledge needed for teaching two secondary specialisations. Each specialisation unit included four units. Pre-service teachers were able to choose from the following secondary specialist units of study: Information and Communication Technology, Language and Literary Studies, Visual Art, Cultural Studies, Performance Studies, Science or Mathematics. In general, program units took one semester to complete, equivalent to 36 hours of face-to-face lectures and tutorials. Most units involved three hours, including a combination of tutorials, online study and or lectures, each week for ten weeks per semester.

### 3.3.3 Primary mathematics education units

*Unit 1A*

Unit 1A was a core first-year unit and was designed to extend pre-service teachers’ focus on their personal learning in the realm of mathematics and community engagement. Pre-service teachers considered what impact their inquiry and learning had on their own practice as they prepared for their first practicum experience in a primary school.
During Unit 1A students studied different topics relating to numeracy, literacy and technology each week, delivered via an inquiry process. The mathematics theory focused on effective mathematics lesson planning, multi-literacies and mathematical thinking, as well as a selection of specific numeracy topics. Different numeracy topics or dimensions from the Victorian Essential Learning Standards (VELS) (DEECD, 2009a), including measurement, chance and data, fractions, area and volume, decimals and place value were investigated each week.

**Unit 1A Assessment Tasks**

Learning Log: This consisted of completing a self-directed learning log, including identifying numeracy areas for improvement and a reflection on their own mathematical understanding and development throughout the semester (60% of mark).

Mathematics in the Community: The pre-service teachers conducted a mathematical investigation relating to their community; they developed a plan and kept a log of their investigation and results. They presented their findings as a multimedia report and presented this to their peers (40% of mark).

Mathematical Skills and Competency (MCSK) test: Assessment of mathematics skills and knowledge (satisfactory completion required). The test paper consisted of seven sections; each section had seven questions and pre-service teachers were required to correctly respond to five of the seven questions for all sections. See Appendix B for MCSK Test 1 (2008). Before completing their program, pre-service teachers were required to pass this test.

**Unit 2A**

Unit 2A was a core second-year unit, designed to encourage pre-service teachers to demonstrate their developing understanding of student learning and assessment in the context of mathematics. Unit 2A was linked to primary practicum teaching experiences in primary schools because it focused on skills of teaching.

**Unit 2A Assessment tasks**

The Early Years Numeracy Interview (EYNI): The pre-service teachers conducted the EYNI (Department of Education Employment and Training (DEET), 2001) with a
primary student from Year 1, Year 2 or Year 3 during their practicum experience. They also completed a report summarising and analysing the mathematical understanding the student demonstrated throughout the interview (50% of mark).

Series of mathematics lessons: For five weeks and with a partner, pre-service teachers taught students at a local primary school. Each week they planned and taught a mathematics lesson to a small group of primary students. They also submitted lesson plans and an evaluation reflecting on their own learning and students’ mathematical outcomes (50% of mark).

Practicum report: A progress report from their teaching practice in a primary school setting (satisfactory completion required; assessed by the primary school mentor teacher).

Practicum Project: This was a report on a project completed during practicum experiences with other pre-service teachers. The project did not have to, but could, relate to primary mathematics teaching (satisfactory completion required).

The MCSK test was completed at the end of the unit and the university lecturers used test data from previous tests to determine the weekly topics and amount of teaching time dedicated to the dimensions of the ACM. For example less teaching and assessment focused on chance and data and more on number and measurement.”

*Unit 2B*

Unit 2B was a core second-year unit in which pre-service teachers were expected to explore the knowledge and understanding of pedagogy and curriculum required for effective teaching of primary mathematics. This unit included a lecture followed by a tutorial and was taught on the same day and immediately following Unit 2A.

Each week a different mathematics topic was presented during the lectures. The main topics were: early number understanding, place value, decimals, fractions, the four operations, space, measurement, structure, chance and data. The lectures focused on developing understanding of the methods used for teaching and how students learn, as well as a discussion about students’ common misconceptions of each topic.
Tutorials were planned to further develop the pre-service teachers’ understanding of the lecture content. The unit also aimed to support pre-service teachers’ development of knowledge needed for teaching in a primary school.

Unit 2B Assessment tasks

Workshop and Report on Teaching: Each week a different group of three pre-service teachers worked together to complete a workshop presentation on a mathematics topic. They each prepared a written report demonstrating their understanding of this topic and how it should be taught in primary school. All pre-service teachers participated in the workshop and were required to submit a list of mathematical and pedagogical questions about the topic (40% of mark).

Two learning trajectory charts: The charts included, for example, numeration and operations for whole numbers and numeration and operations for either fractions or decimals (30% of mark).

Teaching resources: All pre-service teachers constructed, presented and analysed five high-quality primary mathematics teaching resources (30% of mark).

MSCK test: a test similar to that completed for Unit 1A. See Appendix B for sample MCSK Test 1.

Unit 1B

Unit 1B was an elective primary mathematics education unit, usually completed during the second year of the program by pre-service teachers who required additional support to pass an MCSK test. The unit was designed to assist pre-service teachers to develop their understanding of the mathematical concepts needed for teaching primary mathematics. The unit focused on developing understanding and demonstrating competence in the following topics: space, number, common and decimal fractions, measurement, chance and data, structure and working mathematically.

Pre-service teachers who did not pass the MCSK test in Unit 2B were required to complete Unit 1B and then re-sit the MSCK test. These standards were set by the University and endorsed by the overarching professional standards set by the Victorian Institute of Teaching (Victorian Institute of Teaching, 2010).
Unit 1B Assessment tasks

Mathematics Learning File: Pre-service teachers completed a learning log, explaining and recording their growth in mathematics understanding as well as identifying their mathematical errors or misconceptions with reference to the literature (100% of mark).

MCSK Test: Assessment of mathematics skills and knowledge (satisfactory pass).

3.3.4 Teaching practice, Project Partnerships

Pre-service teachers also experienced a range of education settings that provided opportunities to develop their practice in a primary school, as well as understanding of teaching their secondary discipline studies in a secondary school setting. Pre-service teachers’ school and practicum teaching experiences were called Project Partnerships. Pre-service teachers taught collaboratively with their mentor teachers (classroom teachers), to enhance their knowledge of teaching. These experiences, including teaching mathematics in schools, were designed to be centred on the process of praxis inquiry – making connections with the one compulsory education unit of study for each semester of the program (e.g. Unit 1A and Unit 2A). Praxis inquiry is a professional discourse and practice that is reflexive and critical, inquiry and learner responsive, not practice-led (Cherednichenko & Kruger, 2002).

Pre-service teachers were usually assigned to one school for every year of the program. In first, second and fourth year, pre-service teachers were placed in primary schools and during third year they were placed in a secondary school (Table 3.2). They attended their school for their practicum experiences regularly on a Tuesday and also completed full week placements at different times of their program. The total number of days spent in practicum visits (2007-2010) was 144; far in excess of the teacher education course accreditation minimum of 45 days (Victorian Institute of Teaching, 2011).

Pre-service teachers did not commence their Project Partnerships until halfway through first semester of first year, thus fewer practicum days occurred in first year than in later years. About three quarters (71%) of school-based days were primary school experiences. The Victorian Institute of Teaching (2011) did not mandate the type of school experiences pre-service teachers completed, only a minimum number of days of
experiences. Each semester pre-service teachers were required to gain a satisfactory pass for their practicum experience that was assessed by the school mentor teacher.

3.4 Quantitative methods

A quantitative research design was used to respond to the first subsidiary research question because the researcher was interested in discovering what MCK pre-service teachers demonstrated during first and second-year of their teacher education program. A descriptive research design was chosen to describe this outcome of interest and its patterns (Kervin et al., 2006).

Descriptive design is suitable for summarising, organising and reducing large numbers of observations (Kervin et al., 2006; McMillan & Schumacher, 2006). Quantitative research designs use numbers, statistics, structure and control, in addition they can be either experimental or non-experimental (McMillan & Schumacher, 2006). The research design was non-experimental, so as to rank and describe a large number of first and second-year pre-service teachers’ correct and incorrect responses after they had responded to MCSK test items.

3.4.1 Test instruments

Selection of the test instruments

Quantitative techniques need some type of data collection instrument, and the results of analysis depend on the quality of the measurements (McMillan & Schumacher, 2006). This study required a test instrument for measuring pre-service teachers’ MCK and standardised were chosen because of their reliability. Standardised tests are reliable and valid because they have been prepared by experts; trialled and include the same questions each time the test is used (Kervin et al., 2006; McMillan & Schumacher, 2006).

However, it would have been preferred to use the same standardised test instruments for all pre-service teachers but it was unlikely that a large number of first and second pre-service teachers would have agreed to complete a test for this study. The supervisors agreed that it would be difficult to find sufficient participants willing to complete an
MCSK test in addition to the one they would complete in their teacher education program, so the study relied upon existing data. Therefore the researcher chose to use the similar test instrument that was used for each campus as part of the program assessment. During the study (2008) four cohorts of pre-service teachers’ test instruments were collected for analyses, including all first-year (Test 4) and three cohorts of second-year (Test 1, 2 and 3).

The advantages of using similar test instruments were that responses to a greater number of items could be collected for use in the analysis. An analysis of more test items would provide greater scope to identify strength and weaknesses of pre-service teachers’ MCK from more data.

**Validity and reliability of test instrument**

The MCSK instruments were non-standardised and locally devised. Therefore validity and reliability had to be carefully ascertained. For the results of a study to be valid, the data must be collected by a reliable and valid instrument (McMillan, 2004). Valid data will accurately measure the researcher’s interests and if the study were repeated the same results would be reproduced (Betts, Hayward, & Garnham, 2001).

The program mathematical, competency skills and knowledge (MCSK) test instruments in use at the university from mid- to-late-2000s were judged as valid for use in measuring pre-service teachers’ MCK for various reasons. These test instruments had been implemented for more than five years as part of the teacher education program with common administration and scoring procedures. A senior mathematics education lecturer prepared the four MCSK tests (used between 2007 and 2010, and in this study) and ensured their content and level of difficulty were similar. The MCSK items were deliberately varied for each cohort of pre-service teachers; this was done to prevent them obtaining previous years’ tests and gaining an advantage over their peers.

Pre-service teachers completed the test over three hours in exam conditions and no calculators were permitted. They were also encouraged to provide working out and answer all items. The lecturers marked the tests using an answer sheet provided by the senior mathematics education lecturer. All completed tests were remarked and checked before use in this study to ensure validity.
Structure of the test instruments

A wide range of mathematical sources were used when designing the MCSK test items, including items from past Year 5 and Year 7 Achievement Improvement Monitor (AIM) state wide assessment sample tests (Victorian Curriculum and Assessment Authority, 2008). The MCK items varied by year level and difficulty; they generally comprised items examining knowledge and understanding of mathematics for Years 5 to 8, that is, VELS Levels 4 to 5, (Victorian Curriculum and Assessment Authority (VCAA), 2007); or Years 5 to Year 8, Australian Curriculum Mathematics (ACM) (ACARA, 2013). All items required short answers using words or symbols and (as noted earlier) recording of working out was encouraged. (Eight MCSK test items are discussed later (see Table 3.7)).

The internal structure of an instrument refers to how different parts of the instrument are related (McMillan, 2004). The structure of the test instruments for all four MCSK tests were the same, consisting of a paper and pencil test and 49 MCK items; the items were organised into seven sections because these matched the weekly topics that were taught as part of the primary mathematics education coursework during second year.

Number: This section had very similar items in all four MCSK tests: a pattern and sequence item; writing a large number in words; identifying composite or prime numbers; two division items; and one multiplication of a three-digit number by a three- or two-digit number.

Measurement, Chance & Data: All tests had four measurement items and three chance and data items. Second-year, Test 2 and Test 3 had exactly the same items for Measurement, Chance and Data.

Fractions: All tests (first and second-year) included a fraction section that required students to find the difference between two fractions, calculate a multiplication of a common fraction and a two-digit number, use a number line, and solve at least one worded fraction problem.

Space: Shape items in Test 1 (first-year) and Test 4 (second-year) involved mostly closed responses, whereas Tests 2 and 3 provided items related to similar mathematical
knowledge of shape as Test 1 and Test 4 but these items required open-ended responses. These items assessed knowledge of two-and three-dimensional shapes, and all tests had a symmetry item.

**Decimals:** Within the decimal section, Tests 2 Test 3 had exactly the same items. All four tests included a place value item; ordering of decimal fractions to three decimal places; multiplication of two decimal fractions; and a division item, tenths divided by tenths.

**Area & Volume:** The items in the Area and Volume section were the same for Tests 2 and 3. For all tests the Area and Volume section included an estimation of area of a two-dimensional shape; Tests 1 and 4 included a perimeter item, Tests 2 and 3 had an item involving calculating the surface area of a box. All tests had one item about finding an area in hectares given the dimensions in metres of a rectangular field. Tests 1 and 4 had one item comparing volume of two containers, whilst Tests 2 and 3 required students to find the volume of a cube in metres given the dimensions in centimetres.

**Percentage & Ratio.** The final section, Percentage and Ratio, had the same items for Tests 2 and 3, including three percentage items and four ratio problem items. Tests 1 and 4 included two percentage items and four ratio problems. All tests had items that required students to write a decimal as a percentage and solve proportional reasoning problem by calculating which music card is better value.

In summary, the MCSK Tests 1, 2, 3 and 4 consisted mostly of items that measured pre-service teachers’ MCK; a few items could be considered to assess pedagogical content knowledge (PCK). Overall the items in MCSK Test 1 and Test 4 were mostly similar with different number values and likewise the items in MCSK Test 2 and Test 3 were mostly similar MCK items with different values. For example Test 1 Item 8 asked how many millilitres in 2.4 litres; and Test 4 Item 8 asked how many millilitres in 6.35 litres. Also see Appendix B for one MCSK test used in this study, Test 1. Some item designs were conceptually orientated to explore pre-service teachers’ understanding of mathematical principles, ideas and representations of mathematical concepts used in primary mathematics teaching, whilst others were closed item types. The next section
presents the number of items for each test and classifies the items using different frameworks.

**Classifying test instrument items**

The items of the four MCSK tests were classified by mathematical topics; content strand and sub-strand of the ACM (ACARA, 2013) (Table 3.3); item design, including closed, open-ended or multiple choice items (Table 3.3); types of problem items, including routine and non-routine items (Table 3.4); and by cognitive categories (Tatto et al., 2012) (Table 3.5). Test 1, 2 and test 3 were second year test instruments and Test 4 was a first year test instrument. This coding assisted when reporting and describing the results of pre-service teachers’ correct and incorrect responses to the different MCSK test items in Chapter 5.

Table 3.3 presents the number and percentage of items for the four test instruments coded using the ACM content strands and content sub-stra nds (ACARA, 2013). The final column is the total number of all four tests and the number of items coded for each of the ACM content sub-stra nds. Table 3.3 lists the number of items coded for each test and sub-strand; they were generally evenly distributed between tests but not evenly distributed across the content strands or content sub-stra nds.

As Table 3.3 shows, all four tests had more than half of the items coded as the number and algebra content strand; most items were coded real number, with about one tenth coded as pattern and algebra. For measurement and geometry and all tests a bit more than one-third of items were coded at this content strand; more items were coded using units of measurement with less coded location and transformation as well as geometric reasoning. The final content strand, statistics and probability, had a small number of items compared with other content strands. Analyses and comparisons between test responses by ACM content strands and sub-stra nds were possible as items within the tests had a similar distribution by content strand and content sub-strand (ACARA, 2013).
Table 3.3
Description of MCSK Test ACM Content Strands (Number and Percentage of Items per Test)

<table>
<thead>
<tr>
<th>ACM content strands</th>
<th>ACM content sub-strand</th>
<th>Test 1 Yr 2</th>
<th>Test 2 Yr2</th>
<th>Test 3 Yr 2</th>
<th>Test 4 Yr 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number and Algebra</td>
<td>Number and place value</td>
<td>4 (18%)</td>
<td>7 (14%)</td>
<td>7 (14%)</td>
<td>4 (8%)</td>
</tr>
<tr>
<td>Number and Algebra</td>
<td>Pattern and algebra</td>
<td>3 (6%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>3 (6%)</td>
</tr>
<tr>
<td>Number and Algebra</td>
<td>Fractions and decimals</td>
<td>4 (8%)</td>
<td>4 (8%)</td>
<td>4 (8%)</td>
<td>4 (8%)</td>
</tr>
<tr>
<td>Number and Algebra</td>
<td>Real numbers</td>
<td>15 (31%)</td>
<td>14 (29%)</td>
<td>14 (29%)</td>
<td>16 (31%)</td>
</tr>
<tr>
<td>Number and Algebra</td>
<td>Money and financial mathematics</td>
<td>2 (8%)</td>
<td>1 (1%)</td>
<td>1 (1%)</td>
<td>1 (1%)</td>
</tr>
<tr>
<td>Number and Algebra</td>
<td>Total</td>
<td>28 (57%)</td>
<td>26 (53%)</td>
<td>26 (53%)</td>
<td>28 (57%)</td>
</tr>
<tr>
<td>Measurement and Geometry</td>
<td>Using units of measurement</td>
<td>11 (22%)</td>
<td>12 (24%)</td>
<td>12 (24%)</td>
<td>11 (22%)</td>
</tr>
<tr>
<td>Measurement and Geometry</td>
<td>Shape</td>
<td>6 (12%)</td>
<td>5 (10%)</td>
<td>5 (10%)</td>
<td>6 (12%)</td>
</tr>
<tr>
<td>Measurement and Geometry</td>
<td>Location and transformation</td>
<td>1 (2%)</td>
<td>1 (2%)</td>
<td>1 (2%)</td>
<td>1 (2%)</td>
</tr>
<tr>
<td>Measurement and Geometry</td>
<td>Geometric reasoning</td>
<td>0 (0%)</td>
<td>1 (2%)</td>
<td>1 (2%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Measurement and Geometry</td>
<td>Total</td>
<td>18 (37%)</td>
<td>19 (39%)</td>
<td>19 (39%)</td>
<td>18 (37%)</td>
</tr>
<tr>
<td>Statistics and Probability</td>
<td>Chance</td>
<td>0 (0%)</td>
<td>2 (2%)</td>
<td>2 (2%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Statistics and Probability</td>
<td>Data representation and interpretation</td>
<td>3 (6%)</td>
<td>2 (2%)</td>
<td>2 (2%)</td>
<td>3 (6%)</td>
</tr>
<tr>
<td>Statistics and Probability</td>
<td>Total</td>
<td>3 (6%)</td>
<td>4 (8%)</td>
<td>4 (8%)</td>
<td>3 (6%)</td>
</tr>
<tr>
<td>All</td>
<td>Total</td>
<td>49 (100%)</td>
<td>49 (100%)</td>
<td>49 (100%)</td>
<td>49 (100%)</td>
</tr>
</tbody>
</table>
Table 3.4 shows the four MCSK Test items coded by item design, including closed items (excluding multiple choice), open-ended items, and multiple-choice items. An item was considered closed when there was only one possible correct response. The items that were classified open-ended question items had more than one possible correct solution and sometimes required an estimation or written explanation response.

Table 3.4  
*Description of MCSK Test and Item Design (Number and Percentage of Items) 2008*

<table>
<thead>
<tr>
<th>Question items</th>
<th>Test 1 Yr 2</th>
<th>Test 2 Yr 2</th>
<th>Test 3 Yr 2</th>
<th>Test 4 Yr 1</th>
<th>All tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>Closed</td>
<td>41 (84%)</td>
<td>39 (80%)</td>
<td>38 (78%)</td>
<td>39 (80%)</td>
<td>157 (80%)</td>
</tr>
<tr>
<td>Open-ended</td>
<td>6 (12%)</td>
<td>10 (20%)</td>
<td>11 (22%)</td>
<td>8 (16%)</td>
<td>35 (18%)</td>
</tr>
<tr>
<td>Multiple choice</td>
<td>2 (4%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>2 (4%)</td>
<td>4 (2%)</td>
</tr>
<tr>
<td>Total</td>
<td>49 (25%)</td>
<td>49 (25%)</td>
<td>49 (25%)</td>
<td>49 (25%)</td>
<td>196 (100%)</td>
</tr>
</tbody>
</table>

More than three-quarters of the MCSK test items were closed items, and these were evenly distributed across the four tests. Closed questions are easier to score and are suitable for large numbers of items (McMillan & Schumacher, 2006), which might explain why these tests mostly have closed test items. Less than a third of the items were open-ended. Responses to open questions generally require deeper thinking and responses can involve more than stating a fact or reproducing a skill (Sullivan & Lilburn, 1997; Sullivan, Warren, White, & Suwarsono, 1998). Only a small number of items in two of the four tests had multiple-choice items.

Analyses and comparisons between the MCSK test responses by item design could be conducted because the closed MCK items were plentiful and had a similar distribution. In contrast, open-ended items and multiple choice items were relatively rare and fewer comparisons were possible.

Table 3.5 lists the number and percentage of MCSK test items that were coded either routine or non-routine problem items for Tests 1, 2, 3 and 4 used during 2008. Routine items involved one step for calculating or working through the problem to find a correct
solution. Whereas non-routine problems may not be familiar to students and require a greater cognitive demand (International Association for the Evaluation of Educational Achievement, 2011, 2012). An item was coded as non-routine if the item required more than one step for calculating the solution (see example in Table 3.7: Item 12)

Table 3.5
Description of MCSK Test and Problem Items (Number and Percentage of Items), 2008

<table>
<thead>
<tr>
<th>Problem items</th>
<th>Test 1 Yr 2</th>
<th>Test 2 Yr 2</th>
<th>Test 3 Yr 2</th>
<th>Test 4 Yr 1</th>
<th>All tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>Routine</td>
<td>36 (73%)</td>
<td>30 (61%)</td>
<td>30 (61%)</td>
<td>33 (67%)</td>
<td>129 (66%)</td>
</tr>
<tr>
<td>Non-routine</td>
<td>13 (27%)</td>
<td>19 (39%)</td>
<td>19 (39%)</td>
<td>16 (32%)</td>
<td>67 (34%)</td>
</tr>
<tr>
<td>Total</td>
<td>49 (25%)</td>
<td>49 (25%)</td>
<td>49 (25%)</td>
<td>49 (25%)</td>
<td>196 (100%)</td>
</tr>
</tbody>
</table>

About two-thirds of items were coded as routine items and the remainder as non-routine problems. Routine items were evenly distributed between the four tests.

Educational measures can also be classified by cognitive domains and “cognitive focuses on what a person knows or is able to do mentally” (McMillan, 2004, p. 150). The cognitive domains are Knowing, Applying and Reasoning, and have been used to report Year 4 and Year 8 students’ strengths and weakness in mathematics after responding to items in the Trends in International Mathematics and Science Study (TIMSS, 2012). The three cognitive domains were also used to code items in a recent international Teacher Education and Development Study in Mathematics (TEDS-M) (Tatto et al., 2012). For the TEDS-M study, cognitive domains were used to report the MCK of pre-service teachers from 17 countries (Tatto et al., 2012) and this coding was also suitable for reporting the results of this study.

Table 3.6 lists the MCK items of the four Tests coded by cognitive domains (Tatto et al., 2012), including the number and percentage of items for the sub-domains and sample behaviours of each sub-domain.
Table 3.6
Description of MCSK Tests by Cognitive Sub-domains and Sample Behaviours (TEDS-M) (Number and Percentage of Items) 2008

<table>
<thead>
<tr>
<th>Sub-domain</th>
<th>Sample Behaviour</th>
<th>Test 1 Yr2</th>
<th>Test 2 Yr1</th>
<th>Test 2 Yr2</th>
<th>Test 3 Yr1</th>
<th>Test 4 All</th>
<th>All Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowing</td>
<td>Recall</td>
<td>6 (12%)</td>
<td>2 (4%)</td>
<td>2 (4%)</td>
<td>6 (12%)</td>
<td>16 (8%)</td>
<td>49 (100%)</td>
</tr>
<tr>
<td></td>
<td>Recognise</td>
<td>5 (10%)</td>
<td>5 (10%)</td>
<td>5 (10%)</td>
<td>4 (12%)</td>
<td>19 (10%)</td>
<td>49 (100%)</td>
</tr>
<tr>
<td></td>
<td>Compute</td>
<td>11 (22%)</td>
<td>10 (20%)</td>
<td>10 (20%)</td>
<td>11 (22%)</td>
<td>42 (21%)</td>
<td>196 (100%)</td>
</tr>
<tr>
<td></td>
<td>Retrieve</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td></td>
<td>Measure</td>
<td>11 (22%)</td>
<td>10 (20%)</td>
<td>10 (20%)</td>
<td>11 (22%)</td>
<td>42 (21%)</td>
<td>196 (100%)</td>
</tr>
<tr>
<td></td>
<td>Classify/order</td>
<td>1 (2%)</td>
<td>1 (2%)</td>
<td>1 (2%)</td>
<td>1 (2%)</td>
<td>4 (2%)</td>
<td>16 (8%)</td>
</tr>
<tr>
<td>Knowing</td>
<td>Total</td>
<td>34 (69%)</td>
<td>28 (57%)</td>
<td>28 (57%)</td>
<td>33 (67%)</td>
<td>123 (63%)</td>
<td>196 (100%)</td>
</tr>
<tr>
<td>Applying</td>
<td>Select</td>
<td>12 (16%)</td>
<td>12 (16%)</td>
<td>12 (16%)</td>
<td>6 (12%)</td>
<td>42 (21%)</td>
<td>196 (100%)</td>
</tr>
<tr>
<td></td>
<td>Represent</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td></td>
<td>Model</td>
<td>4 (8%)</td>
<td>4 (8%)</td>
<td>4 (8%)</td>
<td>4 (8%)</td>
<td>16 (8%)</td>
<td>16 (8%)</td>
</tr>
<tr>
<td></td>
<td>Implement</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td></td>
<td>Solve routine problems</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Applying</td>
<td>Total</td>
<td>12 (24%)</td>
<td>12 (24%)</td>
<td>12 (24%)</td>
<td>10 (20%)</td>
<td>46 (23%)</td>
<td>196 (100%)</td>
</tr>
<tr>
<td>Reasoning</td>
<td>Analyse</td>
<td>4 (8%)</td>
<td>5 (10%)</td>
<td>5 (10%)</td>
<td>5 (10%)</td>
<td>19 (10%)</td>
<td>19 (10%)</td>
</tr>
<tr>
<td></td>
<td>Generalise</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td></td>
<td>Synthesize/integrate</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td></td>
<td>Justify</td>
<td>0 (0%)</td>
<td>4 (8%)</td>
<td>4 (8%)</td>
<td>0 (0%)</td>
<td>8 (4%)</td>
<td>16 (8%)</td>
</tr>
<tr>
<td></td>
<td>Solve non-routine problems</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Reasoning</td>
<td>Total</td>
<td>4 (8%)</td>
<td>9 (18%)</td>
<td>9 (18%)</td>
<td>6 (12%)</td>
<td>28 (14%)</td>
<td>196 (100%)</td>
</tr>
<tr>
<td>All</td>
<td>Total</td>
<td>49 (100%)</td>
<td>49 (100%)</td>
<td>49 (100%)</td>
<td>49 (100%)</td>
<td>196 (100%)</td>
<td>196 (100%)</td>
</tr>
</tbody>
</table>
When coding the MCSK test items in Table 3.6 descriptions from the TIMMS mathematics framework (IEA, 2012) by cognitive domain were used as they provided a detailed description for each, sample behaviour. For example, Item 20 in Test 4 required pre-service teachers to order fractions and decimal fractions on a number line. The MCSK test item was coded by sub-domain as Applying and sample behaviour as model because pre-service teachers recorded their responses onto a number line and modelled the mathematics.

Tests 1 and 4 had similar coding of items by cognitive sub-domains, as did Tests 2 and 3. For all tests, more than half of the items (63%) were categorised as Knowing: “Knowing, covers the facts, concepts, and procedures” (TIMMSS 2011, 2012, p. 40). For all tests, measure and compute were the most common items for the sub-domain of knowing. This coding of measure (21%) is very similar to the number of items coded as using units of measurement (20%) in Table 3.2.

In the second sub-domain one-quarter of the items (23%) were labelled as Applying: “applying, focuses on the ability of students to apply knowledge and conceptual understanding to solve problems or answers questions” (TIMMSS 2011, 2012, p. 40). Of the Applying items most were coded select because an efficient operation, method or strategy could be chosen to calculate the correct solution. No items were coded for the sample behaviours as represent, implement or solve routine problems.

With regard to the final cognitive sub-domain, 14% of items were categorised as Reasoning; “Reasoning, goes beyond the solution of routine problems to encompass unfamiliar situations, complex contexts, and multi-step problems” (TIMMSS 2011, 2012, p. 40). For the sub-domain of Reasoning, most items were coded analyse because these items involved proportional reasoning and comparison of variables, usually units of measurement, for example millilitres and litres. Within Tests 2 and 3, 8% of items were coded justifying, as they required a written explanation for the response drawing on known mathematical properties. For example, for Test 2 Item 26, pre-service teachers were required to insert a decimal point in the appropriate place in the answer to this multiplication algorithm $534.6 \times 0.545 = 291.357$ and explain their reasoning.
When coding the MCSK test items for Tests 1–4, the researcher sometimes had difficulty identifying which sub-domain best matched the test items. Therefore, the three supervisors checked the final coding for consistency.

Table 3.7 provides examples of the different test items, including coding by content of the ACM (ACARA, 2013), type of problem, routine, non-routine and the cognitive domain. To illustrate, routine items and non-routine items could be open-ended, closed or multiple-choice question items. Item 37 of Test 1 was a non-routine and open-ended problem which required pre-service teachers to find the dimensions of a rectangle given the area. The second step was to use the dimensions of the rectangle to calculate the perimeter.

In summary, the four groupings used to code the items within the four MCSK tests showed that test items were similarly distributed with respect to ACM content strands (Table 3.3), test item design (Table 3.4), type of problem items (Table 3.5) and cognitive sub-domains (Table 3.6). The ACM content strands and cognitive sub-domains were used to analyse and compare first and second-year pre-service teachers’ number of correct responses and were reported by ranking items by three levels of difficulty: least difficult, difficult and most difficult (see later, Figure 3.1)
Table 3.7
Examples of MCSK Test Items and Coding by Content ACM (ACARA, 2013), Question Item, Problem Item and Cognitive Domain

<table>
<thead>
<tr>
<th>Item</th>
<th>Question</th>
<th>Content ACM</th>
<th>Question Item</th>
<th>Problem Item</th>
<th>Cognitive domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Divide 2160 by 20</td>
<td>Number and Place value</td>
<td>Closed problem</td>
<td>Routine</td>
<td>Compute</td>
</tr>
<tr>
<td>10</td>
<td>Estimate the size of the acute angles below.</td>
<td>Geometric Reasoning</td>
<td>Open problem</td>
<td>Routine</td>
<td>Measure</td>
</tr>
<tr>
<td>12</td>
<td>This is a list of the length in metres of snakes in the zoo enclosure.</td>
<td>Statistics &amp; Probability</td>
<td>Closed</td>
<td>Non-routine</td>
<td>Select</td>
</tr>
<tr>
<td></td>
<td>What is the main (average) length of the snakes? 8.37, 5.9, 6.31,</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.08</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>Put these numbers on a number line</td>
<td>Number and Algebra</td>
<td>Closed problem</td>
<td>Non-routine</td>
<td>Model</td>
</tr>
<tr>
<td></td>
<td>$\frac{5}{6}, \frac{4}{3}, \frac{1}{9}, 0.3, \frac{1}{3}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 3.7 (continued)
Examples of MCSK Test Items and Coding by Content ACM (ACARA, 2013), Question Item, Problem Item and Cognitive Domain

<table>
<thead>
<tr>
<th>Item</th>
<th>Question</th>
<th>Content ACM</th>
<th>Question Item</th>
<th>Problem Item</th>
<th>Cognitive domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>28</td>
<td>Which of these nets will NOT make a cube?</td>
<td>Measurement and Geometry</td>
<td>Multiple choice</td>
<td>Routine</td>
<td>Recognise</td>
</tr>
<tr>
<td></td>
<td><img src="image" alt="Net Diagram" /></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>If the area of a rectangular garden is 18 square metres. What could be it dimension? Draw a diagram and label the dimension. Find its perimeter. Length: ___ Width: ___ Perimeter: ___</td>
<td>Measurement and Geometry</td>
<td>Open problem</td>
<td>Non-routine</td>
<td>Measure</td>
</tr>
<tr>
<td>38</td>
<td>3200 square centimetres is the same as ___ square metres</td>
<td>Measurement and Geometry</td>
<td>Open problem</td>
<td>Non-routine</td>
<td>Measure</td>
</tr>
<tr>
<td>49</td>
<td>The scale on the map is 1:1 250 000. What distance in kilometres is 6 cm? (Image not to scale)</td>
<td>Number and Algebra</td>
<td>Closed problem</td>
<td>Non-routine</td>
<td>Analyse</td>
</tr>
</tbody>
</table>
3.4.2 Participants

Prior to commencing this study, ethics approval was sought to collect and analyse pre-service teachers’ MCSK test item responses completed as part of their assessment for the Bachelor of Education program at the beginning of 2008.

Before completing their MCSK test, the pre-service teachers were informed about the study. The researcher met first and second year pre-service teachers at the conclusion of lectures at the four campuses during either first or second semester. All pre-service teachers were invited to complete a consent form and the university lecturers provided the original MCSK test papers for the study after assessment. Pre-service teachers were offered a copy of their test on request.

Two hundred and ninety-seven first-year and 195 second-year pre-service teachers agreed to participate in the study involving their MCSK test analysis. See Table 3.8 for distribution of participants across year levels and the number of MCSK tests that were collected. The second-year MCSK test papers collected during 2008 included 11 of the 17 pre-service teachers who volunteered and participated in the longitudinal study of primary pre-service teachers’ MCK. Section 3.5 provides further information on the longitudinal participants and qualitative methods.

Table 3.8
Convenience Sample of MCSK Test Instrument Collected from Pre-service Teachers during 2008

<table>
<thead>
<tr>
<th>Test Number</th>
<th>Semester</th>
<th>Program of study</th>
<th>Year level of program</th>
<th>Number of test papers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>Unit 2B</td>
<td>2</td>
<td>99</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>Unit 2B</td>
<td>2</td>
<td>47</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>Unit 2B</td>
<td>2</td>
<td>49</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>Unit 1A</td>
<td>1</td>
<td>297</td>
</tr>
</tbody>
</table>

During 2008 MCSK test papers were collected from four cohorts of pre-service teachers. Test instruments were completed in first or second semester depending when pre-service teachers completed the primary mathematics education units of study. All first-year (Test 4) and one cohort of second-year pre-service teachers (Test 1)
completed the MCSK test at the end of first semester, two more cohorts of second-year pre-service teachers (Test 2 and 3) completed it at the end of second semester.

The sample of MCSK test papers had good external validity because most of the first and second-year pre-service teachers’ completed tests were available for the study, and therefore the sample was representative of the population (Betts, Hayward, & Garnham, 2001). The participants were scheduled to complete a MCSK test at a time convenient for the study.

3.4.3 Data Analysis

Two methods of analysis were used for the results of the MCSK tests: descriptive analysis and an in-depth analysis of the MCSK test items participants found most difficult.

Coding of test instrument items

After collecting the MCSK test papers, they were sorted into four groups (Test 1, 2, 3 and 4, according to like tests). All papers were de-identified and labelled by number. The longitudinal study pre-service teacher test papers were labelled using their pseudonym names. Next variables were chosen to code the different measures of the MCSK test papers. When presenting scores, different scales of measurement can be chosen to represent data (McMillan, 2004). Nominal measures have no order and can be used to classify scores to identify the differences between the data, whereas ordinal measures rank the data in order (Kervin et al., 2006). Nominal measures were used for identifying the four MCSK test instruments; the year the test was completed; unit of study (when test was completed); and year of program the pre-service teacher completed the test.

The ordinal coding for all MCK item responses was 0 = no response or error and 2 = correct response. It should be noted that zero was used to code items for which pre-service teachers did not record a response or missed a question, because it was assumed that a blank response indicated they did not know the correct answer. After scoring each item response, the data were entered into an Excel spreadsheet.
Descriptive statistical analysis of test data

The ordinal coding indicating pre-service teachers’ numbers of items with correct responses were first entered into an Excel spreadsheet then exported into SPSS, a statistical software program. SPSS was used to calculate the percentages of correct responses for all items. Items with correct and incorrect responses were reported as means, frequencies and standard deviation.

Ranking by difficulty level

Items were ranked by percentage of correct item responses and coded to identify item difficulty. Grouping MCK items by difficulty involved scoring topics as least difficult, difficult and most difficult.

Figure 3.1 illustrates the coding of difficulty ranking as most difficult, difficult or least difficult. Least difficult MCK items were those for which more than 50% of responses were correct; difficult items were those with between 30% and 50% correct responses; and most difficult items had fewer than 30% correct responses.

Figure 3.1 Ranking of items by percentage of correct item responses including least difficult, difficult and most difficult.
CHAPTER THREE

Analysis of least difficult and most difficult MCK items

As part of this research design, some MCSK test item responses were selected for further analysis. Section 4.5 reports on the least difficult and most difficult MCSK test items, including an in-depth analysis of a most difficult first year MCSK test item. This most difficult MCSK test item was chosen to provide further insights into pre-service teachers’ MCK by grouping and sorting their incorrect and correct responses.

First the mean frequencies of correct responses to items across the four tests were compared to identify the least difficult items. Three similar chance items, Item 12 Test 1, Item 38 Test 3 and Item 12 Test 4, were scored as least difficult. The analysis of results suggests reasons why Item 12, Test 4 was the less difficult for most first-year and second-year pre-service teachers.

Next as a comparison, the mean frequencies of correct responses for items in MCSK Test 4 (the test completed only by first-year students) were ranked to identify the two MCK items first-years found most difficult. Two ratio items were identified as most difficult, and Item 49 (dealing with a ratio scale) was chosen for in-depth analysis. This ratio item was selected because the responses illuminated a range of errors and potential misconceptions, as well as samples of different methods of working out.

A random sample of 20% of the total responses was selected for the analysis of the difficult ratio scale item. This sample size was determined after 62 (20%) responses had been tallied, showing that no new responses were being identified. The proportion of incorrect responses in the 20% sample was thought to be similar to that for the whole cohort. It was concluded that the 20% sample provided an adequate representation of the overall pattern of responses.

The random sample of responses was then grouped into nine categories: correct responses, six categories of common errors or misconceptions, various other incomplete answers, correct answers, and no response recorded. For each category the percentage of responses was calculated, recorded and the error was explained. Coding of results included categories of Chick et al.’s (2006a) PCK framework (see Table 2.1). The codes were useful for describing pre-service teachers’ written responses and to make sense of the range of methods of solutions, including the nature of MCK demonstrated. The in-
depth analysis of the MCSK test item responses was designed to identify pre-service teachers’ strengths and weaknesses in MCK when responding to a most difficult item.

The quantitative methods used in the longitudinal study of pre-service teachers, designed to investigate opportunities and influences for each year of the program, are described next.

### 3.5 Qualitative methods

A qualitative longitudinal study research design was chosen to provide insights into the major influences on pre-service teachers’ MCK. This design aimed to inform the study by extending understanding of MCK that primary pre-service teachers develop during their program experiences, identifying significant factors that enhanced their MCK during the four years of their program. The qualitative component was the major part of the study, because it produced data that assisted with responding to all of the research questions.

#### 3.5.1 Qualitative research design

The qualitative research design aimed to provide a holistic research approach to the MCK demonstrated by the pre-service teachers. The research questions were addressed by analysing different experiences and situations as part of pre-service teachers’ four years of teacher education. Therefore, multiple methods of data collection were needed to provide rich data about different aspects of the teacher education program.

A longitudinal qualitative research design was required to identify factors that influenced development of MCK that could be evidenced through change in the categories of pre-service teachers’ MCK during the four years of their program. The data were coded and classified using different frameworks (e.g. Ball et al., 2008; Chick et al., 2006a; Ma, 1999; Rowland et al., 2009) as appropriate during analysis.

A longitudinal research design enabled identification of how individuals can be influenced and change over years (Kervin et al., 2006). This design also allowed for data to be collected during different situations throughout the education program. The longitudinal design had the advantage that results would be more reliable due to
multiple data collection from individuals over time, as opposed to the recall bias that could occur if participants were interviewed only during fourth year and asked to remember experiences that had occurred during their program. Furthermore, the review of the research literature indicated a need for longitudinal studies of the development of teacher knowledge (Adler et al., 2005; Goos et al., 2008; Mewborn, 2001; Southwell et al., 2004).

An ethnographically informed design was chosen because this included three methods of data collection: observation, interview and analysis of documents (McMillan, 2004). The combination of data collection would provide a rich source of data for the qualitative research design. Other characteristics of ethnographic research are its suitability for “natural” settings, direct data collection, rich narrative descriptions, process orientation, inductive data analysis, participant perspectives and emergent research design (McMillan, 2004). These characteristics are consistent with the context of this study.

Teachers learn in many situations of practice such as across time and across multiple contexts (Peressini, Borko, Romagnano, Knuth, & Willis, 2004). “How a person learns a particular set of knowledge and skills, and the situation in which a person learns, becomes a fundamental part of what is learned” (Putnam & Borko, 2000, p. 4). Therefore, it was planned that during the four years of the longitudinal study data were collected from the participants about their opportunities to learn MCK during coursework and practicum experiences. Due to the large amount of data collected from the participants in the longitudinal study, a situated perspective was useful to guide the research, including the decisions needed for conceptualising the data.

During the second year of data collection, whilst beginning data analysis, patterns began to emerge. The researcher decided that grouping pre-service teachers into study groups according to those passing the MCSK test on their first attempt or those not, as well as pre-service teachers who selected mathematics as their secondary specialist teaching discipline or those who had not. Three study groups were formed (see next, Section 3.5.2) and would assist with reporting the findings.
This design also aimed to provide evidence of situated experiences, in particular by reporting factors such as where and under what situations, commonalities, differences or unusual experiences for generating MCK occurred. A situated perspective is when a study occurs in multiple contexts including the physical and social systems (Peressini et al., 2004). For pre-service teachers in this study, they learnt in two situations - at university and during school practicum experiences.

In an ethnographically informed design the researcher spends extended time in the research setting (McMillan, 2004). During second, third and fourth year, usually towards the end of second semester, the researcher met with the longitudinal study pre-service teachers once each year during their practicum experiences in school settings for two to three hours. Pre-service teachers provided me with copies of assignments and artefacts from their primary mathematics coursework units. Hence, most of the data were gathered from pre-service teachers during interviews and observations in their practicum school settings.

3.5.2 Participants

Initially the participants for the longitudinal component of this study were sought (in second semester, 2007) by emailing an invitation to all first-year pre-service teachers, but there were no volunteers. Next the researcher attended enrolment and lectures at the beginning of the following year (participants’ second year), and by the end of March, 24 pre-service teachers had formally agreed to participate.

The researcher had planned to select a stratified sample of 20 first-year pre-service teachers from the volunteers. A stratified sample allows for enough participants in each of the groups to perform comparisons of variables, for example, gender or age (Burns, 2000; Kervin et al., 2006; McMillan & Schumacher, 2006). The sample aimed to include equal numbers of males and females, and equal numbers of pre-service teachers completing mathematics education as a specialisation for secondary teaching and those not. However, as described above, a convenience sample of volunteers were available for the study. This method of sampling has the most bias because participants who volunteer for the study are likely to have a higher level of motivation than those who do not (Kervin et al., 2006).
In May, 2008 the researcher contacted the participants to determine a date and time for their first interview; seven pre-service teachers withdrew from the study. As a result, 17 pre-service teachers participated in the longitudinal study.

The 17 pre-service teachers in the study represented a small percentage (6%) of the total cohort of pre-service teachers enrolled in second year (N= 300) at the time of the study. However, the researcher was satisfied that they were representative of all second-year pre-service teachers because they included males and females; some mature age (25 years of age and over) pre-service teachers and; some with various secondary discipline specialisations, including mathematics; pre-service teachers who had completed different levels of mathematics during secondary schooling; all had attended different secondary schools; and at least one was studying at each of the four university campuses.

Demographic details

Table 3.9 includes demographic details and pseudonyms of the 17 pre-service teachers participating in the longitudinal component of this study and explains the sample used in the study.
Table 3.9
Demographic and Other Details of Pre-service Teachers Selected for this Study (n=17)

<table>
<thead>
<tr>
<th>Pseudonym</th>
<th>Gender</th>
<th>Age group (years)</th>
<th>Highest level of mathematics passed during secondary school</th>
<th>Passed MCSK test on first attempt</th>
<th>Unit and Year passed MCSK test</th>
<th>Specialising in secondary mathematics</th>
<th>Campus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Con</td>
<td>Male</td>
<td>18-24</td>
<td>Year 12 Mathematics Methods (CAS) Year 12 Specialist Mathematics</td>
<td>Yes</td>
<td>Unit 2B (Yr 2)</td>
<td>Yes</td>
<td>Campus 1</td>
</tr>
<tr>
<td>Don</td>
<td>Male</td>
<td>18-24</td>
<td>Year 10 mathematics(^2)</td>
<td>No</td>
<td>Unit 2B (supplementary Yr 2)(^3)</td>
<td>No</td>
<td>Campus 3</td>
</tr>
<tr>
<td>Elizabeth</td>
<td>Female</td>
<td>18-24</td>
<td>Year 10 mathematics</td>
<td>Yes</td>
<td>Unit 2B (Yr 2)</td>
<td>No</td>
<td>Campus 1</td>
</tr>
<tr>
<td>Emma</td>
<td>Female</td>
<td>25-30</td>
<td>Year 11 mathematics</td>
<td>No</td>
<td>Unit 1B (Yr 1)(^3)</td>
<td>No</td>
<td>Campus 3</td>
</tr>
<tr>
<td>Esther</td>
<td>Female</td>
<td>18-24</td>
<td>Year 11 mathematics</td>
<td>Yes</td>
<td>Unit 2B (Yr 2)</td>
<td>No</td>
<td>Campus 2</td>
</tr>
<tr>
<td>Fiona</td>
<td>Female</td>
<td>25-30</td>
<td>Year 11 mathematics</td>
<td>Yes</td>
<td>Unit 2B (Yr 2)</td>
<td>No</td>
<td>Campus 3</td>
</tr>
<tr>
<td>Janette</td>
<td>Female</td>
<td>18-24</td>
<td>Year 12 Further Mathematics</td>
<td>Yes</td>
<td>Unit 2B (Yr 2)</td>
<td>No</td>
<td>Campus 1</td>
</tr>
<tr>
<td>Jenny</td>
<td>Female</td>
<td>18-24</td>
<td>Year 11 mathematics</td>
<td>No</td>
<td>Unit 1B (Yr 4)</td>
<td>No</td>
<td>Campus 4</td>
</tr>
<tr>
<td>Julie</td>
<td>Female</td>
<td>18-24</td>
<td>Year 10 mathematics</td>
<td>No</td>
<td>Unit 1B (Summer school Yr 2)(^3)</td>
<td>No</td>
<td>Campus 1</td>
</tr>
<tr>
<td>Kerri</td>
<td>Female</td>
<td>18-24</td>
<td>Year 12 Further Mathematics</td>
<td>Yes</td>
<td>Unit 2B (Yr 2)</td>
<td>No</td>
<td>Campus 1</td>
</tr>
</tbody>
</table>

\(^2\) Since 2009, applicants require VCE mathematics Units 3 and 4.

\(^3\) Passed supplementary MCSK test during this unit.
Table 3.9 (continued)

Demographic and Other Details of Pre-service Teachers Selected for this Study (n=17)

<table>
<thead>
<tr>
<th>Pseudonym name</th>
<th>Gender</th>
<th>Age group (years)</th>
<th>Highest level of mathematics passed during secondary school</th>
<th>Passed MCSK test on first attempt</th>
<th>Unit and Year passed MCSK test</th>
<th>Specialising in secondary mathematics</th>
<th>Campus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lisa</td>
<td>Female</td>
<td>18-24</td>
<td>Year 12 Further Mathematics</td>
<td>No</td>
<td>Unit 1B (Summer school Yr 3)</td>
<td>No</td>
<td>Campus 1</td>
</tr>
<tr>
<td>Mathew</td>
<td>Male</td>
<td>18-24</td>
<td>Year 12 Further Mathematics</td>
<td>Yes</td>
<td>Unit 2B (Yr 2)</td>
<td>No</td>
<td>Campus 4</td>
</tr>
<tr>
<td>Michael</td>
<td>Male</td>
<td>18-24</td>
<td>Year 12 Mathematics Methods (CAS)</td>
<td>Yes</td>
<td>Unit 2B (Yr 2)</td>
<td>No</td>
<td>Campus 1</td>
</tr>
<tr>
<td>Peter</td>
<td>Male</td>
<td>18-24</td>
<td>Year 12 Mathematics Methods (CAS)</td>
<td>Yes</td>
<td>Unit 2B (Yr 2)</td>
<td>No</td>
<td>Campus 1</td>
</tr>
<tr>
<td>Rose</td>
<td>Female</td>
<td>18-24</td>
<td>Year 11 mathematics</td>
<td>Yes</td>
<td>Unit 1B (Summer school Yr 2)</td>
<td>No</td>
<td>Campus 3</td>
</tr>
<tr>
<td>Sean</td>
<td>Male</td>
<td>18-24</td>
<td>Year 12 Further Mathematics Year 12 Mathematics Methods (CAS)</td>
<td>Yes</td>
<td>Unit 2B (Yr 2)</td>
<td>Yes</td>
<td>Campus 1</td>
</tr>
<tr>
<td>Shelly</td>
<td>Female</td>
<td>18-24</td>
<td>Year 12 Mathematics Methods (CAS)</td>
<td>Yes</td>
<td>Unit 2B (Yr 2)</td>
<td>Yes</td>
<td>Campus 1</td>
</tr>
</tbody>
</table>

4 Michael initially chose mathematics for secondary teaching as his major, but changed to a different major at the end of the first semester of first year.
**Grouping of participants**

Some ethnographic designs enable comparisons across groups (McMillan & Schumacher, 2006). The pre-service teachers were divided into two groups using the number of attempts taken to demonstrate competency when passing the MCSK test (one, or more than one), and whether they completed mathematics as their secondary elective or chose another discipline. Table 3.10 is a summary of the criteria used for coding the three study groups and the pseudonyms of the 17 pre-service teachers in the study.

Table 3.10
*Criteria of the Three Study Groups and Subsequent Groupings of Pre-service Teachers from the Longitudinal Study (n=17)*

<table>
<thead>
<tr>
<th>Case Number</th>
<th>Criteria</th>
<th>Pseudonyms of pre-service teachers (n=17)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Passed MCSK test on first attempt and selected mathematics as a secondary discipline specialisation</td>
<td>Con, Sean, Shelly (n=3)</td>
</tr>
<tr>
<td>2</td>
<td>Passed MCSK test on first attempt and did not select mathematics as a secondary discipline specialisation</td>
<td>Elizabeth, Esther, Fiona, Jeanette, Kerri, Mathew, Michael, Paul, Rose (n=9)</td>
</tr>
<tr>
<td>3</td>
<td>Did not pass MCSK test on first attempt and did not select mathematics as a secondary discipline specialisation</td>
<td>Don, Emma, Jenny, Julie, Lisa (n=5)</td>
</tr>
</tbody>
</table>

**3.5.3 Data collection**

Multiple data were collected from pre-service teachers about their various program experiences for analysis of MCK and factors contributing to development of MCK. Data collection techniques included an initial questionnaire (Year 2); lesson observations (Years 2 and 4); one-on-one interviews (Years 2, 3 and 4); artefacts and documents from coursework and practicum experiences (Years 1, 2 and 4), and responses to an MCK interview task (Year 4).
Table 3.11 contains a summary of data collected from the study pre-service teachers during coursework and practicum experiences.

Table 3.11
*Sources of Data Collection Including Coursework and Practicum Teaching*

<table>
<thead>
<tr>
<th>Year</th>
<th>Course work</th>
<th>Practicum teaching</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td><strong>Unit 1A</strong></td>
<td>Primary mathematics lesson plans and evaluations</td>
</tr>
<tr>
<td></td>
<td>Praxis inquiry learning log</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mathematical investigation</td>
<td></td>
</tr>
<tr>
<td>2008</td>
<td><strong>Unit 2A</strong></td>
<td>Primary mathematics lesson plans and evaluations</td>
</tr>
<tr>
<td></td>
<td>Early Years Numeracy Report</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Five mathematics lesson plans and evaluations</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Unit 2B</strong></td>
<td>Primary mathematics lesson observation</td>
</tr>
<tr>
<td></td>
<td>Two mathematics learning trajectory charts</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Construct and evaluate five primary mathematics teaching resources</td>
<td></td>
</tr>
<tr>
<td></td>
<td>MCSK test</td>
<td>Interview</td>
</tr>
<tr>
<td>2009</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2010</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Other</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Unit 1B</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mathematics learning file</td>
<td></td>
</tr>
<tr>
<td></td>
<td>MCSK test.</td>
<td></td>
</tr>
</tbody>
</table>

Questionnaire

At the beginning of the study, all participants completed a 15-item questionnaire (see Appendix C) to provide demographic details and information about prior mathematics education. Some of the data gathered from this questionnaire are reported in Table 3.9 and explain the sample used in the study.
Lesson observations

The pre-service teacher lesson observations did not occur until the Department of Education Employment and Training (DEECD, 2007) had given ethics approval for the study. The principals and mentor teachers at the schools where the longitudinal study pre-service teachers were completing their practicum experiences were contacted (2008, 2009 and 2010) and provided with an information sheet explaining the purpose of the study and a consent form. After informed consent had been obtained, each pre-service teacher was contacted to arrange a suitable time and school visit for data collection.

Lesson observations occurred once a year at various stages during the second and fourth year of the program and usually took one hour to complete. The purpose of the lesson observations was to gather data about how pre-service teachers used their MCK when teaching primary mathematics lessons. For example, the researcher focused on what the pre-service teachers said during the lesson that relied on their MCK; the questions and responses they gave children; the materials they choose and the mathematical terminology they used during the lessons. Studying participants in their own settings allows for a richer understanding of the phenomenon (McMillan, 2004).

The researcher did not participate in the lessons and chose to be as unobtrusive as possible so that the pre-service teacher maintained full control over the situation. Nor was there any interaction with the pre-service teacher, the mentor teacher or the students. This provided time to collect data in the form of field notes. In this type of observation, the researcher is described as a passive participant (McMillan, 2004).

All field notes of the observations were written immediately after the lesson to minimise loss of information. The lessons were audio-recorded, capturing the voices of the teacher and students, and transcribed by me for coding and analysis.

Interviews

There are three types of interview approaches: structured, unstructured and semi-structured (Kervin et al., 2006). For this study semi-structured interviews were chosen, which allowed the researcher to gather comparable data from all participants as well as have flexibility when asking questions and in particular, to ask further questions depending on the pre-service teachers’ responses.
The researcher planned and conducted in-depth, audio-recorded one-on-one interviews with participants during the second, third and fourth years of their program. The interviews typically lasted one hour and the researcher subsequently transcribed them for coding and analysis. During second year and fourth year, all interviews were conducted after the lesson observations. (See Appendices D–F for the interview schedules for second-year, third and fourth-year students.)

The interviews had two main purposes. They were designed to gather additional reflections from the pre-service teachers, including data about opportunities, experiences and influences that enhanced their MCK during primary mathematics coursework and teaching practice experiences. Their second purpose was to discuss the lesson the researcher had just observed, specifically identifying what was involved in planning the lesson and whether the pre-service teacher needed to check or revise the MCK needed for teaching the lesson.

*Fourth-year MCK interview items*

In addition to the fourth-year interview, pre-service teachers were asked to complete two MCK interview items. These items were chosen to identify the MCK that pre-service teachers demonstrated in the final year of their program. The responses to the fourth-year interview items were compared with their MCSK test responses in second year.

The mathematical topics and MCK interview items chosen for comparison were (a) fraction and decimals and (b) area and perimeter, because both topics have previously been identified as sources of difficulty for pre-service teachers and middle year students (e.g., Pearn & Stephens, 2004; Stacey et al., 2001; Steinle & Stacey, 1998; Tatto et al., 2012; Ubuz & Yayan, 2010; Widjaja, Stacey, & Steinle, 2011). Also, just over half of the qualitative participants had provided working out in their second-year MCSK test papers that could be analysed to interpret their correct methods of solutions as well as their misconceptions of these topics.

The fourth-year MCK interview tasks were presented to the pre-service teachers during the second part of their fourth-year interview; they took an average of approximately 15
minutes to complete each task. The participants were then asked to explain their thinking or method of solutions.

During the fourth-year interview the researcher took field notes and collected samples of the participants’ work. This part of the interview was also audio-recorded and transcribed for use when analysing pre-service teachers’ responses.

Fraction number line interview task
The fraction and number line task (see Appendix G) was designed to analysis participants’ MCK of the content strand fractions and decimals (ACARA, 2013). The fraction task included four items (see Figure 7.2). First the pre-service teachers were given three pairs of fractions and asked to identify the largest common fraction:

- Item 1: $\frac{3}{5}$ and $\frac{2}{3}$; Item 2: $\frac{3}{5}$ and $\frac{3}{4}$; Item 3: $\frac{3}{5}$ and $\frac{5}{8}$.

For Item 4, pre-service teachers were asked to place common fractions $\frac{2}{3}$, $\frac{3}{4}$, $\frac{3}{5}$ and $\frac{5}{8}$ onto a number line in order and in proportion. For each item they were asked to explain their reasoning and record their responses.

Perimeter and area interview task
The second item, a perimeter and area interview task (Appendix H), was designed to measure the MCK of the content strand measurement (ACARA, 2013). This task (Figure 7.6) assessed pre-service teachers’ MCK of perimeter and area and identified how they explain the difference between perimeter and area, while the second question (adapted from a similar item in Ma’s (1999) study) focused on the relationship between the concepts.

First the pre-service teachers were asked to explain the difference between area and perimeter. The second item required pre-service teachers to imagine that if a student in their class says “I think if the perimeter of a rectangle increases, its area also increases,” and justify their response.
Artefacts and documents
For each year of the longitudinal study the pre-service teachers provided artefacts and documents from their coursework and practicum experiences related to primary mathematics. These sources of data were a primary source of data: they were all first-hand information (McMillan, 2004) and were created or written by participants. They included lesson plans, lesson evaluations, photos of teaching resources and MCSK test responses. All artefacts collected were photocopies of the originals or digital copies sent via email.

3.5.4 Data management and analysis
During the longitudinal study multiple forms of data relating to pre-service teachers’ MCK were collected. Data collection, management and analysis occurred at the same time and included content analysis (Simminoff & Jacoby, 2008) as well as triangulation of the data (Kervin et al., 2006). Qualitative studies often have very large amounts of data (Kervin et al., 2006; Richards, 2006), and content analysis focuses on reducing data to a manageable state using inductive and deductive coding (Simminoff & Jacoby, 2008).

Coding for data management
The qualitative data analysis software NVivo was used for managing and coding the data. The qualitative data entered into NVivo included transcripts of lesson observations and interviews and other artefacts, as described in the previous section. All documents were coded by program year) and like documents (such as second-year assignments, interviews and lesson observations) grouped into folders. All documents were labelled with the participant’s pseudonym. As described earlier (see 3.5.2), the participants were classified into three study groups to facilitate reporting of the data.

As Richards (2006) suggested, nodes and hierarchical coding assists with organising ideas, the project and monitoring the bigger picture. To illustrate, tree nodes were used to code comments from pre-service teachers’ second-year, third-year and fourth-year interviews, as well as comments about coursework during first and second year; lesson observations during second and fourth years were also grouped within tree nodes. Each tree node had subsections; first-year reflections included nodes for comments about
first-year practicum experiences, the first-year practice MCSK test, summer school comments (Unit 1B), reflections about their primary and secondary school experiences, and comments related to their weakest mathematical topics. Another tree node held comments and coding of different resources that pre-service teachers identified as helping them to extend their MCK: lecturers, mentor teachers, textbooks, Department of Education online resources, learning from primary students, observing mathematics lessons, professional learning experiences, tutoring and online quizzes. Other nodes stored the coding of responses to the different fourth-year MCK interview items, evidence of SCK and evidence of procedural knowledge.

Whilst coding, the data were read and reread to identify significant themes and factors for reporting within the findings. This process assisted with refining the subsidiary research questions, and was useful for identifying the opportunities and influences that enhanced pre-service teachers’ MCK during coursework and practicum experiences. Inductive coding assisted with identification of data needed for responding to the qualitative research questions and deductive data analysis.

Analysis of coursework and practicum teaching experiences

The categories used for inductive data analysis of the textual data were derived from the collection of lesson observations, interviews and documents. Inductive data analysis was used to generate the findings (McMillan, 2004) to identify opportunities or influences that enhanced or hindered pre-service teachers’ MCK.

Next, different categories from MCK frameworks (Chick et al., 2006a; Hill et al., 2008; Ma, 1999; Rowland et al., 2009) were chosen to code evidence and identify the differences of pre-service teachers’ MCK from the data. The MCK categories (described below) were suitable for coding evidence of MCK when pre-service teachers responded to coursework tasks, during practicum experiences and teaching a primary mathematics lesson, as well as when responding to interview questions and fourth-year MCK interview tasks.

First and second-year coursework assignments were coded using three of the categories of the Knowledge Quartet (Rowland et al., 2009): foundation knowledge, transformation and making connections (see 2.2.8). These were used to analyse
assignments in order to identify when and how pre-service teachers demonstrated evidence of developing their MCK.

First, second and fourth-year lesson observation and interview reflections on practicum experiences were coded using all four categories of the Knowledge Quartet (Rowland et al., 2009): foundation knowledge, transformation, making connections and contingencies. These were used to identify when pre-service teachers demonstrated evidence of developing their MCK.

Analysis of MCSK test items and MCK interview items

The final analysis involved second-year MCSK test responses and fourth-year MCK interview task responses relating to MCK of fraction and decimals as well as area and perimeter. These topics were chosen for analysis because during second year most of the participants had provided responses for these topics in MCSK Test 2 and recorded their working out. Second-year responses were able to be compared to similar interview items in their fourth-year. The codes for three categories of Content Knowledge in a Pedagogical Context (Chick et al., 2006a) (see Table 2.1) were used to analyse responses to MCSK test items and MCK interview items.

Fraction and decimals coding and analysis

During second year the longitudinal study participants’ written responses to Item 19 MCSK Test 2 (n=9) were coded (a tick indicated a correct response and a cross an incorrect response), as was the method used to describe their verbal responses to the fourth year MCK interview items. The number of correct and incorrect responses (see Table 7.1) were then compared and discussed.

Next, responses to the fourth-year fraction number line task were coded in two parts, signifying success or failure in ordering the set of fractions and decimals and in deciding whether the fractions were in proportion on the number line. A tick indicated a correct response and a cross an incorrect response. The results were also reported by study group (see Table 3.10) so that results could be compared within and across groups.
For the fraction number line task Items 1, 2 and 3, pre-service teachers were required to select the larger fraction from a pair of fractions. These responses were coded after considering the interview responses, and included: (known) fact, drew a linear (strip) model to compare the two fractions, converted to equivalent fractions in order to compare, converted to equivalent decimal and/or percentage to compare fractions, used number sense, or made a correct guess. This coding was chosen to gain insights into pre-service teachers’ strategies and methods of solution to identify evidence of procedural knowledge, mathematical structure and connections (Chick et al., 2006a).

Area and perimeter coding and analysis
The longitudinal study participants’ correct and incorrect responses to Item 48 MCSK Test 2 during second year were coded with a tick indicating a correct response and a cross an incorrect response. For coding and analysis of the fourth-year item, a scoring rubric was designed to code pre-service teachers’ responses for Item 1 and Item 2 (Table 3.12). This coding drew on the categories of procedural knowledge, mathematical structure and connections of Chick et al.’s (2006a) framework Content Knowledge in the Pedagogical Context (see Table 2.1).

Four categories were used to code the perimeter and area responses. Item 1 was scored twice, once for explanation of perimeter and again for explanation of area. Item 2 was scored according to pre-service teachers’ explanation of the relationship between area and perimeter. For example, a score of one related to methods of solution, a score two related to procedural knowledge and a score of three involved elements of mathematical structure and connections.
Table 3.12
Fourth-year Coding for Area and Perimeter MCK Item

<table>
<thead>
<tr>
<th>Description of response</th>
<th>Score = 0</th>
<th>Score = 1</th>
<th>Score = 2</th>
<th>Score = 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unable to provide correct response</td>
<td>Some correct mathematical understanding but incomplete response</td>
<td>Correct response using procedural knowledge or lacking mathematical connections</td>
<td>Correct explanation justifies and/or understands the concept or process</td>
<td></td>
</tr>
</tbody>
</table>

| Example response Question 1 | Area is length plus width. Perimeter is length times width | Perimeter is outside of an object. Area is the inside of an object | Perimeter is adding length and width. Area is length times height | Perimeter is the **distance** around the outside of the shape. Area is the **amount** of space contained within the shape. |

| Example response Question 2 | Accepted student’s hypothesis but did not explain why | Accepted student’s hypothesis, but used diagram/s to justify | Identified student was incorrect and explored area and perimeter of different rectangles to identify one example to show student was incorrect | Knew assumption was incorrect and could justify their response drawing on more than one example |

The analysis of the fourth-year MCK interview tasks identified the different categories (Chick et al., 2006a) of MCK that longitudinal study participants demonstrated: evidence of mathematical structure or connections; procedural knowledge and conceptual understanding need not be evident; or could not demonstrate a method of solution.

The MCK frameworks (Chick et al., 2006a; Hill et al., 2008; Ma, 1999; Rowland et al., 2009) contain different categories of MCK that pre-service teachers demonstrated during their teacher education program experiences. Figure 8.1 shows the similarities and differences between pre-service teachers’ MCK during the study and was designed to identify stages of development of MCK as well as combine categories of theoretical frameworks (Chick et al., 2006a; Hill et al., 2008; Ma, 1999; Rowland et al., 2009). The
deductive data analysis also assisted with identifying factors that contributed to pre-service teachers’ opportunities to develop MCK during coursework and practicum experiences.

3.6 Summary

This study was designed to reveal new understanding and theories about pre-service teachers’ MCK. The research design combined quantitative and qualitative methods. The findings from the different methods were intended to converge and support one another when responding to the major and subsidiary research questions.

The quantitative data collection and analysis was designed to compare the MCSK test scores of different cohorts of first and second-year pre-service teachers from the same university. These findings were compared with the quantitative and qualitative analysis of the longitudinal study pre-service teachers’ responses to MCK interview items during fourth-year. The purpose of the comparison of pre-service teachers’ MCK aimed to identify any change in their MCK from second to fourth-year of the program.

The purpose of the qualitative method was to identify opportunities for pre-service teachers to develop their MCK during their practicum experiences. The different types of MCK that pre-service teachers demonstrated as they undertook units in primary mathematics education and practicum experiences in primary and secondary classrooms were of particular interest. The researcher sought to identify the program elements that influenced or assisted pre-service teachers to develop their MCK. The results of this study are reported in the following four chapters, beginning with the quantitative analysis of the MCSK test in Chapter 4.
CHAPTER FOUR

What did they know?

This chapter reports the results of descriptive analysis of pre-service teachers’ strengths and weaknesses when responding to Mathematical Competency, Skills and Knowledge (MCSK) items. These results also serve as baseline data for the pre-service teachers participating in the four-year longitudinal study, the results of which are reported in the next three chapters. As described in the previous chapter, pre-service teachers normally completed their MCSK test in the second year of their program, but in 2008 both first-year and second-year students completed the MCSK test, providing a much larger sample for analysis.

The first section below contains the mean frequencies and standard deviations of the percentage of correct responses to mathematical content knowledge (MCK) items given by first-year and second-year pre-service teachers. Data were collected from four similar MCSK tests and all MCSK test items were coded by content strands then content sub-strands, of the Australian Curriculum Mathematics (ACM) (ACARA, 2013). The purpose of this analysis was to identify the similarities and differences in the pre-service teachers’ responses to the four MCSK tests reported within this chapter.

The following two sections, describe first-year then second-year pre-service teachers’ MCK using results from analysis of MCSK tests in which test items were grouped according to level of difficulty: least difficult, difficult and most difficult. MCK items were coded as least difficult when more than 50% of responses were correct; difficult if the percentage of correct responses was between 30% and 50% and most difficult if fewer than 30% of responses were correct. After coding, the researcher undertook descriptive analysis, including an in-depth analysis of some of the most difficult MCSK test items. This includes the number of items with correct responses by ACM content strands and sub-strands (ACARA, 2013) and level of difficulty ranking. The total number of MCK items were grouped by sub-strands and reported by number of least difficult, difficult and most difficult items. The numbers of least difficult, difficult and most difficult items with correct responses were reported by cognitive sub-domain and
sample behaviour (Tatto et al., 2012) items. This chapter concludes with an in-depth analysis of correct and common incorrect responses of first-year pre-service teachers’ responses of this most difficult ratio item. These results were described using the categories of Chick, Baker, Pham and Cheng’s (2006a) PCK framework and categorise and interpret their responses to MCSK test items.

**4.1 First and second-year pre-service teachers’ MCK by content strand**

MCSK test item responses were collected during the second year of data collection (2008) from first-year (n=297) and second-year (n=195) pre-service teachers, including longitudinal study pre-service teachers (n=11) who completed the MCSK test during second-year. Four similar MCSK test instruments were used in this study; each consisted of 49 MCSK test items, ranging in difficulty from Year 5 to Year 8 mathematical knowledge (ACARA, 2013).

Table 4.1 contains the means and standard deviations of percentage of MCSK test items answered correctly by one first-year cohort and three cohorts of second-year pre-service teachers; including the number of items with correct responses by content strand and content sub-strand ACM (ACARA, 2013). The first column for each test can be used to compare the number and percentage of items by content sub-strand and the second column compares the mean scores of these items and standard deviation.

The results include pre-service teachers’ percentages of likely frequencies because they were the mean score of usually more than one MCSK test item rather than the mean score of one item. The mean score was calculated by adding pre-service teachers’ total percentages of correct responses for each of the items coded by content sub-strand and then dividing by the number of items per content sub-strands for each MCSK test. All responses were collected during 2008 from four cohorts of pre-service teachers including: MCSK Test 1, second-year pre-service, teachers, Semester 1; MCSK Test 2, second-year pre-service, teachers, Semester 2; MCSK Test 3, second-year pre-service, teachers, Semester 2; and MCSK Test 4, first-year pre-service, teachers, Semester 1.
Table 4.1
Mean Frequencies and Standard Deviations for Pre-service Teachers’ Correct Responses: MCSK Tests 1, 2, 3 and 4 by Content Strands and Sub-strands ACM (ACARA, 2013)

<table>
<thead>
<tr>
<th>ACM content strands</th>
<th>ACM content sub-strand</th>
<th>Test 1 Year 2</th>
<th>Test 2 Year 2</th>
<th>Test 3 Year 2</th>
<th>Test 4 Year 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of Items</td>
<td>Mean scores (%)(SD)</td>
<td>Number of Items</td>
<td>Mean scores (%)(SD)</td>
<td>Number of Items</td>
</tr>
<tr>
<td>Number &amp; Algebra</td>
<td>Number &amp; Place Value 4 (8%)</td>
<td>76% (11)</td>
<td>7 (14%)</td>
<td>79% (12)</td>
<td>7 (14%)</td>
</tr>
<tr>
<td>Number &amp; Algebra</td>
<td>Pattern &amp; Algebra 3 (6%)</td>
<td>65% (43)</td>
<td>0 (0%)</td>
<td>0% (0.00)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Number &amp; Algebra</td>
<td>Fractions &amp; Decimals 4 (8%)</td>
<td>60% (19)</td>
<td>4 (8%)</td>
<td>73% (42)</td>
<td>4 (8%)</td>
</tr>
<tr>
<td>Number &amp; Algebra</td>
<td>Real Numbers 15 (31%)</td>
<td>76% (11)</td>
<td>14 (29%)</td>
<td>67% (23)</td>
<td>14 (29%)</td>
</tr>
<tr>
<td>Number &amp; Algebra</td>
<td>Money &amp; Financial Mathematics 2 (4%)</td>
<td>75% (20)</td>
<td>1 (2%)</td>
<td>72% (0)</td>
<td>1 (2%)</td>
</tr>
<tr>
<td>Number &amp; Algebra</td>
<td>Total 28 (57%)</td>
<td>65% (18)</td>
<td>26 (53%)</td>
<td>71% (22)</td>
<td>26 (53%)</td>
</tr>
</tbody>
</table>
Table 4.1 (continued)
Mean Frequencies and Standard Deviations for Pre-service Teachers' Correct Responses: MCSK Tests 1, 2, 3 and 4 by Content Strands and Sub-strands ACM (ACARA, 2013)

<table>
<thead>
<tr>
<th>ACM content strands</th>
<th>ACM content sub-strand</th>
<th>Test 1 Year 2</th>
<th>Test 2 Year 2</th>
<th>Test 3 Year 2</th>
<th>Test 4 Year 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Number of Items</td>
<td>Mean scores (%)(SD)</td>
<td>Number of Items</td>
<td>Mean scores (%)(SD)</td>
</tr>
<tr>
<td>Measurement &amp; Geometry</td>
<td>Using units of Measurement</td>
<td>11 (22%)</td>
<td>60% (24)</td>
<td>12 (24%)</td>
<td>68% (17)</td>
</tr>
<tr>
<td>Measurement &amp; Geometry</td>
<td>Shape</td>
<td>6 (12%)</td>
<td>52% (24)</td>
<td>5 (10%)</td>
<td>70% (19)</td>
</tr>
<tr>
<td>Measurement &amp; Geometry</td>
<td>Location &amp; Transformation</td>
<td>1 (2%)</td>
<td>71% (0)</td>
<td>1 (2%)</td>
<td>35% (0)</td>
</tr>
<tr>
<td>Measurement &amp; Geometry</td>
<td>Geometric Reasoning</td>
<td>0 (0%)</td>
<td>0% (0.00)</td>
<td>1 (2%)</td>
<td>64% (0)</td>
</tr>
<tr>
<td>Measurement &amp; Geometry</td>
<td>Total</td>
<td>18 (38%)</td>
<td>66% (19)</td>
<td>19 (39%)</td>
<td>67% (19)</td>
</tr>
<tr>
<td>Statistics &amp; Probability</td>
<td>Chance</td>
<td>0 (0%)</td>
<td>0% (0.00)</td>
<td>2 (4%)</td>
<td>40% (7)</td>
</tr>
<tr>
<td>Statistics &amp; Probability</td>
<td>Data Representation &amp; Interpretation</td>
<td>3 (7%)</td>
<td>63% (18)</td>
<td>2 (4%)</td>
<td>39% (6)</td>
</tr>
<tr>
<td>Statistics &amp; Probability</td>
<td>Total</td>
<td>3 (6%)</td>
<td>63% (18)</td>
<td>4 (8%)</td>
<td>79% (13)</td>
</tr>
<tr>
<td>All</td>
<td>Total</td>
<td>49 (100%)</td>
<td>62% (20)</td>
<td>49 (100%)</td>
<td>70% (20)</td>
</tr>
</tbody>
</table>
The four MCSK tests had the same number of items and similar content by ACM (ACARA, 2013). The mean scores show that first-year pre-service teachers had more difficulty responding correctly to their test items, Test 4, when compared with second-year pre-service teachers and Tests 1–3. Second-year pre-service teachers had more difficulty with Test 2 than Test 1 and 3.

In particular, first-year pre-service teachers were likely to have difficulty (ranked 30-50%) with Fraction and Decimal, Real number, Using Units of Measurement, and Shape MCSK test items than second-year pre-service teachers (responding to similar MCSK test items). First and second-year pre-service teachers experienced least difficulty (ranked greater than 50%) with Number and Place Value, Money and Financial Mathematics, and Chance items. Second-year pre-service teachers, Test 2 and Test 3 and first-year pre-service teachers were likely to have most difficulty responding correctly to one Location and Transformation item.

4.2 First-year pre-service teachers: What did they know?

Table 4.1 shows that for the ACM content strand Number and Algebra, just over half of the first-year pre-service teachers answered these MCSK test items correctly. Of the sub-strands for Number and Algebra, to the item with the highest number of correct answers was in Money and Financial Mathematics; more than half were able to solve Number and Place Value and Pattern and Algebra; slightly less than half correctly responded to the Fractions and Decimal items. More (15) test items were coded Real Number, and less than half of pre-service teachers were likely to correctly respond to these items.

Consideration of the responses to the MCSK test items from the second content strand, Measurement and Geometry, suggested that this content strand was difficult for first year pre-service teachers. A little more than one-third of first-year pre-service teachers responded correctly to these items. The most difficult sub-strand (with one item only) was a Location and Transformation item with only a fifth answering correctly. Of the 11 items in Using Units of Measurement, less than half of the pre-service teachers answered correctly. Just over a third of the first-year pre-service teachers correctly
answered the seven shape items. Test 4 had three items relating to Chance and the content strand Statistics and Probability; of these items, more than half of the first-year pre-service teachers responded correctly. Overall the mean percentage of Test 4 items correctly answered by first-year pre-service teachers was just less than half.

4.2.1 Difficulty rank by content strand and sub-strand

Table 4.2 reports the number and percentage of MCSK Test 4 items that first-year pre-service teachers found least difficult, difficult or most difficult. The results are grouped by ACM content strand and sub-strand.

In Table 4.2, less than half of the test items were coded as least difficult, a little less than one-third of the items were coded as difficult (29%) and the remaining items were coded as most difficult (29%) items.

Half of the Number and Algebra items, about one quarter of the Measurement items and all of the Statistics and Probability items were coded least difficult. The difficult items in Test 4 included less than one-third of Number and Algebra items, and a third of Measurement and Geometry items. Less than a quarter of the Number and Algebra items and less than half of Measurement and Geometry (items were coded most difficult items.

These results suggest that first-year pre-service teachers had less difficulty with the small number of Statistics and Probability items in Test 4, in particular chance items. Items coded as difficult and most difficult were the content sub-strands of Real Number, Using Units of Measurement and Shape. The least difficult sub-strands included Place Value, other Real Number items and some items related to Using Units of Measurement.
Table 4.2
First-year Pre-service Teachers: Number (%) of Test Items with Correct Responses for ACM Content Strands and Sub-strands (ACARA, 2013) by Difficulty Ranking (N=49)

<table>
<thead>
<tr>
<th>ACM content strands</th>
<th>ACM content sub-strand</th>
<th>Total number of items</th>
<th>Least difficult n (%)</th>
<th>Difficult n (%)</th>
<th>Most difficult n (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number and Algebra</td>
<td>Number and Place Value</td>
<td>4</td>
<td>4 (100%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Number and Algebra</td>
<td>Pattern and Algebra</td>
<td>3</td>
<td>2 (66%)</td>
<td>0 (0%)</td>
<td>1 (33%)</td>
</tr>
<tr>
<td>Number and Algebra</td>
<td>Fractions and Decimals</td>
<td>4</td>
<td>1 (25%)</td>
<td>2 (50%)</td>
<td>1 (25%)</td>
</tr>
<tr>
<td>Number and Algebra</td>
<td>Real Numbers</td>
<td>16</td>
<td>6 (36.5%)</td>
<td>6 (36.5%)</td>
<td>4 (25%)</td>
</tr>
<tr>
<td>Number and Algebra</td>
<td>Money and Financial Mathematics</td>
<td>1</td>
<td>1 (100%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Number and Algebra</td>
<td>Total</td>
<td>28</td>
<td>14 (50%)</td>
<td>8 (27%)</td>
<td>6 (22%)</td>
</tr>
<tr>
<td>Measurement and Geometry</td>
<td>Using Units of Measurement</td>
<td>11</td>
<td>3 (28%)</td>
<td>4 (36%)</td>
<td>4 (36%)</td>
</tr>
<tr>
<td>Measurement and Geometry</td>
<td>Shape</td>
<td>6</td>
<td>1 (66%)</td>
<td>2 (33%)</td>
<td>3 (50%)</td>
</tr>
<tr>
<td>Measurement and Geometry</td>
<td>Location and Transformation</td>
<td>1</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>1 (100%)</td>
</tr>
<tr>
<td>Measurement and Geometry</td>
<td>Geometric Reasoning</td>
<td>0</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Measurement and Geometry</td>
<td>Total</td>
<td>18</td>
<td>4 (22%)</td>
<td>6 (33%)</td>
<td>8 (44%)</td>
</tr>
<tr>
<td>Statistics and Probability</td>
<td>Chance</td>
<td>0</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Statistics and Probability</td>
<td>Data Representation and Interpretation</td>
<td>3</td>
<td>3 (100%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Statistics and Probability</td>
<td>Total</td>
<td>3</td>
<td>3 (100%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>All</td>
<td>Total</td>
<td>49</td>
<td>21 (42%)</td>
<td>14 (29%)</td>
<td>14 (29%)</td>
</tr>
</tbody>
</table>
4.2.2 Difficulty scoring by cognitive sub-domain

Table 4.3 reports the number and percentage of MCSK Test items that first-year pre-service teachers found least difficult, difficult or most difficult. The results are grouped by cognitive sub-domain and grouped sample behaviour (Tatto et al., 2008a).

The distribution of the total number of items ranked as least difficult, difficult and most difficult items in Table 4.3 was the same as the total in Table 4.2. That is, more items were coded as least difficult when compared to difficult or most difficult items; less than half of the items were coded as least difficult, a bit less than one third were coded as difficult and the remaining items were coded most difficult.

The difficult items for Test 4 included about one-third of the items in the sub-domain of Knowing and smaller proportions of the and Applying items. Items coded as most difficult included half of the Reasoning items and almost one-quarter of the Knowing items and Applying items.

These results show that first-year pre-service teachers had less difficulty with the sub-domain Applying but could respond correctly to more than three-quarters of the Select items. For the sub-domain Knowing they responded correctly to half of the Compute items but had difficulty with most of the Recognise items. About two-thirds of the Measure items were difficult or most difficult. The most difficult Reasoning items were the sample behaviour of Analyse. Note that not all sample behaviours by cognitive domains were represented within Test 4.
Table 4.3
First-year Pre-service Teachers: Number (%) of Test Items with Correct Responses for Cognitive Domain (Tatto et al., 2012) by Difficulty Ranking (N=49)

<table>
<thead>
<tr>
<th>Sub-domain</th>
<th>Sample Behaviour</th>
<th>Total number of items</th>
<th>Least difficult n (%)</th>
<th>Difficult n (%)</th>
<th>Most difficult n (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowing</td>
<td>Recall</td>
<td>6</td>
<td>1 (17%)</td>
<td>1 (17%)</td>
<td>4 (66%)</td>
</tr>
<tr>
<td>Knowing</td>
<td>Recognise</td>
<td>4</td>
<td>1 (25%)</td>
<td>3 (75%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Knowing</td>
<td>Compute</td>
<td>11</td>
<td>6 (55%)</td>
<td>4 (36%)</td>
<td>1 (9%)</td>
</tr>
<tr>
<td>Knowing</td>
<td>Retrieve</td>
<td>0</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Knowing</td>
<td>Measure</td>
<td>11</td>
<td>3 (28%)</td>
<td>4 (36%)</td>
<td>4 (36%)</td>
</tr>
<tr>
<td>Knowing</td>
<td>Classify/order</td>
<td>1</td>
<td>1 (100%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Knowing</td>
<td>Total</td>
<td>33</td>
<td>12 (36%)</td>
<td>12 (36%)</td>
<td>9 (28%)</td>
</tr>
<tr>
<td>Applying</td>
<td>Select</td>
<td>6</td>
<td>5 (83%)</td>
<td>0 (0%)</td>
<td>1 (17%)</td>
</tr>
<tr>
<td>Applying</td>
<td>Represent</td>
<td>0</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Applying</td>
<td>Model</td>
<td>4</td>
<td>2 (50%)</td>
<td>1 (25%)</td>
<td>1 (25%)</td>
</tr>
<tr>
<td>Applying</td>
<td>Implement</td>
<td>0</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Applying</td>
<td>Solve routine problems</td>
<td>0</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Applying</td>
<td>Total</td>
<td>10</td>
<td>7 (70%)</td>
<td>1 (10%)</td>
<td>2 (20%)</td>
</tr>
<tr>
<td>Reasoning</td>
<td>Analyse</td>
<td>5</td>
<td>2 (40%)</td>
<td>0 (0%)</td>
<td>3 (60%)</td>
</tr>
<tr>
<td>Reasoning</td>
<td>Generalise</td>
<td>0</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Reasoning</td>
<td>Synthesize/Integrate</td>
<td>1</td>
<td>0 (0%)</td>
<td>1 (100%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Reasoning</td>
<td>Justify</td>
<td>0</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Reasoning</td>
<td>Solve non-routine Problems</td>
<td>0</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Reasoning</td>
<td>Total</td>
<td>6</td>
<td>2 (33%)</td>
<td>1 (17%)</td>
<td>3 (50%)</td>
</tr>
<tr>
<td>All</td>
<td>Total</td>
<td>49</td>
<td>21 (42%)</td>
<td>14 (29%)</td>
<td>14 (29%)</td>
</tr>
</tbody>
</table>
4.2.3 Summary of first-year pre-service teachers’ MCK

First-year pre-service teachers who responded correctly to least difficult items demonstrated MCK of Number and Place Value items, such as:

- number patterns – finding the next number in a sequence for two-digit numbers and a sequence using halves and quarters;
- writing an eleven-digit number in words;
- renaming fractions, decimals and percentages;
- multiplying three-digit whole numbers and common fractions;
- adding decimal numbers with tenths and hundredths; and
- using proportional reasoning when calculating the better value in dollars.

They also demonstrated MCK of Measurement and Geometry, by:

- using units of measurement to measure the difference in time;
- measuring the perimeter and area of a rectangle;
- correctly converting millilitres to litres;
- using proportional reasoning to solve a capacity ratio item; and
- recognising a net that would not make a cube.

For Statistics and Probability, they were generally able to apply a strategy to calculate probability and the mean and median of a set of numbers.

First-year pre-service teachers who responded correctly to difficult items, most likely responded correctly to the least difficult items by content strands ACM (ACARA, 2013) (Table 4.2) and cognitive domain (Table 4.3). They demonstrated their ability to:

- compute and add common fractions;
- divide by decimal numbers;
- name equivalent fractions;
- multiply three-digit whole numbers and common fractions; and
- use reasoning to write a story problem to demonstrate the meaning of division using fractions.
For Measurement and Geometry, they could:

- solve volume and mass problems;
- estimate the size of an acute angle;
- recognise and name a scalene triangle;
- recall properties of a hexagon; and
- estimate the position of decimals and fractions on a number line.

First-year pre-service teachers had difficulty when answering Number and Algebra items and when Reasoning, using proportional reasoning for calculating scale, distance and capacity, when generating an appropriate model for division of common fractions, and selecting composite numbers from a list of two-digit numbers; these items were most difficult. For Measurement and Geometry items, they had most difficulty when converting and calculating area and measuring volume. For Shape, they had most difficulty knowing how to recall geometric properties for symmetry, naming a rhombus or polygon, and identifying and labelling obtuse angles in polygons.

### 4.4 Second-year pre-service teachers: What did they know?

As shown in Table 4.1, between two thirds and three quarters of second-year pre-service teachers responded correctly to items in the content strand of Number and Algebra. More than two-thirds of second-year pre-service teachers correctly answered items in Number and Place Value, Money and Financial Mathematics, Fractions and Decimals and, for Test 1, Pattern and Algebra. Scores varied for the Real Number items, with pre-service teachers finding Test 3 Real Number items the most difficult when compared with the percentage of correct responses for Test 1 and Test 2 Real Number items.

Over half of second-year pre-service teachers responded correctly to the content strand Measurement and Geometry. More than half of the pre-service teachers responded correctly to the Using Units of Measurement items. Success with the Shape items varied, with just over half of the second-year pre-service teachers responding correctly to Test 1 items whereas around two thirds of responses to the shape items in Test 2 and

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5 These two percentages represent the range of lowest and highest mean score of the three second-year tests by ACM sub-strand in Table 5.1
Test 3 were correct. This suggests Test 1 Shape items were more difficult than those in Test 2. The lowest proportions of correct responses (about one third) were for a Location and Transformation item in Test 2 and 3. Test 2 and 3 both had one Geometric Reasoning item and more than two thirds of the second-year pre-service teachers were able to correctly respond to this item, suggesting that Test 3 had an easier item.

All second-year MCSK test instruments had three or four items relating to the content strand, Statistics and Probability. More than two-thirds of the pre-service teachers correctly responded to these items. This content strand had the fewest items but highest percentage of correct responses from second-year participants.

### 4.4.1 Difficulty ranking by content strand and sub-strand

Table 4.4 reports the number and percentage of items in MCSK Tests 1, 2 and 3 that second-year pre-service teachers found least difficult, difficult or most difficult. The results are grouped by ACM content strand and sub-strand (ACARA, 2013).

In Table 4.4, and for second-year pre-service teachers, more items were coded as least difficult rather than difficult or most difficult. Three quarters of the MCK items were coded least difficult. The least difficult items included more than three-quarters of the number and Algebra items, more than two-thirds of the Measurement and Geometry items and all of the Statistics and Probability items. These items included high proportions of items in the sub-straands of number and place value (94%), real numbers (79%), Using Units of Measurement (72%) and Chance (100%) items. Difficult items included some Real Number (19%), Using Units of Measurement (14%), Shape (31%) as well as Location and Transformation (66%) items.

Very few second-year test items were most difficult. Most difficult items included measurement and geometry items (11%), Number and Algebra (5%) and the most difficult items were Fractions and Decimals (17%) and Using Units of Measurement (14%) items.
Table 4.4
Second-year Pre-service Teachers: Number (%) of Test Items with Correct Response by ACM Content Strands and Sub-strands (ACARA, 2013) by Difficulty Ranking (N=147)

<table>
<thead>
<tr>
<th>ACM content strands</th>
<th>ACM content sub-strand</th>
<th>Total number of items</th>
<th>Least difficult n (%)</th>
<th>Difficult n (%)</th>
<th>Most difficult n (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number and Algebra</td>
<td>Number and Place Value</td>
<td>18</td>
<td>17 (94%)</td>
<td>1 (6%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Number and Algebra</td>
<td>Pattern and Algebra</td>
<td>3</td>
<td>2 (66%)</td>
<td>0 (0%)</td>
<td>1 (33%)</td>
</tr>
<tr>
<td>Number and Algebra</td>
<td>Fractions and Decimals</td>
<td>12</td>
<td>8 (66%)</td>
<td>2 (17%)</td>
<td>2 (17%)</td>
</tr>
<tr>
<td>Number and Algebra</td>
<td>Real Numbers</td>
<td>43</td>
<td>34 (79%)</td>
<td>8 (19%)</td>
<td>1 (2%)</td>
</tr>
<tr>
<td>Number and Algebra</td>
<td>Money and Financial Mathematics</td>
<td>4</td>
<td>4 (100%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Number and Algebra</td>
<td>Total</td>
<td>80</td>
<td>65 (81%)</td>
<td>11 (14%)</td>
<td>4 (5%)</td>
</tr>
<tr>
<td>Measurement and Geometry</td>
<td>Using Units of Measurement</td>
<td>35</td>
<td>25 (72%)</td>
<td>5 (14%)</td>
<td>5 (14%)</td>
</tr>
<tr>
<td>Measurement and Geometry</td>
<td>Shape</td>
<td>16</td>
<td>10 (63%)</td>
<td>5 (31%)</td>
<td>1 (6%)</td>
</tr>
<tr>
<td>Measurement and Geometry</td>
<td>Location and Transformation</td>
<td>3</td>
<td>1 (33%)</td>
<td>2 (66%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Measurement and Geometry</td>
<td>Geometric Reasoning</td>
<td>2</td>
<td>2 (100%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Measurement and Geometry</td>
<td>Total</td>
<td>56</td>
<td>38 (68%)</td>
<td>12 (21%)</td>
<td>6 (11%)</td>
</tr>
<tr>
<td>Statistics and Probability</td>
<td>Chance</td>
<td>4</td>
<td>4 (36%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Statistics and Probability</td>
<td>Data Representation and Interpretation</td>
<td>7</td>
<td>7 (64%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Statistics and Probability</td>
<td>Total</td>
<td>11</td>
<td>11 (100%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>All</td>
<td>Total</td>
<td>147</td>
<td>144 (100%)</td>
<td>23 (76%)</td>
<td>10 (18%)</td>
</tr>
</tbody>
</table>

Second-year pre-service teachers had less difficulty with Statistics and Probability items, in particular Chance items, than other items. They correctly answered higher proportions of Number and Algebra items than Measurement and Geometry items. They found about one-third of the measurement items and fractions and decimal items difficult or most difficult.
4.4.2 Difficulty ranking by cognitive sub-domain

Table 4.5 reports the number and percentage of MCSK Tests 1, 2 and 3 items that second-year pre-service teachers found least difficult, difficult or most difficult grouped by cognitive sub-domain and sample behaviour (Tatto et al., 2008a).

Table 4.5 shows that of 147 second-year MCSK test items, about three-quarters were coded as least difficult; less than one-fifth were coded difficult and a small number were most difficult. It should be noted that not all sample behaviours by cognitive domain were represented within these second-year MCK items.

More than three quarters of MCSK test items were coded as Applying and Knowing, and about two-thirds of the Reasoning items were found to be least difficult. The least difficult items included all of the Classify/Order items, more than two-thirds of the Measure items, and most of the Select, Model and Analyse items.

The difficult items included one-quarter of the Reasoning items (24%), fewer of the Knowing items (17%) and a small proportion of the Applying items (8%). The most difficult items included a small proportion of Knowing (6%), Applying (6%) and Reasoning (9%) items.
Table 4.5
*Difficulty Ranking of Domains for Second-year Pre-service Teachers: Number (%) of Test Items with Correct Responses for Cognitive Domain (Tatto et al., 2012) (N=147)*

<table>
<thead>
<tr>
<th>Sub-domain</th>
<th>Sample Behaviour</th>
<th>Total number of items</th>
<th>Least difficult n (%)</th>
<th>Difficult n (%)</th>
<th>Most difficult n (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowing</td>
<td>Recall</td>
<td>10</td>
<td>6 (60%)</td>
<td>4 (40%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Knowing</td>
<td>Recognise</td>
<td>15</td>
<td>13 (56%)</td>
<td>1 (7%)</td>
<td>1 (7%)</td>
</tr>
<tr>
<td>Knowing</td>
<td>Compute</td>
<td>31</td>
<td>26 (84%)</td>
<td>5 (16%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Knowing</td>
<td>Retrieve</td>
<td>0</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Knowing</td>
<td>Measure</td>
<td>31</td>
<td>21 (68%)</td>
<td>5 (16%)</td>
<td>5 (16%)</td>
</tr>
<tr>
<td>Knowing</td>
<td>Classify/order</td>
<td>3</td>
<td>3 (100%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Knowing</td>
<td>Total</td>
<td>90</td>
<td>69 (77%)</td>
<td>15 (17%)</td>
<td>6 (6%)</td>
</tr>
<tr>
<td>Applying</td>
<td>Select</td>
<td>24</td>
<td>22 (92%)</td>
<td>1 (4%)</td>
<td>1 (4%)</td>
</tr>
<tr>
<td>Applying</td>
<td>Represent</td>
<td>0</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Applying</td>
<td>Model</td>
<td>12</td>
<td>9 (75%)</td>
<td>2 (17%)</td>
<td>1 (8%)</td>
</tr>
<tr>
<td>Applying</td>
<td>Implement</td>
<td>0</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Applying</td>
<td>Solve routine problems</td>
<td>0</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Applying</td>
<td>Total</td>
<td>36</td>
<td>31 (86%)</td>
<td>3 (8%)</td>
<td>2 (6%)</td>
</tr>
<tr>
<td>Reasoning</td>
<td>Analyse</td>
<td>13</td>
<td>11 (85%)</td>
<td>2 (15%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Reasoning</td>
<td>Generalise</td>
<td>0</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Reasoning</td>
<td>Synthesize/integrate</td>
<td>0</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Reasoning</td>
<td>Justify</td>
<td>8</td>
<td>3 (28%)</td>
<td>3 (38%)</td>
<td>2 (24%)</td>
</tr>
<tr>
<td>Reasoning</td>
<td>Solve non-routine problems</td>
<td>0</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Reasoning</td>
<td>Total</td>
<td>21</td>
<td>14 (67%)</td>
<td>5 (24%)</td>
<td>2 (9%)</td>
</tr>
<tr>
<td>All</td>
<td>Total</td>
<td>147</td>
<td>114 (78%)</td>
<td>23 (16%)</td>
<td>10 (7%)</td>
</tr>
</tbody>
</table>
4.4.3 Summary of second-year pre-service teachers’ MCK

Second-year pre-service teachers who responded correctly to least difficult items demonstrated MCK of:

- place value;
- number patterns with whole numbers or decimal fractions;
- order of fractions and decimal fractions, including modelling on a number line;
- writing a ten digit number;
- operations with whole numbers and decimal fractions, common fractions and using scientific notation;
- multiplying a two-digit number by a three-digit number and dividing a four-digit number by a one-digit number;
- converting decimals, fractions and percentages;
- using proportional reasoning to solve capacity, weight, and length problems; and
- using proportional reasoning when calculating the better value in dollars.

For Statistics and Probability, most applied a procedural strategy to calculate the mean and median of a set of numbers.

Most second-year pre-service teachers who responded correctly to difficult items also responded correctly to the least difficult items by content strands ACM (ACARA, 2013) (Table 4.4) and cognitive domain (Table 4.5). For Measurement, these pre-service teachers demonstrated their ability to estimate, calculate, convert and measure when items involved length, area, volume, mass, time and capacity. They could correctly calculate the area of a triangle or the duration of time, but fewer were able to calculate the surface area of a box in square centimetres. For Geometry, they could recall, draw, label and recognise properties of two dimensional shapes such as polygons, quadrilaterals, parallelograms, an isosceles triangle, nonagons and octagons, and could Recall and Recognise a reflex angle, but fewer could draw and label bilateral symmetry.

Finally, the vast majority of participating second-year pre-service teachers demonstrated limited knowledge of multiplication of fractions, decimal fractions and division problems, solving proportional reasoning problems involving distance and time, and
capacity problems. When they correctly responded to *most difficult* items, most second-year pre-service teachers used a strategy to recognise two-digit composite numbers, compare equivalent fractions and convert a decimal fraction to a common fraction or vice versa. For Units of Measurement, most could estimate volume, convert square centimetres to square metres, calculate area in metres to hectares and convert square centimetres to square metres. When responding to Geometry items, most could correctly recall geometric properties and label a reflex angle and quadrilateral, recognise a polygon, describe properties of a parallelogram, and draw and label bilateral symmetry.

In Number and Algebra, most second-year pre-service teachers found generating a model to demonstrate multiplication of fractions, justifying a decimal number sequence, and identifying there are unlimited numbers between two given decimal numbers *most difficult*. With Units of Measurement, most had difficulty converting the volume of a cube from cubic centimetres to cubic metres. Finally, when responding to Geometry items, most found *difficult* identifying a net that does not make a cube.

### 4.5 In-depth analysis of MCSK test items

In this component MCSK test items were ranked by percentage of correct responses. This enabled me to identify the *least difficult* and *most difficult* items of the four MCSK tests (Table 4.1). Table 4.6 presents data for two *least difficult* MCSK Data Representation items for Test 1, 2, 3 and 4. The first item identified MCK for calculating the mean of a set of numbers and the second MCK for calculating the median. The numbers used for mean and median items were different in some MCSK tests, but (for example) in Test 3, pre-service teachers had to calculate the mean of the following numbers: 8, 4, 8, 7, 7, 8, 2 (Item 38) and then calculate the median of the same set of numbers (Item 39).

When analysing the written responses more were coded as the cognitive domain Apply and the sample behaviour Selected, because the pre-service teachers most likely used a method of solution that was familiar to them. Similarly in the TIMMS assessment Applying is used when students use familiar concepts or procedures and select an appropriate method of solution (TIMMS, 2012).
Table 4.6
Percentages of Correct Responses to MCSK Data Representation Items, all Tests

<table>
<thead>
<tr>
<th>Item description</th>
<th>Second Year</th>
<th>First Year</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Test 1</td>
<td>Test 2</td>
</tr>
<tr>
<td>Item No.</td>
<td>Item %</td>
<td>Item No.</td>
</tr>
<tr>
<td>Calculate mean</td>
<td>12</td>
<td>55%</td>
</tr>
<tr>
<td>Calculate median</td>
<td>13</td>
<td>83%</td>
</tr>
</tbody>
</table>

The pre-service teachers who correctly calculated the responses for the items in Table 4.6 may have relied on a procedural method of solution in which conceptual understanding need not be evident (Chick et al., 2006a). However, the test items did not measure pre-service teachers’ conceptual understanding. For example, the mean was calculated by adding up the digits and dividing the total by the number of digits, some pre-service teachers may not understand why they complete these steps and relied on a rule when calculating the answer. When calculating the mean of the set of numbers, pre-service teachers relied on MCK of addition and division, which was not needed when working out the median (the middle measurement when items are arranged in order of size) (De Klerk, 2008). Therefore calculating the median was easier than calculating the mean.

Furthermore, many of the pre-service teachers in this study had recently completed secondary education and five had completed Further Mathematics Units 3 and 4 (see Table 3.9), in which data analysis and statistics were taught (Victorian Curriculum and Assessment Authority, 2010). This recent mathematical education experience may have contributed to the high proportions of correct responses for these MCK chance items.

4.5.1 Analysis of the most difficult items

The aim of this section is to provide insights into pre-service teachers’ methods of solution and common misconceptions when responding to difficult MCK items. A most difficult ratio item was chosen and analysed, because when responding to this item first-
year pre-service teachers provided working out as well as correct and incorrect responses as evidence of their MCK and methods of solution. In this section, the dimensions from Chick et al. (2006a) Pedagogical Content Knowledge (PCK) Framework (see Table 2.3) are used to report the results.

Table 4.7 lists two ratio MCSK test items and the percentages of correct responses by first-year pre-service teachers. Item 38 is a ratio (area) question categorised as Measurement by content sub-strand and Measure by cognitive domain, and Item 49 is a ratio (scale) question categorised as Real Number by content sub-strand and Analyse by cognitive domain.

Table 4.7
First-year Pre-service Teachers’ Responses to Most Difficult Items: Item 38 and Item 49 Test 4 (N=297)

<table>
<thead>
<tr>
<th>Item number</th>
<th>Question</th>
<th>Correct Responses</th>
<th>n (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>38</td>
<td><strong>Ratio Area</strong> (comparison whole to whole) 3 200 square centimetres is the same as __ square metres</td>
<td>28</td>
<td>(9%)</td>
</tr>
<tr>
<td>49</td>
<td><strong>Ratio Scale</strong> (missing value)</td>
<td>33</td>
<td>(11%)</td>
</tr>
</tbody>
</table>

The scale on the map is 1:1 250 000
What distance in kilometres is 6 cm? (Image not to scale)

Item 38 was a comparison whole-to-whole ratio MCSK item and was coded as *most difficult*. For this ratio area item, pre-service teachers had to convert square centimetres to square metres. To do this they had to recall units of measurement, or calculate the ratio for units of area measurement and then find the missing value. They needed to
know that one square metre is equivalent to a square measuring 100 centimetres by 100 centimetres, that is, 10 000 square centimetres, and therefore 3 200 square centimetres is the same as 0.32 square metres. Very few pre-service teachers gave the correct response of 0.32 square metres (N=297), and few provided working out when responding to this test item.

Item 49, a ratio missing value scale problem, was the second-hardest first-year item and was also coded as most difficult when ranked by difficulty. For this item, pre-service teachers had to find the missing value; three of the four values in the ratio were provided (Lamon, 2007). They could have used proportional reasoning as a method of solution for this whole-to-whole ratio problem relating to scaling on a map. They had to convert the map distance of six centimetres to the real distance of 75 kilometres. The scale used in this example was 1:1 250 000, that is, 1 cm on the map represents 1250000 cm (or 12.5 km) on the ground. Only a small proportion of pre-service teachers provided a correct response (see later, Table 4.8).

Both items required knowledge of ratio and measurement. These items could also be identified as a multiple step problem (Siemon, Virgona, & Corneille, 2001) as more than one step was required when calculating the correct response. The items would also be expected knowledge at Year 7, as students recognise and solve problems involving simple ratios (ACARA, 2013).

Some of the first and second-year Real Number MCSK ratio test items were the same, others were similar and some very different. None of the items in the second-year MCSK test were similar to the most difficult item on the first-year Test (Item 49), therefore first-year and second-year pre-service teachers’ responses of this item could not be compared.

4.5.2 Correct responses to the ratio (scale) item

Figure 4.1 illustrates a correct method used for solving Item 49. This common correct response suggests knowledge of conversion of metric units by choosing to convert centimetres to metres and then converting metres to kilometres.
Another correct method (Figure 4.2) used by one pre-service teacher was an algebraic method in which knowledge of an algebraic representation and procedure was used firstly to identify the missing part indicated as $x$. The remainder of the problem was then completed by the pre-service teacher using a common method, the place value algorithm, involving multiplying by six to find the missing part and then converting the answer to the correct unit. Also included in this participant’s response was a graphic organiser used to convert kilometres to centimetres. Other pre-service teachers drew similar graphics.

Pre-service teachers who answered Item 49 correctly demonstrated knowledge of whole-to-whole ratio when calculating the correct solution. These pre-service teachers showed breadth of understanding, not merely procedural knowledge, by connecting their knowledge of ratio and measurement. Ma (1999) described breadth as the capacity
to connect one topic with others of similar conceptual understanding. These pre-service teachers also demonstrated knowledge of measurement facts and the relationship between measurement units.

4.5.3 Incorrect responses to the ratio (scale) item

Table 4.8 provides the distribution of responses to Item 49, using nine sub-headings to classify and describe the misconceptions. Eight common incorrect responses were identified from more than half of a random sample (n=62) of first-year responses and were discussed in greater detail below. A random sample of 62 responses was selected as a pattern of responses had emerged when analysing the responses for Item 49. Most of the random sample (20%) of the first-year pre-service teachers (N=279) were unable to correctly answer item 49.

Correctly identify the missing part: Table 4.7 shows that a small proportion (10%) of the sample (n=62) of first-year pre-service teachers recorded the correct response (75 km). A further one-third (33%) demonstrated some knowledge of whole-whole ratio. Three groups of incorrect answers – 7 500 000 (27%), 75 000 000 (3%) and 7.5 (3%) nevertheless demonstrated understanding that the problem involved a multiplicative relationship and demonstrated a correct multiplication procedure. Altogether, these responses (43%) showed knowledge of how to find the missing part given three parts. The misconception of one-third (33%) of the sample concerned knowledge of the relationship between the units of measurement, that is, not knowing how to convert the answer to the required unit. Ratio knowledge was also demonstrated within other categories of incorrect responses.
Table 4.8  
*Distribution of Responses to Ratio (scale) Question for a Random Sample (n=62)*

<table>
<thead>
<tr>
<th>Answer</th>
<th>Responses (%)</th>
<th>Category</th>
<th>Type of misconception</th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
<td>10%</td>
<td>Correct</td>
<td>Correct response (no misconception)</td>
</tr>
<tr>
<td>7 500 000</td>
<td>27%</td>
<td>Incomplete</td>
<td>Answer not converted from cm to km (correct multiplication used)</td>
</tr>
<tr>
<td>7.5</td>
<td>3%</td>
<td>Incorrect measurement conversion</td>
<td>Misconception with relationship between measurement units; some knowledge of ratio; able to multiply</td>
</tr>
<tr>
<td>75 000 000</td>
<td>3%</td>
<td>Incorrect measurement conversion</td>
<td>Misconception with relationship between measurement units; some knowledge of ratio; able to multiply</td>
</tr>
<tr>
<td>Various (eg.15000 000)</td>
<td>18%</td>
<td>Ratio with large numbers</td>
<td>Misconception of recording of ratio (various answers 15000 000, 1500 000, 150, 15) and misconception with relationship between measurement units; able to multiply (13%), (eg. 250 000 x 6 =15 000 000)</td>
</tr>
<tr>
<td>1250 006</td>
<td>10%</td>
<td>Additive thinking</td>
<td>Additive thinking; no understanding of ratio to show relationship</td>
</tr>
<tr>
<td>Various (e.g. 0.006)</td>
<td>6%</td>
<td>Common incorrect invented strategies</td>
<td>Common invented strategy of multiplying one by six; no understanding of ratio and measurement of units (various answers 0.00006, 0.006, 600, 6000)</td>
</tr>
<tr>
<td>Other various</td>
<td>23%</td>
<td>Other incorrect invented strategies</td>
<td>Range of errors with little knowledge of ratio, multiplication or units of measurement (eg. 25, 100 000 00, 1.25, 150666)</td>
</tr>
<tr>
<td>Blank</td>
<td>8%</td>
<td>No response</td>
<td>Unknown</td>
</tr>
</tbody>
</table>

_Failure to complete multi-step problem:_ The most common error (7 500 000) was provided by more pre-service teachers (27%) than the correct response of 75 km (10%). The pre-service teachers who recorded 7 500 000 multiplied correctly but did not convert their answer from centimetres to kilometres. Their answer was incomplete rather than incorrect. These pre-service teachers correctly interpreted the ratio, understanding the representation and its multiplicative structure, and correctly used multiplication to find the equivalent ratio (6:7 500 000). However, they did not complete the problem, indicating that they did not attempt to make sense of the solution.
and so ignored the connection with the real context of the problem. Figure 4.3 provides an example of the error 7 500 000 and common working. These pre-service teachers were unable to correctly interpret or decode the problem.

![Figure 4.3 Incorrect answer 750 000 km.](image)

Working with formulae and solving multi-step problems has been identified as an area of difficulty for Years 5 to 9 students in recent research (Siemon et al., 2001). Children find worded problems challenging and have difficulty transforming the calculation into numbers (Lawton, 2008). Cockburn (1999) found children can make mathematical errors when they were unable to understand the language used in the question. These pre-service teachers may also have made errors due to one or all of these reasons or due to other misconceptions. Their errors could have been avoided had the question been worded more clearly, such as: What distance in kilometres is represented by 6cm?

Repeated addition: Figure 4.4 shows the work of one student who used a repeated addition method for multiplication to reach the same incorrect answer (7 500 000). This was not a common method used by the pre-service teachers. This method shows that while pre-service teachers may understand whole-to-whole ratio, some do not know the multiplicative facts or have confidence in using them to complete the multiplication algorithm correctly. This method should not be a preferred method for modelling the answer when working with students to develop understanding of ratio.
Misinterpretation of ratio representation: The second largest grouping of incorrect responses (18%) included various answers. Two examples of responses in this category were 15 and 15 000 000 (Table 4.8). These pre-service teachers demonstrated correct knowledge of multiplication of 250 000 by 6 (1 500 000, calculated by 13%) but appeared to have a misconception of how ratios involving large numbers are represented. Pre-service teachers in this group used a range of methods to convert their answer to kilometres, and most converted the units incorrectly. Those recording 15 for their answer were able to correctly convert the units of measurement.

Figure 4.5 provides a common example of this misconception. It is difficult to ascertain from the test data why this error occurred. These pre-service teachers may have seen the ratio as 1:1 followed by 250 000, and made an error when interpreting the question or in their thinking related to proportional reasoning. Another source of error may have been due to a lack of understanding of place value of the digits for 1 250 000. Some students may not have read the digits as one million and two hundred and fifty thousand. The spacing of the digits in the ratio question may have been confusing to some pre-service teachers who may be more familiar with commas when recording large digits. This is consistent with Cockburn’s (1999) claim that errors can occur if the presentation of the task is inappropriate.
Misunderstanding of the structure of measurement units: Small percentages of the sample gave similar but incorrect answers: 7.5 (3%) and 75 000 000 (3%). In these responses the missing part using multiplication was completed correctly but an error occurred with the method for conversion of measurement. In Figure 4.6, the pre-service teacher most likely converted the units in the ratio first before finding the missing part. They calculated 12,500 metres as equal to 1.25 kilometres rather than 12.5 kilometres and did not correctly divide by 1 000, demonstrating a poor knowledge of conversion of measurements. Lawton (2008) suggested reasons students and adults find ratio questions difficult was because a ratio problem may relate to knowledge of fractions, decimals, percentage or measurement. The results from this study support Lawton’s assertion, as many pre-service teachers lacked knowledge of the relationship between measurement units.
Adding thinking: Another grouping of misconceptions was adding thinking (2%), for example adding 6 to 1 250 000 (Figure 7). Use of addition or subtraction is a common mistake in ratio problems (Suggate, Davis, & Goulding, 2006). This error was not very common for this sample of pre-service teachers, although it is a misconception with significant implications for the breadth and depth of their knowledge of mathematics in the middle years. Middle year is Years 5 to Year 9 of schooling.

Figure 4.7 Additive strategy for ratio: 1 250 006 km.

Common incorrect invented strategies: A group of common invented strategies existed for a small percentage of incorrect responses (8%). These invented strategies assumed the ratio was 1:1, and pre-service teachers most likely multiplied one by six and then used a step (or steps) for converting the answer to kilometres; they provided a range of different responses such as 0.00006, 0.006, 600 and 6000. An example of the method that results in an answer of 600 is shown in Figure 4.8, where a graphic organiser was used for converting units of measurement, showing conversion of centimetres to metres as multiplication by 10 then metres to kilometres (k) as multiplication by 100. An answer of 600 km is recorded. This example illustrates little understanding of ratio or relationships between measurement units.
Other incorrect invented strategies: About one quarter provided various incorrect answers, such as 10 000 000 km, 250 000 km, 650 km and 25 km. Also, a small percentage (8%) of pre-service teachers recorded no response. These results were concerning, as the mean nearly one third of pre-service teachers had little or no MCK of the whole-whole ratio concept. Many of these answers used invented strategies and demonstrated a lack of both ratio and measurement knowledge.

A combination of reasons is probably responsible for the high proportion of incorrect responses. These pre-service teachers may have misread the question, had difficulty interpreting how to solve the problem, or they may not have known the mathematics needed for answering the question. It should also be noted that this question was the last test item, and this may have affected the way the participants answered it. For example, if they thought they had already failed the test they might not have spent much time attempting to answer this item; alternatively, they might not have had much time left at the end of the test to devote to it; or had experienced difficulties with previous test items and given up trying when responding to this final item.

Making sense: In the original printed format of the test, Item 49 was presented with the map represented to scale, that is, one centimetre on the map was one centimetre on the test page. Figure 4.8 shows that some distances were recorded, for example, Heywood to Hamilton was shown as 58 km but measured less than 6 cm on the map, the distance that they needed to convert to solve the problem. Therefore a response of 7 500 000 km was not a reasonable answer; these participants did not think about what 7 500 000 km might look like in relation to the map. The distance from Melbourne to Perth is only
3424 km highlighting the absurdity of the answer 7 500 000 km. Reys, Lindquist, Lambdin, Smith and Suydam (2004) suggest that when thinking about problems the use of estimation with ratio can be used to find approximate answers. These pre-service teachers were unlikely to have considered using estimation, therefore were unable to think about the reasonableness of their answer.

After marking ratio Item 49, an examiner could make the generalisation that these pre-service teachers had difficulty answering a ratio question and needed to consolidate their knowledge of ratio and proportion. However, about one third of the random sample group (33%) recorded an error with the conversion of measurement but demonstrated knowledge of this ratio situation. They could multiply by six to find the missing part when given three of the parts. This highlighted an important factor: an incorrect answer may provide evidence of ratio knowledge while the misconception relates to gaps in other mathematical knowledge or connected knowledge (Ma, 1999). This error demonstrated that many pre-service teachers have not made connections between these topics, they lacked understanding of Mathematical Structure and Connections, and/or they lacked a Method of Solution since they did not know the conversion from centimetres to kilometres or could not derive a method to do so correctly.

Pre-service teachers who gave correct responses to Item 49 demonstrated Procedural Knowledge and/or Methods of Solution (Chick et al., 2006a). Some pre-service teachers, notably those who correctly answered both Item 38 and Item 49, might have been able to demonstrate the other three descriptors of content knowledge – Profound Understanding, Deconstructing Content, and/or Mathematical Structure and Connections (Chick et al., 2006a) – but the data did not permit this to be ascertained.

Pre-service teachers who successfully answered Items 38 and 49 were able to interpret and solve multi-step whole-to-whole ratio problems, suggesting the capacity to deconstruct content. If the pre-service teachers used more than one method to achieve their answer they would have demonstrated Deconstructing Content to Key Components and/or Mathematical Structure and Connections (Chick et al., 2006a). Most of the pre-service teachers were not able to identify the various mathematical components within
the ratio (scale) item. They did not understand the concepts of ratio and conversion of measurement units, were unable to deal with the multi-step problem and unable to demonstrate one correct method of solution.

Profound Understanding of Fundamental Mathematics (PUFM) (Ma, 1999) would have been demonstrated if a wide variety of ratio examples were answered and explained using more than one method, such as demonstrating understanding of the three common situations of ratio: part-part, part-whole and whole-whole (Suggate et al., 2006). The structure of Items 49 and 38 did not provide the detail needed for analysing pre-service teachers’ knowledge using all descriptors of Content Knowledge in a Pedagogical Context (Chick et al, 2006a).

Knowing and using mathematics for teaching entails making sense of methods and solutions different from one’s own (Ball et al., 2004). However, the majority of the participating first-year pre-service teachers needed to work on understanding the whole-whole ratio (scale) concept first. Following that they should have developed their MCK of other ratio concepts, as well as strengthened their knowledge of measurement units such as conversions for distance and area. They should also have been encouraged to explore different methods for working out ratio problems. Solving and discussing similar problems and various methods of solutions will help these pre-service teachers to practise solving ratio problems, strengthening and deepening understanding for teaching.

4.6 Conclusion

In this chapter, the results of an in-depth analysis of MCSK test data, identified strengths and weaknesses in first-year and second-year pre-service teachers’ MCK. The MCSK test results confirmed that the participating first-year pre-service teachers entered the program with substantial weaknesses in their MCK. As a result, these pre-service teachers required opportunities during the remainder of their program to learn the MCK needed for primary mathematics teaching.

First-year pre-service teachers scored lower on the MCSK test than second-year pre-service teachers. This difference suggests that second-year program experiences and
opportunities for learning assisted pre-service teachers to develop MCK. Nevertheless, nearly half of second-year pre-service teachers were unable to respond correctly to difficult and most difficult MCSK test items for Number and Algebra and Measurement and Geometry. Therefore further development of MCK was required for half of the pre-service teachers in second-year (2008).

Further results reported the MCK that pre-service teachers demonstrated when responding to MCSK test items by interpreting their methods of solution. When preparing for this test, pre-service teachers may have reinforced their knowledge of rules or procedural knowledge with little regard for developing their conceptual understanding of MCK. The MCSK test question types may have been a factor encouraging pre-service teachers to rely on rule based knowledge because question items were mostly closed question types (see Table 3.4) that could rely on procedural knowledge and/or methods of solution.

The pre-service teachers who participated in the research completed core education units in primary mathematics teacher education during first year and second year and practicum experiences in primary schools during their first, second and fourth years. The next two chapters report the results of the longitudinal study of 17 pre-service teachers’ opportunities and influences for developing their MCK during coursework and practicum experiences.
CHAPTER FIVE

Opportunities to develop MCK: Coursework

The longitudinal qualitative study of pre-service teachers’ mathematical content knowledge (MCK) was designed to investigate the opportunities and influences that enhanced MCK in each year of teacher education. This chapter reports on 17 pre-service teachers’ opportunities to develop different categories of MCK during their Bachelor of Education coursework. The participants completed three core primary mathematics education units during first and second year, with an option to undertake a fourth (elective) unit to assist them to learn the MCK required for passing a Mathematical Competency Skills and Knowledge (MCSK) test.

The process of becoming a teacher can be shaped by prior beliefs about mathematics teaching (Anthony et al., 2012) and shape how pre-service teachers imagine mathematics might be taught (Hodgen & Askew, 2007). Although the researcher collected limited data regarding the longitudinal study pre-service teachers’ prior mathematical learning in primary and secondary school, pre-service teachers’ reflections are reported because these data provide background information on what the participants brought to their program. The longitudinal study pre-service teachers’ attempts at passing the MCSK test are described to compare their MCK with the larger cohort of second-year pre-service teachers reported in the previous chapter. The longitudinal study pre-service teachers’ results are then reported by study groups:

- Study Group 1: pre-service teachers who had chosen mathematics as their secondary specialisation and passed the MCSK test during their first attempt.
- Study Group 2: pre-service teachers who had not chosen mathematics as their secondary specialisation and passed the MCSK test during their first attempt.
- Study Group 3: pre-service teachers who had not chosen mathematics as their secondary specialisation and took more than one attempt to pass the MCSK test.
During second, third and fourth years, the researcher completed a one-on-one interview with each of the 17 pre-service teachers who took part in the longitudinal study. During this annual interview, pre-service teachers provided responses to interview questions (Appendix D, E and F). They also provided artefacts such as assignments related to their coursework units for primary mathematics education. Data were coded using three of the four categories of the Knowledge Quartet: foundation knowledge, transformation and making connections (Rowland et al., 2009) to identify opportunities and influences that enhanced MCK during coursework. (The fourth Knowledge Quartet category, contingencies, was not suitable for coding coursework data because it identifies a teachers’ action when responding to student questions.) Finally, the researcher reports on how one primary mathematics first-year unit began to shape pre-service teachers’ MCK and two second-year units provided opportunities to extend foundation knowledge and make connections. No coursework units related to primary mathematics education were taught during the third or fourth year of the program.

5.1 Diversity of MCK

The previous chapter described first and second-year pre-service teachers’ MCK based on their responses to MCSK test items. The variation demonstrated in the MCSK results of pre-service teachers may have been caused by pre-program identity and mathematical experiences during their schooling.

5.1.1 Pre-program identity

Before commencing their course, the 17 participating pre-service teachers had studied mathematics at primary and secondary school. During their second-year interviews, some pre-service teachers reflected on their experiences of mathematics at school. These experiences would have developed their pre-program identity (Ponte & Chapman, 2008) including a mixture of negative and positive reflections.

Emma (Study Group 3) had negative experiences: “I was choking at maths [sic] in primary school [because] nobody sat down and… taught me a different way.” Emma’s school experiences may have also been a factor as to why she was continuing to have difficulty with her MCK in her second year at university.
Elizabeth (Study Group 2) stopped studying maths in Year 10, explaining, “I lost [forgot] so much of it”, and completed two more years at secondary school before enrolling in teacher education. On the other hand Michael described how he enjoyed mathematics: “I enjoy the challenge. I enjoyed maths when I was at school and I still find it interesting now.”

Lisa (Study Group 3) remembered lacking confidence when learning mathematics, even though she studied mathematics in each year of secondary schooling until Year 12. She needed a tutor to help her learn mathematics.

Before commencing their university teaching program the participants completed Year 12, but only nine of the 17 longitudinal study pre-service teachers had studied mathematics during their final year of secondary school (see Table 3.9). Of the three Study Group 1 pre-service teachers’ Con completed two subjects of Year 12 mathematics and the other two only completed one Year 12 mathematics subject during Year 12. Study Group 2 (n=9) and 3 (n=5) pre-service teachers’ highest level of mathematics completed ranged from Year 10 (n=3), Year 11 (n=5) and Year 12 (n=6) therefore, 14 of the 17 pre-service teachers had completed at least Year 11 mathematics.

Elizabeth (Study Group 2), Julie and Don (Study Group 3) completed mathematics studies to Year 10; Emma (Study Group 3), Esther, Fiona, Jenny, and Rose (Study Group 2) completed mathematics to Year 11; Con, Sean, Shelly (Study Group 1), Janette, Mathew, Michael, Kerri, Peter (Study Group 2) and Lisa (Study Group 3) completed Year 12 mathematics. There were also two mature age pre-service teachers in the study, Emma (Study Group 3) and Fiona study (Group 2), who had bigger breaks between learning mathematics at school and commencing teacher education.

The three longitudinal study pre-service teachers who had studied mathematics to Year 10 level (Elizabeth, Don and Julie) returned mixed results on the MCSK test. Elizabeth passed the MCSK test at her first attempt, but Don and Julie required more than one attempt. In addition, some pre-service teachers enrolled in the program having only passed Year 10 mathematics and were accepted via a special entry pathway. It is not known if any of the longitudinal study pre-service teachers entered the program as special entry students. Furthermore, it should have been expected that all pre-service
134 teachers passed the MCSK test as the level of difficulty was about Year 8 (ACARA, 2013) and all pre-service teachers had completed mathematics up to at least Year 10.

By comparison, 11 of the 14 pre-service teachers who had completed Year 11 or Year 12 mathematics passed the MCSK test at their first attempt. Therefore this result was consistent with the current entry requirements for education courses as selection requirements for undergraduate courses include “a solid foundation in English and mathematics… minimum equivalent of VCE Mathematics Units 1 & 2, not including Foundation Mathematics” (Victorian Institute of Teaching, 2007, p. 7).

Pre-service teachers’ pre-program identities demonstrate their diversity of mathematical experiences. They brought different levels of secondary school mathematics to the program, and not all remembered positive learning experiences; five participants had not completed mathematics study for two or more years before commencing the program, and may have difficulty recalling the mathematics they learnt at school.

5.1.2 Longitudinal study pre-service teachers’ MCK

The 17 pre-service teachers who volunteered and participated in the four-year longitudinal study commenced first year in 2007. During their Bachelor of Education program they completed the same core units of study for primary teaching but different elective units for their secondary teaching. All were required to complete the MCSK test to graduate, and did so at different stages of their program.

Table 5.1 lists the 17 participants by study group, program year and unit of study in which they passed the MCSK test, and the number of attempts required.

Of the 17 participants, 12 (70%) passed the MCSK test at their first attempt. During Unit 1B and Unit 2B pre-service teachers were provided with a second, similar test or supplementary test if they were unable to pass the test during their first attempt; two pre-service teachers passed the supplementary test. Don (Study Group 3) passed the MCSK test during a supplementary assessment for Unit 2B offered about two weeks after the first MCSK test. The other four pre-service teachers in Study Group 3 had difficulty in passing the MCSK test during their program and were required to complete
Unit 1B. Lisa completed Unit 1B at the beginning of third year and passed the MCSK test in Unit 1B.

Table 5.1
Program Year, Number of Attempts and Unit of Study in which Pre-service Teachers Passed the MCSK Test (n=17)

<table>
<thead>
<tr>
<th>Study group</th>
<th>Pre-service teacher</th>
<th>Yr and Unit passed MCSK test</th>
<th>Number of attempts</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Con</td>
<td>Yr 2 Unit 2B</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>Sean</td>
<td>Yr 2 Unit 2B</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>Shelly</td>
<td>Yr 2 Unit 2B</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>Elizabeth</td>
<td>Yr 2 Unit 2B</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>Esther</td>
<td>Yr 2 Unit 2B</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>Fiona</td>
<td>Yr 2 Unit 2B</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>Janette</td>
<td>Yr 2 Unit 2B</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>Kerri</td>
<td>Yr 2 Unit 2B</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>Mathew</td>
<td>Yr 2 Unit 2B</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>Michael</td>
<td>Yr 2 Unit 2B</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>Peter</td>
<td>Yr 2 Unit 2B</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>Rose</td>
<td>Yr 2 Unit 1B</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>Don</td>
<td>Yr 2 Unit 2B 6</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>Emma</td>
<td>Yr 1 Unit 1B 7</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>Jenny</td>
<td>Yr 4 Unit 1B</td>
<td>8</td>
</tr>
<tr>
<td>1</td>
<td>Julie</td>
<td>Yr 2 Unit 1B 8</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>Lisa</td>
<td>Yr 3 Unit 1B</td>
<td>3</td>
</tr>
</tbody>
</table>

When interviewed, Rose (Study Group 2), Emma, Jenny and Lisa (Study Group 3) explained they had completed Unit 1B before second year because they wanted to improve their MCK. Rose and Emma passed Unit 1B at their first attempt, Lisa did not but passed during Unit 2B later in second year. Jenny had the most difficulty and did not pass her test until in fourth year, after having enrolled in Unit 1B in the second and third year of her program.

Jenny (Study Group 3) took many attempts before passing the MCSK test. During her interviews, Jenny reported that she attempted the MCSK at the beginning of second Year (Unit 1B), then during second year (Unit 2B), again during third year (Unit 1B) and passed the test in the final semester of fourth year (Unit 1B). Jenny also explained

---

6 During this unit pre-service teachers passed the supplementary MCSK test
7 During this unit pre-service teachers passed the supplementary MCSK test
8 During this unit pre-service teachers passed the supplementary MCSK test
that she did not attend these primary mathematics classes regularly or study for the MCSK test during first, second and third year because she was busy completing other assignments. If Jenny had not passed the test during her program she would not have graduated.

The 17 pre-service teachers were volunteers from a cohort of second-year pre-service teachers during 2008. During data collection, 283 second-year pre-service teachers completed the MCSK test for the first time for Unit 2B; these results are reported in Chapter 4. Of the 283 second-year pre-service teachers, 195 (42%) had difficulty meeting the assessment criteria required, explained next.

To pass the MCSK test, pre-service teachers were required to answer items in seven sections on different mathematical topics (mainly to do with knowledge of number) and respond correctly to five or more questions in each section. Second-year pre-service teachers (n=119) who had not passed the test by the beginning of third year included two of the longitudinal study participants, Jenny and Lisa (Study Group 3). Therefore, it is likely that Study Group 3 pre-service teachers were representative – with respect to MCK – of half of the pre-service teachers enrolled in second-year at the university at the time of this study who had difficulty when item responses were ranked and reported by difficulty ranking (Chapter 4). In addition, as previously described, finding participants for this longitudinal study was difficult, indicating many pre-service teachers were aware of their lack of MCK and therefore did not choose to agree to participate in this longitudinal study.

One hundred and sixty-four pre-service teachers (58%) had passed their MCSK test by the beginning of third year. No further data were available to identify how many attempts pre-service teachers required before passing the MCSK test, but it is likely that some second-year pre-service teachers required more than one attempt, such as Lisa (Study Group 3). As a comparison, 15 of the 17 longitudinal study pre-service teachers had passed the MCSK test by the end of second year. Hence, the longitudinal study participants had greater MCK than other pre-service teachers.
5.2 Coursework opportunities to learn

During primary school practicum experiences, pre-service teachers had the opportunity to observe and teach primary mathematics lessons to the students in their mentors’ classes. In every year of the program, pre-service teachers were encouraged to experience further teaching responsibility within the classroom and school settings. By their final year, many pre-service teachers were planning and teaching their mentor’s class for a whole week or more.

Pre-service teachers had opportunities to observe and teach mathematics lessons during their 102 days in primary schools. In addition, some pre-service teachers completed mathematics education as their secondary specialisation and experienced secondary mathematics lessons during 42 days in a secondary school (see Table 3.2).

In each semester pre-service teachers completed individual assessment tasks, connecting their experiences and knowledge gained from both university and school-based settings. For example, in Unit 2A, during second year, the pre-service teachers conducted the EYNI (2001). During year two of the program, pre-service teachers were required to teach at least 20 lessons as negotiated with their mentor teacher; this included at least five primary mathematics lessons.

5.2.1 Shaping MCK

During first semester in first year, pre-service teachers completed Unit 1A. This unit was an introduction to primary school literacy and numeracy teaching that aimed to provide an opportunity for pre-service teachers to begin to shape their teacher identity as a primary mathematics teacher. When interviewed for this study in second year, pre-service teachers reflected on their first-year coursework experiences and beginning understanding of MCK needed for primary mathematics teaching. However, the following results were limited, as some of the pre-service teachers found it difficult to remember what they had done.

In first-year pre-service teachers were expected to keep a self-directed learning journal. Don’s (Study Group 3) journal reflection provides evidence of his developing identity
of thinking about the importance of relearning and knowing mathematics even if it was
going to be a challenge for him.

Numeracy is a topic taught in schools everywhere. It is a topic that you either love or hate. Throughout this course, the re-learning of maths and numeracy has been a constant factor of our learning. To understand topics of Maths from my background of creative studies has been very hard for me. In the beginning weeks of the course, the re-learning of maths scared me and had been on my mind for quite a while. Maths is an important subject, no matter if it is hard to understand or if I have to learn it again. This was the challenging bit! (Don)

This experience assisted Don to reflect on his MCK and to consider his foundation knowledge and the purpose of mathematics education a category of the Knowledge Quartet (Rowland et al., 2009).

Con’s (Study Group 1) pre-program identity included the most mathematics completed at secondary school of the pre-service teachers in the longitudinal study. He did not believe the Unit 1A he completed during first year helped him to develop his knowledge of mathematics suitable for teaching primary mathematics.

I just wanted help remembering it all and help practising my lower levels of mathematics. I was concerned that I did not have primary maths knowledge… I just use a calculator and need to learn this again. (Con)

Con wanted to revise his fundamental skills, such as long multiplication, because he had become reliant on a calculator when in secondary school. Hence, he sought his own opportunities to learn, choosing to tutor mathematics with two Year 6 and Year 7 students to practise his MCK.

Julie (Study Group 3) thought Unit 1A in first year should have focused on similar mathematics content to the second-year units she was currently completing. As Julie had completed Unit 1A in first year and Units 2A and 2B, she was able to compare the amount of mathematics she had learnt during these experiences.

They should mould you into maths right from the beginning rather than doing it in second year. [In first year] I chose to do a project on area and perimeter and really until now [in second year] I didn’t know what I was talking about.

I didn’t get much detail out of lectures, it was more ideas for teaching strategies… We did self-directed learning plans. That was good, that [plan] said what I need to learn. But it was up to me. We had to pick three topics and it said what we were going to do to improve these topics and it was up to you if you took it on board. I didn’t have a lot of time to do much… I should have done more but when you go through the year you sort of drift along. (Julie)
The assessment tasks in first year were open-ended and self-directed and did not assist Julie to improve her MCK. Julie had difficulty with these tasks, most likely because of her weaknesses in MCK and attitude towards learning mathematics. Lisa (Study Group 3) elaborated on her mathematics experiences during first year in a similar way, suggesting more revision of MCK was needed during this unit and the tasks were too open-ended.

Three pre-service teachers chose to enrol in the elective numeracy Unit 1B before second year. When completing unit 1B, pre-service teachers revised their knowledge of number, measurement, geometry and statistics. As part of their assessment requirement, pre-service teachers kept a learning log annotating their developing understanding of what they learnt during each week of the program. In particular, they described how the content of this unit had assisted by shaping their thinking about primary mathematics. Rose (Study Group 2) described her experiences:

Numeracy and mathematics [the elective unit] was the most helpful in developing my understanding of maths.

I did summer school maths as I did not want to have that and the education maths units at the same time. I thought that was a smart choice. I could just focus on maths and it was on the holidays and I had plenty of time to study. I think that was the most helpful.

When I learnt maths initially [during her schooling] I was learning facts and rules, I never understood the meaning behind it. This is why I would always forget the rules. The course that we did in summer … it was about hands-on experiences. Doing it themselves physically rather than just doing it on paper.

I feel like we focused more on number because there are so many different areas within number that I think we did… Space I remember as being one of the hardest things for me… I memorised the names of the shapes but now it is fading and I would have to research it again but at least I have all those notes now to go back to. There are so many things and definitions it is hard to remember… (Rose)

Rose provided a number of points that assisted her to extend her MCK. Completing the additional Unit 1B during summer school allowed her to revise her MCK whilst not having to be concerned about other units of study. The additional unit and the lecturer assisted her to develop conceptual understanding by doing the mathematics that was unlike the rule-based learning she remembered from her own secondary school education. Rose explained that her learning log was useful when revising her MCK. A weakness that Rose reported was that most of the topics they studied involved
developing their understanding of number and that when learning the names of shapes she found recalling them difficult.

Emma (Study Group 3) completed Unit 2B during first year and mentioned that she found it helpful to share ideas and strategies for finding the correct answer. Lisa (Study Group 3) commented how she understood what she was learning because “there was [sic] lots of hands-on experiences, [and] it was your basic knowledge, showing us how to work out stuff I had forgotten. It was useful.”

Other first-year experiences assisted some of the pre-service teachers to revise their MCK. Lisa (Study Group 3) mentioned that attending lunchtime mathematics revision lessons offered by the university helped. Don (Study Group 3) recommended reading the unit textbook (Booker, Bond, Briggs, Sparrow, & Swan, 2004), which he considered a vital resource for helping him re-learn and understand concepts of mathematics again. He also asked his lecturer questions, and this combination assisted his MCK.

The three main areas of maths I am currently trying to improve on are Fractions, Measurement and Area and Volume… in the classes with [lecturer], I would ask him questions on concepts I would not understand. Through talking with him, I would use the book at home, look up the concepts, read about them and try to do more practice on the concepts. (Don)

Don identified the topics that he needed to revise. He asked his lecturer to help him during his second year mathematics education units and also completed the recommended reading in his unit guide (Booker et al., 2004).

These results show that first-year pre-service teachers were provided with and/or chose different opportunities that assisted them to extend their MCK. They explained how they needed to unpack their MCK having to develop understanding, and this was described as relearning their mathematics. In particular, pre-service teachers who completed the elective unit explained how they started to shape their MCK. They focused on developing understanding, using resources and considered different strategies that developed categories of Rowland et al.’s (2009) category of foundation knowledge.
5.2.2 Extending foundation knowledge and making connections

Pre-service teachers completed two core education units in primary mathematics in second year. Unit 2A was designed to develop pedagogical content knowledge (PCK) and Unit 2B focused on curriculum knowledge required for teaching primary mathematics. During Unit 2A and Unit 2B, all pre-service teachers completed seven tasks as part of their assessment and these included opportunities to learn MCK. On the same day each week for one semester, they attended two tutorials and one lecture when completing these units.

Two of the four categories of the Knowledge Quartet (2009) – foundation knowledge and making connections – were used to code and report the MCK of the second-year assessment tasks. Seven of the eleven sub-categories of foundation knowledge and all of the nine sub-categories of making connections as listed below.

Foundation knowledge when the pre-service teacher:
- uses appropriate teaching strategies to promote the required mathematical understanding in pupils;
- concentrate on developing understanding rather than excessively on procedures;
- makes use of his/her own resources and teaching strategies rather than adhering to text book;
- shows a good understanding of the processes involved in the four operations;
- demonstrates a knowledge of quick mental methods;
- uses mathematical language correctly;
- demonstrates an accurate understanding of mathematical ideas or concepts. (Rowland, et al., 2009 p. 35)

Making Connections when the pre-service teacher:
- makes links to previous lessons;
- makes links between the mental and oral starter and the main part of the lesson;
- makes appropriate conceptual connections within the subject matter;
- recognises the conceptual appropriateness of mathematical ideas for the students they are teaching;
- asks questions to elicit students’ understanding of connections between mathematical ideas appears to be aware of the different levels of difficulty in a topic
- anticipates the complexity of an idea and break it down into steps that can be understood by the students;
- introduces ideas and strategies in an appropriately progressive order;
- make assessments of students’ understanding and amend their lessons accordingly. (Rowland, et al., 2009 p. 36-37)

The components of each category are used in Table 5.2 to report the different aspects of foundation knowledge and making connections for each second-year assessment task.
The other components of foundation knowledge were less useful for this analysis because they focus on PCK or attitudes and teachers’ beliefs.

Table 5.2  
**Second-year Assessment Tasks Including Coding by Sub-categories of Foundation Knowledge and Making Connections (Rowland et al., 2009)**

<table>
<thead>
<tr>
<th>Unit of Study</th>
<th>Coursework experience</th>
<th>Foundation Knowledge</th>
<th>Making Connections</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit 2A</td>
<td>Conducting a one on one numeracy interview with a primary student. Writing a report.</td>
<td>Concentrating on developing understanding of the students’ mathematical knowledge (2,4)</td>
<td>Make links with student’s response and correct response when identify their strategies (3,5,6,7)</td>
</tr>
<tr>
<td>Unit 2A</td>
<td>Planning a series and delivering a series of five lessons to primary students. Writing lesson plan and student evaluation.</td>
<td>Use appropriate teaching strategies and language to promote students’ understanding (3,6,7)</td>
<td>When planning each lesson make links to the previous lesson building on students’ prior. Amend lessons accordingly (All)</td>
</tr>
<tr>
<td>Unit 2A and Unit 2B</td>
<td>Attending tutorials and lectures related to primary mathematics education</td>
<td>Revise mathematical concepts needed for teaching (All)</td>
<td>Ask questions, engage in discussion, participate in activities</td>
</tr>
<tr>
<td>Unit 2B</td>
<td>Working with other pre-service teachers, conduct a workshop presentation during the tutorial on a mathematics topic. Prepare a written report.</td>
<td>Demonstrate correct mathematical understanding of a topic, model with resources and use correct terminology (1,6,7)</td>
<td>Demonstrate awareness of the different levels of difficult and common misconceptions of a topic (1,2,4,6,7,8)</td>
</tr>
<tr>
<td>Unit 2B</td>
<td>Prepare a learning trajectory chart on a mathematical topic</td>
<td>Demonstrate understanding of a mathematical topic (2,4,6,7)</td>
<td>Demonstrate knowledge of the sequence and identify different levels of complexity in a topic (3,6,8)</td>
</tr>
<tr>
<td>Unit 2B</td>
<td>Construct, present and analysis five high quality primary mathematics teaching resources</td>
<td>Make use of resources for developing mathematical understanding rather than use of worksheets or text books (3,6,7)</td>
<td>Recognise the conceptual appropriateness of a task and its suitability for teaching a concept (3,4,6,7)</td>
</tr>
<tr>
<td>Unit 2B</td>
<td>Pass a Mathematical Competency Skills and Knowledge Test</td>
<td>Demonstrate subject matter knowledge (4,7)</td>
<td>Limited opportunities to make connections</td>
</tr>
</tbody>
</table>
The analysis of second-year assessment tasks shows that pre-service teachers had opportunities to extend their foundation knowledge by concentrating on developing conceptual understanding, extending knowledge of mathematical language and demonstrating accurate understanding of mathematical ideas or concepts. In addition, they were given opportunities to make connections, including recognising conceptual appropriateness, being aware of the different levels of difficulty of topics, anticipating complexity, knowing the order for introducing ideas and strategies, and assessing students’ understanding.

Next, some of the assessment tasks are described in greater detail using examples collected from the longitudinal study participants that demonstrated how they extended their MCK when completing these tasks.

*Early numeracy interview task*

For the Early Numeracy Interview (DEET, 2001) assessment task, pre-service teachers completed a one-on-one interview with a student in Year 1, 2, 3 or 4. First, pre-service teachers were required to identify the mathematical knowledge and difficulties that the student demonstrated, analyse the student’s mathematical understanding and then suggest activities that could support further learning, reporting their errors or misconceptions. In particular, this task assisted pre-service teachers to unpack their MCK by developing understanding of early number concepts. This experience gave them the opportunity to extend their MCK whilst developing understanding of students’ mathematical knowledge. To illustrate, Lisa (Study Group 3) interviewed Charlie (Year 3) and recorded her interpretation of his responses to the interview questions.

Charlie was able to solve mathematical problems mentally by using doubles and repeated addition to solve the equation. Charlie was able to equally distribute the teddies on the mats and used the repeated addition method to solve the multiplication problem. (Lisa, Early numeracy interview report, p. 1)

After identifying the strategies that Charlie used, Lisa demonstrated that she could rely on her developing mathematical language to report the results of the interview and this is evidence of foundation knowledge and making connections.
Planning a series of lessons

For this task, pre-service teachers worked with a partner to plan, teach and evaluate a series of five lessons with a small group of students at a primary school. This experience assisted pre-service teachers to begin to develop skills to make connections with the different levels of difficulty of the topic that they taught.

Table 5.3 is a summary of the primary school year levels and topics that the longitudinal study participants completed when planning their mathematics lessons.

Table 5.3
Year Level and Topic Taught when Planning a Series of Five Lessons for Primary Students during Second-year (n=17)

<table>
<thead>
<tr>
<th>Study Group</th>
<th>Name</th>
<th>Yr level</th>
<th>Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Con</td>
<td>Yrs 5-6</td>
<td>Fraction and decimals</td>
</tr>
<tr>
<td>1</td>
<td>Sean</td>
<td>Yrs 5-6</td>
<td>Fractions and decimals</td>
</tr>
<tr>
<td>1</td>
<td>Shelly</td>
<td>Yrs 5-6</td>
<td>Fraction and decimals</td>
</tr>
<tr>
<td>2</td>
<td>Elizabeth</td>
<td>Yrs 5-6</td>
<td>Fraction and decimals</td>
</tr>
<tr>
<td>2</td>
<td>Esther</td>
<td>Yrs 1-2</td>
<td>Place Value</td>
</tr>
<tr>
<td>2</td>
<td>Fiona</td>
<td>Yr 5-6</td>
<td>Time</td>
</tr>
<tr>
<td>2</td>
<td>Janette</td>
<td>Yrs 5-6</td>
<td>Fraction and decimals</td>
</tr>
<tr>
<td>2</td>
<td>Mathew</td>
<td>Yrs 5-6</td>
<td>Fraction and decimals</td>
</tr>
<tr>
<td>2</td>
<td>Michael</td>
<td>Yrs 5-6</td>
<td>Fraction and decimals</td>
</tr>
<tr>
<td>2</td>
<td>Peter</td>
<td>Yrs 5-6</td>
<td>Fraction and decimals</td>
</tr>
<tr>
<td>2</td>
<td>Rose</td>
<td>Yr 5-6</td>
<td>Time</td>
</tr>
<tr>
<td>3</td>
<td>Don</td>
<td>Yr 5-6</td>
<td>Time</td>
</tr>
<tr>
<td>3</td>
<td>Emma</td>
<td>Yrs 1-2</td>
<td>Place value</td>
</tr>
<tr>
<td>3</td>
<td>Jenny</td>
<td>Yrs 1-2</td>
<td>Place value</td>
</tr>
<tr>
<td>3</td>
<td>Julie</td>
<td>Yrs 1-2</td>
<td>Place value</td>
</tr>
<tr>
<td>3</td>
<td>Kerri</td>
<td>Yr 5-6</td>
<td>Fraction and decimals</td>
</tr>
<tr>
<td>3</td>
<td>Lisa</td>
<td>Yr 5-6</td>
<td>Fraction and decimals</td>
</tr>
</tbody>
</table>

The pre-service teachers experienced teaching different year levels and different mathematical topics when undertaking these lessons. Twelve of the 17 pre-service teachers experienced planning and teaching upper primary year levels, thus receiving more opportunities to extend their foundation knowledge of difficult mathematical concepts and to become aware of weaknesses than the three pre-service teachers who taught lower primary levels.
This experience enabled pre-service teachers to develop breadth (Ma, 1999) of MCK of the topic they taught across five lessons, making connections as they made decisions about the sequence of these lessons. To illustrate, Sean (Study Group 1) planned a series of fraction and decimal lessons with Year 6 students. This assisted with revision of his MCK, and learner identity that provided opportunities to develop MCK including the understanding needed for passing his MCSK test. The discussion chapter (shown in section 8.3.1) maps pre-service teachers’ learner identity during coursework experiences. In addition, working with peers is likely to have supported development of MCK, as pre-service teachers assisted each other to extend their foundation knowledge when planning lessons, teaching together and reflecting on their students’ mathematical learning. Limited results are reported here because the 17 pre-service teachers were not observed teaching these lessons.

Emma (Study Group 3) worked with a Year 1 class on place value of two-digit numbers when planning her five lessons. Emma’s experience would have continued to extend her foundation knowledge, as she was expected to assist students to develop understanding using materials such as tens frames or bundling sticks. This experience would have consolidated her early number understanding, developed as part of the Early Years Numeracy Interview (DEET, 2001) with students. Emma was classified as Study Group 3 because she had difficulty passing the MCSK test as part of her assessment for Unit 2B. When considering development of their MCK Emma and similar pre-service teachers might have benefited from teaching upper year levels, thereby extending their MCK of more difficult concepts required for teaching.

Pre-service teachers were expected to amend lessons each week according to the previous week’s assessment of students’ responses. For this task, pre-service teachers’ were required to complete a summary of their students’ strengths and weaknesses and mathematical understanding during the lesson. Don (Study Group 3) recorded difficulties that his students had demonstrated when calculating duration of time:

The students had trouble with aspects of how to read timetables, work out how to calculate time from timetables. For example: If a train leaves Roxburgh Park train station at 6:50 and arrives at 7:25 how long will the train ride be? There was also confusion with aspects of converting from analogue to digital time. Students were able to interpret half hour timing and work out what time it was on the analogue clock. However, the students had trouble with reading the time on an analogue
clock in quarter and minute time. Students had trouble with reading 7:45 on an analogue clock, as most students thought it was 8:45 as the little hand was almost on the 8. (Don, lesson reflection, p. 3)

This example suggests that Don had difficulty relying on his foundation knowledge and making connections when introducing the mathematical concepts in appropriate progressive order for this lesson. He did not sequence this lesson as might be expected. Students should first know how to read analogue clock times to the nearest five minutes before solving problems involving duration (DEET, 2001).

Finally, Shelly’s (Study Group 1) five lessons were with Year 5 and 6 students and her topic was decimals, fractions and percentages. Figure 5.1 is a copy of Shelly’s lesson plan. Shelly lists her questions, making reference to a revision of Bloom’s Taxonomy (Anderson et al., 2001), and different question types such as knowledge, analysis and comprehension questions.

<table>
<thead>
<tr>
<th>Introduction:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revise previous lesson adding any new concepts, ideas and thoughts to word wall.</td>
</tr>
<tr>
<td>Can you tell us something about fractions that you learnt in our last lesson? (Knowledge.)</td>
</tr>
<tr>
<td>Introduce percentages to the students. Does anyone know another word used to name a half? (Knowledge) If students do not come up with percentage as an answer ask them if they have ever heard of the term percentage (knowledge).</td>
</tr>
<tr>
<td>Link fractions to percentages. Can anyone explain how fractions are linked to percentages? (Analysis.)</td>
</tr>
<tr>
<td>Discuss simple percentages e.g. 0%, 25%, 50%, 75%, 100%, using models, language and symbols, emphasizing that percentage is another name for a fraction explaining that 100% is the same as a whole. What fraction do you think 50% is the same as? Why? (Analysis.)</td>
</tr>
<tr>
<td>Real life fraction or percentage examples. Where have you seen or used fractions or percentages outside of school? E.g. Cooking, sales etc. (Comprehension).</td>
</tr>
</tbody>
</table>

| Main body: |
| Get students to work as a group to complete a table, linking models, language and symbols of both fractions and percentages (see attachments for example). Can you justify your choices? (Comprehension.) |

**Figure 5.1** A selection of the questions listed in Shelly’s fraction lesson plan.

By using the cognitive categories of Bloom’s Taxonomy, Shelly’s questions provide evidence of how she made links demonstrating making connections, between the types of questions she chose and the subject matter. However, some questions concentrated on knowledge of the topic rather than extending students’ conceptual understanding,
suggesting that Shelly was relying on her procedural knowledge of this topic when using questions or was thinking of this knowledge as a known fact, for example, “emphasise that percentage is another name for a fraction, [and] explain that 100% is the same as a whole.”

In the main body of the lesson Shelly chose to link models, language and symbols to fractions and percentages. This suggests that during her coursework she learnt that when teaching this topic it is important to model and develop understanding of fractions by using a variety of models to build students’ understanding of different concepts (Petit, Laird, & Marsden, 2010). By using models she demonstrated how she relied on her MCK and could transform her foundation knowledge with the use of resources (Rowland et al., 2009).

Figure 5.2 is a resource that Shelly made so that students could match equivalent terms and representations for fractions and percentages under the following headings: diagram model, diagram grid, fraction word, fraction symbol, fraction words and percentage symbol.

![Figure 5.2 Shelly’s fraction and percentage resource.](image-url)
Shelly’s fraction and percentage resource demonstrates her understanding of the different representations used to name and model equivalent fractions and percentages. Shelly made connections with symbols and models when designing this resource for her lesson whilst relying on her foundation knowledge of this topic. This resource supports students’ conceptual understanding for making connections between fractions and percentages.

In summary, this assessment task provided opportunities to learn that extended foundation knowledge, demonstrated transformation and making connections. Foundation knowledge is demonstrated by correct use of mathematical language during all levels of primary school teaching. Transformation was demonstrated by the choice and use of equipment or resources the pre-service teachers made for their lessons. This experience increased breadth (Ma, 1999) of mathematical understanding by making connections, building links to previous lessons and making choices about the sequencing of the lessons (Rowland et al., 2009).

*Learning trajectory chart*

Creating a learning trajectory chart enabled pre-service teachers to demonstrate their depth (Ma, 1999) of MCK by preparing a scope and sequence of a topic including models used for teaching the concept showing important stages of learning, as well as the language used to develop student’s understanding. This task also related to knowing the curriculum, which is PCK rather than MCK as a teacher’s PCK is needed for knowing how topics are organised and taught (Shulman, 1987) and will assist teachers to develop mathematical knowledge for teaching (MKT) (Ball et al., 2009). Rowland et al. (2009) agreed, stating “knowledge of mathematics and of mathematical pedagogy are both important aspects of foundation knowledge” (p. 153).

Pre-service teachers were required to complete a learning trajectory chart for numeration, operations for whole number and numeration, and operations for fractions or decimals. Figure 5.3 is an example of a multiplication learning trajectory, prepared by Shelly (Study Group 1).
<table>
<thead>
<tr>
<th>Summary of the sequence of the topic as you move through level 1-4 of VELS.</th>
<th>What are the concepts for this operation?</th>
<th>How are they developed?</th>
<th>Basic Facts, what are the thinking strategies?</th>
<th>How do you develop skills with mental computation?</th>
<th>Algorithm, models and materials, language, symbols</th>
<th>Application and problem solving, difficulties for children/insufficient knowledge of multiplication by powers of 10?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Develop the concepts of multiplication through repeated addition, using array models, and using the X symbol with vertical recording.</td>
<td>VELS Number standard: 2.0 students use repeated addition to compute simple multiplication.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. VELS standard: 2.75 students show multiplication using a rectangular array.</td>
<td>Build up the basic facts of multiplication by securing thinking strategies and focusing on automatic responses.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. VELS standard: 3.0 students have automatic recall of multiplication up to 10 x 10.</td>
<td>1. Using repeated addition to introduce multiplication.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. The idea of thinking in terms of arrays to determine if the operation is independent of addition.</td>
<td>Activities that use the idea of equal sized groups can help the student to develop the initial concept e.g. There are four baskets of apples with 3 apples in each, how many apples are there altogether?</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Two digit number multiplied by one digit number, using mental recording.</td>
<td>2. Introducing students to the use of arrays to determine if the operation is independent of addition.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Two digit number multiplied by one digit number, using mental recording.</td>
<td>Students who use grouping may think that 4 threes are different to 3 threes and so arrays are introduced, they show that multiplication is independent of addition.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. Multiplying large numbers.</td>
<td>3. Introduce the multiplication symbols to represent the array process, focusing on reading from bottom to top.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8. Build strategies for estimation, calculator use and mental computation.</td>
<td>Emphasise that multiplication is commutative downwards.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| 9. Multiplying with decimal fractions. | 4. Build the multiplication language to include ‘multiply by’ or ‘by’.
- Introduce students to the formal language, be careful not to use the term ‘times’ as it does not represent a meaningful process. |
| 10. VELS standard: 3.25 students use arrays to multiply fractions by fractions. | 1. Student’s knowledge of basic facts must be extended to deal with tens. |
| 11. VELS standard: 3.5 students can multiply decimals to one decimal place. | This can be done by using arrays and materials that show that ones multiplied by tens gives as tens. |

**Figure 5.1** Shelly’s multiplication learning trajectory when in second-year Unit 2B.
In her multiplication-learning trajectory, Shelly demonstrated that she could make connections with the theory by providing a detailed summary of the scope and sequence for teaching multiplication across all levels of the primary school curriculum. Evidence of transformation included identifying models that would assist students’ conceptual understanding, such as use of arrays and base ten materials. Models are used by students when focusing on developing new ideas when learning mathematics (Booker et al., 2010). Foundation knowledge was demonstrated by choice of mathematical terminology matching the materials and concepts she identified as important for teaching this topic. The final column included common misconceptions that students demonstrate, and could be used when assessing students’ understanding of this topic and also demonstrated evidence of her developing foundation knowledge.

Shelly has used mathematics references or curriculum documents when recording the sequencing and connectivity of the fundamental mathematical concepts for multiplication. In particular, this task helped Shelly to learn about additive strategies and early strategies that build students’ multiplicative thinking. Additive thinking involves part-part-whole ideas and place value, whereas multiplicative thinking is working with the relationships between numbers (Siemon et al., 2011). This knowledge assisted Shelly to make connections whilst developing her specialised MCK as she recognised the links within this topic, demonstrating her understanding of the difficulties of this topic and order of teaching these concepts.

During this task, foundation knowledge includes extending pre-service teachers’ understanding of the processes involved in the four operations and accurate understanding of mathematical idea. This task also enabled pre-service teachers to begin to make connections and develop an understanding of fundamental ideas and the foundations of students’ mathematical learning with reference to the course text (Booker et al., 2010). Rowland et al. (2009) further defines making connections as evidence when a teacher breaks down mathematical concepts into steps that are understood by younger students when teaching.

MCSK test

The MCSK tests were different but similar (see Section 3.4.1) so that different cohorts of pre-service teachers could undertake tests at different times depending on when they
completed Unit 2B, which was offered during second year in Semester 1 or Semester 2, or Unit 1B which was offered in every semester and year of the program. Thus, the pre-service teachers in this study completed seven different MCSK tests, with 11 of the 17 longitudinal study pre-service teachers completing MCSK Test 1, (Don) Test 2 (Lisa, Kerri, Shelly, Michael, Elizabeth, Sean, Con, Julie and Peter) or Test 3 (Esther); the other six pre-service teachers completed MCSK tests not reported in Chapter 4.

As a comparison of MCK, three pre-service teachers, from different study groups, Sean (Study Group 1), Janette (Study Group 2) and Lisa (Study Group 3) completed the same test (MCSK Test 2), so their results can be compared. MCSK Test 2 consisted of 29 least difficult items, 17 difficult items and three most difficult items. Sean’s (Study Group 1) and Julie’s (Study Group 2) results were similar: Sean correctly answered 44 MCK items, including 26 least difficult items, 16 difficult items, 2 most difficult items. Janette correctly responded to 43 MCK items, including 27 least difficult items, 14 difficult items, and two most difficult items. Lisa (Study Group 3) correctly responded to 16 least difficult items, but could not correctly answer any difficult or most difficult items. Therefore, using the ranking of item difficulty (previously described Figure 3.1) Study Group 1 and Study Group 2 pre-service teachers would be more likely to respond correctly to MCSK test items that were coded as difficult and most difficult, whereas Study Group 3 pre-service teachers would most likely answer difficult and most difficult items incorrectly.

5.3 Conclusion

This chapter reported on the longitudinal study participants’ opportunities to learn MCK as they undertook coursework. The analysis of these opportunities identified factors that contributed to the 17 pre-service teachers’ development of their MCK that will be presented in the discussion chapter (Chapter 8). The factors included pre-service teachers’ identities, quality of teaching, and sustained coursework opportunities to develop, deepen or broaden MCK when in third and fourth years.

First and second-year program experiences included three primary mathematics education units and one elective unit integrating opportunities to learn MCK, PCK and
curriculum knowledge. First-year coursework experiences provided opportunities for pre-service teachers to reflect on their pre-program identity and secondary-school knowledge of mathematics. The pre-service teachers were provided with opportunities to begin to identify their strengths and weaknesses in MCK and for considering how they might identify – and if needed – address MCK weaknesses.

The second-year primary mathematics teacher education units helped pre-service teachers to extend their MCK and develop the knowledge needed for teaching. The range of experiences and assessment tasks enabled pre-service teachers to develop foundation knowledge, make connections and consider how they might transform their MCK when using materials and resources required for teaching primary students.

Foundation knowledge was demonstrated by correct use of mathematical language, revising mathematical concepts and topics and developing understanding rather than relying only on procedural knowledge. Transformation (Rowland et al., 2009) was expected to be demonstrated by the choice and use of equipment or resources the pre-service teachers created or chose when responding to assessment tasks. The analyses of data showed that some pre-service teachers had more opportunity to develop depth of MCK during practicum experiences because they were given higher year levels and difficult topics when completing a series of five lessons. All pre-service teachers had the opportunity to develop breadth of MCK when planning five lessons on one topic for a small group of students but not all pre-service teachers had an opportunity during second year coursework to extended their foundation needed for teaching upper primary school mathematics curriculum when planning a series of lessons.

During their interviews, the 17 pre-service teachers reported different factors that assisted them when revising and developing their MCK during first and second-year. The five pre-service teachers that completed the elective unit identified the lecturer as supportive, assisting with relearning MCK. The remaining 12 longitudinal study pre-service teachers completed self-guided revision, used books and websites, tutored, revised coursework notes and lectures, and sought help from lecturers or mentor teachers at their primary school as preparation for learning the MCK required for passing the MCSK test.
During their program, four of the longitudinal study pre-service teachers chose to complete an elective unit to further address their MCK weaknesses; three of the four passed the MCSK test on their first or second attempts (see Table 5.1). These three pre-service teachers were demonstrating their learner identity because they took the initiative and chose to seek assistance in developing their MCK for teaching, and extended their MCK by completing Unit 1B after identifying weaknesses in their MCK during first-year. In contrast, two of the 17 pre-service teachers were required to complete the elective unit in third year because they had not passed the MCSK test at earlier attempts (in Unit 1B or Unit 2B). They had more difficulties as learners when demonstrating their knowledge during coursework experiences and when responding to the MCSK test items.

In conclusion, core coursework in primary mathematics was limited to the first two years of the program and arguably restricted continued development of pre-service teachers’ MCK. In addition to coursework, pre-service teachers completed practicum experiences in schools each year. The next chapter reports on opportunities and influences on the 17 longitudinal study pre-service teachers’ MCK when they were observed teaching primary mathematical lessons during second-year and fourth-year in primary schools.
CHAPTER SIX

Opportunities to develop MCK: Practicum experiences

The previous chapter described the longitudinal qualitative study of 17 pre-service teachers’ mathematical content knowledge (MCK), including opportunities and influences that enhanced their MCK during coursework experiences. This chapter focuses on the longitudinal study participants’ opportunities to develop MCK during practicum experiences in primary schools. During second-year and fourth-year the researcher observed the participants teaching a primary mathematics lesson during their practicum experiences. After the lesson observation during second-year (Appendix D) and fourth-year (Appendix F) the participants were interviewed about the aspects of the lesson and other program experiences which enhanced their MCK. Interviews were also conducted during third year (Appendix E), when the longitudinal pre-service teachers were completing practicum experiences in secondary schools. Three of the 17 participants chose secondary mathematics teaching during their third-year practicum experiences, the others did not.

This chapter begins with the results of analysis of pre-service teachers’ opportunities to develop breadth and depth of MCK (Ma, 1999). Opportunities to develop depth of MCK were afforded when teaching different year levels and breadth of knowledge when planning and teaching primary mathematics lessons for different mathematical content at the same year level. Next are results relating to pre-service teachers’ opportunities to develop their MCK, including the specialised mathematics content knowledge (SCK) suggested as being unique to teaching (Ball et al., 2008). Rowland et al. (2009) identified different categories of teachers’ MCK when teaching:

- Foundation knowledge – focuses on what teachers know and believe
- Transformation - representing mathematics to learners
- Connection - how mathematical ideas and concepts are linked
- Contingency - knowing what to do when the unexpected happens during a lesson. (p. 29)
These four categories – foundation knowledge, transformation, connections and contingency (Rowland et al., 2009) – are used within this chapter to report on pre-service teachers’ SCK (Ball et al., 2008), as demonstrated during their practicum experiences and when providing reflections during interviews.

6.1 Opportunities to develop breadth and depth

When the 17 pre-service teachers undertook practise teaching in primary schools, different factors contributed to their development of breadth and depth (Ma, 1999) of MCK. For instance, some taught in upper year levels, others taught lower year levels, some taught in country schools or in low-socioeconomic city schools; some taught in composite classrooms with more than one year level or in open-plan classrooms with more than one cohort of students; others taught in schools in which primary students were grouped within their class for mathematics or streamed across different classrooms during mathematics. Some of these factors also represented opportunities for pre-service teachers to extend their MCK.

Opportunities to develop breadth of MCK were identified by comparing the year levels that the 17 participants undertook during each year of practice teaching: experiences in primary schools in first, second and fourth year, and secondary school during third year.

Table 6.1 shows the distribution of pre-service teachers’ practise teaching: lower primary (Preparatory to Year 2), middle primary (Years 3 and 4), upper primary (Years 5 and 6) and lower secondary (Years 7 to 9). The first column identifies the Study Group of each pre-service teacher. Study Group 1 pre-service teachers were completing secondary mathematics elective units of study as their secondary major. Study Group 2 and 3 pre-service teachers were not completing secondary mathematics as their elective unit. Study Group 1 and Study Group 2 pre-service teachers had less difficulty responding to MCSK test items than Study Group 3 pre-service teachers, who needed to sit the MCSK test more than once to demonstrate competency.
Table 6.1
Distribution of Practising Mathematics Teaching During the Teacher Education Program (n=17)

<table>
<thead>
<tr>
<th>Study Group</th>
<th>Name</th>
<th>First-year Primary (20 days)</th>
<th>Second-year Primary (32 days)</th>
<th>Third-year Secondary (42 days)</th>
<th>Fourth-year Primary (50 days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Con</td>
<td>lower primary</td>
<td>upper primary</td>
<td>lower secondary</td>
<td>upper primary</td>
</tr>
<tr>
<td>1</td>
<td>Sean</td>
<td>lower primary</td>
<td>lower primary</td>
<td>lower secondary</td>
<td>upper primary</td>
</tr>
<tr>
<td>1</td>
<td>Shelly</td>
<td>lower primary</td>
<td>lower primary</td>
<td>lower secondary</td>
<td>middle primary</td>
</tr>
<tr>
<td>2</td>
<td>Elizabeth</td>
<td>middle primary</td>
<td>upper primary</td>
<td>Drama &amp; English</td>
<td>middle primary</td>
</tr>
<tr>
<td>2</td>
<td>Esther</td>
<td>middle primary</td>
<td>lower primary</td>
<td>Humanities and Art</td>
<td>lower primary</td>
</tr>
<tr>
<td>2</td>
<td>Fiona</td>
<td>lower primary</td>
<td>middle primary</td>
<td>Visual Arts &amp; English</td>
<td>upper primary</td>
</tr>
<tr>
<td>2</td>
<td>Janette</td>
<td>lower primary</td>
<td>lower primary</td>
<td>Psychology &amp; Textiles</td>
<td>lower primary</td>
</tr>
<tr>
<td>2</td>
<td>Kerri</td>
<td>lower primary</td>
<td>middle primary</td>
<td>Drama &amp; SOSE</td>
<td>upper primary</td>
</tr>
<tr>
<td>2</td>
<td>Mathew</td>
<td>upper primary</td>
<td>upper primary</td>
<td>History &amp; Multi-media</td>
<td>upper primary</td>
</tr>
<tr>
<td>2</td>
<td>Michael</td>
<td>lower primary</td>
<td>upper primary</td>
<td>Outdoor Education</td>
<td>upper primary</td>
</tr>
<tr>
<td>2</td>
<td>Peter</td>
<td>lower primary</td>
<td>upper primary</td>
<td>Geography &amp; Psychology</td>
<td>middle primary</td>
</tr>
<tr>
<td>2</td>
<td>Rose</td>
<td>lower primary</td>
<td>upper primary</td>
<td>Art &amp; Drama</td>
<td>middle primary</td>
</tr>
<tr>
<td>3</td>
<td>Don</td>
<td>Music primary</td>
<td>upper primary</td>
<td>Drama &amp; Media Studies</td>
<td>middle primary</td>
</tr>
<tr>
<td>3</td>
<td>Emma</td>
<td>Pre-school</td>
<td>lower primary</td>
<td>English &amp; ICT</td>
<td>family leave</td>
</tr>
<tr>
<td>3</td>
<td>Jenny</td>
<td>lower primary</td>
<td>lower primary</td>
<td>Outdoor Ed and Drama</td>
<td>lower primary</td>
</tr>
<tr>
<td>3</td>
<td>Julie</td>
<td>lower primary</td>
<td>lower primary</td>
<td>Literature &amp; English</td>
<td>middle primary</td>
</tr>
<tr>
<td>3</td>
<td>Lisa</td>
<td>lower primary</td>
<td>middle primary</td>
<td>ICT &amp; Psychology</td>
<td>middle primary</td>
</tr>
</tbody>
</table>
Most of the participants experienced lower primary in first year, and experienced middle or upper primary in fourth year. In summary during first year seven pre-service teachers experienced lower primary levels, two middle primary and one upper primary (n=15). In their second year seven pre-service teachers experienced lower primary levels, three middle primary and seven upper primary (n=17). In fourth year three pre-service teachers experienced lower primary levels, seven middle primary and six upper primary (n=16). Pre-service teachers specialising in secondary mathematics (Study Group 1) also taught mathematics in junior secondary (Years 7, 8 and 9) schools during third year.

Three pre-service teachers demonstrated foundation knowledge during the fourth-year lesson observation: Elizabeth, Esther (Study Group 2) and Rose (Study Group 3) (see Table 6.2). Overall the nine, Study Group 2 pre-service teachers demonstrated mixed results during their fourth-year lesson observation. They were likely to either not yet demonstrate foundation knowledge or were demonstrating foundation knowledge but had all passed the MCSK test on their first attempt during second year and were not enrolled in mathematics for their secondary specialisation.

Table 6.1 shows that male pre-service teachers were more likely than female pre-service teachers to teach upper primary mathematics lessons during their four years of practicum experiences. Also, two pre-service teachers did not experience primary mathematics in first year: one experienced a pre-school (aged 3 and 4) setting and the other a specialist music (Preparatory to Year 6) practice teaching experience. Therefore, these two pre-service teachers had fewer opportunities to extend their MCK during first-year than the other 15 participants.

In general, pre-service teachers reported that one of the teachers at the school was responsible for allocating the pre-service teachers to different primary classrooms and assigning a mentor teacher, rather than the university or pre-service teacher making these decisions. In other words the university did not request the year level at which pre-service teachers practised their teaching, hence the lack of any pattern in the distribution of pre-service teachers’ practicum experiences. As a result, eleven of the 17

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9 Emma did not complete her teaching practice during fourth year because she had applied for leave from the program and did not know when she would complete her practicum experience.
pre-service teachers did not receive opportunities to develop depth of MCK by teaching lower, middle and upper year levels during their primary practicum experiences.

Study Group 1 pre-service teachers experienced the greatest depth of experiences because they practised teaching lower primary to lower secondary school students. In contrast, three participants were not offered depth during their practice teaching, experiencing only lower year levels in primary schools. Emma (Study Group 3) had not experienced middle or upper primary, but had not completed her final year of the program at the time of this study. Another three pre-service teachers experienced lower primary and middle primary practice teaching only, one experienced upper only, and another only experienced practice teaching upper and middle. Thus these eight pre-service teachers were not given the opportunity to extend their mathematical teaching and learning experiences, limiting their opportunities to develop depth of MCK.

In each year of the program pre-service teachers had a different school experience and mentor teacher to guide their development. Mentor teachers had a wide range of years of teaching experience. For example, during his final year of the program, Sean (Study Group 1) was in a Year 5/6 class with a mentor teacher in her first year of teaching; simultaneously, Esther (Study Group 2) was in a preparatory class and her mentor teacher was retiring after many years of teaching. The researcher was unable to determine if length of mentor’s teaching experience made a difference to pre-service teachers’ MCK, but the findings did identify the quality of mentoring as a factor that inhibited pre-service teachers’ opportunities to extend their MCK (see Section 6.3).

Other studies have concluded that teachers hold their MCK in different ways (e.g. Ma, 1999; Mewborn, 2001) therefore mentor teachers will also have different mathematical knowledge for teaching (MKT) (Ball et al., 2009) because of their varied experiences. However, it is logical that mentor teachers with more breadth and depth of experience are better equipped to assist pre-service teachers to extend their breadth and depth of MCK. To illustrate using the National Professional Standards for Teachers (AITSL, 2012), it is expected that graduate teachers in their first few years of teaching will share their knowledge of content and teaching by demonstrating their knowledge and understanding of mathematical concepts, whereas a highly accomplished teacher should
know how to support the pre-service teacher using current and comprehensive knowledge of their MCK.

In summary, these results identified that pre-service teachers’ opportunities to extend their breadth and depth of MCK during teaching practice were influenced by factors such as the depth of individual school experiences as they undertook teaching practice, as well as by mentor teachers with different amounts of teaching experience and (most likely) different MKT.

6.2 First-year practice teaching experiences

Although, the 17 participants were not observed in primary schools during first year of their program they provided reflections of their experiences during their second year interview. Therefore, when comparing the analysis of first-year with the other three years of data collection the first year analysis was limited.

In first year, pre-service teachers usually completed 20 days of practicum experience in a primary school. These school visits occurred every Tuesday from May until September during first and second semester, and students undertook an additional one-week practicum experience during August. The pre-service teachers were introduced to teaching during practicum experiences in first year. As reported in Chapters 3 and 5, they also studied two core education units that focused on praxis inquiry – the connection between university and school experiences (Arnold et al., 2011).

The analysis of pre-service teachers’ reflections found that during first-year they mainly assisted their mentor when teaching mathematics by working with small groups of students. Lesson planning mainly related to literacy lessons, as expected according to the course requirements, rather than mathematics, indicating the participants gained minimal experience in planning and teaching mathematics lessons. As a consequence, participants had few opportunities to develop their foundation knowledge and transformation (Rowland et al., 2009) during first-year practicum experiences.

Nevertheless, some participants provided examples of experiences that enabled them to begin to shape their foundation knowledge. They could revise and unpack their MCK
when observing their mentor teacher demonstrating mathematics lessons, using and hearing mathematical language in the classroom situation, modelling with mathematical materials, and choosing appropriate strategies for assisting students to develop their mathematical understanding. To illustrate, Esther and Con provided details of their first-year experiences, reflecting on their opportunities to extend their foundation knowledge (Rowland et al., 2009).

When Esther (Case Study 2) assisted a small group of students in a Year 4 class, she also had the opportunity to revise her MCK. She spoke of the challenges of working with these students, stating that one student knew as much mathematics as she did. This was evidence that Esther needed to develop her MCK for teaching Year 4 students. Also, when checking one student’s multiplication calculations, Esther said that she had to work it out with him and this experience helped her to revise how to multiply with larger numbers. As a result, and during first-year, Esther extended her understanding of processes for multiplication, developing her foundation knowledge (Rowland et al., 2009).

Con (Study Group 1) also provided evidence of developing his foundation knowledge (Rowland et al., 2009) during his experience with a Year 1 class. He extended his knowledge of the language needed for teaching division to younger students, as well as broadening (Ma, 1999) his conceptual understanding of division.

For one lesson I did division by sharing and we had so many blocks on each table and you had to share it between so many cards... there is one card, two cards, three cards, four cards, 20 blocks you can share it between two cards or you can share it between four cards or five cards. They had to answer how many. (Con)

This experience would have assisted Con in using his MCK when thinking about modelling early division concepts, the correct strategies and language to introduce division concepts to students, therefore expanding his SCK (Ball et al., 2009). Booker et al. (2010) stated that the division concept should be introduced with the sharing idea and use of stories. Therefore, Con’s practicum experiences were modelling the theory he was expected to develop as part of his coursework experiences. Additionally, when in first-year pre-service teachers, may have practised appropriate teaching strategies by using materials rather than a worksheet to develop students’ mathematical
understanding. These experiences would have assisted pre-service teachers to unpack their MCK and extend their conceptual understanding.

Don and Emma had first-year experiences different from those of the other pre-service teachers in the longitudinal study (Table 6.1). Don (Study Group 3) experienced a music program and assisted his mentor, who taught music across the different year levels of the primary school. He mentioned during his interview, “There was not any maths except maybe counting the beats of the notes.” Emma (Study Group 3) experienced a pre-school program called Kinda Kinda. Kinda Kinda was a literacy-focused play-based learning program, catering for pre-school-aged disadvantaged children who were unable to attend kindergarten (Barry, 2008). Like Don, Emma considered that her first-year practice teaching experience did not include mathematics experiences and stated, “I was in a Kinda Kinda [program] so I did not tackle any mathematics.”

Emma’s comment suggests that she did not recognise that her first-year experiences with pre-school students (aged 3 or 4) could have been an opportunity for play-based learning activities related to mathematics. Research has shown that structured activities through play can provide kindergarten students with opportunities to learn basic arithmetic and geometry (Clements & Sarama, 2013).

Don’s and Emma’s first-year practicum experiences provided relatively limited opportunities to extend their foundation knowledge and transformation needed for primary mathematics teaching. Don and Emma may have been disadvantaged when completing second-year coursework and practicum experiences related to primary mathematics teacher education because they did not have experiences that provided an opportunity to reflect on the role of mathematics in primary classrooms. Therefore, they may have relied on their pre-program experiences, which were likely to be different to current (2008) teaching and learning of primary mathematics.

6.3 Second-year practice teaching experiences

In second year most pre-service teachers usually completed 32 days in primary schools. These school visits were every Tuesday from March until September (during first and second semester), and students also completed a two-week practicum experience during
May. As previously noted, the 17 participants were required to plan and teach at least five primary mathematics lessons as part of their second-year school experience program requirements. This ensured that pre-service teachers had opportunities to extend their foundation knowledge when practising their primary mathematics teaching. Furthermore, evidence of transformation, connection and contingency, the remaining three categories of the Knowledge Quartet (Rowland et al., 2009), was demonstrated during second-year practicum experiences and is reported below.

For one semester in second year, the pre-service teachers were simultaneously completing two units of study for teaching primary mathematics whilst practising their mathematics teaching with primary students. This combination extended opportunities to learn as part of the program structure, assisting the pre-service teachers to make connections with the theory of coursework and practice when teaching students. Pre-service teachers’ coursework should help them to learn from their own teaching experiences in the classroom (Ebby, 2000), providing further opportunities to extend their MKT (Ball et al., 2009).

The 17 pre-service teachers were observed teaching a primary mathematics lesson in their second year. They taught topics such as naming angles, multiplication (upper primary), subtraction, telling the time, area and perimeter (middle primary) and shape (lower primary). Before teaching their lessons they were required to provide their mentor teacher with a lesson plan. The pre-service teachers reported that when they planned their lessons they would search the internet if they needed to check the mathematics before teaching their lesson. These experiences provided opportunities to revise their MCK.

During the lesson observations in second year the 17 pre-service teachers taught a small group of students or the whole class whilst the mentor observed or taught other students at the same time. Some of the pre-service teachers said that this was their first opportunity to teach mathematics. These experiences were important, because they allowed pre-service teachers to use the mathematical language of the topic they were teaching, model the mathematics when teaching, and develop their SCK and foundation knowledge.
In post-lesson interviews, the second-year pre-service teachers who taught lower primary levels said that they did not usually need to check the mathematics before teaching. Shelly (Study Group 1) taught shape to preparatory students and knew the names of the two-dimensional shapes that she used during the lesson. She did, however, look up the Victorian Essential Learning Standards (VELS) (DEECD, 2009a) (the mathematics curriculum used in Victorian schools at the time of the study) to check the content she needed to teach at this level. For this experience she would have relied on her MCK for planning and designing the shape bingo game that she used during the lesson, and this was evidence of transformation.

Pre-service teachers who taught upper primary students were more likely to check their MCK when planning than those who taught lower primary students because the mathematics at these levels was more difficult. To illustrate, Don (Study Group 3) was anxious or nervous because he needed to rely on his MCK when teaching and was concerned that this would be more challenging when teaching Year 6.

This year was the first year I actually taught my very first maths lesson. I remember I was very scared because I was afraid I would stuff up or I was going to get something wrong with the kids. Then they would realise – Oh no you have done that wrong! I have come a long way since when I first started. I have done about 12 maths lessons. The majority of the maths lessons (sic) I have done this year have been maths. (Don)

Don’s regular opportunity to teach Year 6 students assisted his foundation knowledge, including MCK, and self-confidence because of the necessity to revise his MCK when teaching upper primary students.

Whilst observing the pre-service teachers teaching some mentors took notes, so they could provide feedback about the lesson to the pre-service teacher after the lesson. After the lesson the pre-service teachers reflected with their mentor teachers and opportunities to extend their MCK. The pre-service teachers reported that mentors provided a combination of written and verbal reflections, but this feedback focused on lesson structure or classroom management rather than the mathematics content, teaching methods, use of materials or mathematical language used during the lesson.

My mentor told me how to improve the lesson by not having the Year 5 and 6 students sit too long on the mat as they get ratty. (Peter)
My mentor focuses on classroom management... he likes to make sure I do table points [reward system]. Not really on the content. (Janette)

Both of these examples focused on lesson structure or classroom management rather than mathematical content of the lesson, mathematical teaching strategies or procedures, use of materials or mathematical language used during the lesson. This suggests these pre-service teachers were unlikely to extend their foundation knowledge and transformation when reflecting with their mentor teacher on the strengths and weakness of their MCK as illustrated in their lesson plan or when relying on their MCK when teaching a mathematics lesson.

In contrast, Don reported that his mentor teacher shared his knowledge, making suggestions of books or websites that would help Don to revise his MCK. These resources also helped Don when planning his mathematics lessons.

I gave him [the mentor teacher] a goal and I said my goal is by the end of my time here I want to develop and improve my skills in mathematics and I think that has been achieved.

(Don)

In the mathematics lesson observed for this study, Don taught multiplication. During his interview he said he had taught a lot of multiplication lessons. In this situation, teaching Year 6 and seeking help from his mentor assisted Don to develop knowledge of procedures and his foundation knowledge. Additionally, these experiences assisted Don to extend his MCK as preparation for the MCSK test during the middle of second-year, which he failed during his first attempt but passed during a supplementary test.

When teaching this lesson Don completed different multiplication algorithms on the whiteboard. One example included multiplying 735 by 15 (Figure 6.1).

![Field notes of Don’s multiplication algorithm.](image)
This solution, 11 025, suggests that Don could rely on his procedural knowledge when calculating the answer. An example of how Don described this process included, “five times five is 25 and then five times three is fifteen and two is 17…”

Despite Don’s successful calculation, he did not demonstrate place value understanding or SCK (Ball et al., 2009) that might consolidate students’ understanding of this procedure. Booker, Bond, Briggs, Sparrow, and Swan (2010) suggested the following language:

Multiply the ones, five ones by five ones is 25 rename as 2 tens and 5 ones and record. Record the five ones and 2 tens. Multiply the tens, five ones by 3 tens are 15 tens. Rename and record 1 hundred and 5 tens and so on. (Don)

In this teaching situation Don could not rely on the foundation knowledge needed to demonstrate correct and appropriate mathematical language that is appropriate when teaching multiplication of two and three digit numbers with Year 6 students. The results show that although he modelled correct solutions, he used procedural knowledge rather than teaching for understanding.

Next, Rose (Study Group 2) and Lisa (Study Group 3) were chosen to illustrate findings about how two pre-service teachers in different study groups, both hoping to become primary teachers, relied on and demonstrated their MCK when teaching.

6.3.1 Teaching mathematics: Rose (Study Group 2)

When in second year Rose extended her foundation knowledge, demonstrated knowledge of making connections and provided evidence of transformation knowledge (Rowland et al., 2009).

Rose was observed teaching a single-sex class of Year 5 and Year 6 girls. She had planned a measurement lesson that aimed to assist the students to develop their understanding of angles by naming, measuring and constructing angles using protractors.

In her lesson plan (Appendix I), Rose stated the following aims:
Use a protractor correctly to measure angles, name the angles with the proper terms, [and] identify angles and estimating angles. (Rose’s year-two lesson plan, p. 1)

Foundation knowledge

Preparing her lesson enabled Rose to think about its purpose and the MCK she needed for teaching these concepts.

I looked up the definitions and I used them in the lesson. I would have had a rough idea of the definitions for the lesson and now I think I know them off by heart… Before teaching a lesson I usually looked up terms on the internet as part of my planning. (Rose)

Rose checked her knowledge of the different angles and listed the key vocabulary in her lesson plan, including angle, degrees, protractor, reflex and revolution.

For this lesson Rose relied on and demonstrated her procedural knowledge when asking the students questions. She mostly asked closed questions as she questioned the students and recorded the names of the angles onto the whiteboard.

There are certain names for angles; can anyone tell me one?... so which one of these is a right angle?... Yes, I am just going to write a definition for a right angle [wrote: an angle measuring 90°]…Can anyone else name one of these angles? (Rose)

Rose chose not to measure the angles during this part of the lesson with the students, assuming they understood the attribute of the angle size from the diagram. Even though Rose was recalling facts for naming the angles, her decision not to measure these angles or draw attention to their size might have confused students and limited their mathematical understanding of this topic. Using appropriate teaching strategies is a key factor when demonstrating foundation knowledge (Rowland et al., 2009).

These observations suggest that Rose had not yet developed her foundation knowledge sufficiently so that she could design strategies and questioning that support students’ conceptual understanding. Rose has not yet developed her foundation knowledge when using teaching strategies and questioning that will support students’ mathematical understanding when naming and labelling angles.
Transformation

During the lesson Rose helped the students as they found and measured angles in the classroom. Measuring angles is a difficult concept for students to understand (Van de Walle, Karp, & Bay-Williams, 2012). However, in this situation, Rose was able to rely on her foundation knowledge when assisting students one-on-one when compared to teaching the whole class. As a result she was also demonstrating transformation when she provided tasks that assisted the students to extend their skill in using a protractor.

A protractor has heaps of different lines and it shows you the space, what you want to do is line up the lines and it can make the angle. (Rose)

Rose designed a worksheet for this lesson (using information from the internet) so the students could practise measuring different angles. She had drawn different angles and the students had to estimate their size, measure them with a protractor, then name the type of angle. The examples included different sizes and rotations, acute angles and obtuse angles but none greater than 180°, as she had specified in her lesson introduction. The worksheet provided students with a range of carefully drawn angles to practise measuring in degrees (Figure 6.1).

![Measuring Angles](image)

*Figure 6.2 Rose’s second-year measuring angles student worksheet.*
Transformation includes ways of using examples, for instance, when the teacher uses examples in mathematics when teaching or providing exercises for students to practise the ideas being taught (Rowland et al., 2009). Rose demonstrated both methods of transformation during her measurement lesson.

**Connections**
During her interview, Rose demonstrated that she was beginning to make connections (Rowland et al., 2009) as she discussed her understanding of the purpose of student activities and developing their mathematical understanding.

> ...the teacher needs to be able to not just use the rules but do hands-on activities... in the course you get to understand how the VELS Levels work and so you know what to teach certain kids and how to actually put them into groups and find out what they know... (Rose)

Rose extended her foundation knowledge when planning and teaching this lesson by revising mathematical language and the MCK needed for teaching this topic to Year 5 and Year 6 students. During this lesson Rose demonstrated her SCK, including categories of foundation knowledge: using teaching strategies that assisted students to extend their understanding, and demonstrating knowledge of mathematical language and ideas within her lesson. Rose also demonstrated some evidence of making connections and transformation. Although Rose mostly recalled knowledge by naming the angles, she was also able to assist students to correctly measure using a protractor, and provided evidence of transformation and making connections by making links between the lesson introduction and activities she chose for the students to complete during the main body of the lesson.

**6.3.2 Teaching mathematics: Lisa (Study Group 3)**
Lisa’s lesson was observed in second-year and focused on number and teaching subtraction to Year 3 students. The lesson commenced with a bingo game. Lisa verbally gave subtraction problems, such as “thirteen take away nine” and the students crossed off numbers on a bingo board if they had the solution.

For the next activity, Lisa drew a subtraction ladder on the whiteboard. During her interview Lisa explained that she had adapted the idea from a mathematics book her
mentor had shown her. However, during the lesson, Lisa changed how she used the subtraction ladder and appeared to be adapting her ideas as she taught.

Lisa: Here is a subtraction ladder, you have probably never seen this before. We are going to make this work downwards… The first number is going to be ten and I want to put 5 here [the numbers were placed in the first and third spaces]. I want you to tell me what subtraction problem I can make to put a number in between. What is a number between ten and five?

Olivia: Eight.

Lisa: What do I do to get from ten to eight, what subtraction problem?

Darcy: Ten take away two.

Lisa: Who knows how to work that out?

Lisa continued to work through this subtraction method, placing a number (the difference) into the ladder. For the next problem she used the number above it as the minuend to work out the subtrahend. Then, she recorded the subtraction problem to the side. Figure 6.3 illustrates the subtraction ladder at the beginning of the discussion as well as the completed ladder at the conclusion of the discussion.

Lisa continued to work through this subtraction method, placing a number (the difference) into the ladder. For the next problem she used the number above it as the minuend to work out the subtrahend. Then, she recorded the subtraction problem to the side. Figure 6.3 illustrates the subtraction ladder at the beginning of the discussion as well as the completed ladder at the conclusion of the discussion.

Figure 6.3 First subtraction ladder at the beginning and after discussion.

Once the first ladder was completed (Figure 6.3), Lisa suggested that these were pretty easy and proceeded with a new example with larger numbers (Figure 6.4).
Figure 6.4 Lisa’s second subtraction ladder.

For the example in Figure 6.4, Lisa wrote the digits 50 and 35 into the ladder and asked the students to copy it and use strategies to solve the problem.

Lisa: You need fifty at the top and twenty-five halfway down the ladder…… What number might I put here?” [In the second space].

Darcy: Twenty-five. [The student may have been thinking the difference between fifty and twenty-five is twenty-five but Lisa prompted a different number].

Lisa: Maybe count by tens, twenty-five.

Darcy: Thirty-five.

Lisa: Yes. Do you maybe want to put thirty-five in here [she records thirty-five on whiteboard]. OK, fifty take away thirty-five. Write down or draw how you could solve that … You can draw maybe apples. You might have fifty apples. That’s a bit hard. So you might group them… You might want to use a number line.

Listening to different students’ strategies improves students’ mathematical understanding and increases teachers’ mathematical knowledge (Empson & Jacobs, 2008). However, Lisa continued without explaining the answer and asked the students to think of a number to record in the last place on the ladder.

Lisa: Could you put 40 in this box?

Ben: No.

Lisa: Can we do 25 take away 40. No not really. Not properly. We need a number less than 25.
For the remainder of the lesson the students were placed into three groups and completed different tasks to reinforce subtraction skills. Therefore the subtraction ladder activity was not connected to the next part of Lisa’s lesson. This is evidence that Lisa needed to consider her connectivity (Rowland et al., 2009) when thinking about the links and sequence within this lesson.

**Foundation knowledge**

For this lesson Lisa asked the students to solve two-digit subtraction problems, which she was also able to solve correctly. The most difficult example was 50 take away 35 during the subtraction ladder task (Figure 6.3). This understanding drew on her common content knowledge (CCK). In other words, Lisa did not need to learn how to complete these problems as part of her lesson preparation. This was also coded as foundation knowledge, as a teacher needs to rely on accurate understanding of mathematical ideas or concepts (Rowland et al., 2009).

However, after the lesson Lisa was concerned that she had difficulties understanding mathematical concepts, commenting.

…my content is just passing. I think there is a long way to go. After today I know now I needed to do this and I needed to do that and then I will go home and then I will read about it or learn in different ways. Until you are thrown in and experience it, I don’t have an incentive to just read numbers… I think I am just over average. (Lisa)

This reflective comment suggests Lisa was unable able to rely on her foundation knowledge during her primary mathematics teaching experiences. Lisa was also aware she should improve her MCK, but did not clearly articulate what she planned to learn or needed to know. However, later Lisa said she had a Year 5 text-book she was going to use for revision, suggesting she lacked the foundation knowledge she would be expected to teach upper primary students.

Booker et al. (2010) suggested that to develop subtraction facts students require strategies such as “count back” or “thinking of addition” as they develop automatic responses. During this lesson, it was not clear how Lisa’s concept of the subtraction ladder assisted students to scaffold their mathematical understanding of subtraction
concepts. Furthermore, she could not rely on her foundation knowledge when choosing effective teaching strategies.

**Connections**

Lisa attempted to make connections during the lesson but the sequence of her lesson lacked structure. She did make reference to a previous lesson, reminding students they had used a number line, but it was a statement rather than drawing on the previous experience as revision for this lesson.

To facilitate student learning, teachers need to develop understanding by making explicit connections between mathematical topics (Ma, 1999). Booker et al. (2004) stated that children will construct meaning when using materials, as they reflect upon and talk about their ideas. For this lesson and as part of her preparation, Lisa needed to unpack her MCK and consider the sequence for developing subtraction ideas, making connections with her CCK of subtraction and the SCK she needed when teaching subtraction concepts with Year 3 students.

A teacher catering for all learners should rely on their foundation knowledge to target questions to assist weaker learners and scaffold the complexity of the problems to challenge the advanced learners. During this lesson it appeared that Lisa made up her questions whilst she taught. To make connections she needed to prepare her questions before the lesson by referring to a sequence for developing the subtraction concepts (for an example, see Booker et al., 2004, p. 226) as well as to curriculum documents (DEECD, 2006).

**Transformation**

Lisa had difficulty demonstrating transformation. For example, she did not use the subtraction ladder consistently and changed the recording structure when using the second subtraction ladder (Figure 6.4). The lesson began with students using the digits in the ladder to record the minuend and difference of the problem (10-?=8). This method was then switched using the digits to record the minuend and subtrahend (50-35=?). Swapping from a change unknown structure to a result unknown (Carpenter, Fennema, Franke, Levi, & Empson, 1999) would have been confusing for the students.
The ladder was not an appropriate representation for modelling subtraction or difference concepts and was not useful for demonstrating procedures or mathematical thinking.

**Contingency**
Rowland et al. (2009) suggested that when dealing with contingencies, asking questions or responding to students’ questions, one needs to “think on one's feet” (p.37). However, during this lesson Lisa chose simple subtraction problems for the students to respond to and mostly used closed question types therefore not providing opportunities for unexpected responses or contingencies. This observation suggests that Lisa had not developed the confidence to extend her questions when teaching and could not rely on her MCK by asking more difficult mathematical questions when teaching the whole class.

Later in the lesson, Lisa had difficulties when responding to a situation that arose during the lesson that was not expected and this was coded a contingency. When a student asked Lisa about whether 40 could be subtracted from 25 her response was “Not really”. Her response was inappropriate, as 25 subtract 40 equals negative 15 and is a true mathematical statement. Maybe her response was affected by the awareness that this subtraction concept would not normally be introduced at this level and therefore would be too difficult to explain. Ma (1999, p. 3) discussed a similar example, stating that young students’ future learning should not be confused by emphasising a misconception. A better response from Lisa would have been to say, “Yes, it is possible and we can work on that later,” then continue with the lesson. Later, Lisa could talk with the student and possibly explore the concept on a number line.

Overall Lisa demonstrated little evidence of foundation knowledge during this lesson. Furthermore, she had difficulties in demonstrating transformation, connections, as well as in responding to a contingency when an unexpected teaching experience occurred related to the topic of subtraction. The findings from this lesson observation suggest that Lisa could not rely on her MCK when teaching this lesson and therefore she was unable to assist these Year 3 students to extend their mathematical understanding.
6.4 Third-year practice teaching experiences

In third year pre-service teachers usually completed 42 days in a secondary school practising teaching in their secondary discipline (listed in Table 6.1). As previously noted, these school visits occurred every Tuesday from March until September (during first and second semester), and were complemented by a one-week practicum experience during May and a two-week placement in August.

Fourteen of the pre-service teachers (i.e., all of the pre-service teachers in Study Groups 2 and 3) practised teaching secondary disciplines other than mathematics, including subjects in social sciences, the arts, health and physical activity (see Table 6.1). During their interviews, these pre-service teachers reported that they did not teach any mathematics as part of their secondary teaching experiences. These results were similar to Emma’s in first year, in that these pre-service teachers did not recognise any opportunities focused on mathematics within other disciplines. Mathew (Study Group 2) provided the only example, mentioning that his students had to make a timeline as part of their history lesson. He commented that he could rely on his MCK when ordering and checking these dates. On the other hand, three pre-service teachers (Study Group 1) had opportunities to extend their MCK when teaching mathematics with Year 7, 8 or 9 students. Those lessons were not observed as part of this study.

Simon (Study Group 1) reflected on two mathematics lessons he had given to Year 8 and Year 9 students during his third-year experiences. He showed Year 8 students a video of a slam-dunk, basketball game. While watching the video, students tallied each team’s score using their own point system, recorded the data into an Excel spreadsheet, then analysed and shared their findings. The students did not use Excel to generate graphs. Similarly, Year 9 students collected cricket statistics, calculating batting averages of runs and wickets. They also entered data into Excel so they could calculate the statistics, make box plots and report their results.

This opportunity to learn extended Simon’s foundation knowledge beyond primary level mathematics when he was planning and teaching statistics. He was also able to make connections by thinking about different activities that would extend knowledge of this topic for both Year 8 and Year 9 students. The Year 9 students used computer
software to calculate and record their results, which demonstrated Simon’s transformation knowledge for representing the mathematics.

An important characteristic of effective mathematics teachers is breadth and depth of knowing school mathematics (Ma, 1999; Schoenfeld & Kilpatrick, 2008). Study Group 1 pre-service teachers’ practice teaching experiences included mathematics teaching in secondary school settings (unlike those of their counterparts in Study Groups 2 and 3), and hence provided further opportunities to extend their depth of knowing mathematics. These experiences extended pre-service teachers’ foundation knowledge, including understanding of mathematical concepts taught during early secondary school education. Additionally, this assisted development of their SCK and horizon content knowledge (HCK) (Ball & Bass, 2009).

6.5 Fourth-year practice teaching experiences

Unlike in first year and second year, the pre-service teachers did not complete a mathematics coursework unit of study in the fourth year of their program at the same time as they were practising their teaching in primary schools. However, they did continue to extend their knowledge of teaching related to professional preparation and readiness to join the teaching profession as part of their coursework experiences.

Throughout fourth year, pre-service teachers had the opportunity to experience practice teaching in different primary schools and usually with a different year level of students when compared to their first and second-year experiences (Table 6.1). This consisted of 50 days of practice teaching, involving most Tuesdays from March until September (during first and second semester) as well as a four-week practicum experience during April and May and a five-week practicum experience in August and September. Most lesson observations during fourth year were completed near the beginning of second semester. The results of the analysis of those lessons are reported in this section.

Table 6.2 provides a summary of the lessons given by the 17 longitudinal study pre-service teachers during fourth year. The table includes the topic they taught for their observation lesson, teaching approach, strengths and weakness of the lesson when
relying on their MCK, and identifies participants who were not yet demonstrating foundation knowledge, foundation knowledge or SCK.

As previously reported, Table 6.1 shows that more of the 17 pre-service teachers experienced teaching in upper primary year levels or middle primary year levels in fourth year than lower year levels. The topics taught in the observed lessons included number and algebra, statistics and probability, measurement and geometry. Similar to first and second-year, pre-service teachers had the opportunity to extend their breadth of MCK of the year level with whom they were practicing their teaching. Breadth of knowledge is described as knowledge of the curriculum (Ma, 1999; Schoenfield & Kilpatrick, 2008), such as the different topics taught to primary students.
<table>
<thead>
<tr>
<th>Study Group</th>
<th>Name</th>
<th>Yr level</th>
<th>Topic</th>
<th>Teaching approach</th>
<th>Strengths</th>
<th>Weaknesses</th>
<th>MCK</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Con</td>
<td>Yr 5</td>
<td>Making and naming equivalent fractions</td>
<td>Concentrated on developing understanding</td>
<td>Planned a selection of tasks using own resources</td>
<td>SCK</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Sean</td>
<td>Yr 5 &amp; 6</td>
<td>Measurement, area of playground equipment</td>
<td>Concentrated on developing understanding</td>
<td>Planned a series of lessons on this topic</td>
<td>SCK</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Shelly</td>
<td>Yr 4</td>
<td>Place value of 4 digit number</td>
<td>Concentrated on developing understanding</td>
<td>Showed understanding of mathematical idea</td>
<td>SCK</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Elizabeth</td>
<td>Year 3 &amp; 4</td>
<td>Location interpret grid maps</td>
<td>Traditional and worksheet</td>
<td>Planned a series of lessons on this topic</td>
<td>Many lessons included worksheets</td>
<td>Foundation knowledge</td>
</tr>
<tr>
<td>2</td>
<td>Esther</td>
<td>Prep</td>
<td>Teen numbers</td>
<td>Provided hands on tasks for developing counting skills</td>
<td>Planned a selection of tasks using own resources</td>
<td>Needed to revise her MCK before teaching Year 6</td>
<td>Foundation knowledge</td>
</tr>
<tr>
<td>2</td>
<td>Fiona</td>
<td>Yr 5</td>
<td>Learning terms used to describe their chance of occurring</td>
<td>Modelling procedure and worksheet</td>
<td>Knowledge of the mathematical language</td>
<td>Did not concentrate on developing understanding</td>
<td>Not yet demonstrating foundation knowledge</td>
</tr>
<tr>
<td>2</td>
<td>Janette</td>
<td>Prep, Yr 1 &amp; 2</td>
<td>Represent data with objects, make picture graphs and interpret them</td>
<td>Strategies that promote understanding</td>
<td>Used materials (blocks) to make bar graphs</td>
<td>Was concerned that she might have difficulties teaching Year 6</td>
<td>Not yet demonstrating foundation knowledge</td>
</tr>
<tr>
<td>2</td>
<td>Kerri</td>
<td>Yr 5 &amp; 6</td>
<td>Discussion methods of solutions for closed problems</td>
<td>Concentrated on developing understanding</td>
<td>Asked appropriate questions and responded to students strategies</td>
<td>SCK</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Mathew</td>
<td>Yr 5</td>
<td>Calculate volume of prisms</td>
<td>Procedural</td>
<td>Relied on rules for calculating the area of cubes and prisms</td>
<td>Did not concentrate on developing understanding</td>
<td>Not yet demonstrating foundation knowledge</td>
</tr>
</tbody>
</table>
Table 6.3 (continued)
Summary of Pre-service Teachers’ (n=17) Observation Lessons and Categories of MCK Demonstrated during Fourth-year

<table>
<thead>
<tr>
<th>Study Group</th>
<th>Name</th>
<th>Yr level</th>
<th>Topic</th>
<th>Teaching approach</th>
<th>Strengths</th>
<th>Weaknesses</th>
<th>MCK</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Michael</td>
<td>Yr 5</td>
<td>Making and represent fractions on a fraction wall</td>
<td>Relied on processes calculate the length of fraction parts</td>
<td>Attempted to use own resources to develop understanding</td>
<td>Needs to extend choice of teaching strategies</td>
<td>Not yet demonstrating foundation knowledge</td>
</tr>
<tr>
<td>2</td>
<td>Peter</td>
<td>Yr 3 &amp; 4</td>
<td>2 digit division</td>
<td>Limited strategies that promote understanding and worksheet</td>
<td>Modelled division of 32÷4 as an array</td>
<td>Needs to extend choice of language</td>
<td>Not yet demonstrating foundation knowledge</td>
</tr>
<tr>
<td>2</td>
<td>Rose</td>
<td>Yr 3 &amp; 4</td>
<td>Geometry and properties of triangles</td>
<td>Choice of resources to develop understanding</td>
<td>Used open-ended questions to discuss differences and counter examples of triangles</td>
<td>When planning a mathematics revises and checks her MCK before teaching</td>
<td>Foundation knowledge</td>
</tr>
<tr>
<td>3</td>
<td>Don</td>
<td>Yr 3</td>
<td>2 digit subtraction</td>
<td>Modelling procedure and worksheet</td>
<td>Relied on procedural knowledge to model examples</td>
<td>Needs to extend choice of language</td>
<td>Not yet demonstrating foundation knowledge</td>
</tr>
<tr>
<td>3</td>
<td>Emma</td>
<td>Year 1 &amp; 2</td>
<td>Fractions naming and making halves, thirds and quarters</td>
<td>Limited strategies that promote understanding and worksheet</td>
<td>Attempted to use of own resources by modelling fractions with fruit</td>
<td>Needs to extend language and carefully consider resources</td>
<td>Not yet demonstrating foundation knowledge</td>
</tr>
<tr>
<td>3</td>
<td>Julie</td>
<td>Year 3 &amp; 4</td>
<td>Location, extending understanding of terminology</td>
<td>Teaching strategies to promote understanding</td>
<td>Transform what she knows selects activities that engage students when learning</td>
<td>Was concerned that she might have difficulties teaching Year 6</td>
<td>Not yet demonstrating foundation knowledge</td>
</tr>
<tr>
<td>3</td>
<td>Lisa</td>
<td>Year 3 &amp; 4</td>
<td>Perimeter and area of regular quadrilaterals</td>
<td>Modelling procedure and worksheet</td>
<td>Relied on procedural knowledge to model examples</td>
<td>Needs to extend language and carefully consider resources</td>
<td>Not yet demonstrating foundation knowledge</td>
</tr>
</tbody>
</table>

10 Emma did not complete a lesson observation
Three pre-service teachers demonstrated foundation knowledge during the fourth-year lesson observation: Elizabeth, Esther (Study Group 2) and Rose (Study Group 3) (see Table 6.2). Some Study Group 2 pre-service teachers were demonstrating foundation knowledge but others were not. While (by definition) Study Group 2 participants had passed the MCSK test on their first attempt, they were not enrolled in mathematics for their secondary specialisation. These results identify that even though pre-service teachers passed the MCSK test, usually in first or second year they were not all able to retain their MCK and demonstrate foundation knowledge when teaching during fourth-year.

The pre-service teachers described how they sometimes taught alongside their mentor teacher and were responsible for planning more lessons when compared to previous years. They were also required to teach for the whole day, and by their final weeks of their teaching experiences were expected to plan and teach their class for the whole week. When compared to first and second-year, about half of the mentor teachers did not expect pre-service teachers to provide lesson plans. These experiences provided further opportunities for pre-service teachers to extend their foundation knowledge, transformation and connections. However, the observations of the longitudinal participants during their fourth year showed that some were not yet demonstrating foundation knowledge; others were demonstrating foundation knowledge, and others still were demonstrating SCK. Chapter 8 will discuss and consider factors that may have contributed to these results (Section 8.2).

### 6.5.1 Not yet demonstrating foundation knowledge

When observed teaching in fourth year, nine pre-service teachers were not yet demonstrating foundation knowledge: Fiona, Janette, Mathew, Michael and Peter (Study Group 2); Don, Jenny, Julie and Lisa (Study Group 3). These findings suggest that Study Group 3 pre-service teachers, who had the most difficulty passing a MCSK test, were more likely to not yet demonstrate foundation knowledge when compared to Study Group 1 and Study Group 2 pre-service teachers in the fourth year of their program.
Table 6.2 shows that pre-service teachers who were not yet demonstrating foundation knowledge relied on their MCK by demonstrating procedural knowledge rather than teaching for understanding. They were more likely to choose closed question types and have difficulty using correct mathematical language when explaining mathematical concepts; they taught facts, rules or procedures, and/or relied on worksheets during their lessons.

Five of the pre-service teachers relied on worksheets as part of their lessons rather than using resources or mathematical equipment to make mathematical knowledge accessible to the students. Four of these five pre-service teachers concentrated on procedures during the lesson introduction, then for the main body of their lesson gave the students a worksheet to complete.

To illustrate, Fiona (Study Group 2) concentrated on facts and asked the students to help write definitions for chance vocabulary (sometimes, always and never) when introducing her lesson. Next she asked the students to copy the definitions into their maths books and provided a commercially produced worksheet that required her Year 5 students to match chance statements (such as “one in six families have white cars”) with a chance vocabulary (sometimes, always or never).

Similarly, Lisa focused on closed question types when introducing her lesson and rules that demonstrated procedural understanding of perimeter and area. Lisa then provided a worksheet she had designed (Appendix J) for the students which required them to write definitions for height, width, area and perimeter.

Likewise, Peter (Study Group 2) and Don (Study Group 3) concentrated on procedures when teaching students how to solve division and subtraction problems. They did not model correct mathematical terms; they recorded examples of the problems and demonstrated a procedural calculation which most likely relied on their procedural knowledge rather than understanding.

To illustrate, Don commenced his lesson with a demonstration of two-digit subtraction examples, first requiring no renaming (e.g. 35-11=), then requiring renaming (23-17=). This suggests he was making connections with the sequence of learning these ideas,
however he did not model the examples with base ten or other materials, which would support the students’ understanding of the concept or demonstrate his knowledge of how to unpack his understanding of subtraction for developing students’ knowledge. Don also neglected to use the terminology, such as “trade” or “rename one ten as ten ones” (Booker et al., 2010). When explaining and renaming 23 take away 17, he said “Cancel that out and that becomes the one and take the ten over here and that becomes the 13 and 13 take-away seven.”

Don’s mentor teacher assisted some of the students with the two-digit problems by providing a bead frame to count back by ones, which is not an efficient subtraction method. Limited data were collected about the mentor teachers, but pre-service teachers’ theories of what they learn at university and the PCK modelled by the mentor teacher in the classroom may differ, and this confusion may retard the development of pre-service teachers’ MCK and PCK.

As noted previously, Don relied on his procedural knowledge when calculating multiplication problems during second-year (Section 6.3); this suggests that he had not extended his foundation knowledge from second to fourth year and was continuing to rely on procedures. If Don continued to rely solely on procedures, his students were likely to develop misconceptions or errors because they would not understand the mathematical connections behind the mathematics (Booker et al., 2010; Kilpatrick et al., 2001).

Like Don, Mathew (Study Group 2) relied on rules during his lesson. He gave Year 5 students a measurement lesson on the volume of a cube. At the beginning of his lesson he asked the students “What is volume?”, then showed a picture of a block (Figure 6.5).

![The block that Mathew used when discussing volume.](image)
One student suggested “How many blocks make up the cube?”

Rather than elaborating on the student’s response, suggesting he was correct by explaining volume as the amount of material such as blocks within the cube, Mathew referred to a mathematics dictionary (De Klerk, 2008) and wrote the following definition: “The amount of space inside a container.”

Mathew made no connections with his MCK and the student’s definition of volume because he was relying on the formula for calculating the volume and had not made conceptual connections, or demonstrated the connected knowledge used when teaching with understanding.

Another student suggested how to work out the volume of the cube:

The side is 12 and there are six sides so six times 12 is 72 and then in the very middle you count the rest. (Student)

Mathew chose not to deal with this contingency. He continued to have difficulty in making connections and transforming his foundation knowledge when responding to students’ comments.

Another pre-service teacher in fourth year also had difficulty relying on her foundation knowledge when teaching lower primary levels. Jenny (Study Group 3) was unable to discuss and model fractions efficiently with students Year 2 students. During her lesson she cut oranges into thirds, which was difficult to model as three equal parts and may have been confusing for the students. Jenny also had difficulty relying on her foundation knowledge and correct mathematical language, because she renamed one whole as four-four rather than four quarters.

Similarly, when teaching upper primary, Michael (Study Group 2) had difficulty demonstrating foundation knowledge when making fractions. He wanted the students to make fraction walls; he gave the students strips of paper and demonstrated how to use a calculator to work out the measurement of the fraction parts. For example, the paper strip was 21 cm long, the students were asked to make fifths, so each part should have measured 4.2 cm. This was difficult for some students to interpret, calculate and measure. A simpler method would have been to use strategies for folding the paper,
such as halving, thirding and fifthing (Siemon et al., 2011), requiring no measuring. This lesson suggests that Michael had difficulty relying on his foundation knowledge when teaching because of his choice of resources and the strategies he chose for teaching fraction concepts.

During their fourth-year interviews the pre-service teachers were asked, “What would you think if you were asked to teach year six mathematics next year?”

Three pre-service teachers, Esther, Janette (Study Group 2) and Julie (Study Group 3), were concerned about their ability to do this, so were coded as not yet demonstrating foundation knowledge. Neither Esther nor Julie (Table 6.1) had the opportunity to teach Year 5 or 6 students during their program; this would have contributed to their belief and this MCK outcome.

Overall these findings indicated that about half of the longitudinal pre-service study teachers were not yet demonstrating foundation knowledge when in fourth year. Chapter 8 will include discussion of the factors that may have contributed to these results.

6.5.2 Foundation knowledge

Foundation knowledge was evident when these pre-service teachers concentrated on developing students’ understanding when teaching. Most had written a lesson plan; therefore they should have revised the mathematics they required for the lesson. Furthermore, participants who had written a lesson plan were more likely to provide their students with hands-on activities that promoted students’ mathematical understanding, choose appropriate mathematical terminology during their lessons, use resources during their lessons rather than worksheets and demonstrate accurate understanding of mathematical ideas.

A combination of teacher identity and range of teaching experiences at different year levels contributed to pre-service teachers’ MCK. Rose (Study Group 2) demonstrated foundation knowledge during fourth-year; the development of her MCK can be seen in a comparison of her second-year and fourth-year lessons. In second year, Rose chose closed questions when working with the whole class and in fourth year her improved
MCK enabled her to be more open in her questioning. By fourth year Rose demonstrated two categories of foundation knowledge – transformation and making connections. She relied on her foundation knowledge when introducing mathematical ideas to students, enhanced students’ mathematical understanding through her choice of examples, and demonstrated transformation and connections with her MCK when questioning the students and considering the sequence of the lesson. In addition, planning with her mentor and her diligent preparation of her lesson plan supported her MCK when teaching. Rose was not coded as having SCK because she needed to diligently research and revise her MCK before teaching, and she may also have had difficulties dealing with contingencies because she needed to revise her MCK before teaching.

Pre-service teachers who chose to use worksheets were usually coded as not yet demonstrating foundation knowledge because they chose not to transform their MCK by making other choices of representations, but this was not always the case. For instance, Elizabeth (Study Group 2) had planned a series of five lessons on location and mapping for Year 3 and 4 students. For her third lesson she provided the students with a commercially produced worksheet, and students were required to use grid references to mark locations on a map.

> It is just taking step by step and scaffolding the tasks so they [students] can build on their skills each time. (Elizabeth)

Elizabeth chose appropriate tasks for scaffolding student understanding. During her interview, she explained that for the next lesson the students would use the knowledge they had gained from this location lesson by designing their own treasure map using grid references. This shows that Elizabeth could rely on her foundation knowledge and was beginning to make connections by demonstrating links from one lesson to the next.

Similarly, Esther (Study Group 2) – although teaching prep students – demonstrated that she could rely on her foundation knowledge when planning and teaching. She was able to rely on her MCK by using appropriate teaching strategies, correct mathematical terminology, appropriate tasks that assisted students to develop efficient counting strategies and understanding of teen numbers. While these pre-service teachers demonstrated foundation knowledge during their fourth-year experiences, they needed
to rely on their MCK when planning and teaching both upper and lower year levels of the primary mathematics curriculum.

The results of the fourth-year interview indicated that pre-service teachers sometimes needed to revise their MCK before teaching. To illustrate, Rose (Study Group 2) taught a geometry lesson with Years 3 and 4. Before the lesson Rose completed a lesson plan that included teacher tips; she had written short definitions of the terms required for the lesson, such as “equilateral, all three sides and angles are equal; reflex angle, 180-360 degrees.” Rose also mentioned that she relied on her mathematics learning log from first-year when planning her lessons and checking her MCK.

Emma (Study Group 2) was unable to be observed during the final year of the program because she had taken family leave; however, she did meet and complete the interview as part of the fourth-year data collection.

If you are going to teach it you have to know it [mathematics]… there is only so much that you can bluff…[for example] you can say [to the students] have a look on a website on the computer and try to work it out. You can’t do that every time [because] you need to know what you are talking about. (Emma)

Emma was concerned that she had forgotten much of what she had learnt during first and second year when revising her mathematical understanding, but realised the importance of knowing MCK for teaching primary school students. Emma suggested that the program should offer some type of refresher program in fourth-year to assist pre-service teachers to revise their mathematics before leaving university. Some of the other pre-service teachers made similar comments.

### 6.5.3 Specialised content knowledge

Four pre-service teachers demonstrated SCK (Hill, et al., 2009) during their fourth-year lesson: Con, Sean, Shelly (Study Group 1) and Kerri (Study Group 2) (see Table 6.2). Thus, a higher proportion of Study Group 1 pre-service teachers demonstrated SCK (Hill, et al., 2009) than Study Group 2 and 3 pre-service teachers. Study Group 1 pre-service teachers had chosen mathematics for their secondary specialisation; they also passed the MCSK test on their first attempt during second-year.
SCK was evident when pre-service teachers demonstrated breadth and depth (Ma, 1999) of MCK, including categories of the Knowledge Quartet (Rowland et al., 2009). Transformation occurred when pre-service teachers chose their materials with care and purpose, such as when Con (Study Group 1) provided Year 5 students with fraction walls so they could make and name equivalent fractions. In addition, these pre-service teachers made connections by making links between mathematical ideas such as place value concepts. SCK enabled participants to respond appropriately to students’ ideas when discussing mathematical concepts by sharing their understanding with students and modelling correct mathematical language.

To illustrate, Shelly (Study Group 1) explained that her class was focusing on place value; during her fourth-year lesson observation she had assisted the students with reading four-digit numbers, and in her interview she explained that she had also helped the students to read decimals.

“Yesterday I helped them to read decimal numbers with tenths, hundredths and thousandths... one student said four point sixty two rather than four and sixty two hundredths.” (Shelly)

When in fourth-year these pre-service teachers understood clearly why and what they were teaching, demonstrating their foundation knowledge or teacher identity as part of their SCK. Identity and emotion are connected with how teachers might imagine or desire to teach mathematics (Hodgen & Askew, 2007).

During Con’s interview, when reflecting on his observed lesson, he described his understanding of teaching primary mathematics as “Maths is in the world, not maths is in your book,” and went on to explain that he tried to integrate real-life examples into the lessons when he designed activities for the students. For example, as part of a lesson he taught in the week prior to the interview, the students were asked to find the volume of their lunch box. Con also spoke about planning a series of lessons for different topics whilst at the school. These examples also provided evidence of Con’s developing teacher identity, as he demonstrated the knowledge he had gained throughout the four years of the program for effective numeracy teaching and planning.

Shelly (Study Group 1) also demonstrated SCK. During her interview she provided breadth of knowledge when using mathematical terms to describe her place value...
lesson. She demonstrated her beliefs, a category of foundation knowledge, teacher identity and developing SCK when considering the strengths and weakness of how mathematics was taught in her school. Shelly explained how her mentor teacher focused more on literacy than mathematics, and that the classroom had few mathematics teaching resources.

Shelly’s mentor teacher provided her with lessons planned by another teacher (numeracy specialist teacher). Shelly was frustrated by this, explaining she often found it difficult to interpret someone else’s ideas when teaching. This example is evidence that a school situation can hinder pre-service teachers’ opportunities to extend their MCK.

Kerri (Study Group 2) taught Years 5 and 6 during her fourth year. The Year 5 students were preparing for a national assessment program numeracy test (Australian Curriculum Assessment and Reporting Authority (ACARA), 2010). The students had completed practice test questions the day before. Kerri demonstrated how she could question the students, asking them to share solutions and strategies for different test questions, including offering different strategies for the same question. Hence, Kerri was able to make connections and rely on her foundation knowledge. In particular, Kerri demonstrated how a pre-service teacher could rely on their foundation knowledge during a lesson, and contingencies by demonstrating the ability to “think on one’s feet” (Rowland et al., 2009, p. 37) when questioning and responding to students’ methods and thinking.

6.6 Conclusion

This chapter reported on the 17 longitudinal study participants opportunities to learn MCK as they undertook practicum experiences. Most of the participants’ school-based practicum experiences during first, second and fourth year were in primary school settings, providing opportunities to observe and teach primary mathematics lessons. During third year only three of the 17 pre-service teachers (Study Group 1) had chosen mathematics as a secondary specialisation. They had additional opportunities to observe and teach junior secondary mathematics lessons and extend their MCK.
During the program pre-service teachers developed their teacher identity, including identity as a primary mathematics teacher. In first-year most of the 17 participants observed primary mathematics lessons during practicum experiences and by fourth year were able to plan and teach a whole class. Practicum experiences provided pre-service teachers with opportunities to make connections with theory learnt during coursework and their practicum experiences. During Unit 1A (first year) and Unit 2A (second year), the praxis inquiry approach assisted pre-service teachers to develop their teacher identity; including identity as a primary mathematics teacher. However, third and fourth-year coursework program structure limited pre-service teachers’ opportunities to reflect and make connections with primary mathematics teacher education and practicum experiences because other aspects of becoming a primary and secondary teacher were being developed.

Primary school practicum experiences assisted pre-service teachers to review their MCK and foundation knowledge for the students they were teaching, especially during second and fourth years when they were provided with opportunities to plan, teach and reflect on the mathematics lessons they taught. However, mentor teachers were likely to focus on lesson structure or behavioural management rather than development of pre-service teachers’ MCK including the categories of foundation knowledge, transformation, making connections and contingency (Rowland et al., 2009). Eight pre-service teachers did not experience a range of teaching experiences or combination of lower, middle and upper primary mathematics teaching during first, second and fourth years, limiting their opportunities to learn MCK for primary teaching.

Comparing the MCK of the 17 pre-service teachers when they were teaching a primary mathematics lesson during the final year of their program produced mixed findings. In summary, fewer Study Group 3 pre-service teachers experienced upper primary year levels during their primary practicum experiences, and fewer were demonstrating foundation knowledge. The nine Study Group 2 pre-service teachers’ fourth-year results were also mixed: one pre-service teacher demonstrated SCK, three demonstrated foundation knowledge and five were not yet demonstrating foundation knowledge. In contrast, most Study Group 1 pre-service teachers demonstrated SCK when observed teaching a primary mathematics lesson during fourth-year. Their additional program
experiences, including coursework in secondary mathematics and practicum teaching in junior secondary classrooms during third year, provided additional opportunities for these pre-service teachers to experience sustained engagement with mathematics. The breadth and depth of Study Group 1 pre-service teachers’ primary and secondary school practicum experiences undoubtedly assisted development of their SCK. SCK was evident because when observed teaching in fourth year they were able to concentrate on developing student understanding, able to rely on their MCK and chose appropriate materials to promote students’ mathematical understanding, thus demonstrating foundation knowledge, transformation and making connections, and had greater ability to respond to contingencies.

The findings presented in this chapter identified practicum experiences or factors that influenced pre-service teachers’ opportunities to learn MCK during their program. The researcher concluded that nine were not yet demonstrating foundation knowledge, three were demonstrating foundation knowledge and four were demonstrating SCK when teaching during fourth year. Thus the program structure, coursework and practicum experiences did not assist all of the longitudinal study pre-service teachers to develop foundation knowledge and or SCK during fourth year.

The next chapter describes the longitudinal study participants’ opportunities to enhance their MCK during coursework and practicum experiences by focusing on their MCK in second year and fourth year.
CHAPTER SEVEN

Differences of MCK in second year and fourth year

First-year and second-year pre-service teachers’ strengths and weakness in Mathematical Content Knowledge (MCK), as indicated by their responses to Mathematical Competency Skills and Knowledge (MCSK) test items, were reported in Chapter 4. In addition, an in-depth analysis revealed pre-service teachers’ methods of solution (Chick et al., 2006a) used in responding to a difficult first-year ratio MCSK test item. This chapter reports on the 17 pre-service teachers participating in the longitudinal study, comparing their strengths and weakness of MCK when responding to MCSK test items in second-year and MCK interview items in fourth-year.

The purpose of this analysis was to identify transition in methods of solution for two different topics to draw inferences about change in pre-service teachers’ MCK. The results are reported as correct and incorrect responses to fraction and decimal items, as well as correct and incorrect responses to area and perimeter items. In particular, this analysis was designed to gain insights into pre-service teachers’ strategies and methods of solution to identify evidence of procedural knowledge or knowledge of mathematical structure and connections (Chick et al., 2006a). These areas of mathematics have previously been identified as troublesome for pre-service teachers and middle year students (e.g. Pearn & Stephens, 2004; Stacey et al., 2001; Steinle & Stacey, 1998; Tatto et al., 2012; Ubuz & Yayan, 2010; Widjaja et al., 2011).

This analysis was possible because the longitudinal study pre-service teachers provided working out in their MCSK test papers. The working out could be analysed, to interpret pre-service teachers’ methods of solution as well as their misconceptions of these topics when in second year. Then, when in fourth year, similar MCK items were used during a one-on-one interview to compare methods of solution and development of MCK. Various categories of MCK frameworks (e.g. Ball et al., 2009; Ma, 1999; Rowland et al., 2009) are used to report pre-service teachers’ MCK.
The final section of this chapter contains a summary of results, including findings of the three study groups of pre-service teachers’ methods of solution for two different topics. As identified in Chapter 5, Study Group 1 pre-service teachers were completing secondary mathematics elective units of study as their secondary major, while Study Groups 2 and 3 were not. Study Group 1 and Study Group 2 pre-service teachers had less difficulty responding to MCSK test items when compared to Study Group 3 pre-service teachers, usually when in second-year. In fourth year all pre-service teachers had completed coursework and teaching practice experiences as part of their program designed to develop their mathematical knowledge for teaching (MKT).

7.1 Fractions and Decimals

The 17 longitudinal study pre-service teachers did not complete the same MCSK test in second year. Nine pre-service teachers completed MCSK Test 2. Item 19 (MCSK Test 2) required pre-service teachers to order a list of fraction and decimal numbers: 0.42, two fifths, \( \frac{4}{9} \) and 0.399 from least to greatest. Figure 7.1 illustrates the second-year Item 19 as it appeared in MCSK Test 2 and Janette’s (Study Group 2) correct response. Item 19 of MCSK Test 2 was coded as a least difficult item because 66% of pre-service teachers (n=47) responded correctly to this item.

![Image of Janette’s response to Item 19](image)

*Figure 7.1 Janette’s response (Study Group 2) to Item 19 Test 2 MCSK Test 2.*

Of the 80 test items used in MCSK tests in second year, 12 items (15%) were coded fraction and decimals (ACARA, 2013), four were coded difficult or most difficult items, and eight items were scored as least difficult (see, Figure 3.1). These results indicate
that the pre-service teachers demonstrated some MCK of fractions and decimals in second year.

All of the 17 pre-service teachers responded to the fourth-year fraction interview item that was used to compare with their results for the fraction and decimal second-year item. Figure 7.2 shows the comparing fractions fourth-year task, which included four items. For the first three items the pre-service teachers were asked to identify which fraction was larger. For Item 4 the pre-service teachers ordered a list of four common fractions and then were asked to place them onto a number line.

Comparing fractions

Looking at these pairs of fractions, which one is larger? Record your thinking.

- **Item 1** Which fraction is larger \( \frac{3}{5} \) or \( \frac{2}{3} \)
- **Item 2** Which fraction is larger \( \frac{3}{5} \) or \( \frac{3}{4} \)
- **Item 3** Which fraction is larger \( \frac{3}{5} \) or \( \frac{5}{8} \)

**Item 4** Record the fractions on the number line \( \frac{3}{5} \) \( \frac{2}{3} \) \( \frac{3}{4} \) \( \frac{5}{8} \)

*Figure 7.2* Fourth-year comparing fractions interview items and number line.

The pre-service teachers were asked to explain their responses or reasoning when responding to this fourth-year task. During the interview and for Item 4, it was expected that the common fractions would be recorded in proportion on the number line at the bottom of the sheet.

Both of the MCK fraction and decimal items reported in this chapter are part of the mathematical curriculum of Year 6 students; they are expected to be able to “compare fractions with related denominators, locate and represent them on a number line” (ACARA, 2013, p. 42). Hence, graduate teachers who are required to demonstrate knowledge of the content they teach would be expected to respond correctly to both
items and draw on their understanding of the connections or equivalence of fractions and decimals or percentages when finding and explaining their solutions.

Of the 80 Number and Algebra MCSK test items (see, Table 3.3), 12 (15%) were fraction and decimals items (ACARA, 2013); four were ranked difficult or most difficult items; and eight items were least difficult. Therefore, during second year pre-service teachers developed some MCK of fractions and decimals.

7.1.1 Comparison of fractions and decimals MCK

Table 7.1 includes the longitudinal study pre-service teachers’ results for the second-year and fourth-year fraction and decimal items. The second-year results include responses from nine longitudinal study pre-service teachers to the second-year ordering fractions and decimals Item 19 MCSK Test 2 (Figure 7.1), shown as ticks (correct) or crosses (incorrect) (see Table 7.1). No tick or cross indicates that these pre-service teachers (n=8) did not complete MCSK Test 2.

Table 7.1 also includes the fourth-year results for the comparable fraction question (Item 4) for all 17 longitudinal study participants, recorded as ticks and crosses. Responses are reported in two columns, the first showing pre-service teachers who correctly ordered the fractions on the number line and the second indicating pre-service teachers who correctly ordered the fractions on the number line in proportion.

During second year, seven (77%) of the nine longitudinal study participants correctly responded to Item 19 MCSK Test 2. This percentage of correct responses identifies this test item as a least difficult item (Figure 3.1).
Table 7.1

Pre-service Teachers’ Responses to Second-year Fraction and Decimal Item 19 MCSK Test 2 (n=9) and Fourth-year Fraction Task (N=17)

<table>
<thead>
<tr>
<th>Study Group</th>
<th>Name</th>
<th>Second-year task</th>
<th>Fourth-year task</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Item 19</td>
<td>Method</td>
<td>Item 4 Correct</td>
</tr>
<tr>
<td>1</td>
<td>Con</td>
<td>✅ Converted to decimal</td>
<td>✅</td>
</tr>
<tr>
<td>1</td>
<td>Sean</td>
<td>✅ Converted to decimal</td>
<td>x</td>
</tr>
<tr>
<td>1</td>
<td>Shelly</td>
<td>✗ Unable to convert 4/9 to decimal; correctly converted others to decimals</td>
<td>✅</td>
</tr>
<tr>
<td>2</td>
<td>Elizabeth</td>
<td>✅ Converted to decimal; used proportional strategy incorrectly</td>
<td>x</td>
</tr>
<tr>
<td>2</td>
<td>Esther*</td>
<td>Converted to decimals</td>
<td>x</td>
</tr>
<tr>
<td>2</td>
<td>Fiona</td>
<td>Drew fractions circles</td>
<td>x</td>
</tr>
<tr>
<td>2</td>
<td>Janette</td>
<td>✅ Converted to decimal</td>
<td>✅</td>
</tr>
<tr>
<td>2</td>
<td>Kerri</td>
<td>✅ Converted to decimal</td>
<td>x</td>
</tr>
<tr>
<td>2</td>
<td>Mathew</td>
<td>Convert to equivalent fractions</td>
<td>✅</td>
</tr>
<tr>
<td>2</td>
<td>Michael</td>
<td>✅ Converted to decimal</td>
<td>✅</td>
</tr>
<tr>
<td>2</td>
<td>Peter</td>
<td>✅ Converted to decimal</td>
<td>✅</td>
</tr>
<tr>
<td>2</td>
<td>Rose</td>
<td>Compared equivalent fractions</td>
<td>x</td>
</tr>
<tr>
<td>3</td>
<td>Don</td>
<td>Drew fraction regions</td>
<td>✅</td>
</tr>
<tr>
<td>3</td>
<td>Emma</td>
<td>Equivalent fractions and percentages</td>
<td>x</td>
</tr>
<tr>
<td>3</td>
<td>Jane</td>
<td>Partitioned number line</td>
<td>✅</td>
</tr>
<tr>
<td>3</td>
<td>Julie</td>
<td>Compared regions</td>
<td>x</td>
</tr>
<tr>
<td>3</td>
<td>Lisa</td>
<td>✗ Converted to hundredths incorrectly</td>
<td>x</td>
</tr>
</tbody>
</table>

During second-year, three of the nine longitudinal study pre-service teachers did not respond correctly to Item 19: Shelly (Study Group 1), Elizabeth (Study Group 2) and Lisa (Study Group 3). Shelly and Elizabeth (Study Group 2) correctly responded to most of the other test items, passing MCSK Test 2 during second year, demonstrating...

*Esther, Fiona, Mathew, Rose, Don, Emma, Jane and Julie did not provide responses for Item 19 MCSK Test 2 because they completed other MCSK tests.
correct methods of solutions. Lisa was unable to correctly answer Item 19 and many other MCK items and did not pass MCSK Test 2 when enrolled in Unit 2B.

During fourth year, five of the pre-service teachers correctly ordered the fractions and placed them in proportion on the number line: Con, Shelly (Study Group 1), Michael, Don, (Study Group 2) and Jane (Study Group 3). Two pre-service teachers – Janette and Peter (Study Group 2) – could order the fractions correctly but were unable to place them in proportion on the number line.

Four of the nine pre-service teachers correctly responded to both the second-year and fourth-year items, though only two of these students ordered them proportionally. Two responded correctly in second year but could not order fractions correctly for the fourth-year task. One pre-service teacher incorrectly responded to the second-year item but correctly responded to the fourth-year items. Two responded incorrectly to both the second year and the fourth-year items.

These results show a range of outcomes. Four pre-service teachers demonstrated no development in their MCK; two pre-service teachers demonstrated some development in their MCK; and two pre-service teachers demonstrated evidence of MCK during second year but had difficulties relying on their MCK during fourth year.

Table 7.2 records fourth-year pre-service teachers’ correct strategies when responding to the comparing fraction Items 1 to 3 (Figure 7.2). The strategies employed included known fact, renaming as equivalent fractions, renaming as decimal and percentages, drawing and shading fraction circles or linear models to compare and identify the largest fraction, and guessing the answer.
Table 7.2
*Fourth-year Pre-service Teachers’ Correct Strategies (N=17) Used to Compare Fraction Items*

<table>
<thead>
<tr>
<th>Study Group</th>
<th>Name</th>
<th>Item 1</th>
<th>Item 2</th>
<th>Item 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Con</td>
<td>Fact</td>
<td>Fact</td>
<td>Decimal and percentages</td>
</tr>
<tr>
<td>1</td>
<td>Sean</td>
<td>Fact</td>
<td>Fact</td>
<td>Equivalent fraction</td>
</tr>
<tr>
<td>1</td>
<td>Shelly</td>
<td>Equivalent fraction</td>
<td>Equivalent fraction</td>
<td>Equivalent fraction</td>
</tr>
<tr>
<td>2</td>
<td>Elizabeth</td>
<td>Linear model</td>
<td>Linear model</td>
<td>Correct Guess</td>
</tr>
<tr>
<td>2</td>
<td>Esther</td>
<td>Linear model and percentage</td>
<td>Linear model and percentage</td>
<td>Linear model and percentage</td>
</tr>
<tr>
<td>2</td>
<td>Fiona</td>
<td>Fraction circle</td>
<td>Fraction circle</td>
<td>Fraction Circle</td>
</tr>
<tr>
<td>2</td>
<td>Janette</td>
<td>Linear model</td>
<td>Number sense</td>
<td>Equivalent fraction</td>
</tr>
<tr>
<td>2</td>
<td>Kerri</td>
<td>Equivalent fraction</td>
<td>Fact</td>
<td>Equivalent fraction</td>
</tr>
<tr>
<td>2</td>
<td>Mathew</td>
<td>Equivalent fraction</td>
<td>Equivalent fraction</td>
<td>Equivalent fraction</td>
</tr>
<tr>
<td>2</td>
<td>Michael</td>
<td>Equivalent fraction</td>
<td>Equivalent fraction</td>
<td>Equivalent fraction</td>
</tr>
<tr>
<td>2</td>
<td>Peter</td>
<td>Fact</td>
<td>Fact</td>
<td>Equivalent fraction</td>
</tr>
<tr>
<td>2</td>
<td>Rose</td>
<td>Linear model</td>
<td>Fact</td>
<td>Linear model</td>
</tr>
<tr>
<td>2</td>
<td>Don</td>
<td>Linear model</td>
<td>Linear model</td>
<td>Linear model</td>
</tr>
<tr>
<td>3</td>
<td>Emma</td>
<td>Linear model</td>
<td>Fact</td>
<td>Percentage</td>
</tr>
<tr>
<td>3</td>
<td>Jenny</td>
<td>Linear model</td>
<td>Fact</td>
<td>Linear model</td>
</tr>
<tr>
<td>3</td>
<td>Julie</td>
<td>Linear model</td>
<td>Fact</td>
<td>Linear model</td>
</tr>
<tr>
<td>3</td>
<td>Lisa</td>
<td>Equivalent fraction</td>
<td>Equivalent fraction</td>
<td>Equivalent fraction</td>
</tr>
</tbody>
</table>

All 17 pre-service teachers correctly compared pairs of fractions in fourth year (Items 1, 2 and 3, in Figure 7.2). The common correct methods of solutions represented across the three study groups were the application of known facts, evidence of conversion to equivalent fractions or drawing fraction circles or linear models. A common method used by nine of the 14 Study Group 2 and Study Group 3 pre-service teachers was to draw a linear model or a fraction circle.
In summary, more of the nine pre-service teachers responded correctly to a fraction and decimal ordering item during second year than fourth year, but more correctly identified the larger fraction in fourth year. A detailed analysis of pre-service teachers’ responses is provided in the next two sections of this chapter.

7.1.2 Second-year fraction and decimal MCK

Strengths in fraction and decimal MCK

During second year, when responding correctly to the fraction and decimal Item 19, pre-service teachers may have relied on their procedural knowledge rather than conceptual understanding. However, it was difficult to identify whether conceptual understanding was used by the pre-service teachers because their responses did not provide evidence of what they may have been thinking or how they might make connections between concepts and topics when recording their responses.

For example, Janette’s (Study Group 2) correct solution (Figure 7.1) suggests that the fractions were converted to decimals for comparing and ordering. This method was common and was used by seven of the nine longitudinal study pre-service teachers during second year (see Table 7.1). This solution is likely to have relied on procedural methods for renaming the fractions as decimals. Then when ordering the decimal fractions, conceptual knowledge may have assisted pre-service teachers when comparing the size of tenths and hundredths when ordering the decimal fractions from least to greatest.

Figure 7.3 is an example of a correct response, which may demonstrate some conceptual understanding of Item 19.

![Figure 7.3](image)

*Figure 7.3 Item 19 MCSK Test 2 – Elizabeth’s (Group 2) correct response.*
In her response, Elizabeth (Study Group 2) recorded the relationship between $\frac{4}{9}$ and a decimal number. She might have been thinking of the equivalence as “just under 0.5” (Figure 7.3). Steinle and Stacey (1998) described this approach as “truncated thinking” and “rounding thinking”, in which the contexts of money or length could have been used to make sense of decimal notation.

Alternatively, Elizabeth (Study Group 2) may have used a proportional reasoning strategy. Her recording suggests that she was trying to make sense of the numbers, so rather than converting $\frac{4}{9}$ as a recurring decimal she estimated or knew that $\frac{4}{9}$ was “just under 0.5.” It appears that she used half as a reference point and knew the other numbers were bigger than just under one half. If Elizabeth was using a proportional reasoning strategy, this could be described as connected knowledge in that she was thinking about these numbers flexibly and using number sense rather than relying on a procedural method. However, Elizabeth’s test response did not demonstrate whether she knew how to convert a common fraction $\frac{4}{9}$ to a decimal fraction 0.444 (recurring) even though the fractions in her list were in correct order. Further understanding would have been learnt by asking Elizabeth to explain her thinking.

*Weakness in fraction and decimal MCK*

Two pre-service teachers, Shelly (Study Group 1) and Lisa (Study Group 3) responded incorrectly to Item 19 (see Figure 7.1). Especially, Study Group 1 pre-service teachers, who were also studying mathematics as a secondary elective, should have the MCK required for responding correctly to this item. Figure 7.4 demonstrates Shelly’s (Study Group 1) incorrect response to Item 19.

![Figure 7.4](image.png)
Shelly (Study Group 1) was unable to convert $\frac{4}{9}$ to a decimal correctly. Her MCSK test paper contains some evidence of working out (it had been erased), suggesting she had some difficulty with this item. Her recording shows $\frac{1}{9} = 0.102$, which is incorrect. As shown in Figure 7.4, if Shelly knew that two fifths was the same as four tenths, she should have known that tenths were smaller than ninths, therefore noticing that two fifths (four tenths) and $\frac{4}{9}$ were in the incorrect order.

For Item 19, Shelly may have drawn on procedural knowledge to convert two fifths to a decimal, used a known fact for naming one third as a decimal, but lacked procedural fluency by not knowing a method for converting or comparing $\frac{4}{9}$ with the other numbers. Demonstrating procedural fluency is knowing procedures to use and performing them flexibly, accurately, and efficiently (National Council of Teachers of Mathematics, 2000).

During the interview conducted with Shelly in second year, she said:

> With decimals and fractions, I am good with a calculator but not with working out things. I need to learn how to do it without the support of the calculator, definitely. (Shelly)

As Shelly stated, she had become reliant on the use of a calculator, forgetting the MCK needed to convert or compare this fraction. This also indicates an absence of conceptual knowledge, understanding fractions as division. Also in the interview she said,

> I know the process but to actually mentally do it in my head [pause] I need to tidy up.

This statement suggests Shelly knows the steps for calculating the answer but may not know how to make connections between the procedures without a calculator. Her reliance on a calculator probably contributed to her lacking the MCK required for Item 19.

Other pre-service teachers demonstrated weaknesses when responding to Item 19. Lisa (Study Group 3) incorrectly converted vulgar fractions to hundredths; for example, she
recorded $\frac{4}{9}$ as $\frac{45}{100}$ and $\frac{1}{3}$ as $\frac{23}{100}$. This error was difficult to diagnose because she did not record her method for renaming these fractions. During her second-year interview, Lisa confirmed that she has weaknesses in her MCK, including fractions:

My weakest [topics] would be my fractions and volume and shapes and angles… It is the operations with fractions, not so much adding and subtracting [because] I am doing that now in grade six. It is just timesing [multiplying] them I forget, do I need a common factor [multiple] or do I not. (Lisa)

During her interview, Lisa’s use of mathematical language was incorrect (timesing [sic] instead of multiplication). In addition, Lisa suggested that during her school experience she would be able to revise or learn some of her MCK for adding and subtracting with fractions. In other words, while teaching this topic or assisting her mentor to teach the topic of fractions, Lisa would also be learning Year 6 mathematics.

The Australian Curriculum: Mathematics (ACM) states that at Year 6 students’ “understanding includes… representing fractions and decimal in various ways and describing connections between them, and making reasonable estimations” (ACARA, 2013, p.40). These results suggest that during second year, pre-service teachers Shelly, Elizabeth and Lisa lacked the MCK necessary when teaching fractions and decimals in primary mathematics.

At the time of their MCSK test in second year, Elizabeth, Shelly and Lisa would have difficulty teaching this level of the mathematics curriculum because they struggled to order fractions and decimals from least to greatest correctly. This understanding concerns numeration and is the basis for being able to perform fraction computations (Booker et al., 2010). Also of concern is that they might have passed on their misconceptions to the students they were teaching as part of their Year 5 and 6 series of fraction lessons (completed as part of their second-year course work assessment).

**Summary of second-year decimal and fraction MCK**
The results suggest most second-year pre-service teachers chose to convert fractions to decimals before ordering and recording their response for Item 19. This method of solution most likely concentrated on procedure.
7.1.3 Fourth-year fraction MCK

Strengths in fraction MCK

Eight of the 17 pre-service teachers reported in Table 7.2 used known facts or a correct method of solution such as converting to equivalent fractions when comparing the common fractions in Figure 7.2. They most likely relied on their foundation knowledge and use of concentration on procedures.

Table 7.2 shows that Janette’s response to Item 2 was coded as number sense because she could justify her response using her MCK. Janette identified the same numerator and different denominators for the two fractions in Item 2 and knew three-fifths was smaller than three-fourths because fifths are smaller parts than fourths, therefore three-fifths was smaller. Siemon et al. (2011) would describe this strategy of comparing and understanding how many (numerator) and how much (denominator) or the number of parts and size of parts of the different fractions.

Teachers use their knowledge of different strategies and demonstrate mathematical structure and connections as well as having understanding of students’ common misconceptions (Booker et al., 2004). By demonstrating this understanding, pre-service teachers work toward demonstrating SCK.

Con (Study Group 1, see Table 7.2), whilst using known facts could also justify his answers therefore he provided the most evidence of working towards SCK. He used known facts to identify the larger fractions for Items 1–3. He explained his correct responses by making connections between fractions and decimals or fractions and percentages. Con’s explanations in fourth year were similar to his written responses for Item 19 (Figure 7.1) during second year when renaming fractions as decimals. When responding to Item 3, he said:

It is close. This [five eighths] has to be more than point six [six tenths] because one eighth is equal to more than ten percent. One eighth has to be bigger than ten percent. Four eighths is 50 percent or half or whatever and this [three fifths] is sixty percent… so 50 plus more than ten percent is equal to 61 point 8 [61.8%]. I think it is point 888 [0.888%] maybe something like that… I just know it is more than ten percent. (Con)

Con’s explanation drew on extended rational number knowledge by partitioning the fraction and breaking the problem down into steps that helped him justify the answer.
and demonstrate making connections. Rowland et al. (2009) would code this understanding as connected knowledge demonstrating evidence of knowing the complexity of an idea, as well as use of strategies and order of an idea. Similarly, Chick et al., (2006a) would code this as evidence of mathematical structure or connections.

Another Study Group 1 pre-service teacher, Shelly, most likely used a known correct method of solution and relied on her MCK when choosing this method. Shelly commenced her interview by drawing a model to compare the two fractions in Item 1. Unsure, or to check, Shelly then decided to use common denominators to compare the fractions and continued to do this for the following two items. She correctly estimated and recorded the fractions on the number line in proportion. Shelly most likely concentrated on using a procedural method and conceptual understanding was not evident. Moreover, these results did not demonstrate whether Shelly could correctly convert a fraction to a decimal, which was a difficulty she had demonstrated during second year (Table 7.1).

All fourth-year methods of solution to Item 3 in Table 7.2 were correct. The most common method used to solve this item was demonstrated by seven pre-service teachers. They drew on a rote procedure, making equivalent fractions to compare \(\frac{3}{5}\) and \(\frac{5}{8}\) as \(\frac{24}{40}\) and \(\frac{25}{40}\). They used step-by-step procedures and thus demonstrated a correct method of solution. Of the three items, these fractions were also the closest in size therefore other or previous methods that the pre-service teachers had used were less reliable – such as drawing and comparing shaded rectangles.

To compare and order fractions students should develop a range of strategies (Petit et al., 2010). Similarly, teachers should know different strategies to use when teaching students, demonstrating SCK. However, only one fourth-year pre-service teacher – Con (Study Group 1) – in this study had the confidence to use knowledge of fractions, decimals and percentage that demonstrated evidence of making connections and extended fraction MCK.
For Item 3 and the number line task (Item 4), five pre-service teachers – Con and Shelly (Study Group 1) Michael (Study Group 2), and Don and Jane (Study Group 3) – correctly recorded their fractions in order and in proportion on the number line. For their strategies they used other numbers such as zero, one half and one as benchmarks. Lamon (2005) identified such knowledge as rational number sense: the pre-service teachers made conceptual connections with the number line and other fractional numbers. This knowledge is also evidence of specialised MCK, because it is MCK that could be used when assisting students to understand the correct sequence and positioning of the common fractions between zero and one.

**Weakness in fraction MCK**

Fourteen of the pre-service teachers commonly drew a linear model or a fraction circle when comparing two fractions (when responding to Items 1, 2 and 3 during fourth year). This method provided pre-service teachers with a correct solution but is only an estimate of the correct response, and if this method was the only method they understood it would be problematic for other examples, such as Item 3, with fractions of very similar size.

For example, Elizabeth (Study Group 2) drew a linear or strip model to estimate and compare the size of the fractions and could easily compare and identify the largest common fractions for Items 1 and 2. For Item 3 the two fractions $\frac{3}{5}$ and $\frac{5}{8}$ were very similar in size, and Elizabeth drew her models twice before recording the correct response. During these attempts, Elizabeth stated that she would have preferred to use a ruler so she could measure and draw the linear examples exactly so as to compare their size: “This is the easiest way of me thinking about this stuff… obviously if I could do it with a ruler it would be a lot more accurate.”

Using a ruler would be an accurate method for comparing the two fractions, but if Elizabeth used this method with primary students they may have had difficulty drawing and measuring and calculating the fractions accurately.

Siemon (2004) would refer to Elizabeth’s method as a rule-based approach, suggesting her understanding is similar to that of middle-year students with fraction difficulties.
who require a “deeper understanding of how fractions are made, named and renamed” (p.3). This may include renaming fractions based on perceptual recognition; is \( \frac{3}{5} \) closer to zero, half or one? (Siemon et al., 2011).

Elizabeth could not elaborate on her method during her interview, suggesting she continued to have gaps in her MCK of this topic. She did not recall the relationship between decimals and percentages that she had tried to use when responding to Item 19 in the second-year test.

Item 4, involving ordering a list of common fractions, was incorrectly answered by nine pre-service teachers. Three other pre-service teachers had difficulty transforming their knowledge (Rowland et al., 2009) using the number line because they were not able to place the fractions in proportion.

Lisa (Study Group 3) was one of the two pre-service teachers who incorrectly answered the fraction Items during second-year and fourth-year (Table 7.1). Similar to second year, Lisa’s MCK and strategies in fourth year continued to demonstrate misconceptions when converting fractions to decimals and estimating where the fractions should be placed on the number line. Lisa also had the most difficulty in second year when ordering fractions and decimals (Table 7.1) taking longer to respond to the interview question. Whilst converting fractions as decimals during the fourth-year interview, she invented methods to produce the following solutions: \( \frac{3}{5} = 1.2, \frac{2}{3} = 1.1, \frac{3}{4} = 1.1, \frac{5}{8} = 1.3 \). During her interview she explained, “I just went two thirds how many times does two go into three, one and one remainder [and incorrectly recorded 1.1].”

This error relates to incorrectly renaming the fraction. Lisa should have also realised her error and known that two thirds and three quarters are all smaller than one whole.

After several attempts at placing the common fractions on the number line Lisa stopped, realising that she was unable to partition the fractions onto the number line correctly, and said, “I can’t do it that is really bad,” suggesting she was aware that she should know this.
Lisa’s decimal misconception was also of concern as she would have difficulty teaching this topic and may pass her misconception onto her students. Lisa lacked foundation knowledge of this topic, including correct mathematical language (Rowland et al., 2009) when explaining her processes. For example, the term “goes into” might confuse students and restrict their conceptual understanding. In a primary classroom the teacher should model mathematical terminology using “rich words that students appropriate as their own, use as tools for their thinking, and use as tools to communicate their thinking” (Khisty & Chval, 2002, p. 154).

Summary of fourth-year fraction MCK
When comparing the nine pre-service teachers’ growth and change in fraction MCK from second to fourth year, Lisa (Study Group 3) did not have sufficient foundation knowledge to respond correctly to the second-year or fourth-year fraction tasks (Table 7.1). Shelly (Study Group 1) had developed her knowledge of fractions and Con (Study Group 1) and Michael (Study Group 2) had maintained their understanding. Sean’s (Study Group 1), Elizabeth’s, Janette’s, and Peter’s (Study Group 2) foundation knowledge of fraction MCK was demonstrated during second year but not in fourth year.

Of the eight other pre-service teachers, who did not complete Item 19, five pre-service teachers lacked the MCK to correctly respond to Item 4 and only one of these pre-service teachers, Jane (Study Group 3), was able to respond correctly. These results suggest that these pre-service teachers, especially Study Group 1 and Study Group 2 pre-service teachers who responded incorrectly to Item 4, had forgotten the MCK they demonstrated during the second year of their program.

When designing the items in Figure 7.2, the researcher expected that some of the fourth-year pre-service teachers would describe their responses by comparing the parts of how many and how much (Siemon et al., 2011) or by comparing fractions and renaming them as decimals and or percentages, as shown in some of the working-out reported for the second-year Item 19 (Section 7.1.3). In contrast, while the responses were mostly correct, many of the strategies did not demonstrate the mathematical understanding that
would be suitable for modelling and comparing different common fractions when teaching.

### 7.2 Area and perimeter

Figure 7.5 illustrates the second-year item Item 48 as it appeared in MCSK Test 2, and is an example of a correct response from Elizabeth (Study Group 2). For this item pre-service teachers had to calculate the surface area of a box with height 50 cm, width 50 cm and length 50 cm. This item was a closed question and correct responses were recorded using squared units of measurement such as 15 000 cm² or 1.5 m².

![Figure 7.5 Item 48 MCSK Test 2 – Elizabeth’s (Group 2) correct response.]

This item was one of 25 measurement items ranked as least difficult (see Chapter 4), because 51% of pre-service teachers (n=47) answered correctly. Item 48 was coded as ”measure” within the cognitive sub-domain of knowing (Tatto et al., 2012). The analysis of second-year test data found that second-year pre-service teachers correctly answered more measure MCSK test items than first-year pre-service teachers. These results suggest that by the second year of their program, pre-service teachers had developed some understanding of using units of measurement.

The fourth-year area and perimeter task included two items (Figure 7.6). The first item assessed whether or not pre-service teachers could correctly explain the difference between these two measurements, while the second item focused on the relationship between area and perimeter. This task was adapted from a similar item in Ma’s (1999) study of Chinese and American teachers and was used to identify teachers’ profound understanding of mathematical knowledge.
Imagine you are teaching area and perimeter. Can you tell me the difference between the two?

Imagine that a student in your class says, “I think if the perimeter of a rectangle increases, its area also increases.” What would be your response?

——

Figure 7.6 Fourth-year measurement interview task, perimeter and area.

The knowledge, assessed in the second-year area items (Item 48) and fourth-year interview perimeter and area items is expected in Year 5 and 6 students. Teachers of Year 5 would use knowledge of perimeter and area to assist students to explore efficient ways of calculating the perimeter and areas of rectangles. At Year 6, this understanding would assist students to “solve problems involving the comparison of lengths and areas using appropriate units” (ACARA, 2013).

### 7.2.1 Comparison of area and perimeter MCK

Table 7.3 includes the second and fourth-year results by study group for the pre-service teachers participating in the longitudinal study. The second-year results include responses from nine longitudinal study participants. This includes correct (tick) or incorrect (cross) responses for the second-year surface area Item 48 MCSK Test 2 (Figure 7.5). A zero identifies pre-service teachers who did not pass MCSK Test 2 (n=8).

Scores are used to report the fourth-year results for two area and perimeter items. In summary, Score 0 and Score 1 represents incorrect and incomplete responses respectively. Score 2 was used to record a procedural method of solution (Chick et al., 2006a) resulting in a correct response. Score 3 indicates a correct response and that pre-service teachers were able to justify their response, suggesting evidence of mathematical structure or connections (Chick et al., 2006a). The method of scoring fourth-year responses is also described in Chapter 3 (see Table 3.10).
During second year, five (55%) of the nine longitudinal study pre-service teachers, Con, Sean, Shelly (Study Group 1) Elizabeth and Michael (Study Group 2), correctly responded to the surface area item. This percentage of correct responses was similar to the percentage of correct responses for all pre-service teachers’ responses (51%) to this item in MCSK Test 2.

Table 7.3
Pre-service Teachers’ Responses (n=9) to Second-year Surface Area Item and Fourth-year Perimeter and Area Items

<table>
<thead>
<tr>
<th>Study Group Name</th>
<th>Second-year Item 48 Answer</th>
<th>Fourth-year tasks</th>
<th>Item 1 Explanation of perimeter</th>
<th>Item 1 Explanation of area</th>
<th>Item 2 Explanation of the relationship between area and perimeter</th>
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<tr>
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<td>3</td>
<td>3</td>
<td>2</td>
<td></td>
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<tr>
<td>Sean</td>
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<td>3</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Shelly</td>
<td>✓</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Elizabeth</td>
<td>✓</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Esther</td>
<td></td>
<td>3</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Fiona</td>
<td></td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Janette</td>
<td>x</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Kerri</td>
<td>x</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Mathew</td>
<td></td>
<td>3</td>
<td>1</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Michael</td>
<td>✓</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td></td>
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<tr>
<td>Peter</td>
<td>x</td>
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<td>1</td>
<td>1</td>
<td></td>
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<td>Rose</td>
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<td>Emma</td>
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<td>0</td>
<td>2</td>
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</tr>
<tr>
<td>Don</td>
<td></td>
<td>0</td>
<td>1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Jenny</td>
<td></td>
<td>1</td>
<td>1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Julie</td>
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<td>2</td>
<td></td>
</tr>
<tr>
<td>Lisa</td>
<td>x</td>
<td>3</td>
<td>2</td>
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<td></td>
</tr>
</tbody>
</table>

During fourth year, pre-service teachers’ explanations of area and perimeter varied. The distribution of scores ranged from 1 to 3. Ten of the pre-service teachers – Con, Sean (Study Group 1) Esther, Elizabeth, Fiona Kerri, Mathew, Michael, (Study Group 2) Julie and Lisa (Study Group 3) – received a Score 3, justifying correctly their responses.

12 Esther, Fiona, Mathew, Michael, Rose, Emma, Don, Jenny and Julie completed a different MCSK test.
when providing an explanation of perimeter; fewer pre-service teachers, Con and Sean (Study Group 1), received a Score 3 when providing an explanation of area. In other words, eight pre-service teachers could explain the term perimeter but could not justify their response with the same depth (Ma, 1990) of MCK when explaining area.

During fourth-year and for Item 2, four pre-service teachers – Sean, Shelly (Study Group 1) Elizabeth and Mathew (Study Group 2) – received a Score 3 when explaining and justifying the relationship between area and perimeter. Five pre-service teachers, Con (Study Group 1) Janette (Study Group 2) Emma, Julie and Lisa (Study Group 3), received a Score 2 demonstrating correct procedural understanding. Eight pre-service teachers could not correctly explain the relationship between area and perimeter and received a Score 1 – Esther, Fiona, Kerri, Michael, Peter, Rose (Study Group 2) – or Score 0 – Don and Jenny (Study Group 3).

A higher proportion of Study Group 1 pre-service teachers demonstrated understanding of the relationship between area and perimeter and scored 3 for Item 2; other Study Group 2 and 3 pre-service teachers were unable to explain the relationship between area and perimeter for Item 2.

Two of nine pre-service teachers correctly responded to both the second-year and fourth-year items, though only one of these pre-service teachers demonstrated conceptual understanding when explaining the relationship between area and perimeter during fourth year. Three other pre-service teachers responded correctly in second year but had difficulties when asked to provide an explanation for area in fourth year. Three pre-service teachers had difficulty with the fourth-year item or demonstrated only procedural understanding. One pre-service teacher responded incorrectly to all of the fourth-year area and perimeter items.

Similar to the previous section for fractions, these results show a range of outcomes. Three pre-service teachers demonstrated no development in their MCK; three pre-service teachers demonstrated possible development in their MCK; and three pre-service teachers during fourth-year had difficulties with MCK that they most likely could demonstrate during second-year.
7.2.2 Second-year area MCK

Strength in MCK of surface area

Five pre-service teachers used similar methods when correctly responding to the surface area item (Item 48, Table 7.3). Con, Sean, Shelly (Study Group 1), Michael and Elizabeth (Study Group 2) may have relied on a formula (without understanding) when calculating the surface area of a box. They demonstrated foundation knowledge for multiplying and adding when calculating their answer. Similar to the fraction and decimal MCK Item 19, it was difficult to identify if the pre-service teachers used conceptual understanding when responding to a written item.

Con’s (Study Group 1) correct method used few calculations when recording the surface area of the box (Figure 7.7). He chose to record the area of each face of the box in squared metres and multiplied by six for the number of faces. He correctly calculated the decimal operation, recording his response in square metres. This was the most efficient method of solution that demonstrated an accurate understanding of mathematical concepts when converting units in an area context.

Before the program Con had completed the highest possible level of mathematics during secondary school (see Table 3.9), and had selected mathematics as his secondary discipline specialisation, and therefore could rely on his MCK to solve this item (Figure 7.7). This also explains why he used less working than shown in other pre-service teachers’ correct responses (e.g. Figure 7.5).

Weaknesses in MCK of surface area

Janette’s (Study Group 2) response (Figure 7.8) demonstrates her correct working by multiplying and calculating the surface area of three faces of the box, doubled and correctly added together three times, and recorded in square centimetres. However,
Janette demonstrated a misconception when incorrectly converting square centimetres to square metres. Janette was unable to rely on her MCK when working between square units. This result was similar to one-third of the pre-service teachers reported previously in Chapter 4 (Section 4.5.3), who were unable to convert centimetres to kilometres when responding to a ratio scale item.

Figure 7.8 Item 48 MCSK Test 2 – Janette’s (Study Group 2) partially correct response.

Of the four incorrect responses for Item 48 among longitudinal participants, Janette was the only pre-service teacher who converted her response incorrectly to squared metres. In contrast, Kerri, Peter (Study Group 2) and Lisa (Study Group 3) all made errors with their calculations of surface area. For example Kerri incorrectly calculated her response as 150,000 cm$^2$ and 1,500 m$^2$. This suggests that pre-service teachers with incorrect responses could use multiplication facts correctly but demonstrated misconceptions because they incorrectly multiplied by multiples of ten. Similarly, primary students who have difficulties become confused with zeros when multiplying by multiples of ten (Booker, 2011). This may have been Kerri’s error, because she recorded $2 \times (50 \times 50)$ incorrectly as 50 000 rather than 5 000 as part of her working out.

Summary of second-year area and perimeter MCK

For this least difficult area item there were different correct methods of solution. Most second-year pre-service teachers chose to record the steps for calculating the surface area of the different sides of the box by either using additive methods to add the area of the six faces or multiplicative methods to multiply the area of one face by six. This mathematical understanding would assist these teachers when breaking down the problem and explaining the sequencing for solving similar items when teaching Year 6 students.
Con (Study Group 1) had the most advanced MCK, as demonstrated in his written response in Figure 7.7. However, Con’s method of solution did not identify if he could break the problem down or had conceptual understanding of the knowledge needed for effectively teaching this mathematical concept to students.

It was difficult to determine whether conceptual understanding was evident in the pre-service teachers’ responses to Item 48 because written tests provide limited data. Ryan and Williams (2007a), in their study of middle-year students, agree that the use of pen and paper tests to assess measuring skills can be impractical because “they lack the tactile and three-dimensional element that brings measurement activity to life” (p. 101).

Therefore the purpose of the fourth-year interview item allowed me to ask questions to clarify pre-service teachers’ methods of solutions and thinking when responding to the perimeter and area items (Table 7.3).

These results suggest that second-year pre-service teachers not completing mathematics as their secondary elective would be more likely to have difficulty modelling their understanding of surface area to Year 6 students because they could not rely on their foundation knowledge to calculate the correct response. Their incorrect methods demonstrated difficulties in choosing appropriate units of measurement and performing the calculation, which is also knowledge expected of Year 6 students (ACARA, 2013) and therefore of beginning teachers.

### 7.2.3 Fourth-year area and perimeter MCK

**Strengths in MCK of area and perimeter**

Most fourth-year pre-service teachers provided a correct mathematical definition for perimeter, but lacked the foundation knowledge and mathematical terminology required for their explanation of area (Table 7.3). Area refers to the two-dimensional space inside a region (Van de Walle et al., 2012), while perimeter is a measure of length involving the distance around a region (Reys, Lindquist, Lamsdin, Smith, & Suydam, 2012).

Fourth-year pre-service teachers’ responses included mathematical language that classified perimeter and area clearly, with an example of an accurate response or Score
3 being from Sean (Study Group 1): “Perimeter is the distance around the outside of the shape... Area is the amount of space contained within the shape.”

Two pre-service teachers, Con and Sean, (Study Group 1) provided complete explanations for perimeter and area during their interviews. Con and Sean’s correct responses suggest they were starting to make connections. They used correct mathematical language when considering a definition for area, which is important when teaching. Yeo (2008) suggested that teachers need to rely on and demonstrate MCK when they link concepts to students’ experiences when teaching perimeter and area.

Alternatively, Laura (Study Group 3) stated a rule for finding the area of a rectangle, which was given a Score 2. Laura was demonstrating procedural knowledge that relied on her foundation knowledge. Laura’s response confirms Yeo’s (2008) contention that beginning teachers can find it difficult to provide conceptual explanations for the procedural tasks they perform associated with area and perimeter when teaching Year 4 students.

The results for Item 2 in Table 7.3 suggest that Study Group 1 pre-service teachers during fourth-year had developed the MCK used to “identify critical mathematical components within a concept that are fundamental for understanding and applying that concept” (Chick et al., 2006b, p. 299). They were able to clearly state and justify their responses using correct mathematical terminology and diagrams to support their explanation of the relationship between area and perimeter (Figure 7.6).

For Item 2, four pre-service teachers, Sean, Shelly (Study Group 1) Mathew and Elizabeth (Study Group 2), explored a range of examples explaining the relationship between area and perimeter. They identified that the student’s statement (Item 2: Figure 7.6) was incorrect (Table 7.3). These pre-service teachers drew on their MCK to make connections by representing (drawing) rectangles with different dimensions to solve this problem correctly. This is evidence of SCK, as they had started to think about more than one solution and made connections between the area and perimeter of different-sized rectangles. They could rely on their MCK to reason through examples by sketching rectangles to test and check theories.
When responding to Item 2 during the interview, Sean remembered completing a similar problem when assisting a student during his school experience teaching in a primary school and Elizabeth remembered a similar example from her second-year units of study in primary mathematics. These responses suggest they had revised this concept during their teacher education program and were able to apply their understanding to this situation during their fourth-year interview.

Shelly correctly identified the misconception in the student’s conjecture and also provided some appropriate suggestions for assisting the student, demonstrating her connections with MCK and the skills needed for teaching. She said,

Tell them to go and test it... What happens if you change the shape of your rectangle? Maybe give them something to make different shaped rectangles. I think maybe keep the area the same and then change the rectangle around. (Shelly)

This response indicated developing SCK through identification of appropriate teaching approaches and evidence of PCK as she “Deconstructs Content to Key components: Identifying critical mathematical components within a concept that are fundamental for understanding and applying the concept” (Chick et al., 2006a, p. 299) as applied to the area and perimeter of rectangles.

A particular strength of these four pre-service teachers’ responses to Item 3 was that they transformed their MCK through explanation or by drawing an illustration (Rowland et al., 2009). They convinced the researcher of their ability to transform their foundation knowledge by demonstrating their MCK and explaining their thinking. These responses could be described as SCK because they demonstrate a range of mathematical knowledge. Teacher knowledge such as SCK is important for identifying a range of solutions and making connections when working with students, planning lessons and evaluating students’ work (Chick et al., 2006b; Schoenfeld & Kilpatrick, 2008).

*Weaknesses in MCK of area and perimeter*

Several pre-service teachers could not explain the difference between area and perimeter and therefore had therefore difficulties in explaining the relation between area and perimeter.
Two fourth-year pre-service teachers scored zero when explaining perimeter because they either had forgotten or confused the definitions of area and perimeter. Five pre-service teachers scored 1 and provided partial explanations of perimeter because they did not state that it was a measure of the total length. For example, Paul (Study Group 2) said “Perimeter is the outside of an object.”

These errors by fourth-year pre-service teachers suggest difficulties in demonstrating knowledge of the correct mathematical terms needed to define these concepts and a weakness of MCK or foundation knowledge (Rowland et al., 2009).

Study Group 2 and 3 pre-service teachers had the most difficulty with Item 3, Emma and Janette (Study Group 2) Julie and Lisa (Study Group 3). They were able to suggest the student’s assumption was incorrect, and drew two rectangles to explore different perimeters and areas. However, they had difficulties explaining and making connections when describing why this was an incorrect statement. Having the skills to reason and discuss mathematics is an important skill for teachers and Reasoning is taught at all levels as part of ACM, including Foundation to Year 10 (ACARA, 2013).

Six pre-service teachers – Kerri, Michael, Peter and Rose (Study Group 2) – demonstrated a misconception as they accepted the student’s hypothesis and could not correctly respond to this item. These pre-service teachers’ responses indicated a lack of MCK for understanding and making connections with a range of rectangles to solve the problem correctly. They tended to draw rectangles and could not rely on their MCK to make connections to correctly justify their responses for this item.

Summary – area and perimeter MCK in fourth year
The 17 pre-service teachers had mixed results with respect to their growth and change in MCK of area and perimeter during fourth year. Con and Sean (Study Group 1) demonstrated foundation knowledge and making connections when explaining their understanding of area and perimeter. Fiona (Study Group 2) Julie and Lisa (Study Group 3) relied on their foundation knowledge and demonstrated procedural understanding. The remaining 12 pre-service teachers gave incorrect or incomplete responses, demonstrating gaps in their foundation knowledge.
Two pre-service teachers, Shelly (Study Group 1) and Mathew (Study Group 2), could not explain area but could justify their explanation when explaining the relationship between area and perimeter. These results suggest that both Shelly and Mathew understood the meaning of area but were unable to rely on their MCK during the interview to draw on the mathematical terminology needed to explain it, so would confuse students when teaching this concept.

Overall, a higher proportion of the pre-service teachers who were completing mathematics as a secondary elective unit of study (Study Group 1) could demonstrate conceptual understanding and make connections (Rowland et al., 2009). They explained their understanding and the relationship when exploring the properties of perimeter and area of different rectangles. Their secondary experiences and horizon content knowledge (HCK) of this topic, as well as other program experiences reported in Chapter 6 and 7, must have assisted them when responding to these items. In contrast, the other pre-service teachers (Study Group 1 and 2) mostly relied on their foundation knowledge (Rowland et al., 2009) for the perimeter and area items; these pre-service teachers could not demonstrate connected knowledge because they had gaps in their MCK and understanding of area and perimeter. This is of concern, as graduate standards require pre-service teachers to communicate clearly and accurately when designing a lesson and teaching (AITSL, 2011).

The results of this section were similar to those of the TEDS-M report of international pre-service teachers’ MCK and PCK (Tatto, et al., 2012), which included a reference to their understandings about area and perimeter. The report indicated that pre-service teachers would be able to solve “routine problems about perimeter”, but would have “difficulty reasoning about multiple statements and relationships among several mathematical concepts…and [difficulty] finding the area of a triangle drawn on a grid” (Tatto et al., 2012, p. 136). It also determined that while the pre-service teachers “were generally able to determine areas and perimeters of simple figures” (Tatto et al., 2012, p. 136), they “were likely to have more difficulty answering problems requiring more complex reasoning in applied or non-routine situations” (Tatto et al., 2012, p. 137).
Pre-service teachers should bring this mathematical understanding or foundation knowledge to their program rather than exhibiting difficulties with it during second-year and the final year of their teacher education program. The results reported herein are especially disturbing because some pre-service teachers did not improve their MCK of area and perimeter during their program and two pre-service teachers’ MCK actually regressed.

### 7.3 MCK from second-year to fourth-year

Table 7.4 provides a summary of the results of the longitudinal study pre-service teachers’ responses to the fraction and decimal items and area and perimeter items when in second and fourth-year. Three categories identify pre-service teachers’ methods of solution: Score 0 represented incorrect responses, Score 1 incomplete responses, Score 2 a procedural method (conceptual understanding need not be evident), and Score 3 evidence of mathematical structure or connections (Chick et al., 2006a). No score was recorded when the item was not completed by a longitudinal study pre-service teacher when in second year.

Six pre-service teachers (including all three Study Group 3 participants) scored 3, showing evidence of mathematical structure or connections when responding to a fourth-year interview item. Shelly (Study Group 3) was the only pre-service teacher who received a Score 3 for both fourth-year items (Items 2 and 4). The conclusion is that the program opportunities to learn MCK assisted pre-service teachers who completed coursework and teaching practice in primary and secondary teacher education because they demonstrated evidence of mathematical structure or connections (Chick et al., 2006a). These pre-service teachers had transformed their MCK and demonstrated specialised MCK (Ball et al., 2009) when responding to one or more MCK items when in fourth-year of their program.
Study Group 3 pre-service teachers were more likely than Study Groups 1 and 2 pre-service teachers to demonstrate difficulties when responding to MCSK test items during first-year or second-year. Although only nine of the 17 pre-service teachers’ responses are listed in Table 7.4 it would be more likely that Study Group 2 pre-service teachers would respond correctly to MCK Item 19 and MCK Item 48 when in second year because they passed their MCSK test during their first attempt. All four Study Group 2 pre-service teachers mentioned in Table 7.4 responded correctly to one or both second-year items and demonstrated procedural understanding. As would be expected, Lisa (Study Group 3) answered the second-year MCK items incorrectly because she had difficulty passing the MCSK test during her program experiences. Only one Study Group 1 pre-service teacher had data for the second-year MCSK test items presented in this chapter therefore few other comparisons can be made.

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**Table 7.4**

*Pre-service Teachers’ Responses (n=9) to Surface Area Item in Second-year and Fourth-year Perimeter and Area Task*

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<thead>
<tr>
<th>Study Group Name</th>
<th>Fractions and decimals</th>
<th>Area and perimeter</th>
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<tr>
<td></td>
<td>Item 19 Yr 2</td>
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¹³ Esther, Fiona, Mathew, Rose, Emma, Don, Jenny and Julie did not complete MCSK Test 2
Most Study Group 2 pre-service teachers’ scores did not change between second year and fourth year, staying at 0, 1 or 2, suggesting that they had forgotten the MCK they had when in second year. Similarly, Study Group 3 pre-service teachers’ results stayed the same or worsened by fourth-year. Study Group 2 and Study Group 3 pre-service teachers had not transformed their MCK and either had difficulties relying on their MCK or demonstrated procedural knowledge without understanding when responding to one or more MCK items in the fourth year of their program.

7.4 Conclusion

In this chapter the researcher reported on the 17 longitudinal study participants’ strengths and weaknesses in MCK when in second year and fourth year, specifically in knowledge of fractions and decimals and area and perimeter MCK items. Pre-service teachers’ responses to MCSK test items in second year and two MCK interview items in fourth year were reported using aspects of Chick et al.’s (2006a) Pedagogical Content Knowledge (PCK) framework.

Not all pre-service teachers completed the same second year items. When comparing responses for nine of the pre-service teachers’ MCK when in second year with the MCK they demonstrated in fourth year, most Study Group 1 pre-service teachers were shown to have extended their MCK. Study Group 2 and Study Group 3 pre-service teachers’ MCK either stayed the same or regressed. Sustained engagement in mathematics experiences during the program would have contributed to these results, because Study Group 1 pre-service teachers had, as reported in Chapter 6, experienced practicum teaching in mathematics for each year of their program, including primary and secondary mathematics teaching while Study Group 2 and 3 had not.

In this chapter the researcher identified the different categories of MCK that pre-service teachers demonstrated in their responses to MCK interview items in fourth-year. Six of the 17 pre-service teachers, including all Study Group 1 pre-service teachers, demonstrated evidence of mathematical structure or connections (Chick et al., 2006a). One pre-service teacher relied on procedural knowledge and provided no evidence of conceptual understanding. The remaining ten pre-service teachers either gave
incomplete responses or incorrect responses; in other words, they could not demonstrate a correct method of solution (Chick et al., 2006a) that they should know for teaching primary mathematics.

As reported in Chapter 6 Rose (Study Group 2) was able to rely on her MCK when teaching because of her developing teacher identity and lesson preparation, including revision of her MCK before teaching. In this chapter, the researcher described how Rose was unable to respond to the fourth-year items as might be expected, having difficulty relying on her MCK. Pre-service teachers, when unprepared, cannot always rely on the MCK they need for primary teaching and will have difficulty responding to students’ unexpected questions or other contingencies when teaching.

The results reported in this chapter show that program opportunities and influences did not support or scaffold all pre-service teachers in their development of MCK during the four years of their program. Pre-service teachers demonstrated a range of MCK in second year, but due to a lack of program opportunities to continue to learn during third and fourth year, progress in development of MCK by fourth year was minimal. However, pre-service teachers who had opportunities to sustain their practice of mathematics teaching for each year of their program and experienced breadth and depth (Ma, 1999) of teaching experiences, including practicum experiences in lower secondary, were more likely to demonstrate connections within a topic when asked to justify their thinking and responses to the MCK interview items.
CHAPTER EIGHT

Extending MCK and contributing factors

The findings reported in the previous four chapters identified differences between the pre-service teachers’ mathematical content knowledge (MCK) during different situations and stages of their program. In this final chapter the researcher conceptualises the findings whilst responding to the research questions, in particular drawing on the findings of the longitudinal study of pre-service teachers during coursework and practicum mathematics teaching experiences.

The analysis of data reported in the previous four chapters revealed factors that influenced pre-service teachers’ MCK during the four years of their program. This chapter begins with a summary of these findings, and is followed by two sections that respond to the major research question:

What MCK do primary pre-service teachers develop during their teacher education and how is this demonstrated and achieved?

This discussion begins by addressing the first subsidiary research question:

What MCK is demonstrated at different stages of their pre-service teacher education?

The levels of MCK demonstrated at different stages of pre-service teacher education included: 1) not yet demonstrating foundation knowledge; 2) demonstrating foundation knowledge; and 3) specialised content knowledge (SCK). Pre-service teachers begin their program with some MCK then learn and demonstrate other categories of MCK during their program. This discussion includes a map of pre-service teachers’ development of MCK, drawing on the theoretical frameworks used throughout this study (e.g., Ball et al., 2008; Chick et al., 2006a; Ma, 1999; Rowland et al., 2009).

Opportunities to learn are critical to the outcomes of teacher education programs (Blömeke & Kaiser, 2014). The remaining subsidiary questions were:

What opportunities and influences enhanced pre-service teachers’ MCK during units in primary mathematics teacher education?

What opportunities and influences enhanced pre-service teachers’ MCK throughout teaching practice in primary and secondary classrooms during the program?
In response to these questions different categories of pre-service teachers’ MCK is discussed, including their identity such as attitudes, beliefs and perceptions (Lerman et al., 2009) before commencing the program, when learning to teach, and as they developed their professional values as teachers of primary mathematics. Discussion also explores the contribution of sustained engagement and opportunities to extending MCK during coursework and practicum experiences. Finally, the qualities of teaching and learning experiences are discussed, focusing on the influences of primary mathematics lecturers and mentor teachers (classroom teacher in school settings), concluding with a reflection on the methodology, study design, limitations, recommendations and implications arising from the study, including suggestions for future research.

8.1 Review of findings

Chapter 4 reported the strengths and weaknesses of first and second-year pre-service teachers based on their responses to Mathematical Competency Skills and Knowledge (MCSK) test items ranging in difficulty from Year 5 to Year 8 Australian Curriculum Mathematics (ACM) (ACARA, 2013). The items were mostly closed question types that relied on procedural knowledge and/or methods of solution (Chick et al., 2006a). When preparing for the MCSK test, pre-service teachers may have revised their knowledge of rules or procedural knowledge without understanding, because the MCSK items did not require evidence of conceptual understanding or mathematical structure and connections (Chick et al., 2006a). Procedural knowledge enables a teacher to provide a method of solution that relies heavily on procedures to develop student understanding rather than concentrating on demonstrating why the procedures work. Knowing more than procedural knowledge is important because teachers also need to know how and why (Even & Torish, 1995) when teaching different mathematical concepts.

In Chapter 5 the 17 longitudinal study participants’ opportunities to develop different categories of MCK during one elective (Unit 1B) and three core coursework units completed during first (Unit 1A) and second years (Units 2A and 2B) was reported. These experiences, including assessment tasks, provided opportunities for pre-service
teachers to extend their knowledge of the mathematics learnt as part of their pre-program identity. These experiences were designed to address the MCK special to teachers, and included coursework opportunities for pre-service teachers to extend foundation knowledge, make connections when planning and sequencing lessons, as well as developing understanding of transformation by selecting appropriate representations for teaching. However, during third and fourth year no core units in primary mathematics teacher education were offered, limiting their opportunities to learn MCK and foster further development of their learner and teacher identity for primary mathematics teaching.

The findings of Chapter 5 also highlighted pre-service teachers’ identities as learners when learning the MCK for teaching whilst considering the importance of the mathematics they needed for teaching. To illustrate, five of the 17 pre-service teachers who participated in the longitudinal study chose to extend their MCK by completing the additional primary mathematics Unit 1B. Four pre-service teachers took the initiative and chose to complete Unit 1B as an elective before completing their core second-year units in primary mathematics because they were aware of their weaknesses in MCK, although one did not pass until fourth year after repeating Unit 1B several times. By choosing to do an extra unit, they were also demonstrating learner identity. Two pre-service teachers (including the pre-service teacher who did not pass in first year) were required to take Unit 1B after they failed the MCSK test during Unit 2B in second year.

Chapter 6 reported on the 17 longitudinal participants’ opportunities to develop different categories of MCK during practicum experiences in primary and secondary classrooms. The findings of the 16 pre-service teachers’ MCK, based on observations of the pre-service teachers, teaching primary mathematics lessons during the final year of their program, were mixed. Some were not yet demonstrating foundation knowledge, demonstrating foundation knowledge or SCK.

Of greatest concern were those fourth year pre-service teachers who relied on procedural knowledge without understanding or had gaps in their MCK and were not yet demonstrating foundation knowledge. Others demonstrated foundation knowledge and some evidence of concentrating on developing understanding when teaching. Other
pre-service teachers provided evidence of breadth and depth (Ma, 1999) of mathematical understanding and demonstrated foundation knowledge as well as transformation and/or making connections and were grouped as demonstrating SCK. These pre-service teachers could make connections within the lesson because of their questioning, planning or resources they chose. They were more likely to assist students and respond to contingencies because of their extended MCK they had developed during the program.

Sustained engagement in each year of their program assisted the pre-service teachers who demonstrated SCK. A range of teaching experiences, including teaching lower, middle and upper year levels, provided differing opportunities to develop breadth and depth of MCK, including teacher identity when planning and revising MCK before teaching. Some mentor teachers assisted pre-service teachers with classroom management rather than MCK, thereby restricting pre-service teachers’ opportunities to further develop their MCK by not allowing reflection on their MCK in action.

Chapter 7 described the comparison of the 17 longitudinal study SCK participants’ strengths and weaknesses in MCK in second year and fourth year, with particular reference to knowledge of fraction and decimals and area and perimeter. The findings were mixed. In fourth year some longitudinal study pre-service teachers demonstrated evidence of mathematical structure or connections; others relied on procedural knowledge, and conceptual understanding was not necessarily evident while others still gave incomplete or incorrect responses. Some fourth-year pre-service teachers could not demonstrate the mathematical connections necessary for teaching these topics to primary students.

Pre-service teachers who had practicum experiences teaching mathematics in secondary schools demonstrated greater breadth and depth of MCK than pre-service teachers who were not completing mathematics as their secondary discipline specialisation. Program opportunities and influences did not support or scaffold all pre-service teachers’ MCK during the four years of their program. However, various factors contributed to primary pre-service teachers’ development and opportunities to learn MCK and are discussed later in this chapter.
The findings identify the MCK pre-service teachers demonstrated during coursework and practicum teaching experiences. The categories of MCK presented in the review of literature (e.g., Chick et al., 2006a; Hill et al., 2008; Ma, 1999; Rowland et al., 2009; Shulman, 1987) assisted with defining the differences in MCK that the longitudinal study pre-service teachers demonstrated and developed during the four years of their teacher education program. The four results chapters presented the analyses of pre-service teachers’ strengths and weaknesses in MCK when responding to MCSK test items, during coursework and practicum experiences; as well as contributing factors that influenced or enhanced pre-service teachers’ opportunities to learn MCK.

The findings of the longitudinal study pre-service teachers varied. Most of the 17 pre-service teachers relied on procedural knowledge during the final year of their program; few had developed breadth and depth of mathematical understanding, such as demonstrating conceptual understanding and making connections. In the following discussion considers the reasons for these findings.

### 8.2 Mapping MCK understanding

The MCK of pre-service teachers varied across the different stages of their teacher education program. Figure 8.1 presents the sequence and stages used to classify pre-service teachers’ development of MCK. This model includes categories of MCK from theoretical frameworks, presented in greater detail within the review of literature (Chick et al., 2006a; Hill et al., 2008; Ma, 1999; Rowland et al., 2009) and includes the different categories of MCK used when analysing data and reporting findings of pre-service teachers in this study.

Five categories are presented in the centre of the model, starting with methods of solution a method used by pre-service teachers when finding a solution to a problem or a basic understanding of the MCK. Next pre-service teachers were more likely to demonstrate procedural knowledge, but conceptual understanding need not be evident. At this stage they would also rely on their common content knowledge (CCK) (as shown on the right hand side of Figure 8.1) including the mathematical knowledge they had before commencing the program. Program experiences then assisted pre-service
teachers to extend their MCK for teaching and they developed evidence of foundation knowledge. Pre-service teachers knew different methods of solutions for approaching a problem, used mathematical terminology to describe their understanding and demonstrated conceptual and procedural understanding. Rowland et al. (2009) identified foundation knowledge as concentrating on developing understanding as well as beliefs about mathematics and PCK that teachers bring to the teaching situation. The pre-service teachers in this study had opportunities to extend their foundation knowledge during coursework and practicum experiences.

Figure 8.1 Pre-service teachers’ stages of development of MCK combining different categories of theoretical frameworks (Ball, et al., 2008; Chick et al., 2006a; Ma, 1999; Rowland, et al., 2009).

As pre-service teachers continued to develop their foundation knowledge they were demonstrating the specialised content knowledge for teaching mathematics. This stage included breadth and depth (Ma, 1999) of MCK. Breadth included knowing how mathematical topics were connected and depth was when pre-service teachers could understand more difficult mathematical concepts. At the same time pre-service teachers start to make connections and transform what they know when teaching, responding to assessment tasks, planning lessons and sequences of lessons. They rely on their
foundation knowledge when responding to students’ questions, demonstrating evidence of contingencies. While demonstrating transformation, connections and contingency, pre-service teachers are also demonstrating breadth and depth of MCK (as shown on the left-hand side) and SCK (shown on the right-hand side).

Development of breadth and depth, transformation, connection and contingencies and SCK assisted pre-service teachers to demonstrate the MCK of an effective beginning teacher, who has the ability to rely on their MCK in a way that will assist students’ mathematical understanding. An effective numeracy teacher will also demonstrate horizon content knowledge (HCK) because they have the peripheral vision and know how to use their MCK when making decisions for teaching. Once they demonstrate the categories of an effective numeracy teacher with HCK and as they continue to develop the breadth, depth and thoroughness (Ma, 1999) of their MCK, they may demonstrate profound understanding of fundamental mathematics (PUFM). The final stage, PUFM, is evidence of all of the categories within Figure 8.1 as well as the teachers’ ability to demonstrate multiple perspectives, demonstrating a complete knowledge of all of the topics within the primary mathematics curriculum as well as the early secondary mathematics curriculum or depth of MCK.

When commencing their program, pre-service teachers bring their pre-program identity (Ponte & Chapman, 2008) as well as knowledge and beliefs from their life experiences (Beswick & Goos, 2012; Rowland et al, 2009). They may demonstrate their mathematical knowledge using methods of solution or procedural knowledge but conceptual understanding need not be evident (Chick et al., 2006b). They have difficulty knowing why, providing meaningful learning (Even & Tirosh, 1995) or demonstrating efficient methods of solution.

During their program experiences, pre-service teachers started to demonstrate different categories of MCK as they extended their knowledge for teaching primary mathematics (Figure 8.1). The longitudinal study found that not all pre-service teachers demonstrated the same categories of MCK. When in fourth year, 16 of the 17 longitudinal study participants were observed teaching a primary mathematics lesson and all of the 17 pre-
service teachers responded to MCK interview items. The pre-service teachers demonstrated three different categories of MCK:

1) Not yet demonstrating foundation knowledge (n=12);
2) Foundation knowledge\textsuperscript{14} (n=2); and
3) SCK (n=3).

8.2.1 Not yet demonstrating foundation knowledge
Pre-service teachers who were not yet demonstrating foundation knowledge were unable to unpack their MCK as needed. They had difficulties relying on their CCK and were likely to demonstrate procedural knowledge without understanding. CCK is important when teaching and used for calculating an answer (Ball et al., 2008), but an effective teacher demonstrates more than CCK and relies on many other important categories of subject matter knowledge (Ball et al., 2008) or foundation knowledge, as well as beliefs about mathematics and pedagogical content knowledge (PCK) (Rowland et al., 2009).

An emphasis on preparing and passing a MCSK test consisting of mainly closed item types is a factor that plausibly explains why some of the participating pre-service teachers had not demonstrated foundation knowledge. Pre-service teachers developed a method of solution for responding correctly to test items. To illustrate, first-year pre-service teachers of the larger cohort experienced least difficulty with MCK items that were closed number and place value items, involving problems such as writing the name of a number in words, solving division and multiplication algorithms, or data representation items such as calculating the mean or median of a set of numbers. They were less likely to extend their MCK and had most difficulty in interpreting worded problems, such as solving a multi-step ratio and proportion item. They needed to develop their understanding of mathematical structure and their capacity to deconstruct content to key components and make connections between mathematical concepts (Ball et al., 2008; Chick et al., 2006a; Ma, 1999; Rowland et al., 2009). Similarly, Even and Tirosh (1995) identified teachers who could quote definitions but had difficulties when responding to a more complicated situation.

\textsuperscript{14} Emma had missing data in fourth year since she was not observed teaching but was coded as having foundation knowledge on the basis of her responses to the fourth-year MCK interview items.
In fourth year, 12 of the longitudinal study participants relied on methods of solution and could give accurate responses but were not yet demonstrating foundation knowledge because of lack of understanding of mathematical ideas or concepts. These pre-service teachers developed procedural knowledge that may be described as instrumental understanding. Skemp (1976) described instrumental understanding as knowing the starting point but only relying on one possible path to get to the answer. Similarly, Even and Tirosh (1995) explained one aspect of procedural knowledge, “knowing that”, as knowing the rules or procedures.

Nine of the 16 longitudinal study pre-service teachers were not yet demonstrating foundation knowledge when observed practising teaching in a primary school in the final year of the program. To illustrate, one pre-service teacher relied on a rule without fully demonstrating why the formula worked when teaching a measurement lesson concerning volume with Year 5 students. Although he knew how to calculate the volume of a cube, he had difficulty describing the rule with understanding. Nor did he choose to model volume with cubes as a conceptual method for demonstrating understanding and why the rule works. Eight other pre-service teachers demonstrated similar weaknesses.

When responding to the fourth-year MCK interview items, 12 of the 17 longitudinal study pre-service teachers were coded as not yet demonstrating foundation knowledge because they responded incorrectly to one or both MCK interview items and/or relied on procedural knowledge without conceptual understanding. Kerri (Study Group 2) had the most variation in her MCK in fourth-year, demonstrating SCK when teaching but responding incorrectly to the area and perimeter MCK interview task. While teaching she was able to lead a student discussion of their methods of solutions to National Assessment Program – Literacy and Numeracy (NAPLAN) example questions but could not demonstrate the same level of understanding during the interview when justifying the difference between area and perimeter. Similarly, Rose (Study Group 2) had demonstrated foundation knowledge when teaching her fourth-year lesson but had difficulty responding to the MCK interview items.
These findings suggest that pre-service teachers such as Kerri and Rose relied on their MCK when teaching because of the preparation they did before the lesson; they checked the MCK they required for teaching but would have had difficulty responding to contingencies and unexpected questions during a lesson. The remaining ten pre-service teachers were of most concern, because they had difficulty demonstrating foundation knowledge when teaching and when responding to one or both of the MCK interview items.

8.2.2 Foundation knowledge

The second level of MCK demonstrated by the longitudinal pre-service teachers was foundation knowledge, one of the four categories of the Knowledge Quartet that identifies categories of MCK, beliefs about mathematics and PCK (Rowland et al., 2009). Rowland et al. (2009) described foundation knowledge as “the kind of knowledge acquired at school, or in teacher education, sometimes before it is put into use in the classroom” (p. 29).

Pre-service teachers who demonstrated foundation knowledge had an accurate understanding of mathematical concepts and were concentrating on developing understanding rather than procedures. At the same time they were beginning to demonstrate SCK and breadth and depth (Ma, 1999) of MCK and connections (Rowland et al., 2009). Different coursework and practicum experiences (and other factors discussed later in this chapter) contributed to pre-service teachers’ development of their foundation knowledge.

Three of the 16 longitudinal study participants were demonstrating foundation knowledge when observed teaching in fourth year. To illustrate, Rose demonstrated categories of foundation knowledge whilst teaching a geometry lesson with Year 3 and Year 4 students. In particular, she could rely on her MCK to prepare a lesson, which included choices of appropriate resources, evidence of correct mathematical language and tasks that developed students’ mathematical understanding of the properties of different triangles. However, Rose (Study Group 2) was coded as not yet demonstrating foundation knowledge because she was unable to respond correctly to the fraction and decimal MCK interview item and demonstrated procedural knowledge when responding
to the area and perimeter interview item. In contrast, Emma (Study Group 3) did not complete a lesson observation in fourth year but could demonstrate conceptual understanding with the fraction and decimal interview item and procedural knowledge with the area and perimeter interview item and was coded as demonstrating foundation knowledge. Therefore, three of the 17 longitudinal pre-service teachers were demonstrating foundation knowledge during fourth year.

**8.2.3 Specialised Content Knowledge**

The highest level of MCK among the longitudinal pre-service teachers was SCK, one of the three categories of Ball et al.’s (2008) subject matter knowledge framework. SCK developed as pre-service teachers extended their knowledge of the four quadrants of the Knowledge Quartet. Two pre-service teachers relied on their foundation knowledge (as explained in the previous section) but also demonstrated transformation, made connections and/or responded to contingencies and demonstrated SCK.

To illustrate, transformation was evident when these pre-service teachers chose an appropriate form of representation during their lesson or could rely on their MCK to carefully explain a mathematical concept. These pre-service teachers demonstrated making connections when planning and teaching a lesson or series of lessons. They could make links from one lesson to the next and also demonstrated appropriate conceptual connections. In particular, both pre-service teachers who demonstrated SCK could rely on their MCK when thinking about the material they used for modelling mathematical concepts and when posing questions during a teaching situation that promoted students’ mathematical understanding. Few examples of contingencies occurred while pre-service teachers were observed teaching.

“Breadth of understanding is the capacity to connect a topic with topics of similar or less conceptual power…” (Ma, 1999, p. 124). In this study, breadth of knowledge included understanding fractions in multiple ways such as recording a fraction on a number line, ordering a list of fractions from least to greatest, or recording a fraction as a percentage or decimal fraction. Being able to order fractions and “to partition (mentally and physically) provides a firm foundation for connecting fractions representations to multiplication and division” (Siemon et al., 2011, p. 434)
demonstrates further breadth of understanding this topic. Differing from breadth, Ma (1999) also described “depth of understanding as the capacity to connect a topic with those of greater conceptual power” (p. 124).

Ma (1999) described PUFM as understanding mathematics with depth, breadth and thoroughness. Teachers who demonstrate PUFM will assist their students to learn a body of mathematical knowledge rather than isolated topics. The pre-service teachers in the longitudinal study did not demonstrate PUFM, because they needed to extend their foundation knowledge before developing breadth, depth, transformation, connection, contingency and SCK. It is unrealistic to expect these pre-service teachers to demonstrate PUFM, as their program did not provide sustained opportunities to learn MCK each year and PUFM may be acquired after further experiences gained during their teaching experiences after graduation.

When observing the longitudinal study participants teaching a primary mathematics lesson during their fourth year, four of them demonstrated SCK. They provided evidence of their breadth and depth of MCK and readiness to teach primary mathematics. However, only three pre-service teachers were coded SCK after comparing their fourth-year lesson and responses to the interview tasks. Kerri (Study Group 2) demonstrated SCK when teaching but did not demonstrate conceptual understanding when responding to the MCK interview items. The other three pre-service teachers (Study Group 1) demonstrated procedural knowledge with understanding or depth of understanding when responding to the fourth-year fraction ordering interview item. When ordering fractions in proportion on a number line, they knew that but also understood why it (Even & Tirosh, 1995) and did not rely on rule-based knowledge but depth (Ma, 1999) of mathematical understanding.

Study Group 1 pre-service teachers had an opportunity to practise their mathematics teaching in every year of their program with primary or secondary students, a factor that contributed to the differences in their MCK when in fourth year and when compared with the other 14 longitudinal study participants (Study Groups 2 and 3). Study Group 1 pre-service teachers were more likely to know that knowledge of rules rather than why, when observed teaching or responding to MCK items when in fourth-year. This also
implies sustained opportunities to learn MCK during the four years of the program, including primary and secondary practicum experiences, assisted these pre-service teachers to develop SCK and effective mathematics teaching skills.

8.3 Opportunities to learn

Tatto, Lerman, and Novotná (2009) claimed pre-service teachers’ opportunities to learn during teacher education programs can be shaped by the programs’ structure and approaches. The longitudinal study of pre-service teachers highlighted several factors that promoted the growth of their MCK during coursework and practicum experiences as well as in how they approached their learning. Pre-service teachers’ identity, including their pre-program, learner and teacher identity, influenced their opportunities to learn MCK. Other factors were sustained engagement of primary mathematics experiences, quality of teaching and learning experiences, including primary mathematics lecturers, especially for the five pre-service teachers who needed to consolidate their MCK by completing the additional Unit1B and primary mathematics teachers who were the participants’ mentor teachers.

8.3.1 Identity

Identity and emotion are connected with how a teacher imagines teaching or desires to teach mathematics (Hodgen & Askew, 2007) and are influenced by prior knowledge, attitudes, beliefs and teachers’ perceptions of teaching mathematics as well as teacher education experiences (Lerman et al., 2009; Walshaw, 2008). Walshaw (2008) suggests that pre-service teachers enter the program bringing ideas or assumptions of what teaching should be and these ideas can compete with theories presented during coursework and practicum experiences. Ambrose (2004) contends that pre-service teachers often underestimate the importance of MCK because of their beliefs. Conversely, Morris (2001) suggested that some pre-service teachers near the beginning of a one year post-graduate certificate ignored the need to improve their MCK, and it was difficult to address weaknesses later in the program as they were already in school placements.
A role of teacher education is to assist pre-service teachers to shape their beliefs (Anthony et al., 2012). When commencing this study, it was not expected that the findings would include beliefs or the impact of identity as factors that influenced the development of pre-service teachers’ MCK. As a consequence, data specifically about beliefs, pre-program identity, learner identity and teacher identity were not collected; the findings presented here are inferred from other data.

Pre-program identity

The pre-service teachers in the longitudinal study commenced their teacher education with different qualities that shaped their pre-program identity and MCK. Most longitudinal study pre-service teachers began teacher education after completing Year 12. Two pre-service teachers were mature age, with other life experiences that they brought to their program; nine pre-service teachers indicated they wanted to teach primary students rather than secondary students when they graduated; four brought negative mathematical experiences or attitudes to the program when reflecting on their own experiences as learners of mathematics; three pre-service teachers completing mathematics as their secondary discipline specialisation reported that they enjoyed the challenge of mathematics when they were at school.

Three pre-service teachers in the longitudinal study had completed Year 12 but did not study mathematics during Year 11 and Year 12. This number of pre-service teachers was too small to determine whether level of secondary school mathematics study made a difference. It is worth noting that since the commencement and enrolment of pre-service teachers in this study, the Victorian Institute of Teaching (2007) has raised the requirements for entry into teacher education, including and mandating satisfactory completion of Year 11 mathematics. Similar standards are expected in other countries; most of the countries in the TEDS-M study expected primary pre-service teachers to demonstrate upper secondary level mathematics as part of their teacher education program entry requirements (Tatto et al., 2012).

Other studies show that level of secondary school is a factor contributing to the strengths and weaknesses of pre-service teachers’ MCK. A recent Australian study of pre-service teachers’ MCK across seven universities, not including the university where
this study took place, concluded that strong primary pre-service teachers’ backgrounds, including more mathematical secondary education, were associated with more MCK (Beswick & Goos, 2012). TEDS-M findings, across 17 countries, confirmed that high school achievement correlates with MCK (Blömeke, Suhl, Kaiser, & Döhrmann, 2014).

Ponte and Chapman (2008) agreed that multiple influences form a teachers’ identity, including the pre-program identity developed before commencing teacher education as part of their primary and secondary school mathematical experiences. Other studies have also shown that teachers’ beliefs about mathematics usually develop during their schooling (e.g., Beswick, 2012; Handal, 2003). Ambrose (2004) suggested that rather than tearing down pre-service teachers’ existing beliefs we should build on what they bring from their school experiences.

Chapter 4 reported on first and second-year pre-service teachers’ strengths and weaknesses in MCK when responding to MCSK test items. Pre-program identity was a factor contributing to the mathematical understanding of the first-year pre-service teachers, because they had only completed one semester of teacher education but many years of mathematics learning as a student when at primary and secondary school. Differences in MCK were caused by the way mathematics was taught at the primary and secondary schools that these pre-service teachers attended.

The second-year pre-service teachers had less difficulty responding to MCSK test items than first-year pre-service teachers. More than half of first-year pre-service teachers had difficulty correctly responding to the MCSK test items and one third of second-year pre-service teachers had difficulty responding to similar MCSK test items. These findings suggest that – as should be expected – after completing coursework in primary teacher education and practicum teaching in primary schools, second-year pre-service teachers had extended their MCK since commencing the program.

Other researchers have identified diversity in the knowledge that pre-service teachers demonstrate. A recent review of literature, concerning the recruitment of prospective teachers of mathematics confirmed that the quality of pre-service teachers’ MCK is an ongoing debate (Anthony et al., 2012). Although pre-service teachers want to become teachers, we cannot always assume they commence their program demonstrating the
different categories of foundation knowledge needed by pre-service teachers beginning education studies. The findings of this study and other studies have identified pre-service teachers’ weaknesses, such as misconceptions (e.g. Ryan & McCrae, 2005/2006) or reliance on procedural knowledge (e.g. Ball, 1990; Forrester & Chinappan, 2010; Goos et al., 2008; Tobias & Itter, 2007). Many of these weaknesses in MCK are influenced by pre-program identity.

Improving how and what mathematics primary and secondary students learn will increase the MCK pre-service teachers bring to their teacher education. Selecting pre-service teachers who demonstrate foundation knowledge before commencing their program should improve the level of knowledge they demonstrate by the end of their program, because they can focus on developing SCK rather than foundation knowledge or relearning the knowledge they should bring to the program as part of their pre-program identity.

**Learner identity**

Learner identity was inferred from the way in which pre-service teachers might extend their MCK during opportunities to learn. Learner identity assisted pre-service teachers to enhance their MCK and was influenced by their program choices, their identity as learners of mathematics and self-efficacy. Identity is connected with beliefs such as “mathematics is difficult” or “mathematics is all about one answer” (Liljedahl, 2005) and the affective domain; self-efficacy relates to personal beliefs and can influence the choices individuals make and how they engage in tasks in which they feel competent and confident, the length of time they persist on a task, or they may avoid tasks if they are less confident (Pajares, 1996). Some of these factors contributed to how the longitudinal study participants developed their MCK during their program experiences.

The pre-service teachers relied on their learner identity when completing core primary mathematics education units including Unit 1A during first year and Units 2A and 2B during second year. Unit 1B was designed for those pre-service teachers who required extra help developing and demonstrating their MCK. Pre-service teachers also had the opportunity to learn MCK during practicum experiences in primary schools during the first, second and fourth years of their program. Some pre-service teachers also chose
their own methods for revising their MCK such as tutoring students in mathematics, using the internet or reading books. Others chose Unit 1B as an elective primary mathematics unit that assisted further understanding of MCK and knowledge needed for teaching primary mathematics.

Although the pre-service teachers commenced the program with different prior learning or pre-program identity, they were expected to demonstrate knowledge of mathematics up to Year 8 level (ACARA, 2013) and pass a MCSK test as part of the program requirement for Unit 2B. The flow chart in Figure 8.2 identifies the different paths pre-service teachers completed before passing the MCSK Test.

![Flow chart showing the different paths pre-service teachers completed before passing the MCSK Test.]

*Figure 8.2 Pre-service teachers’ learner identity and actions when developing MCK*
Figure 8.2 shows that the longitudinal study, participants brought pre-program identity to their program. They completed Unit 1A in first-year and identified their strengths and weaknesses in MCK by completing a practice MCSK test. At the next stage pre-service teachers chose whether to seek help by enrolling in Unit 1B (elective unit) or by completing self revision. In this study five of the 17 longitudinal study pre-service teachers chose to enrol in Unit 1B. In second year, all of the 17 longitudinal study pre-service teachers completed Unit 2B. Of the 17 longitudinal study pre-service teachers, three pre-service teachers passed the MCSK test during Unit 1B and 12 passed the MCSK test during Unit 2B. Therefore, two pre-service teachers were required to complete Unit 1B as a compulsory unit, providing further development of their MCK needed for passing the MCSK test. Pre-service teachers continued to enrol in Unit 1B until they passed the MCSK test. One longitudinal pre-service teacher passed the MCSK test at the beginning of third year, the other passed the MCSK test at the end of fourth-year. All 17 pre-service teachers in the study passed the MCSK test and therefore passed Unit 2B.

At different stages of their coursework the longitudinal study participants demonstrated competency in mathematics as required by passing the MCSK test. They were also demonstrating their learner identity when revising or developing their MCK. Three pre-service teachers who were completing mathematics as their secondary specialisation sought their own methods of revision, such as tutoring Year 7 students or studying at home in their own time, and passed the MCSK test during Unit 2B at their first attempt. A further 11 pre-service teachers not completing secondary mathematics as their elective unit were also able to pass the test at their first attempt during Unit 1B before completing Unit 2B or during Unit 2B, suggesting they had the self-efficacy or MCK as part of their pre-program identity required for passing the MCSK test.

One pre-service teacher passed the MCSK test during Unit 2B at his second attempt; he required extra time to consolidate his understanding but could also rely on his learner identity to study and pass the MCSK test. Two other longitudinal study pre-service teachers had more difficulty in passing the MCSK test and continued to repeat Unit 1B until they successfully demonstrated competency. These two longitudinal study pre-service teachers who had the most difficulty developing their MCK, and who
represented nearly half of the larger second-year cohort of pre-service teachers reported in Chapter 4, had the most difficulty on the 17 participants in developing and demonstrating their learner identity during their teacher education program experiences.

In addition to the coursework experiences, practicum teaching opportunities assisted pre-service teachers to develop their learner identity. To illustrate, one longitudinal study pre-service teacher explained how his mentor teacher had helped him to revise his MCK in second year, and how learning the mathematics for teaching Year 6 students had enabled him to extend his MCK. As a result he passed the MCSK test during Unit 2B without needing to complete Unit 1B.

Different factors contributed to why some pre-service teachers took longer to pass the MCSK test than others. In summary, knowing more secondary mathematics, self-efficacy, persistence and/or choosing to seek assistance in learning MCK by enrolling in Unit 1B before second year were factors that assisted the longitudinal study pre-service teachers to pass the MCSK test on their first attempt.

By encouraging learner identity and opportunities to address weakness in MCK early in the program, pre-service teachers’ maximum opportunities to learn SCK. On the other hand, if pre-service teachers spend the whole program developing their foundation knowledge, there is no time to extend MCK for teaching. This agrees with Morris’s (2001) assertion that pre-service teachers should address their weaknesses in MCK early in the program.

These findings might have been improved with further data on pre-service teachers’ beliefs. Additional questioning of the longitudinal study pre-service teachers could have provided other evidence of their mathematical beliefs. Grootenboer (2008), for example, in his study of pre-service teachers’ mathematical beliefs, identified three categories of teachers with respect to belief change: those that were non-engaged, those developing a new set of mathematical beliefs, and those that changed their beliefs during their program. Similarly, other studies have considered beliefs as important and a key influence on teachers’ MCK (e.g., Beswick, 2012; Fennema & Franke, 1992; Tattoo et al., 2009).
Some of the longitudinal participants had difficulty maintaining their learning identity. They considered knowing mathematics as important only for passing and responding to MCSK test items rather than continuing to extend their MCK because they needed it for teaching. As reported in Chapters 6 and 7, some of the longitudinal study participants were coded as not yet demonstrating foundation knowledge in fourth year. If all pre-service teachers identified themselves as learners, and extended their MCK throughout their program experiences and valued the importance of doing so, these findings may have been different. More pre-service teachers may have demonstrated SCK and extended their foundation knowledge rather than forgetting the knowledge they were expected to know and demonstrate when responding to the MCSK test.

Teacher identity
Pre-service teachers’ learner identity was influential in forming their teacher identity and readiness to teach primary mathematics. Pre-service teachers who demonstrated SCK in fourth year also valued the importance of knowing mathematics because of their teacher identity. Teacher identity is “assuming the values and norms of the profession” (Ponte & Chapman, 2008, p. 241) and is developed throughout pre-service teacher education. Walshaw (2008) suggested pre-service teachers draw on three sites when constructing their teacher identity: their prior knowledge of teachers and school experience; personal experiences during the teacher education program; and practicum teaching.

During their program, pre-service teachers in the longitudinal study relied on their pre-program and learner identities. When completing their praxis inquiry core units of study (which included a mathematics unit, Unit 2A), they were encouraged to reflect on their practice teaching and coursework experiences as beginning teachers. Unit 1A and Unit 2B assisted pre-service teachers to reflect on their understanding of mathematics and approaches to teaching in primary school, extending their teacher identity during tutorials and when responding to assessment tasks.

Practicum experiences complemented coursework in developing pre-service teachers’ existing beliefs, including knowledge of primary mathematics teaching. Similarly, Ponte and Chapman (2008) described a teachers’ professional community, knowledge
of and beliefs about mathematics and knowledge of mathematics teaching as overlapping to form a teacher’s professional identity. Program opportunities at the university where this study took place have been described as providing a community of inquiry bringing together teachers, pre-service teachers and teacher educators (Arnold et al., 2011). McDonough and Sexton (2011) gave a similar opinion – that partnerships involving pre-service teachers, university lecturers and teacher educators learning together in primary mathematics classrooms assist with building effective mathematics teaching.

Walshaw (2008) explained that pre-service teachers will construct their own identities as they consider different contexts, points of view and choices during program experiences. During the interviews with fourth year longitudinal study participants they reported on their different school settings during their practicum experiences and the units in primary mathematics teacher education they had completed. They provided evidence of their developing teacher identity, describing how they planned with other teachers and taught their mentor teacher’s class and were responsible for teaching lessons for the whole day or consecutive days. They had developed a bank of resources that assisted them when planning, developed understanding of teaching, and could make choices because of their increased confidence as a teacher or in their teacher identity.

Pre-service teachers relied on their MCK to make judgements as they extended their teacher identity during program experiences. To illustrate, a fourth-year pre-service teacher taught a two-digit subtraction lesson to Year 3 students. During the lesson his mentor teacher used a bead frame to assist students when subtracting two digit numbers. This was not an efficient strategy as it relied on counting back by ones and conflicted with the theory the pre-service teacher had learnt during coursework. The pre-service teachers had learnt that students develop understanding of the renaming process by using bundling sticks or other base ten materials because the renaming process can be seen explicitly (Booker et al., 2010). Similarly, Cochran-Smith et al. (2005) reported that across settings, such as coursework and when in school settings, pre-service teachers can receive conflicting messages from the lecturers and mentor teachers; they find it difficult when their beliefs conflict, and therefore be resistant to change.
The pre-service teachers in this study completed more days in school settings than their counterparts in other countries (Tatto et al., 2012) and at other Australian universities (Victorian Institute of Teaching, 2007), providing more practical opportunities to extend their teacher identity. They completed 60 days more than the minimum requirement of 80 days. However, the findings suggested that further review of pre-service teachers’ opportunities to learn MCK during school visits and teaching practice is needed. Lack of sustained engagement and low quality of learning experiences, discussed later explain why pre-service teachers were still not yet demonstrating foundation knowledge or demonstrating foundation knowledge rather than SCK in their fourth year.

In Australia in 2007, few undergraduate programs offered a combined program experience of primary and secondary teacher training, most focusing on primary teacher education or secondary teacher education training (Parliament of Victoria Education and Training Committee, 2005). Fourteen of the 17 longitudinal study pre-service teachers in this study chose this teacher education program because it provided them with the qualifications to teach in primary schools as well as a subject other than mathematics in secondary schools. For some this was problematic because even though they had enrolled in this program they did not want to teach in primary schools; therefore, they may have had limited teacher identity with respect to primary mathematics teaching during their program experiences. One of the pre-service teachers, for example, had enrolled in the program because her choice was to become a secondary outdoor education teacher and this program was offered close to where she lived. This factor possibly contributed to her difficulty in demonstrating her MCK during the program, because she did not imagine herself becoming a primary school teacher, although this program provided her with the qualifications to teach in a primary school.

8.3.2 Sustained engagement

Sustained engagement throughout the program fostering continued development of pre-service teachers’ MCK was important, especially because many pre-service teachers had difficulty demonstrating MCK during the first and second years of their program. However the program structure did not ensure that all pre-service teachers were
provided with sustained engagement and opportunities that continued to extend their MCK. Required coursework experiences related to primary mathematics education only occurred during first and second year and practice teaching in primary schools usually occurred during first, second and fourth year. These experiences provided opportunities to develop or extend foundation knowledge and to also transform what they knew and to make connections within and between lessons (Rowland et al., 2009). However, because most per-service teachers had no opportunity to continue to extend their knowledge of primary mathematics teaching during third-year program experiences, this disrupted learning and limited opportunities to extend their foundation knowledge, including developing and extending their SCK.

A factor that helped three pre-service teachers who had chosen mathematics as their secondary discipline elective to demonstrate SCK during their fourth-year observed lesson was that they experienced sustained engagement, practising mathematics teaching for each year of their program. In other words, these pre-service teachers also practised teaching mathematics during third year, usually with Year 7, 8 or 9 students.

The Prep-Year 12 program structure restricted 14 pre-service teachers’ opportunities to develop SCK because engaging with mathematics was not sustained for each year of the program. Consequently, during their fourth-year interview, some pre-service teachers communicated their concern that they had forgotten much of the MCK they had developed earlier in their program. Also when practising their teaching during fourth year, these pre-service teachers were less likely to demonstrate foundation knowledge and relied on rules or procedural knowledge rather than using their SCK as needed for teaching.

Providing all pre-service teachers a range of practicum experiences, including teaching different year levels, would have assisted them to extend their MCK by making connections when planning lessons. However, the placement of longitudinal study participants during their practicum experiences and opportunities to develop depth (Ma, 1999) of MCK within lower, middle and upper primary levels varied. For example five pre-service teachers did not practice teaching in the upper primary year levels during first, second or fourth year. On the other hand, two pre-service teachers experienced
practice teaching with Year 5 and Year 6 during first, second and fourth year, but not with earlier year levels.

Sustained engagement across different year levels would have provided opportunities to develop breadth and depth of MCK. Pre-service teachers could make connections between the topics they taught, when relying on their MCK for different levels of the primary mathematics curriculum. Practicum experiences were opportunities to develop SCK when practicing their teaching with Prep to Year 2 by assisting students to lay the foundations of mathematical understanding, identifying simple strategies to investigate solutions and strengthen their reasoning by solving personally meaningful problems; during Years 3 to Year 6 they would be expected to assist students to construct key mathematical ideas using models and pictures and introduce them to topics such as fractions and decimals (ACARA, 2013). These experiences would extend their MCK of the different mathematics they were expected to teach and also assist development of their teacher identity.

Little evidence of growth and change was evident in pre-service teachers’ MCK between second year and fourth-year; by the latter, less than half of the longitudinal study participants could demonstrate mathematical structure or connections (Chick et al., 2006a). This finding provided further evidence that the program structure limited pre-service teachers’ opportunities to enhance their MCK. Further coursework related to primary mathematics teaching during third and/or fourth year would have provided greater opportunity to sustain engagement with MCK by assisting pre-service teachers to transform their MCK during each year of the program, hence developing breadth and depth of MCK.

8.3.3 Quality of teaching and learning experiences

During the program pre-service teachers were provided with opportunities that developed their understanding of knowledge and theories for effective primary mathematics teaching whilst under supervision of their mentor teacher during practicum experiences or during lecturers and tutorials when guided by their university lecturer. These experiences were designed to create a partnership between schools and universities, some coursework experiences required engagement in primary schools. In
addition, when developing their identity as primary mathematics teachers, these experiences would have assisted pre-service teachers to make links with the knowledge gained from their lecturers and mentor teachers. These educators could have shared their breadth and depth of MCK, providing further opportunities to develop and extend foundation knowledge, transformation, connection, contingency and SCK, and their quality of teaching and learning experiences they provided pre-service teachers would have been a factor that influenced development of pre-service teachers’ MCK.

Jaworski (2008) suggested that the qualities of teacher educators are similar to the qualities required by mathematics teachers because most likely they have been mathematics teachers themselves. They also play an important role in providing pre-service teachers with opportunities to learn (Blömeke & Kaiser, 2014). No mathematics teacher educators or lecturers were interviewed during this study. However when in second year, Emma and Lisa described the learning they received from their lecturer as unpacking the mathematics they needed for teaching primary mathematics. They developed foundation knowledge such as learning primary mathematical concepts by using concrete materials that developed understanding, hence promoting conceptual understanding. Rowland et al. (2009) suggested that unpacking of the underlying processes of mathematical concepts enables a teacher to consider the best ways to introduce mathematical ideas to a child.

Similarly, mentor teachers can play a vital role in shaping pre-service teachers’ knowledge. An effective teacher knows their content (McDiarmid, Ball, & Anderson, 1989) and should be able to share this knowledge with pre-service teachers. During primary mathematics practicum experiences, pre-service teachers were expected to observe, plan and teach small groups and in second year and fourth year teach a whole class of students. These experiences enabled pre-service teachers to extend their MCK whilst under the guidance of their mentor teacher.

A range of opportunities assisted pre-service teachers to focus on and extend their MCK when planning, teaching and debriefing about their practice mathematics lessons with their mentor teacher. As reported in the findings, for instance, when planning their teaching, mentor teachers suggested resources such as websites or books that assisted
pre-service teachers to check or revise their MCK and they reviewed the pre-service teachers’ lesson plans. Reviewing lesson plans is an opportunity to check for correct MCK, evidence of knowing mathematical language, stages of student learning or appropriate learning goals and teacher actions, such as how the pre-service teacher would demonstrate breadth and depth (Ma, 1999) of their MCK when teaching. However as the mentor teachers were not interviewed during the study it is difficult to identify the aspects of the lesson which mentor teachers focused on when reading lesson plans and debriefing. Nevertheless, during interviews two of the pre-service teachers commented that their mentor teacher focused on behavioural management rather than strengths and weaknesses of pre-service teachers MCK when providing feedback after their lessons.

The pre-service teachers had different mentor teachers and experienced a different teaching situation for each year of their program; therefore experiences and opportunities to learn MCK would differ more when compared with opportunities to learn MCK with peers during coursework as lectures, tutorials and assessment requirements were the same for each unit they completed. One significant difference between school experiences was the expectations from mentor teachers regarding preparation for teaching, including a detailed lesson plan. Some pre-service teachers were expected to write a lesson plan and others were not; in particular, fewer pre-service teachers were expected to prepare lesson plans during fourth year than second year. Considering so many pre-service teachers’ had difficulty with their MCK, all pre-service teachers should have been encouraged to prepare lesson plans to maximise their opportunities to extend their MCK.

Preparing lesson plans assisted second-year and fourth-year pre-service teachers to extend and revise their MCK for teaching. Planning and writing the lesson plans provided pre-service teachers with opportunities to reflect on what the lesson might look like, including the mathematical knowledge the students would learn. Therefore detailed lesson planning assisted pre-service teachers to focus on their MCK, check mathematical terms and language before teaching, and consider the teaching sequence and materials needed to assist students’ understanding.
8.4 Reflections on methodology, study design and implications

A mixed method research design, integrating qualitative and quantitative methods, was required to explore the development of pre-service teachers’ MCK and their opportunities to learn. Quantitative methods were used to investigate the strengths and weaknesses of a large cohort of first and second-year pre-service teachers’ responses to MCSK test items. Qualitative methods were used in a longitudinal study of 17 pre-service teachers’ MCK during their program. In reflecting on the methodology, the following limitations of the study were identified.

8.4.1 Participants

A random sample of pre-service teachers was not possible in the longitudinal study due to the small number of pre-service teachers who agreed to participate. The 17 participating pre-service teachers were a convenience sample, because they were the total number of volunteers. Kervin et al. (2006) stated that convenience sampling involves the most bias because it is assumed participants have a higher level of motivation than non-participants. Accordingly, the longitudinal study participants probably had a higher level of motivation towards learning mathematics than pre-service teachers who did not volunteer.

The longitudinal study participants were not representative of their peers or the larger cohort reported in Chapter 5. Of the 17 pre-service teachers who participated in the longitudinal study, 12 were classified in Study Group 1 or Study Group 2 and were pre-service teachers who passed the MCSK test on their first attempt; five Study Group 3 pre-service teachers did not pass the MCSK test at their first attempt. Thus a higher proportion (71%) of the longitudinal study participants passed an MCSK test in second year during their first attempt than in the larger cohort (about 50%).

Three study groups were used to represent the types of pre-service teacher of the larger cohort and report similarities and differences in their MCK. The longitudinal study participants were grouped according to studying or not studying secondary mathematics as their chosen secondary teaching specialisation as well as the number of times it took
them to pass the MCSK test. The distribution of pre-service teachers in the three study groups was unequal, with small numbers of participants in Study Group 1 and Study Group 3 (which most closely represented the majority of Year 2 students). Care should be taken when drawing conclusions from the longitudinal study about all pre-service teachers, because pre-service teachers participating in the longitudinal study had less difficulty during second year when relying on their MCK.

8.4.2 Data collection techniques

The data were collected after pre-service teachers participated in primary mathematics education units of study and during practicum experiences. The multiple data collection techniques used in each year of the teacher education program provided a detailed understanding of the development of pre-service teachers’ MCK. By following the 17 pre-service teachers for four years, factors could be identified that assisted pre-service teachers to extend their MCK.

Additional data were collected from two larger cohorts of first and second-year pre-service teachers to compare their responses to similar MCSK tests. As part of the study design there was no control over when or which cohorts of pre-service teachers completed the MCSK test, because assessment tasks were part of the teacher education program. The strength of data collection was that test responses provided additional data that may not have been available in other years, contributing to the findings and classification of difficulty ranking. A weakness was that different cohorts of pre-service teachers responded to similar MCSK test items rather than the same test items.

The findings could have been extended by inviting the larger cohort of second-year pre-service teachers to repeat the MCSK test in fourth year to detect differences in MCK. However, it is unlikely that many fourth-year pre-service teachers would have agreed to participate based on the level of difficulty the larger cohort of second-year pre-service teachers had during the study when completing the MCSK test. Similarly, the 17 longitudinal study pre-service teachers could have been asked to complete the MCSK test in fourth year but the researcher considered the fourth-year interview data to provide a richer source of data for the study. Completing a MCSK test during fourth-
year would have also been time consuming for the participants and they may not all have agreed.

Most of the MCSK test items were closed question types and relied on procedural knowledge rather than conceptual understanding. Therefore correct MCSK test item responses were evidence of knowing a method of solutions rather than evidence of conceptual understanding or knowledge of more than one method. Teachers with SCK would know a range of methods of solutions, including conceptual understanding and would be able to explain why rules or procedures work.

The findings identified that pre-service teachers required further development of their conceptual understanding. Factors influencing pre-service teachers’ dependence of procedural knowledge may have been an emphasis on learning or revising MCK required for passing the MCSK test. An emphasis on procedural understanding is of concern because if teachers believe that mathematics relies on rules which have to be remembered, this may impact on how they teach and what their students learn (Goulding, 2007). Booker et al. (2010) stated that students can be taught rules and formulas but those who develop understanding will remember. Similarly, pre-service teachers would most likely remember their MCK if they learned with understanding by making connections and developing understanding of rules and procedures. Therefore the MCSK test should also be designed to assess pre-service teachers’ conceptual understanding, promoting understanding and providing program experiences that extend foundation knowledge as well as SCK.

A final limitation of the study was that the longitudinal study pre-service teachers were observed practising their teaching once during second year and fourth year. The research design would have been improved by including a series of lesson observations that could be used to extend the findings of how pre-service teachers developed their connections and transformed understanding to students across lessons rather than within one lesson.
8.5 Significance of the study

The findings deepen understanding of pre-service teachers’ development of their MCK by identifying contributing factors that influenced opportunities to learn MCK. Previously studies have reported on teachers’ and pre-service teachers’ MCK, but few have reported on pre-service teachers’ MCK over time. Hence, the findings improved upon previous studies by providing an in-depth analysis of pre-service teachers’ MCK over time. In addition, data about pre-service teachers’ MCK were described and interpreted using four categories of the Knowledge Quartet: foundation knowledge, transformation, connection and contingency (Rowland et al., 2009) and other MCK frameworks (e.g., Chick et al., 2006a; Hill et al., 2008; Ma, 2009). These frameworks have been used and reported in previous studies. In this study the similarities and differences of the categories of MCK were used together to describe and extend understanding of the development of pre-service teachers’ categories of MCK for primary mathematics teaching.

The findings contribute to the field of mathematics teacher education research by identifying factors that positively and restricted influenced the development of the pre-service teachers’ MCK. Dependence on procedural knowledge may have been due to an emphasis on learning or revising MCK required for passing a test during second year, a lack of sustained engagement in opportunities to extend MCK, lack of depth of practicum experiences across different year levels, as well as limited opportunities to extend MCK when reflecting on primary mathematics lesson experiences with mentor teachers. Pre-service teachers’ education program designs can be improved by considering the findings and factors or opportunities to extend pre-service teachers’ MCK. In particular, improved program design can assist pre-service teachers who rely on procedural methods of solution and extend their MCK for primary teaching.

Previous studies have focused on the domain of number and less on other domains (Mewborn, 2001; Southwell & Penglase, 2005). This study provided an in-depth analysis of MCSK test items for the domains of measurement, geometry and probability and statistics, including ranking by item difficulty: least difficult, difficult and most difficult items. A second analysis involved coding and reporting items by difficulty
ranking and cognitive sub-domain, including Knowing, Applying and Reasoning. The findings were that first-year pre-service teachers had the most difficulty when using proportional reasoning for calculating scale, distance and capacity when responding to multi-step items. Few researchers have studied pre-service teachers with respect to their MCK and cognitive knowledge.

8.6 Implications for teacher education

The findings of the study will assist with future directions of the Bachelor of Education program where the study was conducted. The following sections contain recommendations that will be valuable for the design and refinement of similar teacher education programs, including primary teacher education programs, and contribute usefully towards the professional development for primary mathematics teachers.

8.6.1 Growth of MCK

All pre-service teachers who participated in this research were required to pass an MCSK test. A quantitative analysis identified the strengths and weaknesses in MCK of a large cohort of first and second year pre-service teachers with respect to number, measurement and geometry, and statistics and probability. The pre-service teachers were less capable in some topics than others, having most difficulty with measurement and geometry items. These findings were similar to those of a previous study of pre-service teachers from seven Australian universities (Callingham et al., 2011).

The percentage of correct item responses were ranked by three levels of difficulty, least difficult, difficult and most difficult by ACM (ACARA, 2013) content strand. The identification of item difficulty facilitated the analysis of the responses, and the findings can be used when designing programs and choosing areas of weakness on which they can focus to improve pre-service teachers’ MCK. Ranking items by difficulty provides lecturers with benchmarks for pre-service teachers’ MCK, so that an appropriate sequence of learning may assist their preparation for primary mathematics teaching. Additional program opportunities to learn would allow pre-service teachers to develop their MCK of different methods of solutions, including common misconceptions, while assisting them to consider the different methods from a conceptual focus rather than rely
on a rule as well as provide ongoing revision so they sustain engagement with their MCK.

The test instrument or the MCSK test was not designed specifically for this study and was problematic due to the test item types and a reliance on procedural knowledge. A recommendation for future test instrument development would be to provide a greater range of test items asking pre-service teachers to explain their thinking or different methods of solutions. Changing the MCSK Test would encourage pre-service teachers to revise and demonstrate more than procedural knowledge when responding to MCSK test items promoting breadth and depth (e.g. Ma, 1999) of MCK.

8.6.2 Structure of the program

Many Australian universities offer Bachelor of Education programs in primary teacher education, but the Prep-Year 12 program that the research participants completed is less common. The findings of this longitudinal study of 17 pre-service teachers suggest that the Prep-Year 12 program structure was problematic for pre-service teachers developing their primary MCK. When interviewed in fourth year, 12 pre-service teachers had insufficient MCK or had forgotten knowledge they had demonstrated in second year (Table 7.4). A factor that contributed to these findings was that the third year of the program focused on theory and practice of their specialisation in secondary teaching, therefore limiting opportunities to revise MCK, except for three pre-service teachers (Study Group 1) who undertook practicum teaching in secondary mathematics classrooms in third year. These pre-service teachers may have extended their MCK by completing the three primary mathematics units during first, second and fourth year or by having an additional unit of study during their final year.

Most Victorian programs offer pre-service teachers coursework in mathematics and practicum experiences during their program (Parliament of Victoria Education and Training Committee, 2005). As a comparison, both nationally and internationally the pre-service teachers in this program had time allocated to developing their primary MCK but as discussed earlier experiences were not always sustained for each year of the program.
Tatto et al. (2012) reported most of the primary mathematics groups in TEDS-M averaged fewer than 100 hours of mathematics coursework during their program. In New Zealand, Burgess and Bicknell (2003) were concerned about the amount of time pre-service teachers were given to learning the content of mathematics when completing their primary teacher education program. Undergraduate primary programs within Victoria can have up to five coursework units dedicated to mathematics teacher education (Parliament of Victoria Education and Training Committee, 2005), totalling about 175 hours. In comparison the pre-service teachers in this study completed 105 hours of mathematics coursework, similar to or more than other countries reported in TEDS-M but less than other Victorian undergraduate primary pre-service teachers.

Tatto et al. (2012) reported that the amount of time spent completing practicum experiences varied across the 17 countries in the TEDS-M study. In Australia, teaching practicum within Victorian programs also varied, ranging between 80 and 180 days supervised teaching practice (Parliament of Victoria Education and Training Committee, 2005). In comparison, the pre-service teachers in this program completed 102 days in primary schools and 42 days in secondary schools or a total of 144 days. The number of days in schools was comparable with other programs and therefore should have been a sufficient opportunity to extend MCK during practicum experiences. However primary school experiences were restricted to first, second and fourth years in this study.

**8.6.3 Breadth and depth of experiences**

The program experiences of the pre-service teachers who completed mathematics as their secondary specialisation, demonstrated greater SCK, including breadth and depth of MCK, during fourth year than pre-service teachers who had chosen other secondary specialisations. Several factors contributed to this finding, such as sustained engagement with mathematics for each year of the program, stronger mathematical pre-program identity, knowing more advanced mathematics, experience that enhance learner identity and teacher identity as a mathematics teacher. However, if pre-service teachers not specialising in secondary mathematics had sustained experiences for each year of their program and had taught secondary mathematics in either Year 7 or Year 8
or upper primary years, these findings may have varied. Future programs should consider providing opportunities for pre-service teachers to extend their MCK and HCK by experiencing practicum teaching in lower secondary mathematics classes. Thus, when teaching Year 6 students, their teachers would know the curriculum and mathematics that their students would learn next. Knowing what and how students learn in previous year levels is also important for SCK and PUFM.

Pre-service teachers’ practicum experiences differed and either limited or extended opportunities to develop breadth and depth of MCK. To illustrate, only four of the 17 longitudinal study participants had the opportunity to practise their teaching in lower, middle and upper primary classrooms in each of their three practicum years, deepening their MCK relative to their peers.

Pre-service teachers who miss opportunities to teach the depth of the curriculum are likely to develop gaps in their SCK of the MCK required for the different stages of teaching mathematics from Prep to Year 6. Based on these research findings, a recommend is that all pre-service teachers be provided with depth in primary mathematical teaching during their practicum experiences.

Different program experiences ensured pre-service teachers revised and developed their MCK, but the level of their MCK during fourth-year varied: 12 pre-service teachers were not yet demonstrating foundation knowledge, two pre-service teachers were demonstrating foundation knowledge and three pre-service teachers demonstrated SCK. Pre-service teachers who developed foundation knowledge and SCK demonstrated conceptual understanding as well as knowledge of procedures and could make connections within their lessons and when planning a series of lessons. Of concern was that, in fourth year, five of the longitudinal study participants had difficulty demonstrating conceptual understanding or connected MCK because they relied on procedural knowledge without understanding. Their dependence on procedural knowledge may have been influenced by the emphasis on learning or revising the MCK required for passing an MCSK test. Mathematics should be taught to students with understanding rather than relying on rules (Booker et al., 2010). Similarly, pre-service teachers can develop understanding by experiencing situations that assist them to
unpack their MCK. Therefore the MCSK test should be designed to assess pre-service teachers’ conceptual understanding as well.

8.6.4 Practicum experiences and missed opportunities to learn

When practising teaching with primary students, the pre-service teachers had an opportunity to increase breadth and depth of MCK. Differences in experiences reported in this study can be used to improve opportunities to learn MCK in future program design by:

- Assisting pre-service teachers to develop their foundation early in program so that they can use other program experiences to develop their SCK (and PUFM).

- Ensuring pre-service teachers practise teaching with different year levels, including upper and lower primary year levels, so as to gain depth of MCK. Practicum experiences could be extended to include experiences in junior secondary classrooms. First-year experiences should include opportunities to experience both upper and lower primary year levels to strengthen pre-service teachers’ teacher identity.

- Reviewing guidelines for and expectations of the content of pre-service teachers’ mathematics lesson plans so that they target their MCK when planning their lessons. Lesson plans should include questions focusing on MCK including correct terminology, anticipation of misconceptions or students’ solutions, and provide details of the mathematics they will model. Furthermore, pre-service teachers with weaknesses in their MCK must be encouraged to write detailed mathematics lessons plans before teaching. They should also include reflection on their MCK, and list what they found difficult or might change if teaching the lesson again.

- Ensuring mentor teachers revise mathematical concepts with pre-service teachers before and after practice teaching experiences. This will assist pre-service teachers to develop their SCK through being shown how to use mathematical resources that will promote students’ conceptual understanding,
and discussing the strategies that students used during the lesson, including common misconceptions or the students’ methods of solutions.

- Providing professional development for mentor teachers and guidelines to extend their skills when mentoring pre-service teachers in their classrooms during primary mathematics lessons.

- Offering an elective unit for pre-service teachers who have strengths in their MCK so they may extend their MCK and SCK by specialising in primary mathematics teaching.

8.6.5 Identifying weaknesses

The findings identified that pre-service teachers in first year can have difficulties with their MCK, and as discussed in this chapter, some of the participants addressed these weaknesses when preparing for the MCSK test but others ignored them (Figure 8.2). Pre-service teachers in first year had the opportunity to identify their strengths and weaknesses in MCK by completing a practice MCSK test. Then they were able to consider their options, and could undertake a unit designed to assist them with the SCK required for teaching primary mathematics.

Not all pre-service teachers identified that they needed to target their weaknesses in MCK; others chose not to address their weaknesses or had difficulties learning MCK. Of concern were those pre-service teachers who continued to demonstrate weaknesses during their program. For example, one second year pre-service teacher with gaps in her MCK was unable to implement a Grade Three subtraction lesson. The pre-service teacher used procedures and closed questions rather than demonstrating teaching strategies with understanding and materials and was unable to promote students’ understanding of subtraction. This situation raises new questions: should pre-service teachers with weaknesses in their MCK be allowed to practise their teaching with primary students? What impact does a lack of teacher MCK have on students’ learning of mathematics? How should this issue be addressed? What is the minimum level of MCK required for pre-service teachers to plan and teach lessons? How many attempts at
passing an MCSK test should pre-service teachers be allowed before unsatisfactory progress provisions apply? Further research is required to answer these questions.

Some of the longitudinal study participants were unable to rely on their foundation knowledge when responding to MCSK items (similar to second-year test items) in fourth year, suggesting they no longer met the graduating standard “teachers know the content and how to teach it” (AITSL, 2011, p. 3). The outcome of this study could be an argument for providing pre-service teachers with an exit MCSK test to ensure they demonstrated the MCK needed for teaching at program completion. In addition the minimum level pre-service teachers demonstrate by the end of their program should be foundation knowledge with evidence of transformation and making connections, but the expected standard should be to demonstrate SCK, the skills of transformation and making connections when teaching, so that mathematics is accessible to students. Finally an improved or adapted MCSK test could be designed to assess these categories.

8.6.6 Future research

The findings of this study need to be tested and validated in primary teaching programs that are structured differently. Future studies could compare the MCK of pre-service teachers completing an undergraduate primary mathematics program with the pre-service teachers in this study or others completing a Prep-Year 12 teacher education program, assessing the benefit of different program structures for pre-service teachers learning to teach primary mathematics. In addition, more might be learnt with a comparison of pre-service teachers completing secondary mathematics in a Prep-Year 12 program with pre-service teachers completing a secondary mathematics program.

The test items in the MCSK tests used in this research were unevenly distributed; more items were related to knowledge of number, measurement and geometry than to statistics and probability. Future studies could implement test instruments that include an even distribution of all primary mathematics curriculum topics, thereby extending these findings. Future studies could also extend these findings by including the different cognitive sub-domains, including Knowing, Applying and Reasoning, again across a more even distribution of mathematics topics. Future researchers should consider how
best to assess pre-service teachers’ MCK and when assessment should occur, either at the beginning of the program, during the program or both.

Pre-service teachers’ MCK could be extended with greater assistance from mentor teachers during practicum teaching experiences. Further studies could focus on data collection from the mentor teachers, such as feedback as to how they assisted pre-service teachers to extend their MCK. Data could include interviews with mentor teachers before and after lessons, as well as observing the interactions between mentor teachers and pre-service teachers when planning, teaching and reflecting on mathematics lessons. Future studies of larger cohorts of pre-service teachers may also consider if the order in which pre-service teachers undertake their practicum experiences makes a difference (such as teaching lower levels before upper primary levels).

Similarly, how university lecturers choose opportunities for pre-service teachers to learn MCK when planning and preparing lectures, tutorials and assessment tasks could be a fertile area of further research. Developing understanding of the complexities of teaching teachers could include observation of pre-service teachers’ learning during lectures and tutorials as well as interviewing lecturers who teach or co-ordinate these units.

8.7 Conclusion

During fourth-year practicum teaching experiences and when responding to MCK interview test items in the final of their program, the pre-service teachers in this study demonstrated different levels of MCK.

*The most accomplished pre-service teachers with the highest level of MCK provided evidence of SCK.*

These teachers were developing breadth and depth of mathematical understanding because they relied on their foundation knowledge and could demonstrate transformation and/or making connections when planning units of work and teaching.
primary mathematics lessons. They relied on their MCK when responding to students’ questions and dealing with contingencies.

*Accomplished pre-service teachers demonstrated foundation knowledge.*

Such pre-service teachers could rely on their MCK after revising mathematical concepts before teaching. They were beginning to make connections when planning and teaching a primary mathematics lesson. They were unlikely to rely on their MCK when responding to students’ questions and dealing with contingencies.

*The least accomplished pre-service teachers were not yet demonstrating foundation knowledge.*

These pre-service teachers had difficulty relying on their MCK, for making connections and deconstructing content to key components between mathematical concepts. They relied on rules without fully understanding why the rule works, and therefore were not yet demonstrating conceptual understanding of a category of foundation knowledge.

During the four-year longitudinal study of primary pre-service teachers’ MCK, the findings showed that factors including developing teacher identity, sustained engagement, and quality of teaching and learning experiences positively influenced pre-service teachers’ MCK. Opportunities to learn MCK were influenced by pre-service teachers’ pre-program identity, learner identity and teacher identity. Pre-program identity influenced pre-service teachers’ MCK, including attitudes and beliefs of how mathematics is taught, as well as the amount of MCK they brought to the program from their primary and secondary school mathematics education. Learner identity was inferred by the way pre-service teachers extended and developed their MCK for primary mathematics teaching, relying on their program and individual choices made as part of their learning experiences. Teacher identity influenced how pre-service teachers shaped and considered their beliefs about teacher education; they reflected on how mathematics should be learnt and taught whilst making connections with their MCK.

A factor that inhibited pre-service teachers’ development of MCK was lack of sustained engagement of primary mathematics teacher education experiences for each year of the program. The teacher education program from which participants were recruited did not
foster ongoing learning of primary mathematics teaching, limiting opportunities to learn MCK during coursework and teaching practicum experiences. Coursework experiences provided a range of assessment tasks that enhanced pre-service teachers’ MCK, but these experiences were limited to the first and second years of the program. Pre-service teachers were taught in a school setting in each year of their program, but during third year most did not teach mathematics in a school.

Coursework and practicum experiences provided opportunities for pre-service teachers to extend their MCK. Course lecturers were also important for assisting pre-service teachers to relearn or unpack the MCK they had forgotten. In addition, during practicum teaching experiences pre-service teachers had further opportunities to enhance their MCK when observing their mentor teacher teaching, planning lessons, when practising teaching mathematics lessons or debriefing with their mentor. However, the findings suggest participants’ MCK could have been extended through more and greater variation in opportunities to learn MCK during practicum experiences, including increasing the depth of mathematical experiences by practising teaching across different primary year levels. Furthermore, improving the quality of mentor teachers’ supervision by assisting mentors to identify how they can further support pre-service teachers to extend their MCK would be very valuable.

This study identified factors that enhance pre-service teachers’ MCK and are therefore important for improving primary mathematics teacher education and in turn the education of primary school children. Ensuring future pre-service teachers enter teacher education knowing sufficient mathematics for primary teaching, are given and can seek opportunities to enhance their MCK, are provided with sustained opportunities to learn during coursework and practicum teaching experiences for each year of their program will promote the development of accomplished pre-service teachers who can demonstrate SCK and rely on their MCK for teaching primary mathematics.
References


REFERENCES


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Maher, N., & Muir, T. (2013). "I know you have to put down a zero, but I'm not sure why": Exploring the link between pre-service teachers' content and pedagogical content knowledge. Mathematics Teacher Education and Development, 15(1), 72-87.


REFERENCES


Appendices

Appendix A

Victorian Certificate of Education Year 11 (Units 1 and 2) and Year 12 (Units 3 and 4) mathematics

*Foundation Mathematics* Units 1 and 2 provide continuing mathematical development of students entering VCE who need mathematical skills to support their other VCE subjects, and who do not intend to undertake Unit 3 and 4 studies in VCE Mathematics in the following year. Foundation Mathematics Units 1 and 2 do not provide a basis for undertaking Unit 3 and 4 studies in Mathematics.

*General Mathematics* Units 1 and 2 provide courses of study for a broad range of students and may be implemented in a number of ways. Students intending to study Specialist Mathematics Units 3 and 4 should be provided with access to a rigorous implementation of General Mathematics Units 1 and 2, which emphasises mathematical structure and the justification of results through general case arguments.

*Mathematical Methods (CAS)* Units 1 and 2 have a closely sequenced development of material, intended as preparation for Mathematical Methods (CAS) Units 3 and 4. Mathematical Methods (CAS) Units 3 and 4 may be taken alone or in conjunction with either Specialist Mathematics Units 3 and 4 or Further Mathematics Units 3 and 4, and provide an appropriate background for further study in, for example, science, humanities, economics or medicine. Further Mathematics Units 3 and 4 are intended to be widely accessible. They provide general preparation for employment or further study, in particular, where data analysis is important. The assumed knowledge and skills for Further Mathematics Units 3 and 4 are drawn from General Mathematics Units 1 and 2. Students who have done only Mathematical Methods (CAS) Units 1 and 2 will also have had access to assumed knowledge and skills to undertake Further Mathematics.

*Specialist Mathematics* Units 3 and 4 are normally taken in conjunction with Mathematical Methods (CAS) Units 3 and 4, and the areas of study extend and develop material from Mathematical Methods (CAS) Units 3 and 4. Specialist Mathematics Units 3 and 4 are intended for those with strong interests in mathematics and those who wish to undertake further study in mathematics and related disciplines. (Victorian Curriculum and Assessment Authority, 2010, p. 8)

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15 Computer Algebra System


Appendix B

Mathematics Competency Skills and Knowledge Test 1 (2008)

NAME __________________________ STUDENT ID _____________________
CAMPUS ________________________ LECTURER ______________________

- TURN OFF YOUR MOBILE PHONE
- You have 3 hours to complete this test
- You are NOT permitted to use a calculator (or mobile phone)
- There are 49 questions. Please write your answer in the space provided.
  Pen and paper working can be included in the space on the page.
- There are 7 sections and 7 questions in each section. To pass this test
  you need to get a minimum of 5 questions correct in EACH section.
- Show all your working

GOOD LUCK!

<table>
<thead>
<tr>
<th>SECTION</th>
<th>No. correct</th>
<th>Satisfactory/Not satisfactory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Section 1: Number</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Section 2: Measurement,</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chance &amp; Data</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Section 3: Fractions</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Section 4: Space</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Section 5: Decimals</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Section 6: Area &amp; Volume</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Section 7: Percentage &amp; Ratio</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OVERALL RESULT</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Section 1: NUMBER

1. What is the next number in this sequence?

   81, 79, 76, 72, 67, 61, .....  Answer __________

2. Write the name of this number in words: 4 236 500 628

   __________________________________________________
   __________________________________________________

3. Circle the composite numbers in the following list:

   46  47  48  49  50  51

4. Divide 4180 by 20

   Answer ______

5. 473 x 19 =

   Answer ______

6. 2180 ÷ 6 =

   Answer ______

7. Write the answer to the following product in words 4 x 10^4

   Answer ________________________________________________
Section 2: MEASUREMENT and CHANCE & DATA

8. How many millilitres in 2.4 litres?  
   Answer

9. Approximately what is the mass of a litre of milk?  
   Answer

10. Estimate the size of the acute angles below.  

   Answer

11. A plane departs Tullamarine airport at 10:40am and arrives in Brisbane at 1:05 pm. How long is the flight?  
   Answer

12. This is a list of the length in metres of snakes in the zoo enclosure. What is the mean (average) length of the snakes?  
   8, 6, 4, 3, 5  
   Answer

13. What is median age of these teachers?  
   27 years, 35 years, 25 years, 28 years, 32 years, 30 years  
   Answer

14. The table below shows VU students who do or do not play netball:  

<table>
<thead>
<tr>
<th></th>
<th>Do play netball</th>
<th>Do not play netball</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female</td>
<td>28</td>
<td>12</td>
</tr>
<tr>
<td>Male</td>
<td>4</td>
<td>16</td>
</tr>
</tbody>
</table>

   What is the probability that a student chosen at random do not play netball?  
   Answer
Section 3: FRACTIONS

15. What is the difference between \( \frac{5}{8} \) and \( \frac{3}{5} \)?

Answer: ______________

16. What is the next number in this number pattern?
   \( 3\frac{1}{2}, 3 \frac{1}{4}, 4, 4\frac{3}{4}, \ldots \)

Answer: ______________

17. Write the answer to \( \frac{5}{6} \times 24 \)

Answer: ______________

18. Which of fraction is closest to 1: \( \frac{34}{35} \) or \( \frac{69}{70} \)?

Answer: ______________

19. Draw a model to illustrate \( \frac{3}{4} + \frac{1}{8} \) and find the solution

Answer: ______________

20. Put these numbers on a number line

\[
\frac{5}{3}, \frac{2}{3}, \frac{1}{6}, 0.6.
\]

Answer: ______________

21. \( \frac{3}{5} + \frac{4}{7} = \)  

Answer: ______________
Section 4: SPACE STRAND

Questions 22 to 26 refer to the shapes above

22. Which of these shapes are polygons?
   Answer ____________________________________________

23. Which of these shapes are octagons?
   Answer ____________________________________________

24. Which polygons include reflex angles?
   Answer ____________________________________________

25. What is the name of shape C?
   Answer ____________________________________________

26. How many lines of symmetry are in shape F above?
   Answer ____________________________________________
27. What type of triangle is the one below?

Answer ______________________

28. Which of these nets will NOT make a cube.

a.  

b.  

c.  

Answer ______________________
Section 5: DECIMALS

29. The number 0.307 is sometimes read as “zero point three zero seven”. Draw a place value chart, put 0.307 in the place value chart and write a meaningful name for this number.

Answer: _____________________________________________________

30. List these decimal fractions in order from least value to greatest value.

4.4, 4.044, 4.04, 4.404

Answer: _____________________________________________________

31. Add 0.2 to 6.08

Answer: ________________________

32. The decimal point on the calculator is not working. It shows that the product of 17.86 x 1.15 is

20539

Answer: _________________________

33. Find the value of 2.4 ÷ 0.4

Answer: _________________

34. Write \( \frac{4}{9} \) as a decimal. Show four decimal places.

Answer: _________________

35. Write 0.125 as a fraction and simplify

Answer: _________________
Section 6: AREA AND VOLUME

Double check that you have included the correct units with your answer!!!!!!

36 Approximately, what is the area in square centimetres of this closed curve?

Answer________

37 If the area of a rectangular garden is 18 square metres. What could be its dimensions? Draw a diagram and label the dimensions. Find its perimeter.

Length: _________________
Width: _________________
Perimeter: _________________

38 How many square centimetres are in a square metre? Draw a diagram to illustrate.

Answer: _________________

39 A rectangular field is 600 m long and 200 m wide. What is its area in hectares?

Answer: __________

40. Find the area of this triangle:

Answer:___________
41. If the water in container A is poured into container B, what height will it reach in container B?

![Diagram of containers A and B with dimensions: A - 8 cm x 2 cm x 2 cm, B - 4 cm x 4 cm x 4 cm.]

Answer: ________________

42. A concrete footpath has dimensions 60cm wide, 8m long and 10cm deep. What volume of concrete is needed to make this path in cubic metres?

Answer: ________________ cubic metres

Section 7: PERCENTAGE AND RATIO

43. What percentage of the rectangle is shaded?

![Shaded grid]

Answer: __________

44. Write 62.5% as a decimal

Answer: __________
45. Write 3.4 as a percentage.  
Answer: ____________

45. Which is better value: 12 songs for $30 or 8 songs for $14?  
Answer: ____________

46. How long will it take to drive 210 km if you are traveling at 100 km per hour?  
Answer: ____________

47. Liquid fertilizer is mixed with water in the ratio of 8ml:5L. How many millilitres of liquid fertilizer should be used for 15 L of water?  
Answer: ________________

48. Cordial drinks are mixed with water in the following ratios. Which is the strongest flavour?

a. 2:7  
b. 3:7  
c. 1:3  
d. 3:8  
Answer: ____________

49. Anh earns 15% commission on each house she sells. If she a house for $460 000 how much commission will she earn?  
Answer: ________________
Appendix C

GENERAL QUESTIONS
FOR PARTICIPANTS
INVOLVED IN RESEARCH

Please answer these general questions about yourself:

1. Your Name: ________________________________

2. Phone Number: _____________________

3. Student Number: _______________________

4. Student Email: _________________________

5. Are you a male or female?  Male  Female

6. Are you completing a Bachelor of Education (Prep- Year 12) degree?  YES  NO

7. Are you a first year pre-service teacher?  YES  NO

8. Your age?  18 to 25 years  25 years or more

9. Which campus are you completing your major at?
______________________________

10. Is mathematics one of your general elective studies?  YES  NO

11. Did you complete the VCE?  YES  NO
12. If yes,

   a. Did you complete and pass VCE Mathematics Methods? YES NO

   b. Did you complete and pass VCE Further Mathematics? YES NO

13. What year did you complete secondary school?

14. Have you completed any other tertiary study? Please explain.

15. Do you have any questions about the study you would like answered before you participate?
Appendix D

Longitudinal study: Second-year interview questions

1. Do you enjoy teaching primary mathematics lessons?

2. Are you intending to work in a primary or secondary setting when you finish your course? Have you changed your mind since beginning the course?

3. What do you think mathematical content knowledge for teaching is?

4. Can you describe the mathematics you think a primary teacher requires for teaching in a primary school?

5. Since beginning the course have your ideas changed about the mathematics content knowledge you need for teaching in a primary school? How and when did this occur?

6. How would you describe your own understanding of mathematical content knowledge?

7. Which areas of mathematics content knowledge do you feel strongest with and why?

8. Which areas of the course have helped you to develop your mathematical content knowledge? (You might like to reflect on Project Partnership experiences, Praxis Inquiry or course work.)

9. I have just seen you teach a primary mathematics lesson.
   a. What were the strengths of the lesson?
   b. How would you change the lesson next time?
   c. If you think the lesson catered for all learners how do you know this?
Or if the lesson did not cater for all learners how do you know this?

d. What was the main mathematical idea or skill that was the focus of this lesson?

e. What mathematical knowledge do children need to know before completing your lesson?

f. What mathematics knowledge did you need to know or learn when planning this lesson?

g. What was involved in planning this lesson?

h. What did you find difficult when planning this lesson?

i. Why did you choose this particular explanation/approach for the lesson?

j. In what other ways could you have explained/approached this lesson?

k. What do you think the next mathematics lesson should be about?

l. What would be the next stage in the mathematics learning for these students?

10. How has your mentor teacher helped you with this lesson/developing your mathematics content knowledge?

11. What other comments would you like to make?
Appendix E

Longitudinal study: Third-year interview questions

SECTION ONE

Think about your own learning of mathematics during the course focusing on Project Partnership experiences during third year.

Have you taught secondary mathematics in your Project Partnership School this year?

If yes please provide some information explaining your experiences;

1. What year levels have you taught and what were your teaching responsibilities?
2. Have you struggled with any of the mathematical content knowledge required of you during your secondary placement? Explain.
3. Have you developed a greater understanding of mathematics topics during this years’ Project Partnership? Please explain a topic you learnt the most about and why.
4. Can you remember a time during your secondary primary school experience this year when a mathematical idea just clicked for a student you were working with? What was it, where were you and what happened?
5. If you were to describe your own mathematical content knowledge for secondary teaching, what are your strengths?
6. If you were to describe your own mathematical content knowledge for secondary teaching, what are your weaknesses?

SECTION TWO

The following questions are applicable to everyone; please answer with reference to your Project Partnership experiences this year.

1. During your teaching as part of your Project Partnership when did you use mathematical content knowledge? Please explain and add as many examples as you can think of.
2. Was there a teaching experience that used mathematics and you assisted students develop their understanding? Please provide details.
3. Was there a teaching experience that used mathematics and you found the concepts too difficult and were unable to assist the students? Please provide details.
4. Will you consider teaching secondary mathematics in the future and to what level would you feel comfortable teaching? Why or why not?

What other comments would you like to make?
Appendix F

Longitudinal study: Fourth-year interview questions

Where are you at?

1. Do you enjoy teaching primary mathematics lessons? Why-why not?
   a. Is this more or less than in first or second year or the same?
2. Are you intending to work in a primary or secondary setting when you finish your course?
3. Have you changed your mind since beginning the course? If so why?

Reflections about your primary mathematics lesson

4. What were you thinking when you planned the structure of this lesson, what was involved?
5. What mathematics knowledge did you need to learn when planning this lesson?
6. What do you think the next mathematics lesson should be about in order to move the students to the next stage of learning?

What next?

7. Thinking about the units you have studied throughout the course what was the most helpful in developing your mathematical content knowledge? What happened, where were you and when did this happen?
8. Now you have nearly finished the course what would you have done differently when thinking about learning the mathematics you need for teaching in a primary school?

9. What would you think if you were asked to teach year six mathematics next year? Why?
10. Next year if you needed help with teaching a mathematics concept, who or where would you go for help?

11. If a parent said to you during a parent teacher interview, “Maths is a subject dominated by memorised facts and rules.” How would you respond?

MCK Interview questions

- Comparing fractions (Appendix G)
- Perimeter and area (Appendix H)

Conclusion:

12. Is your partnership project related to mathematics?

13. Is there anything else you can tell me that you think would help me prepare pre-service teachers for their work in a primary maths classroom?

Thank-you, for participating in this project.

Before the end of the year if you have any mathematics’ lesson plans or assignments can you please email these to me.
Appendix G

Longitudinal study: Fourth-year MCK interview question

Comparing fractions

Looking at these pairs of fractions, which one is larger?
Record your thinking.

3/5 and 2/3

3/5 and ¾

3/5 and 5/8

Record the fractions onto the number line
Appendix H

Longitudinal study: Fourth-year MCK interview question

Perimeter of a rectangle

Imagine you are teaching area and perimeter.
Can you tell me the difference between the two?

Now imagine that a student in your class says, “I think that when the perimeter of a rectangle increases, its area also increases.” What would be your response?

Perimeter of a rectangle

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

Perimeter = 16cm
area = 16 square cm

perimeter = 24cm
area = 32 square cm

“As the perimeter of a rectangle increases, its area also increases.” (Ma, 1999)
## Appendix I

**Rose’s second-year lesson plan (2008)**

<table>
<thead>
<tr>
<th>School: XXXX Primary school</th>
<th>Year Group: 5 and 6</th>
<th>Day: Wednesday</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date: 28/5/08</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Topic:
- Lesson number one-
- Angles:
  - Naming angles
  - Using a protractor to measure angles
  - Estimating angles
  - Ordering angles
  - Identifying angles

### Aims:
- Use a protractor correctly to measure angles
- Name the angles with the proper terms
- Identify angles
- Estimating angles

### VELS: Strands, Domain, Foci and Standards
- Level/progression point 3.25-3.5
- Recognition of angles between lines particularly when lines are parallel or perpendicular
- Classification and sorting of 2D shapes using lines, orientation, angles, greater than 90 degrees.
- Construct a shape given details about angles.

### Location / Setting:
- Sitting at desks in a social arrangement.

### Organisation / Student:
- Whole class at desks

### Classroom management strategy:
- Suggestions box
- If distracted or it gets noisy motivate them and remind them of the set task.
- Student of the day

### Key Vocabulary:
- Angle
- Degrees
- Protractor
- Acute
- Obtuse
- Reflex
- Revolution
- Straight line
- Perpendicular
- Parallel

### Materials, Resources and Equipment:
- Dictionary (definition and examples)
- White board and white board markers
- Protractors
- Large protractors
- Matching angle game
- 27 Work sheets (measuring angles)

### References/Sources:
- Teacher’s books for grade five and six students.
- [http://www.primaryresources.co.uk/mathsmathsE7.htm](http://www.primaryresources.co.uk/mathsmathsE7.htm)
<table>
<thead>
<tr>
<th>INTRODUCTION Connecting, Engaging and Modelling Inquiry</th>
<th>MAIN BODY Guiding Inquiry and Practise</th>
<th>CONCLUSION Sharing, Explaining and Reviewing Inquiry</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Explanation:</strong></td>
<td><strong>Explanation:</strong></td>
<td><strong>Discussion of what angles students found around the room</strong></td>
</tr>
<tr>
<td>Today we are going to be identifying and measuring angles</td>
<td>On this sheet I would like you to estimate the angle. Then I would like you to measure the angle with a protractor. Then I would like you to name the angle (obtuse, acute, right angle). After this is done You can cut them out and put them in order of biggest to smallest.</td>
<td>Quick quiz on the names of the different angles on the board.</td>
</tr>
<tr>
<td><strong>Discussion/ Questions</strong> (have angles on board to write definition under and for examples to measure/estimate)</td>
<td><strong>Activities:</strong></td>
<td></td>
</tr>
<tr>
<td>-What is an angle? (Write definition on the board)</td>
<td>measuring angles worksheet/ordering</td>
<td></td>
</tr>
<tr>
<td>-What are the different types of angles? (Write -different angles on the board)</td>
<td>make own angles</td>
<td></td>
</tr>
<tr>
<td>-Can anyone see any angles in the room?</td>
<td>find angles in the room and measuring them with larger protractor</td>
<td></td>
</tr>
<tr>
<td>-could anyone see any angles at federation square yesterday? (hold up picture to trig memories)</td>
<td>Card game and extra sheets to go on with.</td>
<td></td>
</tr>
<tr>
<td>-Why might more things be at right angles in the room? (to use space, level/balance-think of city/houses) give example of what would happen if it was more or less than 90 degrees with a ruler.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-What unit do you measure? (degrees, not degrees Celsius like the weather) – finding space between two lines in relations to a whole 360° circle.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
- How do we measure angles (have protractor A3/A4 sheet and do an example)
- arc lets us know where to measure.
- How could we estimate? (think of circle, and specific angles- 90, 180, 270, 360)

**Reflection**

Today’s lesson went quite well. The students were set on the task and when I observed their work I could see a lot of them were getting the hang of it. A few students may need further practise in this area and seemed to have little practise in this area.

Next lesson I will break them off into smaller groups and get them on some more challenging tasks.

I need improvement with my timing and confidence in the lesson.
Appendix J

Lisa’s worksheet (2010) provided to students during her fourth-year practicum experience

MEASUREMENT INVESTIGATION Name:

PART A
In Measurement the word height means

In Measurement the word width means

In Measurement the word area means

In Measurement the word Perimeter means

PART B
Find 5 items around the classroom and complete the table below

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<th>Item</th>
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<th>Width</th>
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