INVESTIGATIONS INTO HYDRODYNAMICS AND HEAT TRANSFER IN VACUUM FLUIDISED BEDS

by

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M.Tech.

Submitted in fulfilment of the requirements for the degree of
Doctor of Philosophy

Deakin University,
July, 2014
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Abstract

Fluidised bed reactors are widely utilised due to their high heat and mass transfer capability. Industries such as pharmaceutical, gasification, chemical, surface treatment and mineral processing require efficient mixing environment that is readily available in gas-solid bubbling fluidisation. In recent years, certain thermolabile substances that need safe processing environment have been fluidised under vacuum conditions. In addition, vacuum conditions also reduce the operating temperature for many chemical processes. Despite these advantages, the vacuum fluidisation is not widely used due to its poor fluidisation quality. Lack of experimental data also deter any optimisation of its operating parameters.

The present work therefore attempts to understand the effect of vacuum pressure on the thermal performance during fluidisation. Despite the poor fluidisation quality under vacuum, the heat and mass transfer capability can be readily optimised if the effect of vacuum pressures on hydrodynamics and heat transfer is understood properly. Recent work on hydrodynamics have shown possibility to operate the vacuum fluidised bed in regions with minimal loss of quality. In addition, fluidisation maps have been of great assistance in locating optimal bubbling space that enhances the thermal performance. The results reported in the thesis would further assist a thermal engineer to design and operate a vacuum fluidised bed. Effect of pressure on fluidisation quality, segregation, bubble characteristics and heat transfer has been examined with a view of facilitating optimisation of pressure, flowrate and location of an immersed object under vacuum conditions. Moreover, models to numerically predict the fluidisation quality and hydrodynamics using CFD have been proposed. This includes a pressure gradient model integrated with the famous Gibilaro-Rowe model to predict the travelling of fluidisation interface in a vacuum fluidised bed. In addition, the Gidaspow drag model is modified to assist numerical simulation of hydrodynamics in slip flow regime. The results indicate that hydrodynamics can be predicted accurately and can be utilised in the design process with confidence.

In general, the thesis has added knowledge in the field of fluidisation in areas of quality, bubble and heat transfer characteristics along with proposing models for predicting the hydrodynamic behaviour under vacuum conditions. These details will be helpful in optimal heat and mass transfer processes.
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INTRODUCTION

Fluidisation is the achieving of a fluid like behaviour for any solid granular entities with the help of a fluidising media, such as gasses and liquids. The fluidising medium displaces and suspends the solid granular particles from their static position, transferring the momentum and resulting in fluid like motion of the particles. These particles being fluid borne are free to move from one location to another in a seemingly random manner. It is this random motion of the particles that resembles the flow of a fluid and hence the name fluidisation. A commonly used phenomena in industries such as chemical, mineral cracking, heat treatment, and surface engineering; fluidisation offers a processing environment with a wide spectrum of advantages. The ability to achieve uniformity of temperature, high solid-fluid mixing leading to high heat and mass transfer and continuous operation, makes the use of fluidisation quite appealing [1].

The basic principle of fluidisation is the suspension of bed of solid granular particles in a fluidising medium by virtue of the momentum transfer from the fluid which is balanced by the weight of the particles. This momentum exchange rate is the drag force which acts on the periphery of the solid particles and is counteracted by their weight. Under these conditions the particles which were initially resting on each other are minimally suspended in the fluidising medium. This condition is the criterion for fluidisation beyond which the particles are accelerated and displaced further till the point where the gravitational force again balances the fluid-particle interaction force. This expands the bed and the particles are displaced uniformly within the bed and suspended in the fluidising medium. Further increase in the fluid velocity gives rise to an interesting phenomenon where the fluid either begins to escape from the
Introduction

interstices between the particles, or initiates bubbles that erupt at the surface of the bed. The latter causes considerable particle mixing and gives the bed the characteristics of a boiling liquid. This quality of fluidisation is known as aggregate, or bubbling, fluidisation whereas homogeneous fluidisation is characterised by the flow of fluid through the interstices and the particles are displaced uniformly. Bubbling fluidisation is generally seen in cases where the density difference of the solid and fluid is quite large; e.g.: gas-solid fluidisation where $\rho_{\text{solid}}/\rho_{\text{liquid}}>>1$. Homogenous fluidisation occurs mostly in liquid-solid fluid beds where $\rho_{\text{solid}}/\rho_{\text{liquid}}\approx 1$. Fig.1 shows the different flow patterns in a fluidised bed. At low velocity, the fluid merely percolates through the void spaces between stationary particles and is called a fixed bed (Fig.1a). With increasing flowrate the fluidised bed expands and reaches a minimum fluidised state (Fig. 1b). For fine particles or for liquid-solid fluidisation, the bed expands smoothly for a range of flowrates after it is minimum fluidisation condition and is known as homogenously fluidised (Fig.1c). At higher flowrates, bubbling begins with increased agitation and mixing. This is known as heterogeneous fluidisation (Fig.1d). A further increase in

Fig.1: Different flow patterns in a fluidised bed[1]
flowrate introduces slugs or voids of size of the fluidised bed (Fig.1e). Turbulent fluidisation (Fig.1g) and lean-phase fluidisation (Fig.1h) takes place at very high fluidisation velocity.

Heat transfer is one of the primary uses of fluidised beds in industry. The underlying mechanism of heat transfer is conduction and convection in the gas and emulsion phase, accompanied by radiation at high temperatures (800-1000°C). The heat transfer coefficient (h), expressed as heat flux per unit temperature difference, is composed of conduction in the particle phase, and convection from the bulk movement of the particle phase and the gas (or bubble phase) [3] (eq. 1).

\[ h = f h_{bubble} + (1 - f)h_{emulsion} \quad \text{(eq. 1)} \]

where, \( f \) is the fraction of the heat transfer surface covered by bubbles.

Heat transfer is a complex phenomenon and depends upon various operating parameters in the fluidised bed. The fluidising velocity, operating pressure and temperature of the system, particle size, gas density, and shape and size of the immersed surface are important parameters that affect the heat transfer from an immersed surface. These parameters have been found [3] to influence the quality of convection and conduction of the gas and emulsion phase.

An important operating parameter that influences the heat transfer is the operating pressure. It is known that increasing the operating pressure of the fluidised bed greatly alters the heat transfer characteristics and has a varying influence depending on the particle characteristics like particle size and density [1]. For instance, it is generally found that increasing the operating pressure for Geldarts Group A particles [2] (see Fig.2) doesn’t significantly affect the heat transfer whereas for coarser and heavier particles the heat transfer is greatly enhanced [4].
is mainly due to the fact that particle convection is a more dominating factor than gas convection for fine particles.

In recent years, fluidised beds have been operated under vacuum conditions for various purposes such as drying of porous materials, pyrolysis of hydrocarbons, and coating processes like chemical vapour deposition (CVD). Vacuum conditions offer safe operating atmosphere for many thermolabile substances and have been found to be advantageous in mineral cracking and drying technology [5-7].

Fluidisation in vacuum conditions is characterised by slip flow regimes that exist due to increased mean free path of the fluid ($\lambda$). The mean free path is the average distance travelled by particles before mutual collision. The Knudsen number ($Kn = \frac{\lambda}{d}$) is a non-dimensional number which characterises the slip flow regime and is the ratio of the mean free path of the fluid to the appropriate length scale in the vacuum system. In fluid mechanics, an increase in
mean free path gives rise to various flow regimes, such as molecular flow \((Kn \gg 1)\), transitional flow, slip flow \((Kn \approx 1)\) and continuum flow \((Kn \ll 1)\) [8]. Thus, the fluidisation undergoes these regimes with reduction in pressure and affects the hydrodynamics of the bed.

Although literature reports prior work on fluidisation at low pressure, there are certain gaps in knowledge that need to be addressed for the benefits of low-pressure fluidisation to be utilised optimally. In particular, the effect of vacuum pressure on hydrodynamics of the fluidised bed such as the fluidisation quality, powder characteristics and bubble dynamics. Knowledge of heat transfer from immersed surface and the effect of axial location under vacuum fluidisation is essential for optimising thermal performance. In addition, prediction of hydrodynamics using CFD models need modifications to account for slip flow regime. This thesis aims to answer these research question in order to enhance our knowledge of low pressure fluidisation.

A critical literature review follows in **Chapter-2** to understand the existing depth of knowledge relevant to the present project. The Literature review covers important aspects of hydrodynamics, heat and mass transfer capabilities of fluidised beds and their use in high and vacuum pressures. The understanding of these areas of fluidisation will be used to find fundamental gaps of knowledge and thus define the aim and scope of the present project.

**Chapter 3** (Published paper) experimentally investigates the effect of the segregation and pressure gradient on the quality of fluidisation in a cylindrical vacuum fluidised bed. The results attempt to estimate the relative contribution of the segregation of particle and significant pressure gradient on quality.

**Chapter 4** proposes an analytical model based on the expanding gas theory to estimate the fluidisation interface in the bed under vacuum conditions. Utilisation of this model leads to representation of fluidisation maps for different particle size and pressures.
Chapter 5 presents experimental heat transfer results for the optimal powder characteristics and quality as estimated by Chapter 3. A parametric study is carried out to examine the effect of location of immersed object, pressure and particle size on heat transfer at low operating temperatures.

Chapter 6 (combination of published paper and conventional chapters) proposes a new slip flow drag model expected to accurately predict the bubble characteristics in comparison to the existing drag models. In addition, an experimental validation of the slip flow drag model along with study of bubble characteristics as size, growth and velocity is carried out in a pseudo two dimensional vacuum fluidised bed.

Chapter 7 attempts to combine the results of the previous chapters to answer the research question. Conclusion and suggestions for future work is provided in this chapter.

References


CHAPTER 2

LITERATURE REVIEW

This chapter will introduce the basic principles behind the hydrodynamics and heat transfer in fluidised bed reactors followed by a critical review of the recent research results concerning high pressure and vacuum pressure effects. Gaps in knowledge will be summarised towards the end of the present chapter along with related research questions.

1. Hydrodynamics in fluidised beds

Fluidised bed reactors (FBR) have been used in heat and mass transfer applications as they offer a wide array of advantages arising from the complex multiphase hydrodynamics. Liquid-solid, liquid-solid-gas and solid-gas fluidised beds are used in industries as processing environments to carry out surface treatment of metals, catalytic cracking, coating of pharmaceutical products, coal gasification and many such heat-mass transfer processes. The presence of fluid bubbles and the assisted mixing of solid particles produces high heat and mass transfer capability of fluidised beds [2]. Consequently, studies of hydrodynamic fluidised beds are of prime importance. Achieving the optimal performance of FBRs requires the understanding of many parameters - minimum fluidisation velocity, bubble shape, size, and rise velocity, the expansion of bed weight, and the effect of particle shape, size and morphology. The following sections discuss the various analytical and semi-empirical correlations available in the literature that incorporate various operating parameters and are often used in the design and operation of FBRs.
1.1 Minimum fluidisation velocity

Minimum fluidisation velocity ($U_{mf}$) denotes an operational variable relative to which other hydrodynamic parameters such as bubble rise velocity, size and growth and bed expansion are defined. $U_{mf}$ is defined as the velocity at which the particles in the fluidised bed are minimally suspended against their weight and when the normal compressive force between the adjacent particles vanishes [2].

Thus, $(drag \ force \ by \ upward \ moving \ fluid) = (weight \ of \ particles)$

$$\Delta P A = AH_{mf}(1 - \varepsilon_{mf})\left[(\rho_s - \rho_g)g\right] \quad (Eq. \ 1)$$

Where, $\Delta P$ : Pressure drop through the bed, N/m²

$A$ : the cross-sectional area of the bed, m²

$H_{mf}$ : Height of bed at minimum fluidisation, m

$\varepsilon_{mf}$ : void-fraction at minimum fluidisation.

$\rho_s$ : density of solid, kg/m³

$\rho_f$ : density of fluid, kg/m³

g : acceleration due to gravity, m/s²

Experimentally, $U_{mf}$ is obtained from pressure drop vs. superficial velocity plots. As the flowrate increases, the pressure drop of the fluidising fluid increases linearly as the momentum transfer increases to the particles. The velocity at which the pressure drop equals the bed weight is known as the minimum fluidising velocity where the bed is suspended against its weight. The $U_{mf}$ depends on various operational parameters such as particle size, operating pressure and temperature. Operating temperature and pressure changes the density of the inlet gas which affects the fluidisation velocity. Thus, it has been observed that $U_{mf}$ decreases with increase of
pressure above atmospheric and increases with decrease of pressure below atmospheric [2, 19, 20]. Similarly, since density of gas is inversely proportional to the temperature, the $U_{mf}$ decreases as temperature increases [2].

Particle size influences the minimum fluidisation velocity due to the resistance offered by the particle surface (drag). Larger the particle size, higher the $U_{mf}$ requirement due to increased drag force. It is known that finer particles (Geldarts Group A) do not show dependency on operating pressure and temperature while coarser particles (Group B) are significantly affected [21]. Geldarts group is discussed in the following section.

Additionally, the wall effect has also been observed to affect the minimum fluidisation velocity of particles and are of significance when a pseudo two dimensional fluidised bed is used. Glicksman and McAndrews [22] observed that the minimum fluidisation velocity is increases with reduction of bed thickness which is confirmed by Saxena and Vadivel [23] who observed that decreasing the bed thickness offers greater resistance to particle motion especially if the particle size are relatively coarser.

### 1.2 Particle classifications

Powder groups are generally denoted by their mean diameter wherever they are used for classification. In reality, powders have a non-uniform distribution of particle size. Different particle size and distribution measurement techniques are available such as particle sieving, laser diffraction, image analysis, sedimentation, acoustic spectroscopy etc. [8, 24]). The particles size distribution is represented by gauss normal and log normal plots [15, 25].

The particles size and morphology play a very important role in determining the fluidisation characteristics. In order to systematically study the effect of particle characteristic on fluidisation, the powders are often classified according to their density, size (Geldarts groups) [5] and the hydrodynamic and thermal property of the fluidised bed (Saxena’s groups) [26].
Geldarts classification: From observations of different particle size and densities, Geldart proposed four distinct particle behaviours [5]. With increasing particle size they are:

- **Group C**: These are very fine cohesive powders and do not fluidise normally as the inter-particle forces overcome the drag force by the gas.

- **Group A**: These are aeraable and have small mean particle size and low particle density (less than 1.4 g/cm³). The Group A powders show smooth fluidisation followed by bubbling at higher flowrate.

- **Group B**: These are the most favourable group of powders due to their vigorous bubbling at all the fluidisation velocities. The particle are in size range of 40-500 μm and density of 1.4-4 g/cm³.

- **Group D**: These are larger dense particles that often result in spout beds. They behave erratically and results in spouting behaviour.

The Geldart group (Fig.1) is the most widely used powder classification. The classification however was based on experimental data of powders at atmospheric pressures. Powders are known to change their fluidisation characteristics with different operating pressures [20, 27].

Saxena’s group: The thermal and hydrodynamic properties of the fluidised beds were considered in classifying different particles according to the Archimedes (Ar) and Reynolds (Re) number [26]. Archimedes number denotes the ratio of gravitational force to viscous force and relates the density difference between the fluid and particles. Reynolds number on the other hand relates the inertial forces to viscous forces and is used to characterise the flow as laminar or turbulent. Particle size is used as the characteristic length to calculate the Reynolds no.
**Group I** : The particles with $1 \leq \text{Re} \leq 10$ and $3.55 \leq \text{Ar} \leq 21,700$ belong to Group I and the flow around the particles are laminar. The voidage at minimum fluidisation conditions, $\varepsilon_{mf}$, remains constant with changing $\text{Re}_{mf}$ and $\text{Ar}$.

**Group IIA** : $10 \leq \text{Re} \leq 40$ and $21,700 \leq \text{Ar} \leq 130,000$ classify the particle as Group IIA where the laminar boundary layer becomes increasingly turbulent with wake formation along the downstream of the particles. The $\varepsilon_{mf}$ decreases with increasing $\text{Re}_{mf}$ and $\text{Ar}$.

**Group IIB** : Particles have $40 \leq \text{Re} \leq 200$ and $1.3 \times 10^{5} \leq \text{Ar} \leq 1.6 \times 10^{6}$ with complete turbulence with the separation point present along the downstream of the equatorial plane of the particle diminishing the wake size. The $\varepsilon_{mf}$ increases monotonically with increasing $\text{Re}_{mf}$ and $\text{Ar}$.

**Group III** : Particles are characterised by $\text{Re} \geq 200$ and $\text{Ar} \geq 1.6 \times 10^{6}$. The $\varepsilon_{mf}$ remains constant with a complete turbulent flow in the bed.
1.3 Particle size

Most of the particles used in fluidised beds are not spherical, and therefore, an effective diameter or size is defined. An equivalent spherical diameter ($d_{\text{sph}}$) is defined as the diameter of the sphere having same volume as the particle. Further, a parameter known as sphericity ($\phi$) can be defined which compares the surface area of the sphere and the particle with equal volume.

In order to estimate the distribution of particle size in a given sample of powder, various techniques are available such as image analysis, sedimentation technique, laser diffraction etc. Every powder sample contains a statistical distribution of particles of different size [24]. The particle size is commonly represented either as a frequency distribution curve or a cumulative (undersize) distribution curve such as:

- Number weighted distributions
- Volume weighted distributions
- Weighted distributions
- Intensity weighted distributions.

A range of statistical parameters can be utilised to interpret particle distributions. Commonly used are mean, median and modes [28] (Fig. 2). For a symmetric distribution of particle size (as in a Gaussian curve) these parameters are equivalent, however they are discreet values for a skewed distribution. Most commonly used means are:

- Number length mean ($D[1,0]$) : It is often referred to as arithmetic mean and can be used only when the total no. of particles are known.
- Surface area moment mean or Sauter mean diameter ($D[3,2]$): It is defined as the size of a sphere that has the same ratio of volume/surface area as a particle of interest.
- Volume moment mean or De Brouckere Mean diameter (D[4,3]) : It is the weighted average volume diameter, assuming spherical particles of the same volume as actual particles.

Fig. 2: A particle size distribution showing the mean, median and mode for a symmetric and skewed distribution [8]

The choice of the mean diameters depends on the reason of interest. For instance (Fig.3), D[4,3] is used to monitor the size of the coarser particles that makes up the bulk of the powder whereas D[3,2] is most appropriate for monitoring proportions of fines in a sample [28]. The number length mean (D[1,0]) is not used in most calculations since the number of particles is an essential information. The moment means do not require the number of particle information.
For volume weighted particle size distributions, percentiles are used to monitor significant changes in the main particle size along with changes at the extremes of the distributions. Oversized particles and agglomerates can be identified easily with percentile diameters. $D(0.1)$,

Fig.3: A particle size distribution showing Sauter mean diameter ($D_{3,2}$) and volume moment mean diameter ($D_{4,3}$) [4]

Fig.4: The percentile diameters representing the particle size distribution for a modal and bimodal distributions [4]
D(0.5) and D(0.9) (Fig.4) and the widely used percentiles which define 10, 50 and 90% of the particle sizes are below the percentile. D(0.5) is also known as a median.

Another important parameter that is widely utilised is the width ratio of the distributions, which is the ratio of standard deviation (σ) and the representative diameter (d_{rep}) of the distributions. For segregation studies using continuous range of particles, segregation index has been correlated with the \( \frac{\sigma}{d_{rep}} \) ratio [15, 25]. Gaussian and Lognormal curves are often used to represent the particle size in segregation studies. Chew et.al.,[15, 25] studied the axial segregation in bubbling fluidised beds with Gaussian and Lognormal distributions of Geldart Group B powders and concluded that wider Gaussian distributions show increased segregation. Also, for segregation studies, Gaussian curves are preferable than Lognormal as the segregation index shows a monotonic behaviour for Gaussian curves (Fig.5).

The Gaussian and lognormal distributions are denoted as:

\[
 f_{m, \text{Gaussian}} = \frac{1}{\sigma \sqrt{2 \pi}} \exp \left[ -\frac{(x-d_{mean})^2}{2\sigma^2} \right] 
\]

(eq. 2)

\[
 f_{m, \text{Lognormal}} = \frac{1}{\lambda \sigma \sqrt{2 \pi}} \exp \left[ -\frac{(\ln(x)-d_{mean})^2}{2\sigma^2} \right] 
\]

(eq. 3)

1.4 Bubble characteristics

Bubbles in solid-gas fluidisation enhances the heat and mass transfer capability and hence knowledge of their characteristics are of prime importance. Bubble size and its growth, bubble velocity, eruption frequency, maximum size are some of the parameters that has been utilised to understand and optimise the design of the fluidised beds [6]. Most of the correlations predicting the bubble properties in a fluidised beds are empirical and semi-empirical with fitted parameters from the experimental data. Both intrusive and non-intrusive methods have been
utilised to measure the bubble properties. Optical, pressure, acoustic, electric sensors intrude the bubbles rising in the fluidised bed to gather information while techniques such as high speed photography [29], X-ray [30], Electrical Capacitance Tomography (ECT) [31] and recently magnetic resonance [32] have been used without disturbing the flow. Each method of measurement have their advantages over the other. The intrusive probes have generally been criticised for smaller values of bubble size mostly due to the fact that the probe may not always measure the maximum vertical dimension [33-39]. They are however a cost effective option for bubble measurements. Imaging/photography techniques, on the other hand, provide a comprehensive insight into the bubble dynamics but involve costly technology. The photography techniques are limited to two dimensional version of the fluidised beds. Electro-capacitance tomography (ECT), X-ray and NMR are hence the most popular choice to image the interior bubbles in a three dimensional fluidised beds [40-48]. Their application is however limited due to size restrictions and inability to distinguish between overlapping bubbles [49]. Fig 6 shows the different aspects of a bubble that can be measured by the intrusive probes and the imaging techniques. Glicksman et.al [50] determined the relation between the data obtained from fibre optic probe and video recordings.

Fig.5: Plot showing the correlation between segregation index and $\sigma/d_m$ for Gaussian and lognormal distribution [15]
Davies and Taylor [51] were the first to model the mechanics of bubble rise in liquids and most of the correlations used currently for bubble rise are based on their pioneering work. Table 1 lists the various correlations used to estimate bubble rise velocity in a fluidised bed along with correlations for bubble size. Knowledge of these bubble characteristics thus assists the understanding of bubbling fluidisation: the most desirable fluidisation for heat and mass transfer.
Table: 1. Correlations from literature on bubble characteristics

<table>
<thead>
<tr>
<th>Correlations for bubble size in fluidised bed</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $y = 3.274 \rho_p d_p \left( \frac{u_0}{u_{mf}} - 1 \right)^{0.63} h$ [52]</td>
</tr>
<tr>
<td>2. $y = 1.4 \rho_p d_p \left( \frac{u_0}{u_{mf}} \right) h$ [53]</td>
</tr>
<tr>
<td>3. $y = 0.34 \left( \frac{u_0}{u_{mf}} \right)^{0.33} h^{0.54}$ [54]</td>
</tr>
<tr>
<td>4. $d_f = -4.08 + 0.0534h + (0.173h + 2.2) \frac{u_0}{u_{mf}}$ [55]</td>
</tr>
<tr>
<td>5a. Porous distributor: $y = 0.64(U_0 - U_{mf})^{0.4} + 0.019h(U_0 - U_{mf})^{0.94}$ [46]</td>
</tr>
<tr>
<td>5b. Perforated distributor: $d_f = 1.43g^{-0.2} \left( \frac{u_0 - u_{mf}}{n} \right)^{0.4} + 0.019h(U_0 - U_{mf})^{0.94}$ (Geldart, 1972)</td>
</tr>
<tr>
<td>6. $\frac{d_{bm}-d_v}{d_{bm}-d_o} = \exp \left( -\frac{0.3h}{D} \right) d_{bm}$ [56]</td>
</tr>
<tr>
<td>7. $d_v = d_0 \left[ 1 + 0.272(U_0 - U_{mf}) \right]^{\frac{1}{2}}(1 + 0.0684h)^{1.21}$ [57]</td>
</tr>
<tr>
<td>$d_0 = 0.61$ for Geldart A</td>
</tr>
<tr>
<td>$d_0 = 0.853$ for Geldart B</td>
</tr>
<tr>
<td>$d_0 = 1.23$ for Geldart D</td>
</tr>
<tr>
<td>8. $d = \frac{(u_0 - u_{mf})^{1/2}(h+h_o)^{3/4}}{g^{1/4}}$ [58]</td>
</tr>
<tr>
<td>9. $d_v = 0.54g^{-0.2}(U_0 - U_{mf})^{0.4} (h + 4A_0^{0.5})^{0.8}$ [59]</td>
</tr>
</tbody>
</table>
10. \( d_v = d_0 + \frac{k_3}{11.13 m} h^5 d_0^{0.5} + \left( \frac{k_3}{12.6 m^{(m-1)}} \right) h^{2s} \); \( k_3 = 82, m = 10, s = 0.4 \) [60]

11. \( F(d_v) - F(d_{b0}) = \frac{(h-h_{b0})^{13/4}}{4 (g_{mf})^{5/4}} \) [50]

\[
F(d) = \left( \frac{0.71 \sqrt{gd}}{U_0 - U_{mf}} + 1 \right) ^{\frac{9}{4}} \left( \frac{0.71 \sqrt{gd}}{U_0 - U_{mf}} - \frac{4}{9} \right) ^{\frac{9}{4}} + \frac{4}{9} ; C = 0.3
\]

12. \((U_0 - U_{mf})(d_v - d_0) + 0.474g^{0.5}(d_v^{1.5} - d_0^{1.5}) = 1.132(U_0 - U_{mf})h \) [61]

**Correlations for single bubble rise velocity in the fluidised bed:**

13. \( U_{br} = 0.71 \sqrt{gd_b} \) [62]

14. \( U_{br} = 0.71 g^{1/2} V_b^{1/6} \) [63]

15. \( U_{br} = 0.74 g^{1/2} V_b^{1/6} \) [64]

16. \( U_b = 0.35 \sqrt{gd_b} \tan^{0.555} \left( 3.6 \left( \frac{d_b}{D} \right)^{0.9} \right) \) [65]

17. \( U_{br} = k_p \sqrt{g \frac{d_b}{2}} \) [44]

\[
k_p = 1.38 - 0.00182 d_p
\]

**Correlations for bubble velocity in a fluidised bed**

18. \( U_b = U_{br} + (U_0 - U_{mf}) \) [62]

19. \( U_b = \varphi \sqrt{gd_b} \) [57]

For Geldart A (FCC): \( \varphi = \begin{cases} 
1 & D \leq 10 \\
0.396D^{0.4} & 10 < D < 100 \\
2.5 & D \geq 100 
\end{cases} \)
1.4.1 Bubbling fluidisation

Heterogeneous fluidisation is characterised by the presence of bubbles in a particulate bed. These bubbles are regions of higher void fraction and the denser regions are known as the emulsion phase where the solid phase is dominating. A bubble is generally considered only if its size is more than 10 times the diameter of the particles [1]. The bubbles originate at the surface of the distributor plate and grow through the bed height due to coalescence. According to Harrison [1], the type of fluidisation (homogenous or bubbling) depends upon the conditions prevailing for formation of a minimum size bubble. When the circulation velocity of the fluid

\[
\begin{align*}
\text{For Geldart B (Sand)} & \quad \varphi = \begin{cases} 
0.64 & D \leq 10 \\
0.254D^{0.4} & 10 < D < 100 \\
1.6 & D \geq 100
\end{cases} \\
\text{For Geldart A and } D \leq 100 \text{ cm:} & \quad U_b = 0.34 \left( (U_0 - U_{mf}) + 14.1(d + 0.5) \right) D^{0.33} + U_{br} \\
\text{For Geldart B and } D \leq 100 \text{ m:} & \quad U_b = 0.0032 \left( (U_0 - U_{mf}) + 11.3d_b^{0.5} \right) D^{1.35} + U_{br} \\
\text{The wall effect is important for } db/D < 0.125: & \quad U_{br} = 0.711\sqrt{gd_p} \\
\text{And for } 0.125 \leq db/D < 0.6: & \quad U_{br} = (0.71\sqrt{gd_b})1.2\exp(-\frac{1.49d_b}{D})
\end{align*}
\]

For \(db/D \geq 0.6\) slugging is more probable [2].

Where \(y\) : pierced length (Fig.6); \(U_o\): superficial gas velocity (cm/s); \(h\): distance from distributor (cm); \(d_f\): frontal diameter (cm); \(d_{bm}\): max. possible bubble diameter (cm); \(d_v\): volume avg. eq. bubble diameter (cm); \(U_{br}\): single bubble rise velocity (cm/s); \(V_b\): bubble volume (cm\(^3\)); \(D\): fluidised bed diameter (cm)
within the bubble is less than the terminal fall velocity, bubble growth occurs. However, when the circulation velocity exceeds the terminal fall velocity of the particles, the particles from the wake are dragged inside the bubble, resulting in the bubble dissolution [1].

The velocity at which fluid starts to bubble in a fluid bed is known as the minimum bubbling velocity, $U_{mb}$. For powders belonging to different groups characterised by size ($d$) and density ($\rho$) (Geldart’s Group), $U_{mb}$ has varying characteristics. For instance, a fluid bed composed of Group A powders the bed expands uniformly prior to bubble formation and produces a “smoother” fluidisation quality when fluidised beyond minimum fluidisation conditions [2]. However, Group B particles bubble instantaneously at the minimum fluidisation condition [2].

The fluid flow through the bubbles have a significant effect on the quality of solid-gas interaction, in particular, the movement of the emulsion phase either in the wake or around the

![Fig.7: A two-dimensional bubble](image1)

![Fig.8: (a) Streamline around the bubble. (b) Particle movement around the approaching bubble (dark line numbered 1-4)](image2)
bubble. Unlike a liquid gas, bubbles in a fluid bed are elliptical in 2D beds, and or a distorted spherical shape in 3D beds where the base of the sphere is indented [1].

Fig. 7 shows a two-dimensional bubble and the particle flow around it. It is observed that the flow of particles around the bubble is similar to the flow of a fluid around a rigid spherical body with streamlines passing smoothly around the bubble. From the frame of reference of the bubble, the particle motion is revealed (Fig. 8b). When the bubble approaches a axial location in the bed particles that surround the bubble drift sideways perpendicular to the direction of flow of bubble. As bubble travels away, the particles return to their respective radial position but at a lower height. This process mixes the particles from a higher to lower axial location. In Figure 8a the streamline is shown from the emulsion frame of reference and Figure 8b shows the particle path from the bubble frame of reference. It can be seen that the particles moves sideways (along the horizontal axis) and finally form a looping motion (dark line numbered 1-4) resulting in a net upward displacement after the bubble has passed [6]. This is one of the basic mechanisms of mixing inside a fluid bed that creates a high degree the

Fig. 9: a) Bubble flow pattern in a shallow bed; b) Bubble flow pattern in a deep bed [6]
uniformity through a bed, which is desired in heat transfer applications [6].

The bubble path shows a strong dependence on the aspect ratio (AR) of the bed [2, 6]. In shallow beds (AR<2), the bubble flow takes place away from the centre line and the emulsion phase flows down the centre line and has an upward motion near the end walls. However, in deep beds (AR>2), the bubbles converge and grow along the centreline and are accompanied by the emulsion phase in its ascent along the centre line, giving rise to downward flow of the emulsion near the walls. Figure 9 summarises the macroscopic flow of bubbles and the emulsion phase in fluid beds with varying AR [6].

### 1.5 Segregation in fluidised beds

Segregation in a fluidised bed is defined as separation of a single type of particle from the general populace. The separation occurs due to size or density difference and occurs as soon as the fluid passage enables the movement of the particles in the fluidised beds. Both gas-solid and liquid-solid fluidised bed experience segregation of particles but for different reasons. The movement of bubbles is mainly responsible for segregation in gas-solid fluidisation, while density difference is operative in liquid-solid fluidisation [1]. Initial experiments by Nienow and Cheesman [66], Nienow and Chiba [67] and Altenkirch et.al., [68] used words like “flotsam” and “jetsam” to describe the top and bottom particle layers after segregation. The heavier or larger particles settle at the bottom of the bed while a lighter and finer particles float at the top of the bed. An inverse phenomenon also occurs called “layer inversion” in liquid-solid fluidised bed where flotsam and jetsam switch their defined location after mixing at higher flowrates [69].

Size segregation has been found to increase by increasing the bed height, increasing gas flow, widening the size distribution by decreasing the size of fines and increasing the mean size of powders [70]. Experimental works on size segregation usually present the volume/weight
fraction of jetsam and flotsam particles along the height of the fluidisation column for different operating parameters. The fluidised bed is divided into horizontal segments and the powders in each section is collected and analysed [71]. The data is presented in a visual form of variation of concentration from the mean concentration. Widely used analysis parameters such as coefficient of segregation \( C_v \) and the mixing index \( M \) are used to quantify and compare segregation of different fluidisation systems [71] and are defined as:

\[
C_v = \frac{X_B - X_T}{X_B + X_T} \times 100 \quad (\%)
\]

Where \( X_B \) and \( X_T \) are the concentration of material of interest in the botton and top halves respectively,

\[
M = \frac{X}{\bar{X}}
\]

Where \( X \) is the concentration of the larger/denser particles at the top of the bed and \( \bar{X} \) is its average concentration in the bed. Binary powder samples have been recently classified according to their segregation behaviour by Rao et.al. [13] using density, size and velocity ratio \( \frac{u_{min,flotsam}}{u_{min,jetsam}} \) to define a segregation index \( S \);

\[
S = \left( \frac{x_i - x_m}{1 - x_f} \right)
\]

(\text{eq. 4})

Where, \( x_i \) is the mass fraction of jetsam in each horizontal layer, \( x_m \) is the final flotsam fraction in the bed at the end of experiment, and \( x_f \) is the initial composition of the jetsam in the bed.

The binary mixtures can be classified as type A-D according to [13] and are represented graphically in Fig. 10:

- **Type A**: Particles with very large particle size ratio \( (d_i > 4.5; U_i > 8) \) which fluidize at two distinct points when fluidised from a completely segregated stage. The pressure
drop profile for an initially mixed state shows a linear increase followed by an erratic
Fig. 10: Classification of binary mixture on the basis of level of segregation [13]
region. On the other hand, the pressure drop curve for an initially segregated powders show two distinct linear profiles, denoting the fluidisation of flotsam and jetsam particles sequentially. There is a distinct change of slope which denotes the partial fluidisation of the bed. Upon complete fluidisation the pressure drop profile remains constant. The velocity required to fluidise the bed is slightly larger than the minimum fluidisation velocity of the jetsam alone. The segregation index is seen to decrease and then increase after a critical flowrate. This occurs due to larger bed expansion of flotsam than jetsam at higher flowrates.

- **Type B**: Particles having significant level of disparity in size and density ($\rho_r > 3$ or $4.5 > d_r > 3.3; 4.2 < U_r < 8$) belong to Type B who have similar pressure drop profiles as Type A. The segregation index however shows a monotonic decreasing trend with flowrate increase.

- **Type C**: These powders have intermediate level of disparity in size and density ($2 < \rho_r < 3$ or $2 > d_r > 3.3; 2.5 < U_r < 4.2$). The pressure drop profile shows a linear increase with a single peak behaviour as shown in the Fig. 10. The segregation index is large at lower velocity followed by a very low constant value at higher flowrates representing a complete mixing stage.

- **Type D**: Mixtures having minimal disparity in particle size and density ($1 < \rho_r < 2$ or $1 > d_r > 2; 1 < U_r < 2.5$) behave like a single component bed that fluidised at a single point. They show good mixing even at lower velocities.

Continuous particle distribution powders occur more commonly than single component mixtures. Segregation in a distributed powder depends on the width of the particle distributions often represented as a Gaussian or Log normal curve [25]. The width of
the particle distribution also influences the bubble properties in the bed namely frequency, and velocity. The local particle distribution of any powder sample in the bed is known to be similar to the overall powder distribution.[15]. Similar to binary or ternary mixtures, larger and finer particles separate at the bottom and top of the bed, respectively.

Various mechanism have been proposed and reported that contribute to the segregation process. Consequently, a number of analytical and numerical models have been proposed in the literature that are used to predict the concentration profiles for different operating parameters. The Gibilaro-Rowe (GR) [10] model is the fundamental model used to estimate the segregation distribution for binary mixtures in a gas-solid fluidised bed. It is a steady state differential formulation of physical mechanisms of segregation that accurately predicts the segregation profiles for the binary mixtures. The parameters used in the GR model have been studied by Naimer et.al, [72] and Tanimoto et.al., [73] to link them to the fluidisation behaviour. The basic mechanisms that affect segregation in gas-solid fluidised beds are:

- **Circulation**: Solids that are carried up by the wake of the bubble contributes to the circulation. \( w \) is the volumetric flowrate \( (L^3/T) \) with which the jetsam travels from the bottom of the bed to the top resulting in a total circulation of \( wC_B(z) \) of the bulk phase and from top to the bottom with a circulation of \( wC_w(z) \). \( C_B \) and \( C_w \) are the volume fraction of jetsam in bulk and wake phase, respectively.

- **Exchange**: The exchange of solids between bulk and wake phase contributes to separation of jetsam and flotsam. The rate of exchange is defined as the total volumetric rate of exchange per unit height of bed \( (L^2/T) \).

- **Axial mixing**: A pseudo-diffusional coefficient is defined to account for axial displacement of the solids due to movement of bubbles. A parameter \( r \) with dimensions \( L^4/T \) describes the total axial mixing of the jetsam as \( (r/H)(dC_B/dZ) \).
**Segregation:** The segregation coefficient accounts for the separation of particles on the basis of their physical characteristics such as density and size. A parameter \( k \) with dimensions \( L^3/T \) is used to define segregation of particles.

Gibilaro and Rowe [10] formulated a governing equation (Fig. 11) by considering the above mechanism which can be solved analytically and numerically to yield the concentration profile of the jetsam. These governing equations are:

\[
\frac{r}{H} \frac{d^2 C_B}{dz^2} + (w + k - 2kC_B) \frac{dC_B}{dz} + qh(C_w - C_B) = 0 \quad \text{(eq. 5)}
\]

and

\[
w \frac{dC_w}{dz} - qh(C_B - C_w) = 0 \quad \text{(eq. 6)}
\]

![Fig. 11: The Gibilaro and Rowe model [10]](image-url)
The drawback of the analytical solution to these governing equations is the use of approximate parameters which are semi-empirical in nature with fitted coefficients from various experiments and thus have limitations. Numerical solutions on the other hand yields more accurate concentration profile. Hoffman et al [74] was the first to solve the governing equations for cases without axial mixing numerically. Leaper et al [75] solved the complete Gibilaro and Rowe model numerically with accurate predictions. These solutions are however valid only for binary mixtures. No attempt has been made to consider continuous powder distributions. CFD solutions, however, have been utilised to predict the hydrodynamics of fluidised beds with single and multiple particle sizes. The numerical solutions are based on the first principles of conservation of mass and momentum and hence are more accurate than the analytical models. Nevertheless, even the numerical models depend on the semi-empirical models to provide proper closures to certain indeterminate terms in the governing equations as solid effective stress tensors and the interaction forces shared between the phases. DQMOM (Direct Quadrature Method of Moments) have been used to solve polydisperse fluidised beds [76] as they provide efficient method to solve the population balance models.

1.6 Numerical solution of fluidised bed hydrodynamics

To understand the complex multiphase flow behaviour inside gas-solid fluidised beds the mathematical models proposed mainly falls under four groups depending on how they treat each phase and the magnitude of the length scales. These are (1) Discrete Bubble model, (2) Two-fluid model (TFM), (3) Discrete Particle model and (4) Molecular Dynamics mode, where the gas-solid phases are considered to be either Eulerian or Lagrangian [77]. Selection of these models depends mostly on the geometry to be modelled and the available computing resources. For instance, the Eulerian-Eulerian models are used readily for lab/pilot scale fluidised bed models whereas due to limitation on computational resources the Lagrangian-Lagrangian...
models are limited to miniature scale models due to requirement of input of number of particles y. Of these, most popular are the Two-Fluid models (Eulerian-Eulerian) where the two phases are modelled as interpenetrating continua. Each of the two phases are modelled as separate fluid (gas and solid) and is solved by Eulerian method (classical Navier-Stokes equation) [30]. The solid-fluid coupling is given by drag force that appears in the momentum balance equation for each phase and is equal in magnitude but opposite in direction. Anderson and Jackson [78] formulated by the overall averages of local mean variables of each phase to translate the Navier-Stokes equation for the fluid directly into mass and momentum continuum equations for the solid and gas phase. The following set of equations completely define the mass and momentum differential equations for the solid and gas phase[30, 78]:

**Continuity equations**

*Gas phase:*

\[
\frac{\partial (\varepsilon_g \rho_g)}{\partial t} + \nabla \cdot (\varepsilon_g \rho_g \vec{v}_g) = 0 \quad (eq. 7a)
\]

*Solid phase*

\[
\frac{\partial (\varepsilon_s \rho_s)}{\partial t} + \nabla \cdot (\varepsilon_s \rho_s \vec{v}_s) = 0 \quad (eq. 7b)
\]

**Momentum Equations**

*Gas phase*

\[
\frac{\partial (\varepsilon_g \rho_g \vec{v}_g)}{\partial t} + \nabla \cdot (\varepsilon_g \rho_g \vec{v}_g \vec{v}_g) = -\varepsilon_g \nabla P_g + \nabla \cdot \tau_g + \varepsilon_g \left( \vec{v}_g - \vec{v}_s \right) + \varepsilon_g \rho_g \vec{g} \quad (eq. 8a)
\]

*Solid phase*
\[
\frac{\partial (\varepsilon_s \rho_s \bar{v}_s)}{\partial t} + \nabla \cdot (\varepsilon_s \rho_s \bar{v}_s \bar{v}_s) = -\varepsilon_s \nabla P_g - \nabla P_s + \nabla \cdot \tau_s - F_s (\bar{v}_g^s - \bar{v}_s^s) + \varepsilon_s \rho_s \bar{g} \quad \text{(eq. 8b)}
\]

where \( \tau_g = \mu_g \left[ \nabla \bar{v}_g^s + \nabla^T \bar{v}_g^s \right] - \frac{2}{3} \mu_g (\nabla \cdot \bar{v}_g^s) I \) \quad \text{(eq. 9a)}

\[
\tau_s = \mu_s \left[ \nabla \bar{v}_s^s + \nabla^T \bar{v}_s^s \right] + \left( \lambda_s - \frac{2}{3} \mu_s \right) (\nabla \cdot \bar{v}_s^s) I \quad \text{(eq. 9b)}
\]

and \( 1 - \varepsilon_g = \varepsilon_s \)

For the problem to be completely defined the governing equations requires closures for the solid-phase pressure \( (P_s) \), solid-phase shear viscosity \( (\mu_s) \) and the solid-phase bulk viscosity \( (\lambda_s) \). These constitutive equations are derived from kinetic theory of granular flow and are presented in Table.2. Apart from these closures, kinetic theory of granular flow requires the solution to the transport equation for the granular temperature (Eq.10). Granular temperature, \( \Theta \), signifies the random motion of the solid particles and is analogous to temperature definition according to Kinetic theory for dense gases [79].

\[
\frac{3}{2} \left[ \frac{\partial (\varepsilon_s \rho_s \Theta_s)}{\partial t} + \varepsilon_s \rho_s \bar{v}_s \Theta_s \right] = (-P_s I + \tau_s) : \nabla \bar{v}_s^s + \nabla \cdot (k_s \nabla \Theta_s) - \gamma + \phi_s \quad \text{(eq. 10)}
\]

Solutions to the mass and momentum equations have been carried out for numerous cases in fluidised beds [80-86]. A two dimensional bed with bubble injection by a jet is the most popular numerical model to be solved by numerous researchers [80-86]. The advantage of knowledge of definite bubble shape and size from the experimental data enables validation of numerical solutions. A number of numerical studies have improved the application of TFM to accurately describe the hydrodynamic behaviour of a fluidised bed [87-90].

In recent years, hybrid models involving CFD and DEM have been tested to improve the accuracy of the numerical predictions and avoid the complex empirical closures to traditional TF models [91, 92]. The major disadvantage with a complete discrete solution is the availability
of computing resources and time. Most of the DEM studies involve a very small scale models of the fluidised bed due to the limitation of number of particles. The solutions are however highly accurate [93-95].

Table 2. Closure Equations used in governing equations

\[ p_s = \varepsilon_s \rho_s \Theta_s + 2(1 + e)\varepsilon_s^2 g_0 \rho_s \Theta_s \]

\[ \mu_s = \frac{4}{5} \rho_s d_s g_0 (1 + e) \left( \frac{\Theta_s}{\pi} \right)^{1/2} + 1.01600 \frac{5}{16} d_s^{1/2} \left( \frac{\Theta_s}{\pi} \right)^{1/2} \left( 1 + \frac{4}{5} (1 + e) \varepsilon_s g_0 \right) \left( 1 + \frac{8}{5} \varepsilon_s g_0 \right) \]

\[ \lambda_s = \frac{4}{3} \rho_s d_s g_0 (1 + e) \left( \frac{\Theta_s}{\pi} \right)^{1/2} \]

\[ g_o = \frac{1}{\varepsilon_g} + \frac{3d_{sm}}{2\varepsilon_g^2} \sum_{k=1}^{M} \frac{\varepsilon_k}{d_{sk}} \]

Srivastava and Sundaresan [13] frictional model:

\[ p_c(\varepsilon_s) = \begin{cases} F_r \left( \frac{\varepsilon_s - \varepsilon_{s,min}}{\varepsilon_{s,\text{max}} - \varepsilon_s} \right)^n, & \varepsilon_s > \varepsilon_{s,min} \\ 0, & \varepsilon_s > \varepsilon_{s,min} \end{cases} \]

\[ \mu_s^f = \frac{p_c(\varepsilon_s) \sqrt{2} \sin \phi}{2\varepsilon_s \sqrt{\bar{D}_{ij} : \bar{D}_{ij} + \frac{\Theta_s}{d_{sg}^2}}} \]

where \( D_{ij} \) is the strain rate and \( \phi \) is the internal angle of friction

\[ \gamma = 3(1 - e^2)\varepsilon_s^2 \rho_s g_0 \Theta_s \left( \frac{4}{d_s} \left( \frac{\Theta_s}{\pi} \right)^{1/2} - \nabla \cdot \frac{\varepsilon_s}{\pi} \right) \]
The sub-atmospheric fluidisation pose an interesting challenge to the validity of the CFD models in the slip flow regime. At atmospheric pressures, the Eulerian-Eulerian models consider the particles and the gas medium to the inter-penetrating continuum medium and thus are within the continuum approximation. Under vacuum (especially slip flow regime), the continuum approximation begins to break down and thus it is not known if the existing fluidisation models would predict the hydrodynamics accurately.

1.7. Hydrodynamics in high pressure fluidised beds

Understanding hydrodynamics of fluidised beds is important for the prediction of the heat and mass transfer characteristics. The particle mixing, bubble formation, formation of slugs (gas bubble occupying the entire width or diameter of the fluidised bed), affect the heat and mass transfer processes directly. The operating pressure is an important parameter that affects the hydrodynamics of fluidised beds and hence affect the heat and mass transfer capacity of the beds. Gas-solid fluidisation has been studied over a wide pressure range, from sub-atmospheric [18, 20, 96] to atmospheric to high pressures [97, 98]. With increase of operating pressures, several parameters such as minimum fluidisation velocity and minimum bubbling velocity are affected and are discussed below.

1.7.1 Minimum fluidisation velocity

For powders belonging to Geldart’s group A, an increase in pressure has no effect on minimum fluidisation velocity since for small particles the fluid-solid interaction is dominated by fluid viscosity which remains constant with pressure (flow is in laminar regime as characterised by Re (<1)). However, for larger particles belonging to Group B, the inertial forces becomes predominant which are influenced by the operating pressures[21] (flow regimes transits into turbulent flow [99]). Fig. 12 summarises the effect of pressure on different particle sizes [9]. For larger particles, increasing pressure decreases the minimum fluidising velocity A similar trend is observed in particles belonging to Group D. The effect of Reynolds number (Re=$\frac{\rho ud}{\mu}$)
for laminar range (Re < 0.5) has no effect on the minimum fluidisation velocity, whereas a monotonic decreasing trend in turbulent flow regime (Re > 500) can be observed [100].

1.7.2 Minimum bubbling velocity and voidage

The occurrence of bubbling fluidisation is a sign of instability of the equilibrium state of the particle bed and is desirable for high particle mixing. The application of pressure above atmospheric extends the region of stability and delays the occurrence of bubbles for finer particles belonging to Group A. Thus, the region of stability extends to higher velocity and consequently leads to higher voidage as governed by the Richardson-Zaki equation (eq.11).

The velocity of fluid (u), as described by the Richardson-Zaki equation [99]:

\[ u = u_t \varepsilon^n \]  

(eq.11)

Where, \( \varepsilon \) is the voidage of the fluidised bed, \( u_t \) is the terminal velocity of the particles and n is an exponent whose value varies from 4.8 for laminar flow to 2.4 for turbulent flow and is dependent on Re. For instance, Group A powders are stable up to a pressure of 90 bar at the voidage of 0.7 instead of 0.45 at atmospheric pressures [101]. For Group B particles, the bubbles occur at minimum fluidisation at atmospheric pressure [2]. They are more sensitive to
rise in pressure and a modest increase of pressure makes the fluid bed behave similar to Group A particles with increased stability [101]. Fig.13 demonstrates the behaviour of Group A and B with pressures by plotting the variation of terms T1 and T2 in eq.12 with pressure and voidage [27]. Stability is achieved when T1>T2.

\[
\left[ \frac{g d_p (\rho_p - \rho)}{u_l^2 \rho_p} \right]^{0.5} - 0.56n (1 - \epsilon_{mb})^{0.5} \epsilon_{mb}^{n-1} = \begin{cases} \text{positive,stable} \\ \text{negative,unsatble} \end{cases}
\]

(eq.12)

With T1 and T2 being the first two terms on LHS of eq.12.

1.7.3 Fluid bed dynamics at higher pressure

The effect of pressure also affects the dynamics of fluid beds. Bubble characteristics are found to be sensitive to an increase in bed pressure [2]. However the effect varies with particle size and density [27]. For instance, for Group A particles, there exists a maximum size of bubbles in the bed during their evolution which is then followed by its disassociation for atmospheric operating pressures. The limitation of bubble size increase beyond a maximum occurs due to the circulation caused by the shear force on the rising bubbles by the falling particles in the

![Fig.13: Variation of T1 and T2 in eq.12 with pressure and voidage [27]](image)
emulsion phase. As a result, a circulation velocity, $u_c$, is set up inside the bubble that is approximately equal to the bubble rise velocity, $u_b$. When $u_c$ exceeds the terminal fall velocity, $u_t$, of a single particle (constant velocity attained due to balance of drag force and weight of particle), the particles from the wake are drawn into the bubble causing it to break [1]. For small and lighter particles, since $u_t$ is smaller, they show a smoother fluidisation. Since the effect of pressure is to reduce $u_t$, the fluidisation remains smooth even at higher pressures for Group A particles as bubbles formation is restricted.

Another reason attributed to the breaking of a bubble is the decrease in the kinematic viscosity of the dense phase, which is a strong function of voidage, which in turn is a function of pressures for finer particles [102]. Therefore, a decrease in kinematic viscosity, due to increase in voidage caused by increase of pressure for Group A particles, would result in rapid breaking of bubbles. Viscosities of particles greater than $100 \, \mu m$ are practically insensitive to increase of pressure [103]. The bubble breakage is also caused due to the falling of particles from the bubble roof. The falling of particles is more pronounced at higher pressures [27].

The bubble behaviour of Group B particles under increasing pressure is similar to Group A particles. However, it is also observed in addition that the rise velocities of the bubbles increase despite reduction in their size. Increasing pressure condenses the distribution of bubbles in the bed and limits it only along the vertical axis of the bed [21]. A central channel, therefore, can be seen due to rapid increase in bubble coalescence at higher pressures. In contrast, a fluidised bed at atmospheric pressure would exhibit a uniform distribution of bubbles.

1.8. Hydrodynamics of sub-atmospheric fluidised beds.

In the literature, fluidisation under vacuum conditions has been examined by a few researchers who have reported general observations on hydrodynamics of fluidisation [17, 104-108], heat transfer [7, 29] along with theoretical analysis of operating parameters and their effect on fluidisation [96, 107]. An advantage often cited for use of vacuum conditions is the existence
of safe operating conditions for certain thermolabile substances often used in pharmaceutical industry. Apart from this, vacuum conditions also offer possibility of reduction of operating temperatures and thus opportunity for reduction of energy for certain chemical reactions in extractive metallurgy such as in reduction of magnesium from dolomite by ferrosilicon [20]. In addition, there is a possibility to utilise the advantage of low mass consumption (in kg/s) of fluidising media at sub-atmospheric pressures for thermo-chemical treatments of metals found from preliminary results of unpublished work of Fabijanic et.al [109]. This aspect of the vacuum fluidised bed is further dealt with in Chapter 3 in context of heat treatment/thermochemical processing of metals in FBR.

Vacuum fluidisation was first carried out by Kawamura and Suezawa [108] who studied the mechanism of fluid flow at reduced pressures (0.133-13.33 kPa) for particulate beds of Group B powders (sand, silica gel and glass beads). These researchers reported similar fluidisation behaviour as found in atmospheric conditions for all bed material. Subsequently, experiments were carried out by various researchers [20, 104, 107, 110, 111] who attained lower sub-atmospheric pressures and examined various particle sizes under vacuum conditions and these results are discussed in the following sections.

1.8.1 Effect of slip flow on fluidisation

A distinct feature of the fluidised bed at sub-atmospheric pressure is the significant pressure drop through the bed in comparison to the operating pressure which gives rise to an increasing velocity gradient along the bed height and thus results in differential nature of fluidisation [20]. At atmospheric pressure, this phenomenon is absent and hence no effect is observed on quality of fluidisation. However, a quiescent layer and a fluidising layer co-exist in vacuum conditions due to the presence of significant pressure drop and intermediate slip flow characterised by $\text{Kn} = \left( \frac{0.001334}{P_d p} \right)$ (where $d_p$ is the particle diameter) in the range of 0.01–1. The existence of slip flow also means a reduction in solid-gas interaction force that contributes to the
inhomogeneous fluidisation. The quality of fluidisation is one of the area of concern that deters its utility. The existence of dual layer of fluidisation narrows down the window of flowrates where the bed can be operated in a bubbling zone. This is because the vacuum bed either bubbles & remain quiescent or bubbles & slug at bottom and top of the bed, respectively, with increasing flowrates. A similar behaviour is also observed during segregation caused by differential particle size in the bed [74]. It is not known how the effect of segregation and the significant pressure drop effects the quality of the fluidisation under vacuum.

Kusakabe et.al [107] theoretically analysed the existence of slip flow and its effect on the minimum fluidisation velocity, $U_{mf}$. The existence of slip flow reduces the drag force and results in an increased $u_{mf}$ in vacuum conditions. Kusakabe et.al [107] combined the expression for throughput of gas under viscous flow regime and molecular flow regime as proposed by Dushman [112] and expressed the $U_{mf}$ as:

$$U_{mf} = U_{mfv} \frac{P_o}{P} \left[ 1 + \frac{k_2(1-\epsilon_{mf})}{\epsilon_{mf}} \times \frac{2RT}{\pi M \phi d_P p_o} \right]$$

(eq. 13)

Where, $U_{mfv}$ is the minimum fluidisation velocity at atmospheric conditions; $P_o$ is the pressure at the bottom of the bed; $P$ is the pressure at any axial location of the bed; $M$ is the molar mass of the gas, $\epsilon_{mf}$ is the void fraction at $U_{mf}$, $k_2 = 50$ [112], $\mu$ is the dynamic viscosity of the gas, $d_p$ is the particle diameter; $T$ is the temperature of gas.

Later on, Llop et.al [96] modified Kusakabe’s attempt by including the effect of viscous and turbulent flow regime and proposed a general equation for prediction of $U_{mf}$ for all pressure ranges (from sub-atmospheric to atmospheric). According to Llop et al [96], $U_{mf}$ (or alternatively, $Re_{mf}$) is expressed as:

$$Re_{mf} = \begin{cases} \left[ \left( \frac{0.909}{Kn+0.0309} \right)^2 + 0.0357Ar \right]^{1/2} - \left( \frac{0.909}{Kn+0.0309} \right), & \text{for } \phi > 0.8 \\ \left[ \left( \frac{1.9}{Kn+0.0492} \right)^2 + 0.0571Ar \right]^{1/2} - \left( \frac{1.9}{Kn+0.0492} \right), & \text{for } 0.5 < \phi \leq 0.8 \end{cases}$$

(eq. 14)
Where $Ar$ is the Archimedes number \( \left( \frac{d_p^3 \rho ( \rho_p - \rho ) g}{\mu^2} \right) \); $Re \left( \frac{\rho d_p}{\mu} \right)$ is the Reynolds number and $Kn$ is the Knudsen number.

The increase of $U_{mf}$ with increasing sub-atmospheric pressure also suggest that the superficial mass consumption of the gas reduces with pressure. Due to the presence of vacuum conditions, the density of the gas reduces significantly as compared to atmospheric conditions which causes a natural increase in velocity to conserve continuity of mass flowrate. Thus, although the $U_{mf}$ increases with reduction of pressure, the mass consumption reduces. This fact is not highlighted in literature and bears greater advantages for use in surface treatment of metals where the quantity of consumption of gases is significantly higher.

Eq. 14, predicts the minimum fluidisation velocity from slip flow to viscous and turbulent flow regimes accurately. It has been compared successfully with equations from Wen and Yu [113] and Kusakabe et.al [107], who approach these flow regimes separately. A limitation of Eq.14 is that it does not take into account the inter-particle cohesive forces that becomes predominant for very fine particles. Wank et.al.,[114] proposed a modified form (eq.15) of Llop’s equation which included the cohesive force term.:

\[
1.75 C_1 Re_{mf}^2 + Z Re_{mf} - N_{FL} = 0 \quad \text{(eq. 15)}
\]

Where, $C_1$ and $Z$ are constants[114]; $N_{FL}$ is a cohesive-force number.

### 1.8.2 Effect of particle size

Kusakabe et.al [107] used a wide range of particle diameters of different material such as glass beads (139-445μm), Fe (150μm), Al (130μm), PVC (130μm) and sub-micron particles such as Al$_2$O$_3$ (0.41μm), TiO$_2$ (0.14μm), SiC (0.12μm) and Ni (0.02μm). These particles fall under a wide range of Geldart’s group A, B and C group, respectively. Agglomerates with mean size of 150-200μm were found to form for Group C particles. It was observed that the fluidisation
of ultra-fine particles took place without difficulty under vacuum pressure (1-100 kPa). The fluidisation was carried out for shallow beds only. The formation of agglomerates is a property of the solid and was found to independent on pressures. An upper and lower fluidisation section existed for all particle sizes.

In another vacuum FB study by Mawatari et.al [17], Group C particles were studied for vibration induced fluidisation. Glass beads of 6\( \mu \)m were used and different flow structures were observed under vibration and varying gas velocity. Fig. 14 summarises the observation made by Mawatari et.al [17]. The flow behaviour was not smooth. At lower gas velocity, channels were observed which started to break when vibration was introduced. With increase of velocity, gas bubbles began rising from the upper portion of the bed and erupted at the surface. A flatter bubble size was observed for Group C particles as compared to atmospheric. Further, Mawatari et.al [18] fluidised Group A particles down to a pressure of 100 Pa. It was observed that the
homogeneous fluidisation region becomes larger as the particle diameter decreased from 100 to 40\(\mu\)m. It was also inferred by the authors that the interaction forces between primary particles and subsequent cluster formation were responsible for the experimental value of \(u_{mf}\) to be larger than the value predicted by expression developed in literature [96]. Thus, it is necessary to account for inter-particle forces when dealing with fine powders. The effect of particle size on minimum fluidisation and minimum bubbling velocity (\(U_{mb}\)) can be inferred from this. It was seen that for Group A particles there was significant increase in \(U_{mf}\) with decrease of pressure. In comparison, these particles, have no effect on \(U_{mf}\) when pressures are increased above atmospheric (see section 1.7.1 and Fig. 12). The minimum bubbling velocity decreases with increase of pressure and the region of homogenous fluidisation decreases subsequently. The homogenous region, however, increases with decrease of particle size. Fig. 15 shows the effect of pressure on \(U_{mb}/U_{mf}\) [18]. Similar effect is seen for rise of pressures beyond atmospheric [27].
Group D particles were experimented by Kozangalu et al. [105, 106] for fluidisation in vacuum pressures. The void fraction at minimum fluidisation was seen to remain constant with decrease of pressures. However, no specific details were provided on the fluidisation characteristics like spouting etc with reduction in pressure.

All the above mentioned works had employed particle size with a particle distribution and thus it is not clear if the quality of fluidisation (as discussed in section 1.8.1) is due to segregation or amplification of the pressure gradient in comparison to the operating pressure. It is noteworthy that a wide and narrow range particle distribution can have varying effect on quality of fluidisation [13]. In addition, there is no work that has described the bubble characteristics in vacuum bed.

2. Heat transfer in fluidised beds

Heat transfer is one of the primary uses of fluidised beds in industry. Availability of a perfect mixing medium characterised by isothermal bed conditions enable applications as surface treatment of metals by thermochemical processes, coating of pharmaceutical products, plasma treatment of nano-particles as well as gasification of coal for power generation possible. The underlying mechanism of heat transfer is conduction and convection in the gas and emulsion phase, accompanied by radiation at high temperatures (800-1000°C). The particle convection, especially, enables a continuous “scourging” of the thermal boundary layer on immersed surfaces thereby reducing the thermal resistance and enhancing the heat transfer. This is the reason why the heat transfer in a solid-gas fluidised bed is multiplied several times to ordinary gas convection heat transfer. The heat transfer coefficient (h), expressed as heat flux per unit temperature difference, is composed of conduction in the particle phase and convection from the bulk movement of the particle phase and the gas (or bubble phase) [6] (Eq. 16).

\[ h = f h_{\text{bubble}} + (1 - f) h_{\text{emulsion}} \]  

(eq. 16)
where, $f$ is the fraction of heat transfer surface covered by bubbles at any given instant [2].

Heat transfer is a complex phenomenon and depends upon various operating parameters in the fluidised bed. The fluidising velocity, operating pressure and temperature of the system, particle size, gas density, shape and size of the immersed surface are important parameters which affect the heat transfer from an immersed surface. These parameters have been found [6] to influence the quality of convection and conduction of the gas and emulsion phase. A few of these are discussed below due to their relevance to the present project.

2.1 Parameters influencing heat transfer in fluidised beds

2.1.1 Effect of particle size

Heat transfer between bed to surface during fluidisation for larger and finer particles are summarised in Fig.16 [14]. It is mainly limited by the thickness of the gas film near the surface. For finer particles, the particle convection dominates due to insignificant film thickness as compared to a larger particle. Also, smaller thermal time constant ($t = \frac{\rho_p \rho_{d} \frac{d^2}{k_g}}$) in comparison to the renewal frequency for a smaller particle size warrants consideration of the bed properties instead of only the first layer of particles as in case of larger particles. Therefore, particle replacement time greatly affects the heat transfer for finer particles whereas for larger particles, heat transfer is relatively independent of the particle residence time (a function of bed property) and is thus more dependent on the gas velocity [14]. It is noted in the literature [100, 115] that the trend of the total heat transfer coefficient is to decrease with increasing particle size and then increase after a critical particle size (Fig.17). For fine particles, the total heat transfer coefficient is independent of increase in pressure (in the range of 1-300 atm) [11]. This is because any increase in pressure influences the density of the gas which convective heat transfer of gas contributes to more than 50% of total heat transfer [1, 2, 6]. So does heat transfer increase with pressure for coarse particles?
2.1.2 Effect of thermo-physical properties of the gas and particles

Density, specific heat and thermal conductivity are the critical thermo-physical properties that influence heat transfer in fluidised beds. The effect of the density of the gas is to contribute to convective heat transfer [27]. The specific heat capacity of the fluid influences heat transfer only with an increase in pressure. However, the specific heat capacity of the solid particles ($C_s$) influences the heat transfer at any pressure as, $h \propto (C_s^n)$, where $n$ ranges from 0.2 to 0.8 depending on the material [1]. The thermal conductivity of a solid particles has practically no influence on heat transfer. It is the gas thermal conductivity that has the maximum influence.
Literature review

Depending on the gas used, the thermal conductivity can increase the heat transfer coefficient by a power of $\frac{1}{2}$ to $\frac{2}{3}$. The effect of temperature is, thus, to vary the thermal conductivity and density of the gas [1, 2].

2.1.3 Effect of and the immersed surface in a the fluidised bed

As discussed in section 1.1, the bubble flow profile is influenced by the aspect ratio (AR) of the fluidised bed which has a pronounced effect on the movement of the emulsion phase and hence the particle mixing in the bed. Therefore, the location of the immersed surface has an important role in heat transfer by determining how frequently a bubble contacts the surface and how frequently the renewal of the emulsion phase takes place. It is observed that in deep beds, heat transfer along the axis is maximum and decreases away from the axis. However, for an immersed object in the same relative location in a shallow beds, the heat transfer is reduced owing to the less frequent exposure to bubbles at the central axis [6].

Fig.17: Variation of heat transfer coefficient with particle size between a surface and the bed [2] on the heat transfer [1, 2]. Depending on the gas used, the thermal conductivity can increase the heat transfer coefficient by a power of $\frac{1}{2}$ to $\frac{2}{3}$. The effect of temperature is, thus, to vary the thermal conductivity and density of the gas [1, 2].
2.2 Heat transfer models

2.2.1 Film theory

Film theory was the first attempt by Dow and Jacob [19] to understand heat transfer between a fluidised bed and an immersed surface. Heat transfer was proposed through a finite thickness of gas film adjacent to a wall that hosted descending solid particles. The solid particle to surface contact was neglected in this model and heat transfer was from the surface to the air film that was scoured continuously by the solid particles. Leva. et.al.,[116] extended the film theory of Dow and Jakob by incorporating the effect of solid particles velocity and viscosity of air on the film thickness, initially absent in the original model. This was later modified by Levenspiel and Walton [117] who suggested a discontinuity in the gas film due to the contact of solid particles with the surface. This approximation reduced the heat transfer resistance. A drawback of the film theory lies in the fact that the thermal capacity of the solid particles was neglected and emphasis was laid only on thermal conductivity of the gas. The reason for enhanced heat transfer from the surface was explained to be a consequence of the scouring effect of the solid particles that reduced the gas film and hence decreased the resistances. Experiments by Mickley and Trilling [118] showed otherwise and emphasised that the heat transfer was mainly due to the heat transport by the solid particles by conduction at the immersed surface.

2.2.2 Packet theory

Mickley and Fairbanks [119] first pointed out that the heat transfer to and from the immersed surface is a case of unsteady heat transfer by the renewal of packets of emulsion-phase material that acts as a source/sink for energy transfer. These “packets” contains the solid particles in some random configuration loosely locked and engulfed by the gas. The movement of these packets to and from the surface induced by the bubble movement is the chief mechanism of heat transfer. The instantaneous heat transfer coefficient was predicted as:
where, $k_p$, $\rho_p$, $C_p$, are thermal conductivity, density and heat capacity of the packet respectively and $\tau$ is the time during which the packet remains in contact with the surface. Although a better model of heat transfer than the film theory, the heat transfer predicted by this model was high at the initial contact time than what the experiments showed and was valid only for large contact points. Another drawback of this model is that it considered homogeneous bed properties similar to the packet.

Works of Baskakov [120] and Gorelik [12] refined the packet theory by considering varying porosity in distinct zones near the surface. Baskakov [120] introduced an additional resistance to heat transfer due to the variation of porosity near the surface. The additional resistance was derived to be:

$$R = \frac{d_p}{\pi k_g \left[ \ln \left( \frac{k_s}{k_g} \right) + 1 \right]} \quad (eq. \ 18)$$

Where, $d_p$ is the particle size, $k_s$ and $k_g$ is the thermal conductivity of the solid particles and gas respectively and $C$ is the constant.

Later on, Gorelik [12, 121] to propose two distinct layers of properties due to the presence of different but constant porosity in two distinct layers near the heat transfer surface (see Fig.18.
Kubie & Broughton [122] adopted the Micley and Faribanks [119] model to include a continuously varying porosity in the bed beginning from the surface-bed interface, in contrast to the constant porosity considered by Baskakov and Golerik. The major drawback of this model lies in the assumption that an effective bed thermal conductivity is applicable to unsteady heat transfer. In addition, the convective heat transfer was neglected.

Many researchers [10, 123-125] extended the packet theory, however, failed to improve the accuracy in comparison to Mickley and Fairbank [119].

2.2.3 Particle theory

A new perspective of heat transfer (in 1963) was contributed by Botterill and Williams[21] who considered heat transfer to a row of particles perpendicular to and in contact with the surface. They approximated heat transfer to take place by conduction between the surface and an isolated particle surrounded by a thin layer of gas (taken as 0.2 times the particle diameter, \(d_p\)), initially at the temperature of the bed. Due to shorter contact times, heat penetrates only to the first row of particles. Radiation and convection was neglected in their numerical solution to the model. Improvements in the particle model was later brought by Gabor [126] who
considered an alternative-slab model where heat flows through a series of solid and gas phases which represents a string of spherical particles. This model treated the bed as a series of particles beginning with initial resistance of the wall and the first layer of particles. Other models considered only the initial contact resistance with the wall and the rest of the bed as single phase medium described by an effective thermal conductivity.

Ganzha et.al [127] extended the particle model and considered an orthorhombic arrangement of particles on the surface. The convective and conductive heat transfer were considered in the solution for large particles. It was seen that this model predicted well the dependence of heat transfer coefficient on Reynolds number (Re).

Recently, Gao [3] had proposed a Discrete particle and porous layer model (DPPM) to predict heat and mass transfer to an immersed surface (Fig.19). A porous medium of uniform porosity was assumed away from the wall and a rhombohedral arrangement of particles was considered near the surface. Governing equations for the proposed model was numerically solved using a commercial software FLUENT. The predictions of the heat transfer coefficient were compared with those existing in literature [6, 128] and the results lied in the range of ±25% of the experimental values.

Fig. 19. Double particle-layer and porous medium (DPPM) model proposed by Gao[3]
2.3 Effect of increase of pressure on heat transfer in fluidised beds

The main effect of pressure on heat transfer is to alter the thermo-physical properties of the fluid, in addition to changing the fluid bed hydrodynamics [27]. It is seen in literature [6, 11, 27] that an increase in pressure has no effect on heat transfer for fine particles belonging to Group A or finer Group B. Similarly, experiments by Botterill [6, 11, 27] showed insignificant increase in heat transfer for coarser particles as compared to fine particles (Fig.20). Although, an increase in pressure produces smooth fluidisation [21], its effect on fluidisation quality for fine particles is minimal. The density of gas which bears the maximum effect due to increase in pressure is found to have no effect on the heat transfer characteristics in fine particles since only particle convection affects heat transfer for such particles [6]. However, for coarser particles, gas convection is significant and hence any variation of gas properties alters the heat transfer significantly [6, 100].

Fig.20. Effect of pressure on heat transfer from (a) Coarser particle (625μm) and (b) Fine particle (150μm) [11]
2.4 Effect of vacuum pressure on heat transfer in fluidised beds

Only two studies have been carried out to investigate heat transfer in vacuum conditions. Bhat and Whitehead [29] carried out heat transfer on a horizontal tube inside a vacuum fluidisation chamber with constant superficial velocity. It was seen that the heat transfer coefficient remained same with increase of pressure from 8 kPa to 100 kPa. Tests were carried out for Group B particles of size 120 μm sand particles for three different superficial velocities that remained constant for all pressure ranges (bed temperatures were below 200 °C).

Shlapkova [7] heated a vertical tube inside vacuum and fluidised 120 μm sand in the pressure range of 0.2-100 kPa and concluded a decreasing trend of heat transfer coefficient (Fig.21). The Knudsen number considered in this work was in the range of 0.0001-0.05, i.e, viscous (100 kPa) to slip flow (0.2kPa). The reason attributed to the above observation is the decrease of the thermal conductivity of the air under vacuum conditions that is characterised by the presence of slip flow. It is, however, well known that the thermal conductivity of the gas is independent of pressure, which is valid as long as the pressure is above the molecular flow range (Kn ≥ 1) [112]. In addition, it is also known, that for fine particles, the gas density and thermal conductivity do not contribute to the total heat transfer (see section 2.1.1) due to which the heat transfer coefficient remains independent of pressure [11]. Therefore, to attribute a decrease of the thermal conductivity of air as a reason to explain reduction of heat transfer for a bed of fine particles is open to question. A possible reason for the reduced heat transfer coefficient could be the placement of the immersed surface or a significant change in the hydrodynamics of the bed under vacuum. Shlapkova [10] used a vertical surface for the experiment whereas Bhat and
Whitehead [9] used a horizontal tube where the probability of bubbles hitting the surface is higher than in the vertical placement and hence will give better heat transfer. A parametric study is thus needed to understand the effect of location of the surface on the heat transfer. Interestingly, it can also be observed in comparison of Fig.20a and 21 that the heat transfer for fine particles in vacuum (200 \( \mu \text{m} \) sand particles) resembles the characteristic of a coarse particle (625 \( \mu \text{m} \)) at atmospheric pressures, suggesting that the particle convection is significantly dependent on vacuum pressures. Thus, the existing heat transfer results under vacuum conditions help raise questions on the effect of immersed shape, orientation and size; effect of variation of hydrodynamics, effect of location of the immersed surface and how does radiation heat transfer is affected with reduction of pressures; to improve the depth of knowledge in this area of fluidisation.

Fig. 21. Variation of heat transfer coefficient with fluid velocity for different pressures

for 200 \( \mu \text{m} \) sand particles[7]
3. Summary and gaps in knowledge

From the above discussion, it is clear that the work reported in literature in the area of hydrodynamics, heat and mass transfer in vacuum fluidised bed is limited. Several fundamental gaps of knowledge is found from the review of existing literature. These are:

a) Hydrodynamics of fluidisation in vacuum conditions is not well understood especially from point of view of bubble formation, its size and growth, and effect of segregation on quality in the fluidised bed. It is not known what affect vacuum conditions has on the bubble size variation during its travel in significant pressure gradient in the bed which allows expansion of gas. Knowledge of bubble dynamics is essential to understand the heat transfer effects when an object is immersed inside the bed since the bubble flow greatly affects the emulsion flow around the object. Work reported in literature for bubble dynamics at elevated pressure is high and so knowledge about heat transfer at high pressures in properly understood. This is lacking in the vacuum region of fluidisation.

b) The heat transfer results available in literature for vacuum conditions do not provide adequate information on the effect of vacuum on heat transfer for immersed objects. For instance, it is not known conclusively if the heat transfer increases or decreases with decrease of pressure since the results available are inconclusive. Effect of axial location of the immersed object, particle characteristics, aspect ratio of the fluidised beds, high temperature heat transfer in vacuum conditions are some of the key areas which needs to be attended.

3.1 Research question and scope

Based on the critical literature review, the present work aims to answer if the vacuum fluidised beds can be optimised in order to utilise the heat transfer capability efficiently and take advantage of the lower mass consumption even though the quality of fluidisation degrades rapidly under vacuum. Due to the axial variation of fluidisation quality, for certain optimal
Literature review

flowrates, axial location and powder size distribution, vacuum fluidised bed should offer optimal thermal performance. The abovementioned gaps in knowledge are addressed by investigating:

1. The hydrodynamics of the fluidised bed in sub-atmospheric conditions as the literature reports limited or no study in this area thus limiting any attempt to optimise the operating parameters for enhanced heat transfer. This includes investigating the effect of pressure on the quality of fluidisation, powder characteristics and finally bubble dynamics such as size, growth and velocity of the bubbles under vacuum conditions.

2. Investigating experimentally the heat transfer from an immersed heat sources under vacuum and the effect of the operating parameters as pressure, particle size and location of immersed surface.

3. The limitation of the existing continuum models (as Eulerian-Eulerian model) for accurate prediction of nature of hydrodynamics. This is essential due to presence of slip flow regime at higher vacuum where the continuum approximation begins to break down.

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Literature review


Literature review


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Literature review


Chapter 3

Investigating the effect of sub-atmospheric pressures on fluidisation quality

Section 1 (Published paper):


Section 2:

b. Appendix I: Malvern Mastersizer results of the powder size distribution used in the present study.

This chapter presents the experimental results to determine if the degradation in quality of fluidisation under vacuum condition is due to combined effect of particle segregation and the pressure gradient. This chapter is essential in order to estimate optimal powder characteristics to operate the vacuum fluidised bed utilising the bubbling regime efficiently. The results from this experimental work will be used to carry out heat transfer experiments in the coming chapters. The Appendix I presents the size distribution results estimated by Malvern Mastersizer. In addition, the Optical flow technique is not elaborated in the present work to maintain brevity. This technique is elaborately explained in the literature.
Authorship declaration

This is to certify that the first author, Mr. Apurv Kumar, of the paper titled "Investigating the effect of segregation of particles and pressure gradient on the quality of fluidisation at sub-atmospheric pressures" has done more than 85% of the work which includes conducting experiments, analysis and preparation of the first draft of the manuscript. Other authors have been instrumental in providing guidance, discussion and final preparation of the manuscript.

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Investigating the effect of segregation of particles and pressure gradient on the quality of fluidisation at sub-atmospheric pressures

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A B S T R A C T

Degradation in quality of fluidisation for Group B powders under vacuum conditions occurs due to an existence of a fluidisation interface that separates the bubbling and static bed near the minimum fluidisation conditions. The significant pressure gradient existing due to bed weight and the particle segregation mainly affects the quality. The aim of the present work is to investigate the relative contribution of the pressure gradient and the presence of the segregation on the quality of fluidisation in vacuum conditions. Further, the effect of morphology of the particles on fluidisation quality is also studied. In addition, fluidisation maps are also obtained that reveal the optimal area of operation for enhanced heat and mass transfer processes. The results indicate that the travel of interface in the bed is affected only when the level of segregation in the bed is high. For intermediate and minimal disparity in size, the quality degradation is caused by the presence of sharp pressure gradient. Changing the morphology of the particle also altered the fluidisation characteristics of the bed and improved the quality significantly.

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1. Introduction

Fluidized beds have been readily operated under varying pressure conditions apart from the atmospheric pressure. High pressures in fluidized beds affect the minimum fluidization velocity, bubble size and bubble dynamics in the bed [1–3]. Increased heat and mass transfer (relative to atmospheric) have also been observed in a bed of coarse particles when operated at high pressure [4]. In addition, many researchers have studied vacuum fluidisation, which is characterised by the presence of slip flow where the Knudsen number \((K_n = \lambda/d_p)\) lies in the range of 0.01–1 [5–11]. The presence of slip flow affects the quality of fluidisation for powders with different particle density and size, generally classified under Geldart’s groups. In low-pressure conditions, the existence of non-homogeneous fluidisation where an interface exists that differentiates the quiescent bed and the bubbling bed in proximity to the minimum fluidisation conditions has been reported [12–14]. Despite these observed quality issues, fluidisation under vacuum has been shown to be advantageous in various applications such as drying of pharmaceutical products, extractive metallurgy, CVD [10,11,15].

Segregated fluidisation in atmospheric conditions occurs when a bed has a broad particle size range, forming a horizontal interface that separates the lighter (flotsam) and the heavier particles (jetsam) [16]. The bed thus fluidises non-homogeneously with increase of flow rate.

Group B powders (size, density) under vacuum have been found [7] to exhibit dual fluidisation zones (quiescent and fluidising) separated by a fluidisation interface in proximity to the minimum fluidisation conditions. In contrast, a bed Group A powders (size, density) fluidised under low pressure conditions exhibit smooth and interface-free fluidisation [7,17].

The existence of the horizontal interface in vacuum fluidisation is also attributed to the presence of a similar order of magnitude of the top pressure and the pressure drop through the bed [8,11]. The pressure gradient inside the fluidised bed (due to the weight of the particles) becomes significant when the operating pressures are reduced below atmospheric and this pressure gradient results in an acute velocity gradient causing the fluid to expand inside the bed [11]. Non-homogeneous fluidisation has therefore been ascribed to the inefficiency of the fluid to transfer momentum to the particles. However, numerous vacuum fluidisation studies have used powders with a continuous range of particle sizes [8,11–13,17]. Therefore, it is difficult to ascertain if the observed low quality fluidisation under vacuum is due to the segregation phenomenon magnified by the presence of vacuum or an inherent incapability of vacuum condition to fluidise the entire bed simultaneously or a combination of both.

The aim of the present work is to investigate the relative contribution of powder segregation and the bed pressure gradient to the sub-atmospheric fluidisation quality. Alumina powders consisting of narrow and wide size distribution classified as Geldart’s Group B were used in the present investigation. In addition, porous alumina is investigated in order to understand the effect of particle morphology on the fluidisation quality under vacuum. Quantification of the fluidisation
quality of the powders was achieved by monitoring the progression of the fluidisation interface through the bed at different sub-atmospheric pressures. A proposed fluidisation quality index compares the behaviour of the various powders under vacuum condition. Further, fluidisation maps were constructed for all the powders, which are used to study the variation in the available optimal bubble space with pressure. These maps are highly useful for carrying out heat and mass transfer processes in cases where visualisation of the fluidisation phenomenon is unavailable due to the setup limitations.

2. Experimental method

2.1. Low-pressure fluidisation

A cylindrical fluidised bed made of polycarbonate and of size 50 mm ID and 1 m length was used having a porous sintered steel disc as a distributor plate (Fig. 1). The vacuum in the chamber was controlled by using bypass needle valves. Variable area flow meters with an accuracy of ±3% were used to measure air flow rate to the vacuum chamber. To ensure correct mass flow meter readings a pressure of 50 KPa was maintained on entry and exit side of the chamber. In order to study the effect of segregation on the quality of fluidisation, the particles are classified as Group B powders. The porous alumina powders had narrow size distribution and varying median diameters, d(0.5) of 80, 103 and 170 μm. Fluidisation studies were performed on these powders and on mixtures of these powders (Mixture 1 and Mixture 2) to study segregation effect on quality. The size distribution for these powders accompanied by their respective SEM image is shown in Fig. 2. The properties of the powders and inlet conditions are given in Table 1. It can be seen that the alumina powders are irregular faceted particles and are classified as Group B powders. The porous alumina powders have minute pores throughout the structure ranging from 4 to 10 nm. This porosity greatly reduces the density of the alumina particles, relative to the monolithic alumina. Correspondingly, the weight of the bed is reduced; how this affects the pressure gradient in the bed and the resulting fluidisation quality is of interest.

2.2. Powder characteristics

Porous alumina and alumina powders were used for the current investigation, which are widely used for heat transfer applications in solid–gas fluidisation. Three sets of alumina powders were used that had narrow size distribution and varying median diameters, d(0.5): 80, 103 and 170 μm. Fluidisation studies were performed on these powders and on mixtures of these powders (Mixture 1 and Mixture 2) to study segregation effect on quality. The size distribution for these powders accompanied by their respective SEM image is shown in Fig. 2. The properties of the powders and inlet conditions are given in Table 1. It can be seen that the alumina powders are irregular faceted particles and are classified as Group B powders. The porous alumina powders have minute pores throughout the structure ranging from 4 to 10 nm. This porosity greatly reduces the density of the alumina particles, relative to the monolithic alumina. Correspondingly, the weight of the bed is reduced; how this affects the pressure gradient in the bed and the resulting fluidisation quality is of interest.

2.3. Powder segregation classification

In order to study the effect of segregation on the quality of fluidisation, the particles are classified by using the pressure drop curves as Type A–E according to the segregation behaviour reported by Rao et al.[18]. Here powders are characterised by the density and size ratios of the jetsam and flotsam particles found in a mixture. In the present analysis, the jetsam and flotsam particles are considered as particles of size corresponding to the average of d(0.9) and d(0.5); d(0.5) and d(0.1), respectively. Similar averages of particle diameters have been used earlier [19,20] as jetsam and flotsam diameters in segregation analysis of particle size distribution powders. The result discussed in Section 4.1.2 justifies the classification of powders based on Rao method as minimal, intermediate and high segregation. The density and size ratios are thus defined as \( p_1 = \frac{\rho_{flotsam}}{\rho_{jetsam}} \) and \( d_1 = \frac{d_{flotsam}}{d_{jetsam}} \), respectively.

Velocity ratio is defined as \( U_s = \frac{U_{flotsam}}{U_{jetsam}} \). As all alumina particles belonging to Geldart’s Group B have similar density, \( p_1 \) is unity in the present case. The size ratio, \( d_1 \), for 80, 103 and 170 μm is 1.58, 1.527 and 1.547, respectively. \( d_1 \) for Mixture 1 is 2.22. According to Rao et al. [18] the following are the powder classification (Type A–D):

Type A mixtures: Very large particle size ratio (\( d_1 > 4.5, U_s > 8 \))

<table>
<thead>
<tr>
<th>Powder type</th>
<th>d(0.1) (μm)</th>
<th>d(0.5) (μm)</th>
<th>d(0.9) (μm)</th>
<th>Diameter ratio (d1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alumina</td>
<td>49.284</td>
<td>79.199</td>
<td>125.427</td>
<td>1.59</td>
</tr>
<tr>
<td>Porous alumina</td>
<td>55.032</td>
<td>84.056</td>
<td>127.922</td>
<td>1.52</td>
</tr>
<tr>
<td>Mixture 1</td>
<td>59.404</td>
<td>116.806</td>
<td>275.145</td>
<td>2.22</td>
</tr>
<tr>
<td>Mixture 2</td>
<td>62.124</td>
<td>153.69</td>
<td>550.148</td>
<td>3.46</td>
</tr>
</tbody>
</table>
Fig. 2. SEM images and size distribution of Group B alumina and porous alumina powders used in the present study.
Type B mixtures: Significant level of disparity in particle size and density

\( (\rho_r > 3 \text{ or } 3.3 < d_r < 4.5 : 4.2 < \text{Ur} < 8) \)

Type C mixtures: Intermediate level of disparity (2 < \rho_r < 3 or 2 < d_r < 3.3; 2.5 < \text{Ur} < 4.2)

Type D mixtures: Minimal level of disparity in size and density

(1 < \rho_r < 2 \text{ or } 1 < d_r < 2; 1 < \text{Ur} < 2.5)

The \( \sigma/d(0.5) \) value for a Gaussian fit on the particle size distribution for narrow size particles yields a value of 0.33, 0.29 and 0.32 for 80, 103 and 170 \( \mu \)m particles. Wider range mixture, Mixture 1 has a value of 0.44. It is to be noted that the percentage increase in \( \sigma/d(0.5) \) and \( d_r \) for wider range mixture with respect to narrow size particles is similar (~44%).

2.4. Fluidisation studies

The operating pressures were reduced from atmospheric to 15 mbar and the pressure drop through the bed of particles was measured by using the pressure transducers. Pressure drop vs. superficial flow rate curves were obtained for each operating pressure in the defluidisation mode. Defluidisation mode has been ascertained to be a better procedure to measure the minimum fluidisation velocity than the fluidisation mode as it subverts the occurrence of the pressure drop “hump” while predicting similar magnitude of \( U_{mf} \) [11,13]. It is to be noted that all the experiments were carried out at constant H/D ratio (= 8).

The defluidisation front was observed to appear near the distributor plate, which travels upwards to the bed surface as the flow rate is reduced. Fig. 3 depicts a typical interface travel through the bed (Fig. 3(a)) along with a schematic of the different phase of fluidisation observed during the defluidisation process (Fig. 3(b)). Due to the presence of a fluidisation interface and a rapidly developing slugging regime under vacuum pressures, it is imperative to obtain a fluidisation map.
which can be utilised to improve the performance for heat and mass transfer processes. The fluidisation interface is marked by using visual observation and the process is validated for accuracy by quantifying the interface through an image processing technique called optical flow [21]. Due to experimental complexity in marking the interface for the entire pressure range for different powders and the decreasing flow rate, only visual observation is used.

The optical flow technique is employed to distinguish the static and dynamic portion of the bed by calculating the velocity of the objects in two subsequent images. Video images of the fluidised bed was taken using macro lens at 50 fps and the velocity vectors were calculated by using Lucas–Kanade algorithm [22] in Computer Vision Toolbox in Matlab R2013. A time-averaged plot of velocity vectors was obtained for a video run time of 20 s, which reveals the interface that separates the static and dynamic portions of the bed. Two examples of interface tracking is shown in Fig. 4 where case (a): binary mixture of 300 μm (blue colour) and 75 μm (white colour) (Fig. 4(a)–(c)) where the interface is visible visually and case (b) Mixture 2 (Fig. 4(d)–(i)). In addition,

(a) Original image of segregated bed at atmospheric pressure (blue region is static and made of coarser (Video-1) particles)

(b) Contour plot of time averaged velocity (in pixels/sec) using optical flow

(c) Binary image of time averaged velocity plot showing the interface

(d) Original image of Mixture 2 at 350 mbar and 3000 cc/min (video-2)

(e) Time averaged contour plot (in pixels/sec) for Mixture 2 at 350 mbar and 3000 cc/min

(f) Binary image for Mixture 2 at 350 mbar and 3000 cc/min

(g) Original image of Mixture 2 at 350 mbar and 2500 cc/min (video-3)

(h) Time averaged contour plot (in pixels/sec) for Mixture 2 at 350 mbar and 2500 cc/min

(i) Binary image for Mixture 2 at 350 mbar and 2500 cc/min

Fig. 4. Optical flow analysis results for determination of interface in vacuum fluidised bed.
a video sequence of case (a) and case (b) (Fig. 4(d–i)) is provided as a supplementary data, which shows the velocity vectors calculated (yellow coloured lines), by the Lucas–Kanade algorithm for a run time of 5 s. Video 1 compares the original images of a segregated bed and the processed images showing the calculated velocity vectors (yellow lines). Videos 2 and 3 are comparison of original images of fluidised bed under vacuum (350 mbar) for 3000 and 2500 cc/min. The binary image was obtained by defining a threshold value equivalent to the noise (0.03 pixels/s) when the bed was completely static. The interface height from the top of the image is then marked from the binary image with the condition that at least 90% of the bed must be static. An average error of ±10% is found between visually observed and image processed tracked interface. As seen in Fig. 4(a–c), the segregation interface is tracked by optical flow analysis very accurately. This validates the present technique, which is then applied to the present case of different size range particles where the interface is caused by combination of pressure gradient and size segregation.

Fig. 3(b) shows a typical fluidisation map for alumina particles in vacuum pressure. The region between the fluidisation interface curve and the slugging curve is where the entire bed is bubbling homogenously and therefore this is an optimal area for any application of fluidised bed requiring good mixing. Knowledge of such optimal areas at different operating pressures and flow rates is highly useful when the fluidisation is taking place in closed columns. The regions A1, A2 and A3 denote the quiescent, bubbling and slugging regions in the bed, respectively. The total area A1 (A1 + A2 + A3) is the total area under the bed height curve. Bubbling area ratio (BR = A2/A1) can be defined as the relative area of the bubbling region as compared to the entire area under the bed surface curve and thus quantify the bubbling region. Generally, as the flow rate is increased during fluidisation the bed fluctuates due to erupting bubbles and slugs and hence the bed height used in the present investigation is the minimum bed height observed during the defluidisation experiments.

The onset and termination of interface is shown in the map (Fig. 3(b)) and the corresponding flow rate is defined as \( U_{\text{onset}} \) and \( U_{\text{terminate}} \). A quality index \( q \) is used to quantify the interface travel through the bed and is defined as \( q = \frac{U_{\text{onset}} - U_{\text{terminate}}}{U_{\text{onset}}} \) which denotes the fraction of the flow rate \( U_{\text{onset}} \) required for the interface to travel through the bed.

Quantification of the fluidisation quality in vacuum has been reported by Llop and Jand [7] who used the power density function of the pressure measurements as a quality index and have found the quality of fluidisation to degrade rapidly after an initial improvement when pressures are reduced below atmospheric. This work, although significant, shows the different nature of the fluidisation. The existence of a fluidisation front was not taken into account, which is essential for Group B powders since the knowledge of the high-rate space between the bubbling and slugging regimes is optimal for heat and mass transfer processes.

The quality of fluidisation was studied by separately fluidising each powder sets (80, 103 and 170 μm alumina powders) in vacuum conditions and calculating the quality index. The powders used have a narrow size distribution thus eliminating the effect of segregation. In order to study the effect of segregation on the quality of fluidisation, two powder mixtures were prepared. Mixture 1 was prepared by mixing the narrow size distribution particles (80, 103 and 170 μm) by 1/3 proportion (by vol.). Mixture 2 was prepared by adding a coarser particle size powder (200–500 μm) to the narrow size distribution powders in 1/4 proportion (by vol.). The main aim of mixing the particles is to obtain a wide range of particle size distribution. The median particle diameter \( d_{50}(0.5) \) and other related parameters are present in Table 1. The size distributions for both the mixtures are plotted in Fig. 2.

3. Results

3.1. Mass consumption in vacuum fluidisation

There are numerous advantages of vacuum fluidisation: low pressure reduces the operating temperature of thermolabile substances and allows faster vapourisation, enhancement of drying for porous materials, lower entrainment and elutriation due to lower gas density [10,11,15,23]. There also exists another important and rather unnoticed advantage: reduced mass consumption of the fluidising medium. This affects the operational costs associated with fluidising gases such as nitrogen, argon, ammonia, propane and carbon dioxide, which are used intensively in many heat and mass transfer processes. Although the minimum fluidising velocity increases with reduction in pressure (below atmospheric), the superficial mass consumption reduces significantly. This is because the density of gas decreases more rapidly than the increased requirement of fluidisation velocity. Therefore, a lower superficial velocity supplied at atmospheric pressure results in increased velocity inside the vacuum chamber, thus reducing the actual requirement of the gas consumption with decrease of pressure, as dictated by the continuity conservation principle.

The superficial mass consumption for powders in vacuum was measured at inlet conditions given in Table 1. Fig. 5 shows the superficial minimum fluidisation flow rate requirement for the alumina and porous alumina powders obtained from the pressure drop vs. superficial fluid rate curves. The values shown are average \( U_{\text{onset}} \) calculated from three sets of experiments. Noting the logarithmic scale, there is a rapid decrease in mass consumption with reduction of pressure. Additionally, the porous alumina exhibited a reduction in minimum fluidisation flow rate of 70–85% as compared to 80 μm monolithic alumina powder. The experimental values of flow rate in this work are strongly correlated to theoretically derived flow rate values using the velocity expression proposed by Llop et al. [5] (see Fig. 5). Bhat and Whitehead [9] have shown that the heat transfer in a fluidised bed remains constant with reduction of pressure, which further corroborates the benefit of using vacuum conditions for gas–solid fluidisation along with the advantage of reduced mass consumption of fluidising media.

3.2. Interface onset and termination flow rates

The powders in Group B (narrow and wide range distribution) were separately fluidised under vacuum conditions. The interface was tracked with reduction of flow rate and the height of the interface was noted. The corresponding flow rates of onset and termination of the interface for all the powders are presented in Figs. 6 and 7. Fig. 6 shows the variation of the \( U_{\text{terminate}} \) and \( U_{\text{onset}} \) for 80, 103 and 170 μm powders. Mixture 1 and Mixture 2 interface flow rates are shown in Fig. 7. The results report the average values along with the standard deviation calculated from three sets of experiments. Visualisation of the interface and corresponding flow rates was difficult for small diameter powders at high vacuum pressures. Therefore, the standard deviation for 80 μm alumina particles was very high at pressures less than 100 mbar (Fig. 6). All other particle sizes show smaller standard deviations. However, despite the higher variation in results for lighter particles, the trend of the flow rates with pressure remains unchanged.

It is seen from the plots that the flow rates for all powders decreases monotonically with decrease of pressure. However, there is change in gradient in all the three powder sets when the Knudsen number \( (Kn) \) changes from transition \( (Kn = 0.001–0.01) \) to slip flow \( (Kn = 0.01–1) \) regime. It can also be seen that the critical pressure for this transition increases with increase in particle size. This plot suggests that the slip/transition flow regime affects the characteristics of fluidisation.

Porous alumina powders were fluidised under vacuum pressures and the pressure drop and the bed height was measured. The powder
showed homogeneous fluidisation with smooth expansion of the bed prior to transition into bubbling regime. No existence of interface was observed during fluidisation. A drop in bed height was observed as the bubbling initiated inside the bed. The flow rate

Fig. 5. Variation of minimum fluidisation flow rate for porous alumina and alumina powders with pressure.

Fig. 6. Variation of interface onset & termination flow rate with pressure.
corresponding to this bed height drop is called the bubbling flow rate ($U_b$). Fig. 8 shows the observed minimum fluidisation flow rate and the bubbling flow rate for the porous powder for different vacuum pressures.

### 3.3. Pressure drop curves

The pressure transducers located at the bottom and top of the bed measured the pressure drop through the bed. The values were obtained at a rate of 10 Hz using the data logger. Figs. 9 and 10 show the averaged pressure drop curves obtained during defluidisation of the powders for the narrow and wide range particles, respectively. It can be noted from these plots that the pressure drop curve shapes vary significantly with reduction of pressure, denoting the changes in flow characteristics.

Utilising the pressure drop curves and the tracked interface through the bed, fluidisation maps were obtained as shown earlier (Fig. 3) and will be discussed in Section 4.2.
4. Discussion

4.1. Quality of fluidisation in vacuum

The following section discusses the quality of fluidisation calculated from the results of $U_{onset}$, $U_{Umf}$ and $U_{terminate}$ flow rates for the narrow and the wide size range powders. The effect of segregation and change in particle morphology is also discussed.

4.1.1. Narrow size distribution

Fig. 11(a and b) shows the results of $U_{onset}/U_{Umf}$ and $U_{terminate}/U_{Umf}$ for the narrow range Group B alumina powders. The particle size affects the travel of the interface inside the bed and it varies significantly with variation of vacuum pressures. Higher $U_{onset}/U_{Umf}$ ratio for coarser particles at all vacuum pressures indicates that the interface onsets farther from the $U_{Umf}$ and a higher flow rate consumption (relative to $U_{Umf}$) is required for coarser particles as the pressures are reduced. Interestingly, an opposite trend is seen for $U_{terminate}/U_{Umf}$ where, although the ratio is higher for coarser particles, the ratio decreases with decrease in pressure indicating that the interface terminates farther from minimum fluidisation flow rate. Thus, the difference of the ratio of $U_{onset}/U_{Umf}$ and $U_{terminate}/U_{Umf}$ denotes the resistance encountered by the interface during its movement towards the top surface. This difference is therefore defined as a measure of quality. Ideally, for a mono-sized particle bed, the Group B powder bubbles at $U_{Umf}$ [24], denoting a very good quality fluidisation with $\alpha = 0$.

It is to be noted that the size distribution for the powders is narrow and hence there is minimal disparity of size in the bed signifying that the effect of segregation is insignificant, which will be confirmed in coming sections. The pressure gradient inside the bed therefore plays a key role in the progress of interface.

Further, $U_{onset}/U_{Umf}$ and $U_{terminate}/U_{Umf}$ ratios are seen to remain constant down to pressures of 90, 150 and 250 mbar for powder sizes 80, 103 and 170 μm, respectively (Fig. 11) and then a significant change in their
gradient is observed. These critical pressures lie in the slip/transition regime ($Kn \approx 0.01–0.05$) as seen in Fig. 6 for each of the powders. In addition, the total variation of the ratios for the finer particles are affected the most with the reduction of vacuum pressures — 95.6% and 46.8% change in $\frac{U_{mf}}{U_{c}}$ and $\frac{U_{mf}}{U_{b}}$ is seen, respectively for 80 μm particle as compared to 34.8 and 44.4% ($\frac{U_{mf}}{U_{c}}$) and 75 and 86% ($\frac{U_{mf}}{U_{b}}$) for 170 and 103 μm, respectively.

Fig. 12 reports the calculated quality index variation with the operating pressure and the particle size. The variation of the quality index with pressure is identical to the $\frac{U_{mf}}{U_{c}}$ ratio for all the particles. In addition, the quality remains constant with initial reduction of pressure but then rapidly degrades. In other words, the interface experiences higher resistance in its travel from the bottom of the bed towards the top as the defluidisation is taking place at sub-atmospheric pressures signifying that the interface moves rather slowly under vacuum pressure. The presence of dual fluidisation regions is therefore present over a large range of flow rates. From a practical point of view, this result signifies that the bed transits from bubbling/slugging regime to a quiescent regime rather slowly under vacuum pressures thereby reducing the opportunity to utilise the bubbling region for heat and mass transfer applications. It should be noted that due to the presence of vacuum, the present size of the fluidisation column gave rise to slugging at the top of the container along with a bubbling regime simultaneously at high vacuum pressures. A fluidisation map indicating the fluidisation and slugging interface variation with flow rate is therefore essential for optimal usage of the vacuum fluidisation.

The quality index ($q$) increases with increase in particle size denoting better fluidisation quality for finer particles. The effect of particle size on quality can be understood as follows. Ideally, for a mono-sized Group B particle (segregation absent), the fluidisation quality is due to the presence of the pressure gradient caused by bed weight that is similar in order of magnitude to the top pressure. As a result, the fluid experiences a significant reduction in pressure as it moves through the bed. For instance, at minimum fluidisation condition for a bed weight pressure of 5000 Pa and operated at a top pressure of 100 Pa, the fluid experiences significant change in pressures resulting in substantial reduction in the fluid density. This causes the fluid to expand and increase the velocity to conserve the mass flow rate [17]. Thus, during defluidisation, the bottom of the bed begins to settle down while the top portion of the bed bubbles/slugs since the velocity in this region is sufficient to not only minimally fluidise but also bubble/slug the upper portion of the bed. With increase of particle size, the velocity requirement for $U_{mf}$ increases [5] thereby increasing the velocity requirement for onset and termination of the interface.

In the case of particle size distribution, the inlet velocity not only needs to provide momentum to fluidise the bed against the pressure gradient but also to fluidise various sizes of particles. In the case of high disparity in particle sizes, the bottom of the bed is occupied mostly by the coarser particles (jetsam), thus increasing the velocity requirement for particle suspension in this region as compared to a mono-sized particle bed. The velocity at which the interface terminates at the top of the bed during defluidisation depends on the size distribution of the particles that the interface experiences during its travel. For a high disparity in particle size, a higher range of velocity is required to defluidise the bed and become quiescent thus creating high resistance for the interface.

Focusing on the segregation effect on narrow size range particles, the Rao classification [18] (Section 2.3) categorises the present narrow size range powders at atmospheric pressure as Type D mixtures where the disparity in size is minimal, and the bed fluidises at a distinct flow rate. The pressure drop curves for these powders also classify them as Type D (Fig. 5) confirming that the segregation is minimal. However, as the operating pressure is reduced, the pressure drop curve shape changes significantly. The sharp gradient change of pressure drop curve transforms to a gradual smoother change of slope. Such pressure drop curves (Fig. 9(a): 25, 75 mbar; 9(b): 35, 75 mbar; 9(c): 25, 45 mbar) are similar to pressure drop curves for Type B segregation group [18]. It is however interesting to note that the variation of the segregation parameters: $p_{b}$, $d_{t}$, and $U_{b}$ (Fig. 13) with reduction of pressure does not reflect the Type B parameters: $p_{b} > 3$ or $4.5 < d_{t} < 3.3 ; 4.2 < U_{b} < 8$. This implies that the values of the segregation parameters do not characterise the powders to Type B, although the shapes of pressure drop curves may suggest otherwise. Type B powders have higher values of $U_{b}$ than the Type D powders and are made of powders with very high disparity in particle size or density.

A possible explanation of the effect of segregation on quality is as follows.

The pressure drop varies linearly with increase of flow rate up to the point where the weight of the bed is balanced. If the powders consist of mono-sized particles, the pressure drop remains constant with further increase of flow rate and increases only when slugging occurs [24]. In case of the presence of jetsam and flotsam particles, the pressure drop increases gradually as the flotsam particles begin to fluidise earlier than the jetsam particles, thereby lowering the pressure drop slope [25]. In the present case, where the segregation parameters — $p_{b}$, $d_{t}$, remain unaltered and $U_{b}$ decreases with pressure, the change of pressure drop shape may not be occurring due to amplification of segregation phenomenon. Alternatively, the degradation of quality of fluidisation is caused more due to the existence of a sharp pressure gradient in the bed caused by the decrease in operating pressure for the narrow size distribution particles.

### 4.1.2. Wide size distribution

Classifying the mixture particles (Mixture 1 and Mixture 2) based on the segregation parameters ($d_{t}$ and $U_{b}$) (Fig. 13 and Table 1) and the pressure drop curves (Fig. 10). Mixture 1 corresponds to Type C (intermediate level of segregation) and Mixture 2 corresponds to Type B (significant level of segregation) groups according to Rao et al. [18].

The effect of wide particle size range on the quality is revealed by the fluidisation maps (Fig. 14) of Mixture 1 and 2 that indicates two slopes of the interface during its progression towards bed surface. The maps are presented for normalised velocity ($U/U_{mf}$) and height ($H/H_{static}$). Comparison of the fluidisation maps of Mixture 2, Mixture 1 and 80 μm particle at 350 and 55 mbar shows that the interface travels very slowly in the bottom region of the bed up to a height of approx. 200 mm for Mixture 2 and approx. 50 mm for Mixture 1 and then the slope increases significantly. The narrow size range particles, however, shows a constant slope of the interface throughout the fluidisation.
region. This corroborates the effect of segregation on the interface travel, as discussed in Section 4.1.1. During fluidisation experiments of Mixture 2, the bottom portion in the bed was mostly occupied by coarser particles. Bubbles were seen only at very high fluidisation flow rates in the jetsam region.

Comparison of variation of $U_{\text{onset}}/U_{\text{mf}}$ and $U_{\text{terminate}}/U_{\text{mf}}$ for Mixture 1 and Mixture 2 with their narrow size range particle counterparts (103 and 170 μm) (Fig. 11) reveal that only highly segregated mixture affects the ratios. The variation of ratios with pressure is similar to narrow size range particles. However, Mixture 1 ratios are very close to 103 μm ratios. It is to be noted that Mixture 1 is classified as Type C powder where intermediate effect of segregation is present. Nevertheless, there is no significant effect found on either of the ratios. This substantiates that the segregation effect is negligible in this case and that the degradation of quality is taking place mainly due to the presence of pressure gradient inside the bed. The quality index for Mixture 1, as seen in Fig. 12, also confirms this fact. The quality index variation for Mixture 1 and 103 μm particle is very close in magnitude over the entire range of pressures considered in the present work.

While the ratio $U_{\text{onset}}/U_{\text{mf}}$ for Mixture 2 behaves similarly to 170 μm ratio, the $U_{\text{terminate}}/U_{\text{mf}}$ exhibits dissimilarities (Fig. 11). The $U_{\text{terminate}}/U_{\text{mf}}$ is found to be very low as compared to Mixture 1 and the narrow size range particles. This signifies that the interface experienced a very high resistance in its movement through the bed. The fluidisation maps confirm this fact, where the slope of the fluidisation interface is found to be very low (Fig. 14(e and f)). There was also a very clear segregation observed during experiments. As a result, the quality of fluidisation is found to be very low for Mixture 2 particles at atmospheric and vacuum pressures (Fig. 12). Presence of intermediate segregation did not affect the quality of fluidisation, while a highly segregated mixture resulted in a very high

Fig. 14. Fluidisation maps showing fluidisation and slugging interface for 55 and 350 mbar pressures.
degradation of the quality. This suggests that beyond the intermediate disparity in size, segregation of particles play an additional role in quality of fluidisation along with the pressure gradient.

4.1.3. Effect of morphology: porous alumina

The porous alumina particle contains nanostructured porosity that significantly changed the fluidisation characteristics in comparison with monolithic alumina. 4–10 nm pores are present throughout the structure of the porous powders. It was observed that the interface did not exist when the powder was fluidised in vacuum, as was the case with narrow size particle distribution (80 μm). Fig. 15 shows the $U_b/U_{mf}$ ratio calculated from Fig. 8. The bubbling flow rate tends to approach the minimum fluidisation flow rate as the pressures are reduced below atmospheric. This shows that the changed morphology of the porous alumina powders affect the fluidisation characteristics of the powder in not only improving the quality of fluidisation but also the bubbling onset inside the bed.

4.2. Bubbling area ratio

A bubbling area ratio ($A_2/A_1$) (BR), defined as the ratio of bubbling area to the total area below the bed height curve in Fig. 3(b) is used to quantify the optimal bubbling area available under vacuum pressures. BR decreases with reduction of pressure below atmospheric for the Group B powders (Fig. 16). As discussed earlier, this is mainly due to the degradation of quality. It is interesting to see that the finer particles offer better bubble area than the coarse particles. Additionally, it is observed that by increasing the particle size distribution range, the bubbling area ratio increases, at least by 10–15%. This increase is observed for both the wide range mixtures (1 and 2). This arises mainly because a larger length of the interface is present for the same height travel, thereby increasing the optimal area of fluidisation. However, a very narrow space is seen which can be utilised between the fluidisation interface and the slugging interface (Fig. 14(e) and (f)).

5. Conclusions

A cylindrical column fluidised bed was operated by varying pressures from atmospheric to vacuum to investigate the fluidisation quality existing in vacuum by using powders with wide and narrow size range. Fluidisation characteristics were quantified in the present work to compare the behaviours of different powders. The main conclusions from the present work are:

a) A reduced mass consumption exists under vacuum conditions for fluidisation that has been overlooked in the literature.

b) The degradation of quality in vacuum is caused mostly by the existence of a pressure gradient inside the bed for powders having minimal or intermediate particle size disparity (up to a size ratio, $d_r$ of 3.3). Quality is affected by size disparity only when highly segregated powders are used ($d_r > 3.3$).

c) By changing the morphology of the powders (by using porous powders), the fluidisation quality can be improved significantly. No interface exists during fluidisation of porous alumina powders and the $U_b/U_{mf}$ tends to reach unity with reduction of pressure.

d) The fluidisation maps revealed that the bubbling area ratio increases by increasing the disparity in particle size suggesting a higher bubbling flow rate space under vacuum conditions.

Achieving improved quality of fluidisation (by proper choice of powders) while reducing gas consumption is important in order to design vacuum fluidised beds for heat and mass transfer applications. Further investigation needs to be carried out to study the effect of interface on heat and mass transfer in vacuum conditions.

Nomenclature

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<tr>
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<th>Description</th>
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<tr>
<td>$A$</td>
<td>area under curve in Fig. 3(b)</td>
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<tr>
<td>$d, D$</td>
<td>diameter, m</td>
</tr>
<tr>
<td>$H$</td>
<td>height, m</td>
</tr>
<tr>
<td>$Kn$</td>
<td>Knudsen number ($\lambda / d_p$)</td>
</tr>
<tr>
<td>$q$</td>
<td>quality index</td>
</tr>
<tr>
<td>$U$</td>
<td>superficial flow rate, cc/min</td>
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Greek letters

<table>
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<tr>
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<th>Description</th>
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<tr>
<td>$\rho$</td>
<td>density, Kg/m$^3$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>mean free path of air, m</td>
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</table>

Sub-scripts

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<tr>
<td>b</td>
<td>bubbling</td>
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<tr>
<td>mf</td>
<td>minimum fluidisation</td>
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<tr>
<td>p</td>
<td>particle</td>
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<tr>
<td>r</td>
<td>ratio</td>
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<tr>
<td>term</td>
<td>termination of interface</td>
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<td>total</td>
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Supplementary data to this article can be found online at http://dx.doi.org/10.1016/j.powtec.2013.12.014.
References


Appendix I
Result Analysis Report

Sample Name: sample1_alumina_53-75mic - Average
Sample Source & type: sample1_alumina_53-75mic - Average
SOP Name: Measured by: malvern-user
Result Source: Averaged
Measured: Tuesday, 11 December 2012 10:48:03 AM
Analysed: Tuesday, 11 December 2012 10:48:05 AM

Particle Name: Alumina
Particle RI: 1.780
Dispersant Name: Water

Table:
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<th>Span : 1.032</th>
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<tr>
<td>Specific Surface Area: 0.0819 m²/g</td>
<td>Vol. Weighted Mean D[4,3]: 84.305 um</td>
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<tr>
<td>d(0.1): 49.284 um</td>
<td>d(0.5): 79.199 um</td>
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<tr>
<td>d(0.9): 125.427 um</td>
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</table>

Graph:
- Particle Size Distribution
- Sample 1_alumina_53-75mic - Average, Tuesday, 11 December 2012 10:48:03 AM

Operator notes:
Result Analysis Report

Sample Name: sample3_alumina_1.78 - Average
Sample Source & type: 
Sample bulk lot ref: 

Accessory Name: Hydro 2000S (A)
Absorption: 0.1
Dispersant Name: Water
Dispersant RI: 1.330

Analysis model: General purpose
Size range: 0.020 to 2000.000 um
Weighted Residual: 0.832 %
Result Emulation: Off

Concentration: 0.2089 %Vol
Span: 0.876 um
Uniformity: 0.27
Result units: Volume

Specific Surface Area: 0.0614 m²/g
Surface Weighted Mean D[3,2]: 97.773 um
Vol. Weighted Mean D[4,3]: 108.648 um

Operator notes:

---sample3_alumina_1.78 - Average, Thursday, 22 November 2012 12:47:36 PM---
Result Analysis Report

Sample Name: sample1_alumina_125mic - Average
Sample Source & type: 
Sample bulk lot ref: 
SOP Name: 
Measured: Thursday, 29 November 2012 2:21:04 PM
Analysed: Thursday, 29 November 2012 2:21:05 PM

Particle Name: Alumina
Particle RI: 1.780
Dispersant Name: Water
Dispersant RI: 1.330
Specific Surface Area: 0.0367 m²/g

Accessory Name: Hydro 2000S (A)
Absorption: 0.1
Dispersant Name: Water
Dispersant RI: 1.330

Analysis model: General purpose
Size range: 0.020 to 2000.000 um
Weighted Residual: 1.034 %
Result Emulation: Off

Concentration: 0.3497 %Vol
Span: 0.878
Uniformity: 0.274
Result units: Volume

d(0.1): 109.160 um
d(0.5): 170.638 um
d(0.9): 264.064 um

---sample1_alumina_125mic - Average, Thursday, 29 November 2012 2:21:04 PM---

Operator notes:
Result Analysis Report

Sample Name: Alumina_mixture-4 - Average
Sample Source & type: Water
Sample bulk lot ref: 0.0589 m²/g

Concentration: 0.2567 %Vol
Specific Surface Area: 0.0589 m²/g

Dispersant Name: Water
Dispersant RI: 1.330
Absorption: 0.1

Particle Name: Alumina
Particle RI: 1.780
Accessory Name: Hydro 2000S (A)

Result Source: Averaged

Analysis model: General purpose
Sensitivity: Normal

Size range: 0.020 to 2000.000 um
Weighted Residual: 2.779 %
Result Emulation: Off

Volume In % d(0.1): 59.404 um
d(0.5): 116.806 um
d(0.9): 275.145 um

---Alumina_mixture-4 - Average, Monday, 4 March 2013 4:09:23 PM---

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Operator notes: Monday, 4 March 2013 4:09:24 PM
Result Analysis Report

Sample Name: wide_range_240-150-100-80-60_mix -
Sample Source & type: 
Sample bulk lot ref: 

Result Source: Averaged

Particle Name: Alumina
Particle RI: 1.780
Dispersant Name: Water
Dispersant RI: 1.330

Accessory Name: Hydro 2000S (A)
Absorption: 0.1
Dispersant Name: Water

Analysis model: General purpose
Sensitivity: Normal
Size range: 0.020 to 2000.000 um

Specific Surface Area: 0.0489 m²/g
Concentration: 0.2713 %Vol

Surface Weighted Mean D[3,2]: 0.0489
Volume Weighted Mean D[4,3]: 122.614 um

d(0.1): 62.124 um
d(0.5): 153.69 um
d(0.9): 550.148 um

Operator notes:

Particle Size Distribution

---wide_range_240-150-100-80-60_mix - Average, Wednesday, 6 March 2013 3:26:05 PM---
Result Analysis Report

Sample Name: sample1_Porous - Average
Sample Source & type: 
Sample bulk lot ref: 

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<td>Concentration:</td>
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<td>Specific Surface Area:</td>
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Vol. Weighted Mean D[4,3]: 79.705 um  
Surface Weighted Mean D[3,2]: 88.389 um

Evaluation

- Uniformity: 0.272 %
- Obscuration: 84.056 %
- Weighted Residual: 0.722 %
- Concentration: 0.1697
- D(0.1): 0.272 um
- D(0.5): 0.867 um
- D(0.9): 8.456 um

Operator notes:

- Sample Name: sample1_Porous - Average
- SOP Name: 
- Measured: Friday, 18 January 2013 10:59:45 AM
- Measured by: malvern-user
- Analysed: Friday, 18 January 2013 10:59:46 AM
- Result Source: Averaged

- Volume (%)

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- Size range: 0.020 to 2000.000 um
- Specific Surface Area: 0.0753 m²/g

- Absorption: 0.1
- Dispersant RI: 1.330
- Dispersant Name: Water
- Concentration: 0.1697

- Hydro 2000S (A)
- Averaged
- Malvern, UK
- Serial Number: MAL101100
- Mastersizer 2000 Ver. 5.22
- Result Source: porous alumina.mea

File name: porous alumina.mea
Record Number: 4
10/02/2014 12:06:19 PM
Chapter 4

Analytical model to predict fluidisation interface in vacuum fluidised bed

Section 1 (Published paper):


This chapter presents a simple analytical model based on the expanding gas theory responsible for the presence of fluidisation interface in vacuum conditions. The model proposed in this work can be readily utilized for drawing fluidisation maps of the powders and hence facilitate efficient utilization of the bubbling regimes for heat transfer analysis.
Chapter 4

Analytical model to predict fluidisation interface in vacuum fluidised bed

Section 1 (Published paper):


This chapter presents a simple analytical model based on the expanding gas theory responsible for the presence of fluidisation interface in vacuum conditions. The model proposed in this work can be readily utilized for drawing fluidisation maps of the powders and hence facilitate efficient utilization of the bubbling regimes for heat transfer analysis.
Authorship declaration

This is to certify that the first author, Mr. Apurv Kumar, of the paper titled “Analytical model to locate the fluidisation interface in a solid-gas vacuum fluidised bed” has done more than 85% of the work which includes conducting experiments, modelling, programming, analysis and preparation of the first draft of the manuscript. Other authors have been instrumental in providing guidance, discussion and final preparation of the manuscript.

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Deakin University

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Senior Research Fellow, Institute for Frontier Materials
Deakin University.

Dr. Subrat Das,
Lecture, School of Engineering,
Deakin University.

Dr. Weimin Gao,
Research Fellow, Institute for Frontier Materials
Deakin University.
Analytical model to locate the fluidisation interface in a solid–gas vacuum fluidised bed

Apurv Kumar\textsuperscript{a,⁎}, Peter Hodgson \textsuperscript{a}, Daniel Fabijanic \textsuperscript{a}, Weimin Gao \textsuperscript{a}, Subrat Das \textsuperscript{b}

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\textsuperscript{b} School of Engineering, Deakin University, 3216, Australia

Abstract

Vacuum fluidised beds have a distinct advantage of being operated with reduced mass consumption of the fluidising media. However, a low quality of fluidisation reduces the opportunity to utilise the bubbling regime in vacuum fluidised beds. Fluidisation maps are often used to depict the interface between the quiescent, bubbling and slugging regimes inside a fluidised bed. Such maps have been obtained by visual observations of the fluidisation interface in transparent fluidised beds. For beds which are visually inaccessible fluidisation maps are difficult to obtain. The present work therefore attempts to model the interface travel in a vacuum fluidised bed. The pressure gradient due to the bed weight has been determined to be a main contributor for fluidisation/defluidisation under vacuum. A simple analytical model based on the pressure gradient (PG model) is developed to predict the interface location in a vacuum fluidised bed. For a segregated bed, the Gibilaro–Rowe (GR) model is modified and used to predict the jetsam layer growth along with the fluidisation interface. The predictions are compared with the experimental data for minimal- and highly segregated particles and it is seen that for non-segregated powders the predictions are quite accurate. Lack of sufficient knowledge of bubble characteristics, however, impeded accurate prediction of the jetsam growth especially at high flow rates. However, an approximate complete fluidisation interface is successfully predicted using the GR–PG model.

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1. Introduction

The operation of fluidised beds under vacuum conditions rapidly degrades the quality [1,2] due to the existence of an interface that separates the dynamic and static regions near the minimum fluidisation conditions. The presence of the fluidisation interface is similar to the interface between the jetsam and flotsam in a binary segregated mixture. However, in vacuum conditions, even for a non-segregated mixture, the cause is the presence of a high pressure gradient inside the bed [1]. Due to the sub-atmospheric pressures, the bed weight induces a significant pressure gradient that causes the velocity of the fluidising medium to increase significantly while traversing the bed [3,4]. Differential fluidisation behaviour is therefore always observed where the bed either slugs vigorously at the top and bubbles at the bottom or bubbles at the top and remains quiescent at the bottom regions of the bed as the fluidisation velocity is decreased. This reduces the available optimum bubbling space of the fluidised bed for any heat and mass transfer application. Nonetheless, vacuum fluidised beds have a distinct advantage of reduced mass consumption [1]. This necessitates the study of fluidisation quality and Obtain optimised operating parameters to utilise the economic benefits of the vacuum fluidised beds.

Fluidisation maps (Fig. 1) describing the interface between the quiescent and bubbling regimes obtained are generally used to operate vacuum fluidised beds to avoid unfavourable regions during heat and mass transfer processes [1,4]. However, it becomes challenging to operate a fluidisation column where direct visualisation may not be possible due to setup limitations. Normalised fluidisation maps from transparent columns generally fail to accurately predict the interface position in an opaque column of different size due to various scale-up factors such as particle size, bed diameter, operating pressures and inlet velocity. Very few studies on the effects of these factors on fluidisation quality in vacuum are available in the literature [1,2]. Various models exist in the literature [5,6] that predict the segregation profile for binary mixtures. However, these models do not account for pressure gradients and hence cannot be used for interface prediction under vacuum conditions. It has been shown earlier that the pressure gradient plays an additional role in the interface travel for a segregated mixture under vacuum conditions [1]. The main objective of the present work is, therefore, to analytically model the travel of the interface inside a fluidised bed (for segregated and non-segregated mixtures) taking into account the pressure

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2. Interface prediction model

The defluidisation interface in a vacuum fluidised bed near minimum fluidisation conditions separates the quiescent and bubbling region. Originating near the distributor plate the interface travels upwards as the supply of fluidising gas is reduced. The following section derives a simple analytical model to predict the interface location inside the bed for a segregating and non-segregating bed after the bed acquires steady state conditions.

Rao et al. [7] have characterised segregation of particles based on the size and density ratio. A non-segregated (or minimally segregated) bed is defined as $1 < \rho_s / \rho_f < 2$ or $1 < d_r < 2; 1 < U_r < 2.5$, where $\rho_s = \rho_f \left( \frac{\rho_s}{\rho_f} \right), d_r \left( = \frac{d_f}{d_b} \right)$ and $U_r \left( = \frac{\rho_f}{\rho_s} \right)$ are the density, size and velocity ratios. All the powders used in the present study classify as a minimally segregated powder as per Rao et al. classification ($d_r = 1.1–1.2$). Similar powders have been used in our earlier study [1]. Ideally, for a mono-sized Group B particle (segregation absent), the fluidisation quality is due to the pressure gradient caused by bed weight that is similar in order of magnitude to the top pressure. As a result, the fluid experiences a significant reduction in pressure as it moves through the bed. For instance, at the minimum fluidisation condition for a bed weight pressure of 5000 Pa and operated at a top pressure of 100 Pa, the fluid experiences significant change in pressures resulting in substantial reduction in the fluid density. This causes the fluid to expand and increase the velocity to conserve the mass flow rate [3]. Thus, during defluidisation, the bottom of the bed begins to settle down while the top portion of the bed bubbles/slugs since the velocity in this region is sufficient to not only minimally fluidise but also bubble/slug the upper portion of the bed. With an increase of particle size, the velocity requirement for $U_{mf}$ increases [8] thereby increasing the velocity requirement for onset and termination of the interface.

2.1. Pressure gradient (PG) model

For analysis, consider a cylindrical column of size D and height $H$ with a bed made of particles of size $d_p$. Fig. 2 shows the fluidised bed in the column with height $h$ with an interface at a location $z$ from the bottom of the bed operating at a pressure $P_T$ at the top of the bed (at $z = H$). A mass flow rate balance analysis at the interface will yield the following expression for the velocity $v_z$ of the fluidising media.

$$\rho_z v_z = \rho_{z-dz} v_{z-dz}.$$  

If $P_b$ is the pressure due to the bed weight and $h$ is the bed height, the total pressure acting at any location within the bed is therefore,

$$P_z = (P_T + P_b) - \left( \frac{P_b}{h} \right) z.$$  

And the corresponding density of the fluidising media (assuming ideal gas behaviour) is therefore,

$$\rho_z = \frac{P_z}{RT}.$$  

Since the interface separates a fluidising and quiescent bed, the velocity at the interface is:

$$v_z \geq v_{mf}.$$
Combining Eqs. (1), (3) and (4)), and equating the differential mass flow rate to inlet mass flow rate (\(\rho_{\text{inlet}}V_{\text{inlet}}\) per unit area), we obtain

\[
v_z = \left(\frac{P_f + P_{j}}{P_{j}}\right) \cdot \frac{V_{\text{inlet}}}{V_{\text{inlet}}}.
\]

And using Eq. (2) and applying the limiting condition (4):

\[
z = \left(\frac{P_z + P_{j}}{P_{j}}\right) \cdot h_{j} \cdot \left[1 - \frac{V_{\text{inlet}}}{V_{\text{mf}}}\right] + z^*.
\]

For a binary fluidised bed made up of flotsam and jetsam, the interface position in the jetsam and flotsam layer separately (due to different pressure gradient) can be estimated, from the first principles (from Eq. (2)) as:

\[
z_{j} = \left(\frac{P_f + P_{j}}{P_{j}}\right) \cdot h_{j} \cdot \left[1 - \frac{V_{\text{inlet}}}{V_{\text{mf}}}\right] + z^*,
\]

\[
z_{f} = \left(\frac{P_f + P_{j}}{P_{j}}\right) \cdot h_{f} \cdot \left[1 - \frac{V_{\text{inlet}}}{V_{\text{mf}}}\right] + z^*.
\]

where

- \(P_j, P_f\) are the pressure in the bed due to jetsam and flotsam layer, respectively, and
- \(z^*\) is the pure jetsam layer height.
- \(h_j, h_f\) are the jetsam and flotsam layer height at complete segregation.

In order to estimate the minimum fluidisation velocity in Eqs. (6)–(8), the general expression by Llop et al. [8] can be used for any operating pressure \(P_f\), which is expressed as a function of Knudsen no. (\(Kn_{p}\)) and Archimedes no. (\(Ar\)).

\[
U_{\text{mf}} = \frac{H}{P_{f}} Re_{mf}
\]

where \(Re_{mf} = \left(\frac{z^*}{K_1}\right)^2 + \frac{U_{\text{mf}}}{K_2} \cdot \frac{1}{K_3} \cdot \frac{1}{\sqrt{\rho_{\text{j}}/\rho_{\text{mf}}}}\); while \(K_1, K_2, C_1, C_2\) and \(C_3\) being constants [8].

Thus, utilising Eqs. (6)–(8), the location of the interface can be predicted for cases satisfying the following assumptions:

a) the powder consists of a uniform particle size with no size distribution and hence only the pressure gradient is responsible for interface travel,

b) the particles are spherical with size \(d_{p}\),

c) and the height of the bed during travel of interface from bottom to top is \(h_{mf}\).

A recent study on fluidisation quality in vacuum has shown that the interface travels linearly in a minimally segregated bed while with different slopes in a segregated bed [1]. As the jetsam particles settle down with defluidisation of the bed, the interface during its upward movement travels with different rates which vary according to the particle size. Fig. 3 shows the differential rates of travel of the interface for the segregated and non-segregated particles under vacuum conditions (reproduced here from [1] for convenience). The height of the jetsam layer can be seen to remain constant for all the vacuum pressures. Therefore, in order to predict the interface location inside the bed, the jetsam/flotsam concentration distribution inside the bed should first be estimated followed by the prediction of the interface location using Eqs. (6)–(8).

2.2. Gibilaro–Rowe model

The Gibilaro–Rowe (GR) [5] model is the fundamental model used to estimate the segregation distribution for binary mixtures in a gas–solid fluidised bed. It is a steady state differential formulation of physical mechanisms of segregation that accurately predicts the segregation profiles for the binary mixtures. The parameters used in the GR model have been studied by Naimer et al. [6] and Tanimoto et al. [9] to link them to the fluidisation behaviour. The present work therefore solves the GR model [5]. Wake phase corresponds to the particles present in the wake of a bubble and the bulk phase is the region occupied by rest of the particles in the bed. The differential equation that defines the concentration of jetsam in the bulk and wake phase, respectively, is thus given by:

\[
e \frac{d^2C_B}{dz^2} + \left(1 + \frac{1}{\lambda} \frac{dC_B}{dz}\right) \frac{dC_B}{dz} - \frac{q_B h}{w} (C_W - C_B) = 0
\]

and

\[
w \frac{dC_B}{dz} - q_B h (C_W - C_B) = 0
\]

where

- \(w\) solids circulation rate, \(L^3/T\)
- \(k\) segregation coefficient, \(L^3/T\)
bulk/wake exchange coefficient, $L^2/T$

axial mixing coefficient ($r$/$wh$), $L^4/T$

total bed height, $L$

$w/k$

volumetric fraction of jetsam in the wake

volumetric fraction of jetsam in the bulk.

The numerical solution to which is (by second order Taylor series approximation):

$$C_{B_i+1} = \frac{1}{\Delta z} \left[ C_{B_i} \left( \frac{2E}{\Delta z} + 1 + \frac{(1-2C_i)E}{x} + \frac{q_{wh} z \Delta z}{w} \right) - \frac{C_{B_i+1} E}{\Delta z^2} \frac{q_{wh} z}{w} \right]$$

and

$$C_{W_i+1} = \frac{q_{wh} (C_{B_i} - C_{W_i})}{w} \Delta z + C_{W_i}. \quad (13)$$

In the present simulation, axial mixing was neglected ($E = 0$) (see Section 4.3).

The mass balance for the jetsam in the bed is:

$$f_j = Z' + \int_0^Z C_{w'j} \, dZ \quad (14)$$

where

$Z'$ is the non-dimensional jetsam layer and

$f_j$ is the initial jetsam volume fraction in the bed,
\( C_{\text{avg}, Z} = (1-f_w)C_{B, Z} + f_w C_{W, Z} \). 

The following boundary conditions are used [5]:

\[
C_{B, Z} = 1 \quad \text{and} \quad C_{W, Z} = 1 \quad \text{at} \ Z = 0
\]

The initial condition used in the present simulation is:

\[
C_{B, Z} = 1 \quad \text{and} \quad C_{W, Z} = 1 \quad \text{for} \ Z = Z'.
\]
iteratively evaluating the bulk and wake phase jetsam concentration (CB and CW) and correcting the jetsam layer height to satisfy the jetsam concentration balance given in Eq. (14). The fluidised bed is divided into n segments and the local jetsam concentration is determined using Eqs. (12) and (13). Finally, the overall jetsam mass balance is carried out and the jetsam height is corrected. The simulation is run iteratively till a convergence criterion of \((Z_{\text{prev}} - Z_{\text{present}}) < 0.0001\) is satisfied.

The simulation flowchart (Fig. 4) is used to obtain the segregation profile for the powders used in the present work. Table 1 describes the various parameters used to determine the segregation coefficients used in Eqs. (10) and (11).

3. Experimental study of segregation in vacuum

3.1. Experimental method

A cylindrical fluidised bed made of polycarbonate (50 mm ID and 1 m length) was used with a porous sintered steel disc as a distributor plate (Fig. 5). The vacuum in the cylinder was controlled using bypass needle valves. Variable area flowmeters with an accuracy of \(\pm 3\%\) were used to measure air flow rate to the vacuum chamber. To ensure correct flowmeter readings, a pressure of 50 kPa was maintained on the entry and exit sides of the flowmeter using a two-stage needle valve configuration. Pressure transducers with an accuracy of 0.25% were used to measure the pressures at two locations inside the chamber: 5 mm above the distributor plate and 5 mm below the exit of the chamber. In order to maintain consistency in results, similar inlet conditions (Table 2) were used in all experiments. The pressure data were acquired (10 Hz) using an ALMEMO 2590 data logger and were analysed offline using a personal computer. The sieved powders were analysed for size distribution by a Malvern Mastersizer 2000, which utilises laser diffraction to measure the size of particles. It does this by measuring the intensity of light scattered as a laser beam passes through a dispersed particulate sample. This data is then analysed by the device to calculate the size of the particles that created the scattering pattern.

The particles were fluidised in air for various sub-atmospheric pressures and the pressure drop was measured during the defluidisation along with the interface tracking. Once the fluidisation maps of the segregated powders were obtained by interface tracking and the pressure drop vs. flow rate data, the jetsam concentration was determined for two flow rates (2Umf and 4Umf). In order to measure the jetsam distribution, the bed was divided into 10 equal segments and powder samples were collected from them using a vacuum collection device, which was made up of a vacuum pump and a particle filter to collect the powders. During the experiment, the bed was run for 30 min to obtain a steady state and then the air supply was immediately turned off in order to freeze the mixing pattern. A vacuum hose suspended from the top of the bed traversed vertically down to collect the powders from each segment. Care was taken to avoid mixing within the subsequent segments. After collection, the powders were sieved to separate the jetsam and weighed to calculate the jetsam mass fraction in each segment. The coarser particles were also coloured to visually observe the jetsam layer using a camera.

The fluidisation interface location in a fluidised bed can be measured using an image processing technique called optical flow [10] that

![Original image of segregated bed at atmospheric pressure (blue region is static and made of coarser particles)](image1)

![Contour plot of time averaged velocity (in pixels/sec) using the optical flow technique](image2)

![Binary image of time averaged velocity plot showing the interface](image3)

Fig. 6. The prediction of fluidisation interface using optical flow velocity vectors for a segregated bed.

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Inlet conditions and powder characteristics.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet pressure</td>
<td>50 kPa</td>
</tr>
<tr>
<td>Inlet temperature</td>
<td>27 °C</td>
</tr>
<tr>
<td>Inlet flow rate</td>
<td>100–10,000 cm³/min</td>
</tr>
<tr>
<td>Bed height</td>
<td>400 mm</td>
</tr>
<tr>
<td>Bed diameter</td>
<td>50 mm</td>
</tr>
<tr>
<td>Vacuum pressure</td>
<td>1000–50 mbar</td>
</tr>
<tr>
<td>Alumina powder density</td>
<td>3800 kg/m³</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Powder</th>
<th>d(3,2) (μm)</th>
<th>d(0,5) (μm)</th>
<th>d(4,3) (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alumina: 80 μm</td>
<td>80</td>
<td>86</td>
<td>90</td>
</tr>
<tr>
<td>Alumina: 95 μm</td>
<td>95</td>
<td>103</td>
<td>108</td>
</tr>
<tr>
<td>Alumina: 160 μm</td>
<td>160</td>
<td>170</td>
<td>180</td>
</tr>
<tr>
<td>Alumina: 313 μm</td>
<td>313</td>
<td>332</td>
<td>353</td>
</tr>
</tbody>
</table>
distinguishes the static and dynamic portion of the bed by calculating the velocity of the objects in two subsequent images. Video images of the fluidised bed was taken using a macro lens at 50 fps and the velocity vectors were calculated using Lucas–Kanade algorithm [11] in the Computer Vision Toolbox in Matlab R2013. A time-averaged plot of velocity vectors were obtained for a video run time of 20 s, which reveals the interface that separates the static and dynamic portions of the bed. The binary image was obtained by defining a threshold value equivalent to the noise (0.03 pixels/s) when the bed was completely static. The interface height from the top of the image is then marked from the binary image with the condition that at least 90% of the bed must be static. An average error of ±10% was found between visually observed and image processed tracked interface. This method has been previously validated to accurately predict the interface in vacuum fluidised beds [1]. The time-averaged velocity contours were calculated for a segregated bed made up of 310 and 80 μm particles (Fig. 6) where the interface between the coarse and fine particles can be seen very clearly from the binary image.

3.2. Powder characteristics

In order to compare the predictions of the interface-tracking model, four powder samples were used in the present study: Alumina powders belonging to Group B having a Sauter mean diameter \( d_{3,2} \) of 80, 95 and 160 μm and a segregated mixture of 80 and 310 μm. The particle distribution of the powders used is shown in Fig. 7. The powders were analysed for size distribution by Malvern Mastersizer 2000. Table 2 reports the particle sizes for various powders used in the present work.

4. Results and discussion

4.1. Minimal segregated powders

The pressures were reduced from atmospheric to vacuum and the interface was tracked during the defluidisation process. Figs. (8)–(10) show the interface tracked and the comparison with the predicted interface using Eq.(6). Repeated tests (three) were done only for the 80 μm powder (due to experiment complexity) to estimate the accuracy of the interface location measurement and Fig. 10 depicts the standard deviation as error bars. For the minimally segregated powder, the
interface travels vertically with a constant rate due to a narrow size range. The predicted interface location is accurately predicted by Eq. (6) (with an average standard error of ±12% as compared to the experimental data), which was based only on the pressure gradient. Although the slope of the interface is predicted accurately, the onset and termination of the interface are not precisely predicted. This is due to the sensitivity of the pressure gradient model to the particle diameter which assumes the particles to be of uniform spherical shape. In addition, the particles have a size range distribution due to which coarser particles settle at the bottom during de-fluidisation. The PG model assumes a uniform particle size in the powder. Fig. 11 shows the variation of the interface prediction for 95 μm particles at atmospheric pressure for various characteristic particle diameters such as median d(0.5), surface weighted mean d(3,2), and volume weighted mean d(4,3). The Sauter mean diameter (d(3,2)) is used since it relates to the fines present in a powder sample and the weight fraction of flotsam is greater than 0.5 for the binary mixtures used in the present study. In the present case, an average difference of 20 μm was observed between the d(3,2) and d(4,3) particle size. Hence for analysis purpose, only the d(3,2) particle size is used for all powder samples.

4.2. High segregated particles

It was observed during the de-fluidisation test for the segregated powders that the growth of the jetsam layer and the interface movement took place simultaneously for all operating pressures. For atmospheric pressures, the jetsam and de-fluidised interface coincided. However, under vacuum the two interfaces were quite distinct. The de-fluidisation interface, along with having a distinct growth rate and originated after the jetsam layer, starts to build. In other words, there was always a lag between the jetsam layer and de-fluidisation interface which increased with reducing pressure. Therefore, in order to use the pressure gradient model to predict the interface travel in highly segregated mixtures, it is necessary to predict the jetsam layer growth first with decreasing flow rate. The interface location can then be estimated based on the jetsam layer height. The growth of jetsam layer can be seen in Fig. 12 for all the pressures considered in the present work.

Further, at lower flow rates, the jetsam layer stayed at a plateau (with reducing supply of air) for a while after which the growth continued. This occurs because the separation of coarser and finer particles within the jetsam size range occurs as the supply velocity is reduced and causes jetsam layer growth after reaching a plateau.

The GR model determines the steady state jetsam layer for the vacuum pressures and then the interface is predicted by Eqs. (7) and (8) in the segregated layers. The de-fluidised jetsam layer is calculated by the segregation pressure gradient (PG) model, which is then used as the bubble initiation point during the simulation. The jetsam layer height and de-fluidised interface is corrected iteratively in the simulation till the jetsam mass balance is satisfied. In earlier works [5,6,12] the entire pure jetsam layer was always considered to be de-fluidised. However, it is seen in the present work that the de-fluidised layer lags the jetsam layer under vacuum conditions and hence this assumption is not valid anymore. A lower concentration of jetsam was predicted in vacuum when the pure jetsam layer was considered completely de-fluidised and the jetsam height was taken as the bubble initiation point. This was subsequently used to estimate bubble size at different heights of the fluidised bed. In order to estimate the fraction of de-fluidised jetsam layer the jetsam layer weight fraction was calculated as \( f_j - C_{avg,mix} \), where \( C_{avg,mix} \) is the average weight fraction of the mixed layer which was then used in Eq. (7).

---

**Fig. 9.** Comparison of fluidisation interface prediction by the pressure gradient (PG) model and the experiments for d(3,2): 95 μm.
In order to simplify the solution, the assumption made in the boundary condition (Eqs. (16) and (17)), that no exchange of particles between bubble and bulk phase takes place in the pure jetsam layer is preserved even though the jetsam layer and defluidised layer do not coincide. During calculation, care was taken to convert the superficial velocities to actual chamber velocities, which considerably increased due to the presence of vacuum. Fig. 13 compares the prediction of jetsam weight fraction for the segregated mixture, Mixture 1, to the data obtained from experiments. The pure jetsam layer height is predicted with an average of 10% accuracy in comparison to the experimental data. Since the experimental bed was divided into only 10 segments (simulation considered 100 nodes), the transition interface between mixed and pure jetsam is not captured accurately by the experiment. Apart from this, the prediction by the model is not accurate for higher flow rates under vacuum pressures, although the model performs accurately near the minimum fluidisation velocity ($\approx 2U_{mf}$). For vacuum pressures it was observed that the jetsam weight fraction was under-predicted by at least 25% for a velocity of $4U_{mf}$. Neglecting axial mixing can be a reason for this inaccuracy as it may play a significant role under vacuum pressures. However, in the present case lack of information on bubble characteristics and size variation under vacuum pressure can be attributed to the inaccuracy in the current model as effect of axial mixing is negligible even under vacuum conditions (see Section 4.3). The initial bubble diameter and the subsequent bubble growth are predicted using the empirical models that have never been tested in vacuum pressures. The bubble growth is expected to be higher under vacuum as the gas expands in the increased pressure gradient atmosphere. The inaccuracy in bubble size prediction affects other related parameters such as bed height expansion, which is critical in determining the jetsam height. The comparison between experimental bed height and the bed height model used in the present simulation (Table 1) is seen in Fig. 14. As the pressures are reduced below atmospheric, bed height at higher velocities are inaccurately predicted. Detailed investigations are therefore needed to improve our understanding of bubble characteristics under vacuum conditions.

Fig. 10. Comparison of fluidisation interface prediction by the pressure gradient (PG) model and the experiments for d(3,2): 80 $\mu$m (error bars indicate standard deviation).

Fig. 11. Sensitivity of the interface prediction to the powder size for d(3,2): 95 $\mu$m powder.
different operating pressures to obtain the interface growth in the segregated bed. Fig. 15 shows the prediction of the jetsam layer growth with defluidisation and the interface travel in the bed and its comparison with the experimental data for atmospheric and vacuum pressures. The growth of jetsam and the subsequent interface location is predicted accurately (average standard error of 15%) by the Gibilaro–Rowe and pressure gradient (GR–PG) model for atmospheric pressure. However, as the pressures were reduced the error increased monotonically with a maximum standard error of 50% for 55 mbar at higher flow rates \((U/U_{mf} > 2.5)\). This is mainly due to inaccurate prediction of bubble characteristics at higher velocities. However, for velocities near the minimum fluidisation velocity \((U/U_{mf} \approx 1)\), the predictions were quite accurate with the maximum standard error of 10%. The GR–PG model predicts the growth of the jetsam and interface only up to the jetsam plateau since the size distribution was not considered in the numerical simulation. As the bed defluidised beyond the jetsam layer, only the PG model was used for interface location prediction in the flotsam as seen in Fig. 15. Combination of these two models approximates the complete interface location inside a segregated bed during defluidisation.

4.3. Axial mixing

In order to study the effect of axial mixing, another set of simulations were run by considering \(E > 0\). Table 1 gives the correlation used to
measure $E$ for the segregated bed. The actual $E$ was of the order of $10^{-4}$ for the velocity considered in the present case. Therefore, a parametric study was carried out by varying $E$ from 0 to 10 to study the effect more clearly. Fig. 16 shows the variation of the jetsam concentration in the bed for various $E$ values for atmospheric and 250 mb pressures. No significant difference is obtained for $E = 0.1$ and the results show erroneous values for $E > 1$. Axial mixing therefore can be safely assumed to play no role at all in determining the jetsam concentration using the GR model. This is consistent with the results of other researchers [6,12,13].

5. Conclusions

Knowledge of the interface in vacuum greatly enhances the opportunity to utilise the bubbling regime for heat and mass transfer processes. Prediction of the defluidised bed interface with reducing supply of fluidising media was carried out in the present work for minimally and highly segregated fluidised beds under atmospheric and vacuum pressures. The presence of a pressure gradient operating under vacuum is the main contributor to the defluidisation of the bed and a simple analytical model was proposed in the present work to predict the location of the defluidised interface. The pressure gradient model, in combination with the Gibilaro–Rowe model, predicted the fluidisation interface accurately for a minimally and highly segregated bed, with maximum accuracy for a narrow size range of particles. It was seen that the prediction deviated from the experimental data mainly due to the lack of information on the bubble characteristics under vacuum conditions. In addition, the GR model needed modification in its basic assumption of a completely defluidised jetsam layer, which was observed to be incorrect for vacuum conditions. With reducing supply of air, only a fraction of the jetsam layer was observed to be defluidised in the experiments. With improved correlations of bubble diameter and growth, the present pressure gradient model can be used to predict the fluidisation interface for all sets of powders.

References

Fig. 16. Parametric study for variation of E for different pressures.
Chapter 5

Heat transfer investigations in vacuum fluidised bed

1. Introduction

Heat transfer forms the main application of a fluidised bed and vacuum fluidisation has potential to offer economic benefits due to reduced mass consumption as indicated in the Chapter 1. Despite Group B powder’s poor quality at high vacuum, proper knowledge of hydrodynamics enables optimum utilisation of vacuum fluidisation even though Group B powder is known to exhibit poor quality [1]. However, Group A powders have good quality of fluidisation even under vacuum. Therefore, motivation to study heat transfer aspects under vacuum fluidisation is highly justified since powder behaviour can be optimised.

Limited observations in literature are present on heat transfer aspects under vacuum. Shlapkova [2] found that the heat transfer declines with pressure reduction and degrades rapidly after 13.3 kPa. Results from Bhat and Whitehead [3] suggest that heat transfer remains constant in range of 10-100 kPa. These studies were limited in their scope and no parametric study was carried out to study the dependence of heat transfer on other operating parameters as operating temperature and location inside the bed. It is known that the particle size range and the expanding gas due to pressure gradient contributes to the quality of fluidisation [1, 4] and hence can affect the heat transfer considerably. In addition, inclusion of radiation heat transfer can also affect the overall heat transfer and may not depend on the vacuum conditions.

The work in the present chapter addresses two critical questions:

1) How does the heat transfer vary under vacuum conditions with location of the heat source inside the bed?
2) What is the effect of particle size on heat transfer under vacuum conditions?

Answers to these questions will reveal a better picture of heat transfer characteristics than is present in the literature currently. Use is made of information gained from Chapter 1 and 2 on the quality of fluidisation and fluidisation maps in order to carry out the parametric study.

2. Lumped capacitance method

In order to carry out a comparative study of heat transfer with pressure, estimation/measurement of heat transfer coefficient is necessary. Many researchers [5-8] have used a heat transfer sensors made of assembly of heaters, metal block and thermocouples. With knowledge of operating conditions and transient heat transfer characteristics, the local temperature on the immersed surface can be predicted. In the present work, to study the effect of pressure on heat transfer, a lumped capacitance method is used to estimate the local heat transfer coefficient based on the local temperature profile of the immersed surface. A heat transfer sensor made of a copper block, surface & bed thermocouples and a heater is used (Fig.1(a)). A lumped capacitance method applies to a system with no spatial variation of temperature and estimates the heat transfer coefficient for a transient process [9]. This method has been previously applied to many heat transfer analysis as free and forced convection in packed beds, pipes, heated surfaces etc and has been a very effective method [9]. Its use in fluidised bed has, however, been practically non-existent to the best of author’s knowledge.

The overall heat transfer is known to be made of three major heat transfer modes, namely (a) gas convection, (b) particle convection and (c) radiation. Thus,

\[ h = h_{g, \text{conv}} + h_{p, \text{conv}} + h_{\text{rad}} \]  

(eq. 1)

Any change in the heat supply to the source would disturb the thermal equilibrium of the bed and initiate cooling/heating process that depends directly on the fluidisation characteristics. Therefore, study of the history of temperature variation of the heat source can be utilised to
compare the heat transfer at different pressures. A lumped capacitance analysis can estimate the transient heat transfer from/to the heat source which is a direct function of the fluidisation characteristics. Biot number \( (Bi = \frac{hL}{k_c}) \) is less than 1 (Bi ~ 0.01) for heat transfer coefficient of the order of 500-800 W/m² K (typical values for bed made up of alumina particles of size 100-500 μm for a copper block and is 30mm in length) and hence use of the lumped capacitance method is justified. In addition, the steady state temperatures that the heat source attains also reflects the cooling/heating strength of the fluidised bed operating at any given pressure. These parameters can be effectively used for a comparative study.

2.1 Mathematical formulation of lumped capacitance method

Let us consider a heat source inside a fluidised bed at location L (from the distributor plate) being supplied by heat energy Q (watts) and a temperature, \( T_s \). A change \( +/-\Delta Q \) in the heat supply initiates the cooling/heating of the source by the fluidised bed until a new thermal equilibrium is attained in time \( t \). A transient energy balance (for a cooling environment) at the source boundary will yield (see Fig. 1(c)),

\[
\rho m c_{p,s} \frac{dT_s}{dt} = -h A_s (T_s - T_{ambient})
\]  

(eq.2)

With the initial condition (for cooling of the copper block),

\[ T_s = T_{s, \text{max}} \text{ at } t = 0 \]

The solution to Eq.2 is:

\[
(T_s - T_{ambient}) = (T_{s, \text{max}} - T_{ambient}) \times e^{-\frac{hA}{m c_{p,s}} t}
\]  

(eq. 3)

Which can also be written in the form:

\[ T = e^{-\frac{t}{\tau}} \]  

(eq.4)

Where \( \tau = \frac{(T_s - T_{ambient})}{(T_{s, \text{max}} - T_{ambient})} \)
and $\tau = \frac{mc_p}{hA}$, also known as the time constant.

The time constant, $\tau$, will depend on the quality of fluidisation and hence this parameter can be utilised to compare the heat transfer characteristics at different pressures.

In the present work, the lumped capacitance method is used to study only the cooling capacity of the fluidised bed as this will enable calculations without taking into account the heat energy supplied.

Fig.1: Schematic of the copper block assembly (a); the vacuum fluidised bed setup (b) and the Lumped capacitance analysis on block surface (c)
3. Experimental method

A copper block (φ20mm and 38mm length) inserted with an electric heater (φ6 and length 36mm) is used as a probe to study the heat transfer in a vacuum fluidised bed (Fig.1 (a)). Equipped with surface and ambient thermocouples (3 each separated by 120° angle), the copper block was inserted vertically inside the bed. The surface thermocouple (K type-0.75% accuracy) were made from polyamide/fibreglass-junction insulated lead wire and was pasted on the copper surface by a silicone glue. The maximum operating temperature for the surface thermocouples were 260° C. The ambient thermocouples (K type-0.5% accuracy) were 3 mm diameter mineral insulated 310SS with closed junction. The air thermocouples were inserted through a bore and safely bent to lie adjacent to the surface thermocouple at a distance of 8mm from the surface. The maximum operating temperature of ambient thermocouples are 1000° C. Dalton’s wattflex electric heater with a maximum wattage of 170 W and 120 V was used. The heater of size 6.38 mm diameter and 31.75mm length was used as the heat source. The top and bottom surface of the copper block was insulated using a ceramic cement of 2mm thickness.

A metallic cylindrical column of diameter 56mm and length 960 mm having a porous sintered steel disc as a distributor plate was used to carry out the fluidisation tests (Fig.1(b)). 4 sets of flowmeters with different flow scales (0-100 cc/min, 0-1000 cc/min, 0-4lpm and 0-10 lpm) each with an accuracy of 3% was used to meter the air flowrates. To ensure correct mass flowmeter readings a pressure of 50 KPa was maintained on entry and exit side of the flow meter using a two-stage needle valve configuration. Pressure transducer with an accuracy of 0.25% was used to measure the pressure at the top of the column which was connected to a vacuum pump. The pressure data were acquired (1 Hz) using ALMEMO 2590 data logger and were analysed offline using a personal computer. A vacuum pump was used to create the sub-atmospheric pressures. The fluidisation column was sealed using a ceramic cement at the lid
through which the heater and thermocouple leads were taken out from the chamber. This proved to be a low cost alternative for a feed through for the lead wires and sealed the chamber effectively.

Two cases of experiments were carried out to investigate heat transfer under vacuum: Case (a) heat transfer at constant heat input and Case (b) heat transfer at constant flowrate. In case (a), the flowrate was changed usually from $U_{mf}$ to a flowrate which ensured complete fluidisation of the bed and devoid of slugs, as determined from the fluidisation maps after the block reached a desired temperature while maintaining the heat input constant. Whereas, in case (b) the copper block was heated at a certain flowrate and then the heat input was switched off and the temperature profile was recorded. Time $t=0$ corresponds to time when the power input is switched off or the flowrate is increased. The temperature plot shows an instant decline in magnitude at $t=0$.

![Fig.2: Typical temperature profiles of the copper block and the bed for (a) constant flowrate and (b) constant heat input](image)

Objective of these cases was to observe the cooling capacity of the fluidised bed under various conditions and then quantify the heat transfer (using Lumped capacitance method for case (a))
data. The results of different powder size were then compared which helped observe the effect of pressure on heat transfer as a part of the initial investigation. Fig. 2(a) shows a typical temperature profile for a case (a) experiment. Both heating and cooling of the block are shown along with the air temperature. Case (b) was used to estimate the heat transfer coefficient at different flowrates (from 0.5 $U_{mf}$ to 4-8$U_{mf}$). Fig. 2(b) shows the temperature profile obtained while heating and cooling the block at constant flowrate. The block was heated with an input power $Q$ and then it was switched off at either steady state temperature or 150°C, whichever appeared first. The air and surface temperature were then used to fit the exponential curve and the value of $h$ was deducted. All the fitted curves in the present analysis had $R^2 > 0.80$. Table 1 gives the physical properties of the copper block.

The experiments were carried out by varying the following parameters:

- Particle size – 3 sizes (100, 180 and 250 μm)
- Heat source location – 3 location (Bottom, middle and top)
- Pressure– 7 pressures (Atmospheric, 550, 250, 95, 75, 55 and 35 mbar).

The results for particle size 100 and 180 μm were repeated 3 times to verify the repeatability and obtain the standard deviation for the experimental data. The density of the powder is

<table>
<thead>
<tr>
<th>Material</th>
<th>Thermal conductivity (W/mK)</th>
<th>Specific heat (J/kgK)</th>
<th>Density (gm/cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper</td>
<td>401</td>
<td>390</td>
<td>8.96</td>
</tr>
<tr>
<td>Alumina</td>
<td>40</td>
<td>930</td>
<td>3800</td>
</tr>
</tbody>
</table>

Copper block weight : 0.1 Kg

* Note: The alumina particle properties are obtained from the MSDS of the manufacturer.
maintained constant with the use of only monolithic alumina. Study of porous particles and its effect on heat transfer is beyond the scope of the present work. Porous alumina with a significantly less density has shown good quality fluidisation in Chapter 3. It is expected that use of porous particles will improve heat transfer performance of the fluidised bed.

4. Results and discussion

4.1 Case (a): Constant heat input

The steady state temperatures obtained by the copper block for different particle size, location and pressures are shown in Fig. 3-5.

4.1.1 Effect of particle size

The average temperature with respect to different location inside the bed is observed to increase with reduction in pressure. Increasing the particle size similarly has a degrading effect when

![Fig.3 : Steady state temperature during cooling of the copper block for constant heat input (Case(a)) for 100 μm alumina powder](image)
the size is increased from 100 to 250 μm (Fig.6). The heat transfer at atmospheric pressure is a function of the particle size and tends to reduce the heat transfer when the size is coarse for Group B particles [10]. For vacuum pressures, the average temperature for the three locations inside the bed is in similar way a function of the particle size.

4.1.2 Effect of pressure and location
Studies on heat transfer between immersed surface and the bed indicate a moderate dependence on the location, both axially and radially [11, 12]. Similar moderate variation in steady state temperature can be also observed at atmospheric pressure (Fig.6) with the three locations tested (Bottom: 50 mm; Middle: 230 mm and Top: 400 mm, from the bottom of the bed). The temperature was lowest at the middle and highest at the top. However, as the pressures are

Fig.4 : Steady state temperature during cooling of the copper block for constant heat input (Case(a)) for 180 μm alumina powder
reduced below atmospheric a very interesting trend is observed for all the particles. Higher temperatures were recorded for all the powders as the pressures were reduced below atmospheric. With particle size increase, the temperatures increased along with an increase in the magnitude of \( (T_{\text{top}} - T_{\text{middle}}) \). This behaviour is seen to magnify after 100 mbar (approx.) below which all the particles size enter the slip/transitional flow regime (Fig.7). The finer powder is seen to be maximally affected by the reduction in pressure below 100 mbar.

This behaviour under vacuum offers a clear insight into the mechanism of heat transfer especially for the particle convection. While it’s well-known that the heat transfer from

Fig.5 : Steady state temperature during cooling of the copper block for constant heat input (Case(a)) for 250 μm alumina powder

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an immersed surface to the bulk of the bed depends entirely on bubble and particle characteristics, the variation of their relative contribution is difficult to ascertain due to interaction of different operating parameters. In general, the particle convection is the dominant mode of heat transfer in fluidised beds. The periodic movement of the particles “scourge” the gas film layer, thus reducing the boundary layer thermal resistance and increasing the heat transfer [10]. At atmospheric pressures, in the bubbling regime, the particle convection is significantly higher than gas convection. It is this particle movement that gives high heat transfer capability to the fluidised bed. However, it is not yet known how sub-atmospheric
pressure influences the particle convection heat transfer. Upon reasoning based on known principles, the particle convection depends on the bubble size and velocity. It is known that displacement of particles is caused only due to bubble movement [13]. The particle movement that participates in heat transfer from the immersed surface is particularly due to the horizontal movement caused by the low pressure region in the wake of the passing bubble [14]. The bubbles can be seen as a source of kinetic energy for the particles whereas the emulsion is the sink. The apparent viscosity can be understood to offer resistance to solid circulation and is a characteristics of particle morphology, size and density. The wake of the bubbles creates the pressure differential that draws in the surrounding fluid along with the particles. In absence of any immersed surface these drawn particles is carried vertically upwards by the bubbles. When an immersed surface is present, the particles strike the wall instead of plunging inside the bubble. This kinetic energy transfer from bubble to particles depends on the bubble size, its velocity and the drag force between the gas and particles. In light of present study on pressure affect, any changes in the momentum transfer capability of the fluid as well as bubble size will alter the particle convection.

Fig. 7: Variation of $T_{\text{top}} - T_{\text{bottom}}$ and Kn with pressure for 100, 180 and 250 $\mu$m
Heat transfer coefficients are derived from case(b) experiments. Steady state temperatures were used in initial study for qualitative investigations. The results on effect of location on steady state temperatures attained by the immersed surface reveal a very interesting information on physics of heat transfer. The steady state temperatures obtained during the cooling of the block reflects the heat transfer capacity of the bed. Higher temperatures indicate lower heat transfer and vice versa when operated under similar conditions. Under normal atmospheric conditions, the results (Fig.6) indicate that the bubble and particle characteristics don’t vary significantly enough to alter the heat transfer along the vertical axis of the bed. Although, there are changes in the bubble size, shape and velocity, the heat transfer is minimally affected at atmospheric pressure. The variation in particle velocity with height is negligible and hence the heat transfer remains unaltered with location. This is however, not true for vacuum fluidisation. A delta (Δ) effect can be observed for heat transfer where its magnitude increases from the bottom of the bed up to middle and then plummets towards the top. There can be two possibilities that can explain the variation of heat transfer with location. The presence of vacuum can cause heat transfer being diminished at the top and/or being enhanced at the middle. Due to large pressure variation in the bed, gas expansion takes place rapidly which affects the bubble characteristics thus altering the local heat transfer with height. A larger bubble size decreases the renewal frequency of the particles contacting the immersed surface [10]. This effect can be understood to contribute towards weakening of heat transfer at the top. This is further discussed in chapter 6.

In the slip flow regime, the momentum exchange capacity of air is significantly lower than the atmospheric conditions which explains the overall decrease in the magnitude of the heat transfer with pressure in the entire bed including the top portion. This is quite similar to loss of heat transfer capacity in micro-channel flow due to slip flow. These two effects complement each other and the resultant effect causes the heat transfer to vary along the vertical axis.
An increase in heat transfer from bottom to middle location is generally caused by the increasing particle velocity due to the presence of stable and greater bubble size towards the top. At atmospheric pressure the bubble travels in a nearly constant pressure atmosphere, whereas the vacuum pressure offers a highly varying pressure conditions. This may not allow the bubble to acquire a stable size which is observed in atmospheric conditions [15]. Instead a continuously varying bubble size causes the heat transfer to increase while travelling from bottom to middle and then decrease it from thereupon.

### 4.2 Case (b): Constant flowrate

Experiments were conducted to quantify the heat transfer and study its variation with pressure, location and particle size. The temperature profiles during cooling of the block and air for 100 and 180 μm for all the three location, different pressures and flowrates obtained from one pair of thermocouples are shown in Fig. 8-9. The surface temperature declines in a near exponential manner till it equals the bed temperature. When static, the bed temperature is seen to increase due to the proximity of the thermocouple to the block. In order to gain more information from these curves, the temperature profiles were non-dimensionalised according to Eq.3 (of the form $T = e^{-Ax}$) and exponential curves were fitted using the Origin plotting software (Fig.10-15). The results for different pressure and location is shown for 100 and 180 μm. The curve fitting report generated by Origin is also shown in the graphs. All the fits had $R^2 > 0.8$. This is acceptable as per different studies especially for lumped capacitance model in conduction and convection heat transfer [9,13]. The corresponding fluidisation maps are also provided for each powders to show the distribution of the exponential gradient. This aids to observe the results along with the essential knowledge of fluidisation quality in the bed. The magnitude of the exponential gradient, $A$, shows a very good correlation with the quality of fluidisation. The region where the bed is static and where the bed slugs have a very low magnitude of $A$. Whereas, bubbling regime shows higher values. The results show an increasing trend with
flowrate till slugging occurs. A better understanding of the variation of heat transfer coefficient can be obtained from the plots for different location and flowrates (Fig.16). These values are obtained by substituting the surface material properties from Table.1
Fig. 8: Temperature profiles during cooling of the block for 100 µm powder for different pressure and locations.
Fig. 9: Temperature profiles during cooling of the block for 180 μm powder for different pressure and locations
Fig. 10: The exponential fit on cooling temperature profile for 100 μm and atmospheric pressure for different flowrates and location.
Fig. 11: The exponential fit on cooling temperature profile for 100 μm and 250 nmbar pressure for different flowrates and location.
Fig. 12: The exponential fit on cooling temperature profile for 100 \( \mu \)m and 3.5mbar pressure for different flowrates and location.
Fig. 13: The exponential fit on cooling temperature profile for 180 μm and atmospheric pressure for different flowrates and location.
Fig. 14: The exponential fit on cooling temperature profile for 180 μm and 250 mbar pressure for different flowrates and location.
Fig. 15: The exponential fit on cooling temperature profile for 180 μm and 250 mbar pressure for different flowrates and location.
Plots indicate the standard deviation for the three thermocouples. The variation of heat transfer is significantly different for the three locations when the flowrate is changed. The bottom heat transfer increases continuously whereas heat transfer at middle and top see a maximum at lower flowrate which then drops with any further increase. Maximum reduction is seen for the top location as compared to the middle. This is mainly due to larger bubble size at the top. The atmospheric magnitude of heat transfer is quite similar for all the three locations. The variation trend at sub-atmospheric pressure is identical to the atmospheric pressure, though magnified. The maximum variation is seen for very high vacuum pressure. Fig.16 also depicts the variation of maximum heat transfer at each pressure for 100 and 180 μm. The heat transfer degrades rapidly after 100 mbar. These results are in complete correlation with the steady state temperatures obtained with case (a) results. The degradation of heat transfer at the top needs to be better understood by systematic hydrodynamic study of the bubble characteristics under vacuum. Computational fluid dynamics (CFD) has emerged as an important tool to numerically solve the multiphase Navier-stokes equations. Chapter 4 will deal with proposal of a new drag model which is essential to predict the drag force in slip/transitional regime.

5. Conclusions
Heat transfer from a cylindrical immersed surface was studied under vacuum conditions in a solid-gas fluidised bed. The main aim of the present chapter was to understand the variation of heat transfer with axial location and particle size. Use was made of lumped capacitance method which directly yields the heat transfer coefficient from the temperature history of the surface and bed temperature. The exponential fits were highly accurate with $R^2 > 0.8$. The results indicated a strong dependence on axial location when the pressure is reduced especially below
Fig. 16: Variation of heat transfer coefficient with flowrates and maximum heat transfer coefficient with pressure for all locations.
the slip/transitional regime. A delta effect (Δ) was observed where the heat transfer increased
from the bottom of the bed towards the middle and then plummets towards the top of the bed.
This is mainly understood to be caused by the continuous variation of bubble size due to gas
expansion, which is non-existent at atmospheric pressures. Use of fluidisation maps helped
understood the variation of heat transfer with flowrates and is an important step in conducting
heat/mass transfer process at vacuum conditions. This fact emphasises the significance of the
interface model developed in Chapter 2.

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Technology, 6 (1972) 231-238.


Chapter 6
Study of hydrodynamics in vacuum fluidised bed

Section 1 (published papers):


b. Drag model comparison by single bubble injection in vacuum fluidised bed, ICMF, South Korea, 2013.

Section 2 (convectional chapter):

a. Validation of slip flow drag model

This chapter proposes a new slip flow drag model to facilitate CFD simulations for Eulerian-Eulerian model under vacuum conditions. The numerical solutions are compared with contemporary drag model. In section 2, an experimental validation of the drag model is carried out along with few relevant bubble characteristics which explain the reason behind the heat transfer results in Chapter 5.
Authorship declaration

This is to certify that the first author, Mr. Apurv Kumar, of the paper titled "Numerical solution of gas–solid flow in fluidised bed at sub-atmospheric pressures" has done more than 85% of the work which includes simulations, modelling, analysis and preparation of the first draft of the manuscript. Other authors have been instrumental in providing guidance, discussion and final preparation of the manuscript.

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Authorship declaration

This is to certify that the first author, Mr. Apurv Kumar, of the paper titled “Drag models comparison by single bubble injection in vacuum fluidised beds” has done more than 85% of the work which includes simulations, modelling, analysis and preparation of the manuscript. Other authors have been instrumental in providing guidance, discussion and review of the manuscript.

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Numerical solution of gas–solid flow in fluidised bed at sub-atmospheric pressures

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Abstract

Fluidised beds are characterised by excellent thermal and chemical uniformity and have a wide application range including heat and surface treatment, ore roasting and catalyst production. However, compared to other gas-based systems, to fluidise a particulate mass, a significant quantity of gas is required. To conserve gas there is potential to operate the fluid bed under low-pressure conditions. It is also observed that heat transfer remains constant with reduction in pressure. The present work has numerically studied the nature of hydrodynamics in fluidised bed at sub-atmospheric conditions and a new drag law is proposed to account for the increased mean free path of the fluid. A wide range of sub-atmospheric pressures were considered such that slip flow regime, which is characterised with Kn<1, is applicable. An open source code (MFIX) is used to numerically solve the multiphase problem of a jet in the fluidised bed column with an immersed surface at vacuum pressure conditions. Bubbling fluidisation in shallow and deep beds are also solved. The new drag model takes into consideration the effect of slip flow to model drag force on the particles and the results of velocity distributions in the column and around the submerged surface is presented. The results of velocity distributions from the slip flow model are compared with the existing Gidaspow’s model. Significant differences were observed in the simulation results of velocity distributions and flow structure in the fluidised bed under vacuum conditions.

1. Introduction

Fluidisation is the achieving of a fluid like behaviour for any solid granular entities with the help of a fluidising media, such as gasses and liquids. The fluidising medium displaces and suspends the solid granular particles from their static position, transferring the momentum and resulting in fluid like motion of the particles. These particles being fluid borne are free to move from one location to another in a seemingly random manner. It is this random motion of the particles that resembles the flow of a fluid and hence the name fluidisation. It is a commonly used phenomenon in chemical industries and is used in processes such as mineral cracking, heat treatment, surface engineering etc. Fluidisations phenomenon offers a processing environment with a wide spectrum of advantages. The ability to achieve uniformity of temperature, high solid–fluid mixing leading to high heat and mass transfer and continuous operation, make the use of fluidisation quite appealing.

To understand the complex multiphase flow behaviour inside gas–solid fluidised beds the mathematical models proposed mainly fall under four groups depending on how they treat each phase and the magnitude of the length scales. These are (1) Discrete Bubble model, (2) Two-fluid model, (3) Discrete Particle model and (4) Molecular Dynamics model. In other words, each of these models considers the gas–solid phases to be either Eulerian or Lagrangian [1]. Selection of these models depends mostly on the geometry to be modelled and the available computing resources. Of these, most popular are the Two-fluid models where the two phases are modelled as interpenetrating continua. Each of the two phases are modelled as separate fluid (gas and solid) and is solved by Eulerian method (classical Navier–Stokes equation). The solid–fluid coupling is given by drag force that appears in the momentum balance equation for each phase and is equal in magnitude but opposite in direction.

Operation of fluidised bed at various pressure ranges (sub-atmospheric to high pressures) offers advantages that make the use of fluidised bed reactors (FBR) even more appealing. In the literature, fluidised beds at high pressures have been studied extensively. A comprehensive review is given by Yates [2] on the effect of pressure and temperature on gas–solid fluidisation. Coal combustion and gasification are the primary areas of interest for high-pressure fluidisation. Although, high pressures greatly effects heat and mass transfer rates, certain heat sensitive materials such as thermolabile substances, used in pharmaceutical industry for coating and drying purposes, cannot be used at high pressures.
An alternative is to use sub-atmospheric pressure conditions as it reduces the possibility of partial degradation of thermolabile substances and the process is made safer by operating outside the flammability ranges. Bhat and Whitehead [3] carried out experiments to study the effect of sub-atmospheric pressures on heat transfer from an immersed surface in fluidised bed reactors. Cooling water tubes were passed across the fluidised bed and the overall heat transfer coefficient was determined for various operating pressures and superficial velocity. At constant superficial velocity, the heat transfer coefficient remained constant with decrease of pressure, thus establishing the advantage of low-pressure fluidisation.

Suezawa and Kawamura [4] were the first to study the mechanism of fluid flow in fluidised bed at reduced pressures. It was observed from the fluidisation of sand, silica gel and glass beads (at pressures 0.133–13.33 kPa) that the behaviour of fluidised bed at reduced pressure resembled the bed at atmospheric pressure. In experiments (pressure range of 0.533–4 kPa) by Germain and Claudel [5], a coexistence of upper fluidised layer and a bottom quiescent layer was observed for deep beds. Llop et al. [6] gave an expression for minimum fluidisation velocity which accounts for the operating pressures. This expression predicted the minimum fluidisation velocity in vacuum conditions as well as higher pressures accurately. Expression for minimum fluidisation velocity at atmospheric pressures given by Wen and Yu [7] and Ergun [8] were compared with Llop’s equation and a significant difference in prediction of the velocity with change in pressure at vacuum conditions was found. Llop’s equation accounts for the increase of mean free path of the fluid particles and its effect on the pressure drop in the bed and hence deals with the physics of fluidisation more comprehensively. However, this equation does not accurately predict the minimum fluidisation velocity for fine particles. Wank et al. [9] accounted for the inter-particle cohesive forces in Llop’s equation which predicted the minimum fluidisation velocity of fine boron nitride powders accurately. Thus, most of the work done in low pressure fluidised beds found in literature deals mainly with basic experimental and theoretical study of hydrodynamics of fluidised bed. There has been no work reported in literature that numerically simulates and studies the hydrodynamics of low-pressure fluidised bed. Therefore, the scope of the present work is to numerically solve the hydrodynamics of a gas–solid fluidised bed at sub-atmospheric pressures using the Two-fluid model by incorporating the effect of slip flow and compare it with the widely used drag model of Gidaspow [10]. Two cases of gas–solid fluidisation are modelled: case (a) fluidised bed with an immersed surface and case (b) bubbling fluidisation in shallow and deep beds. The numerical solution to the two-phase model is carried out by an open source code, MFIX.

2. Two-fluid model and drag laws

Two-fluid model considers each of the phase to be interpenetrating continua and the governing equations of mass and momentum conservations are solved for each of the phases which are local mean averages of the point fluid and particles variables. Descriptions of the mass and momentum governing equations are presented extensively in MFIX theory guide [11] and in the Supplementary section.

For the problem to be completely defined the governing equations requires closures for the solid-phase pressure (\(P_s\)), solid-phase shear viscosity (\(\mu_s\)) and the solid-phase bulk viscosity (\(\lambda_s\)). These constitutive equations are derived from kinetic theory of granular flow and are presented in Table 1 of the Supplementary information section. Apart from these closures, kinetic theory of granular flow requires the solution to transport equation for the granular temperature. Granular temperature, \(\Theta\), signifies the random motion of the solid particles and is analogous to temperature definition according to the Kinetic theory [12]. In order to account for the friction between the solid particles when the void fraction in the bed approaches the packing limit, a frictional stress model

**Fig. 1.** Comparison of simulated equivalent bubble diameter in the fluidised bed with Kuipers [16] experiment.
that forms a part of the solid shear stress tensor is generally used. In the present work, the frictional stress model of and Srivastava and Sunderasan [13] is used, which has been shown in literature to accurately predict the shape and size of a bubble in fluidised bed [14].

Coupling between the solid and fluid phase in Two-fluid model is through the interphase momentum exchange coefficient, $F_s$. Several semi-empirical closures exist in literature to define $F_s$. Of these, the Gidaspow’s model [10] is the widely used drag law and is a combination of drag law by Ergun [8] and Wen and Yu [7].

In vacuum conditions, as the pressures are reduced below atmospheric, the mean free path of the fluid particles increases. This is characterised by Knudsen number, which is the ratio of mean free path ($\lambda = \frac{RT}{np}$) to a length scale, $d$. Knudsen number increases with decreasing pressures that changes the nature of the fluid flow. Molecular flow ($Kn >> 1$), intermediate or slip flow ($Kn \sim 1$) and laminar flow ($Kn << 1$) are found to exist in vacuum conditions. Llop et al. [6] derived pressure drop equations for flow through fluidised bed and predicted the minimum fluidisation velocity ($u_{mf}$) by considering slip flow regime [7]. In the present work, the Gidaspow’s drag law is modified to include the effect of slip flow regime by incorporating the effect of Kn on fluid flow and its effect on gas–solid fluidisation is studied.

### 2.1. Modified drag law

The pressure drop equation through a fluidised bed which accounts for operating pressures is given by Llop et al. [6].
This can be extended to derive the interphase momentum coefficient, $F_s$, since the pressure drop in a fluidised bed can also be expressed as [10]:

$$\frac{dp}{dt} = \frac{\mu}{\frac{u}{2\pi}} \cos^2 \psi \frac{u^2}{(1-\epsilon)} \sqrt{\frac{2}{\rho_d \nu_d} + \frac{2}{\rho_s \nu_s}} + 1.75 \left(1 - \epsilon_s\right) \frac{\rho u^2}{\epsilon_s \phi_d d_s}$$  \hspace{1cm} (1)

$$\frac{dp}{dt} = F_s \frac{\bar{v}_g - \bar{v}_s}{\epsilon_s}$$  \hspace{1cm} (2)

This upon comparison with Eq. (1) and substituting the constants given by Llop et al. [6] yields the following expression for $F_s$, the interphase momentum exchange coefficient:

$$F_s = \frac{(1 - \epsilon_g)}{0.1356 \phi_d d_s \sqrt{\frac{2}{\rho_d \nu_d} + \frac{2}{\rho_s \phi_d d_s} (1 - \epsilon_s \phi_d d_s)^2}} + 1.75 \frac{\rho_s (1 - \epsilon_g)(\bar{v}_g - \bar{v}_s)}{\phi_d d_s}$$  \hspace{1cm} (3)

**Fig. 5.** Contours of time averaged velocity for shallow beds predicted by the drag models. (a) Modified drag law at 1000 Pa (b) Gidaspow drag model at 1000 Pa (c) Gidaspow drag model at atmospheric pressure.
Eq. (3) accommodates the effect of slip flow and is equal to the Ergun’s equation [8] in the limit \( P \to 1 \).

3. Problem description and numerical simulations

The sub-atmospheric pressure conditions in a fluidised bed, where slip flow is predominant, were solved numerically using the modified drag law (Eq. (3)) derived in earlier section. Different cases are studied under vacuum conditions which includes a two dimensional rectangular column of size 0.57 x 1 m with a central jet (0.015 m) and an immersed rectangular surface (0.1 x 0.2 m). Apart from this, a fluid bed of same dimension without the immersed surface but with varying aspect ratio of the bed was solved additionally in the present numerical study. Glass beads of size 500 \( \mu \)m were used in the simulation. Simulations were carried out by an open source code (MFIX) and the initial and boundary conditions for the bed with immersed surface are given in the Supplementary information section. For the case where immersed
surface and the jet were absent, a uniform inlet velocity of $1.5U_{\text{mf}}$ was applied in order to simulate bubbling fluidisation. A uniform velocity of $1.5U_{\text{mf}}$ in the bed and $1.5U_{\text{mf}}$ in the freeboard region was specified as initial conditions for bubbling fluidisation model. A uniform grid of size 0.01 m was considered along the length of the column and 0.0075 m was considered along the width of the bed for the model of immersed surface. A grid independent study revealed that fine mesh was successful in simulating meso-scale structures for the case of fluid beds without the immersed surface. Due to limitation of computational resources, a non-uniform grid size of $10^{-3}$ along the width and 0.01 along the length of the fluid bed was selected with finer mesh in the bed region. The Gidaspow's drag law was modified by replacing the interphase momentum coefficient by Eq. (3) for dense regions in the bed. A time step of $10^{-3}$ s was used in all simulations and a residual limit of $10^{-4}$ for continuity, momentum and granular temperature terms was used. No slip wall-boundary conditions was used for the gas-phase whereas partial slip wall-boundary condition given by Johnson and Jackson [15] was applied to solid-phase with stick-slip coefficient as 0.5 and particle–wall coefficient of restitution ($e$) as 0.9.

4. Results and discussion

In order to validate the numerical model, the fluidised bed is operated at ambient pressure conditions and the Gidaspow's drag model is used. The equivalent bubble diameter is compared with the experimental results of Kuipers [16]. The equivalent bubble diameter is defined as the diameter of the circle enclosed by $d_{\text{p}}$ and the Gidas-pow's model over-predicts the magnitude of $F_s$ as predicted by the Gidaspow's model especially towards the top of the bed. In addition, the modified drag law (Eq. (3)) is compared with the Gidaspow's model over-predicts the magnitude of $F_s$ as predicted by the modified drag law was significantly lower than the Gidaspow's model especially at the top of the bed.

4.1. Case (a): Immersed surface with central jet

A number of simulations were run in order to compare the effect of the bubble size and its movement through the bed predicted by the modified drag law with those predicted by the Gidaspow's model. The first term in the denominator of $F_s$ (Eq. (3)) becomes comparable to the second term only at high vacuum pressures. It is seen that this occurred at a pressure of 1000 Pa for the present set of conditions. Since a linear pressure gradient exists in the bed, which is tantamount to the bed weight, the magnitude of $F_s$ as predicted by the modified drag law was significantly lower than the Gidaspow's model especially at the top of the bed. Fig. 3 shows the percentage difference in the time-averaged values of $F_s$ as predicted by the Gidaspow's model and the modified model for operating pressure at 100 Pa. All time average results are for a period of 3 s. As seen from the contour plot, a maximum difference of 70% and an average difference of 18% in the bed region exist between Gidaspow's model and the modified model, with the Gidaspow's model predicting a higher value. In addition, the difference increases linearly and the maximum occurs near the interface of bed and the freeboard region. The modified slip-flow drag law also predicted a lower bed height.

Due to lower values of $F_s$ predicted by the modified drag law, the velocity of air was found to be greater than those predicted by the Gidaspow's model especially towards the top of the bed which can be seen in Fig. 4. This is due to the inefficiency of the air to transfer momentum to the solid particles. Consequently, the velocity of solid particles in the bed was found to be lower in
the simulations where the modified drag law was used. The bubble size at the bottom of the bed was of similar size and shape when the modified drag law was used in simulations. This is because the pressures at the bottom of the bed were of equal order of magnitude (around 8800 Pa) and therefore the predictions of velocity and $F_s$ were of same order of magnitude at the bottom of the bed.

4.2. Case (b): Bubbling fluidisation in shallow and deep beds

Two cases of bubbling fluidisation with different aspect ratio (AR = L/D) were numerically solved: shallow (AR = 0.5) and deep beds (AR = 1). Bubble flow profile under vacuum conditions was observed in each of the simulations. Figs. 5 and 6 shows the time averaged $x$ and $y$ velocity contour plots (for a period of 3 s) for shallow beds and deep beds at atmospheric and sub-atmospheric conditions as predicted by the modified and Gidaspow’s drag model. It is seen that the general flow profile of the emulsion phase is preserved with decrease of pressure – the emulsion phase rises in the core and descends at the walls. This nature of hydrodynamics is typical of fluidised beds [17]. In addition, the emulsion phase was seen to ascend uniformly in the bed region near the distributor at atmospheric pressure. The modified drag model predicts a similar profile whereas the Gidaspow’s drag model predicts a profile that is concentrated in the middle of the bed (Fig. 5b).

The circulation pattern of emulsion phase can be determined by observing together the $x$ and $y$ velocity profiles (Figs. 5 and 6). In the shallow beds (Fig. 5), the circulation pattern reverses at vacuum pressures and the emulsion phase moves towards the core near the distributor plate while rising upwards whereas the emulsion phase moves towards the wall at atmospheric pressures in the same region. However, for the deep beds no such reversals in flow structure were seen with decrease of pressure. It can also be observed that for deep beds, both the drag models predict a different circulation pattern as compared to atmospheric conditions. As seen from Fig. 5 (a and b), the positive and negative velocity (the red and the blue region) are one above the other instead of being opposite, as seen in all other cases (shallow and atmospheric deep beds). It is seen in the case of the deep beds in vacuum pressure that a larger

Fig. 8. Volume fraction of air for deep beds at 3 s for 1.5Umf. (a) Atmospheric conditions (b) at 10^4 Pa using modified drag law (c) at 10^4 using Gidaspow drag model (d) at 10^3 Pa using modified drag law (e) at 10^3 Pa using Gidaspow drag model.
region of maximum velocity of descent (region with blue colour in Fig. 6) of the emulsion phase near the walls was predicted by the Gidaspow's drag model. In all the simulation at vacuum pressures, an average difference of 20% is observed in prediction of the maximum emulsion phase rise velocity near the distributor plate by the two drag models. This result is consistent with those obtained from the case of an immersed surface in the fluidised bed.

Fig. 7 shows the velocity profiles at y = 0.05 m in the shallow bed and at y = 0.1 m in deep beds. It can be seen that the prediction of velocity profiles by the drag models at atmospheric and vacuum pressures are similar for deep beds. However, for the shallow bed the Gidaspow's drag model does not predict a uniform velocity profile whereas the modified drag model predicts a profile similar to the atmospheric conditions.

The volume fraction of air at different pressures (10^3, 10^4 and 101.325 Pa) and in deep bed (aspect ratio, AR = 1) is shown in Fig. 8 which are predicted by modified and the Gidaspow's drag model at the end of simulation at 3 s. Bubble size is seen to increase with reduction in pressure for both deep and shallow beds. This trend is opposite to what is observed in high-pressure fluidisation where the bubble size reduces with rise of pressure above atmospheric [2]. Gidaspow's model and the modified model both predicted this trend. It was seen at 10^3 Pa that the Gidaspow's model predicted larger bubble size and the bed seemed to slug at this pressure. This was not the case with the prediction from the modified drag law. There were no differences seen in prediction of the maximum emulsion phase rise velocity near the walls was predicted by the Gidaspow's drag model does not predict a uniform velocity profile whereas the modified drag model predicts a profile similar to the atmospheric conditions.

5. Conclusion

The drag term appearing in the Two-fluid model governing equation was modified in the present work to incorporate the effect of slip flow, which becomes predominant when the fluidised bed is operated at high vacuum pressures, where Kn approaches a value of unity. Two cases were solved numerically: fluidised bed with immersed surface and bubbling fluidisation in shallow and deep beds. Upon comparison with the existing Gidaspow's model, the velocity distributions in the bed were predicted significantly higher by the slip flow drag law, due to which the velocity of solid particles were predicted to be slower than the Gidaspow's model in the fluidised bed with immersed surface. Significant differences were observed in the bubble flow structure and bubble size for bubbling fluidisation in the shallow and the deep beds. In addition, under vacuum conditions, a reversal of flow structure of the emulsion phase was observed and both the drag laws used in the present study predicted it consistently.

Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.apt.2012.04.010.

References

Drag models comparison by single bubble injection in vacuum fluidised beds

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Keywords: vacuum fluidisation, drag models, single bubble injection, CFD

Abstract

Single bubble injection simulations inside a minimally fluidized bed have been studied widely and are often used to validate the accuracy of different numerical models. Bubble shape, size and voidage distribution are the important parameters that are validated from the experiments. In the present work, the most widely used drag model (Gidaspow’s drag model) is compared to a new proposed slip flow drag model which takes into account the presence of the slip flow regime, often encountered in vacuum fluidized beds and characterised by Knudsen no. (Kn). Shape and size prediction of the bubble evolution inside the bed is carried out numerically by using the two fluid model, comparing the results predicted by the drag models. It is seen that the predictions are different for the two drag models only under high vacuum conditions corresponding to Kn in slip/transition flow regime. The predictions are also found sensitive to pressure gradient in the bed and fluid velocity.

Introduction

The reduced pressure conditions in fluidised beds results in different flow regimes (slip, transition and molecular). It has been shown that these flow regimes affect the minimum fluidisation velocity of the fluidized beds (Germain and Claudel, 1976; Kawamura and Y. Suezawa, 1961; Wraith and Harris, 1992). The well-established correlations often used to predict the minimum fluidisation velocity have been found to be inaccurate in their prediction in the slip and transition flow regimes (Kusakabe, K et al., 1989; Llop et al., 1996). This is mainly because the earlier correlations(such as Wen & Yu (1966)) have failed to account for the increased mean free path of the fluid under reduced pressure conditions, which results in slip flow at any solid-fluid interface. However, extended versions of these correlations (Llop et al., 1996) have been used to accurately predict the experimental minimum fluidisation velocity under reduced pressure conditions. Recently, Kumar et al., (2012), had extended the work of Llop et al., (1996) and proposed a new drag model which can be used under reduced pressure conditions to account for slip flow and have used it to predict bubbling fluidisation in vacuum using CFD.

In recent years, CFD methods have become widely used in solving the classical Navier-Stokes equation for solid-gas fluidisation in order to predict the complex interaction of the bubbles with the emulsion phase. Of the many models which exist, the kinetic theory of the dense gas model, known also as the two-fluid model is widely used along with several semi-empirical constitutive models (Gidaspow, 1994a). Over the years, the Eulerian-Eulerian continuum model has been evaluated and compared with experimental results and it has been seen that the continuum models predict the bubble characteristics such as size and shape accurately as compared to the experimental results (Goldschmidt et al., 2004). In addition, numerical models of 2D beds also predict the general behaviour of bubbles similar to the numerical 3D beds (Xie et al., 2008). In the literature, many new numerical models proposed have often used the symmetrical fluidisation problem of a single bubble injection (Gidaspow, 1994a; Kuipers et al., 1992). Controlled injection parameters and a resulting definite predictive path and size of the bubble enables the use of this simple model very efficiently in validating the numerical models (Patil et al., 2005).

The scope of the present work, therefore, includes the comparison of the slip flow drag model and the widely used Gidaspow’s drag model (Gidaspow, 1994a) under reduced pressure conditions for a single injected bubble in a minimum fluidised bed. Bubble shape and size is compared for various reduced pressure conditions and different injection jet velocities.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>d/D</td>
<td>diameter, m</td>
</tr>
<tr>
<td>e</td>
<td>restitution coefficient</td>
</tr>
<tr>
<td>F</td>
<td>momentum exchange coefficient</td>
</tr>
<tr>
<td>g</td>
<td>acceleration due to gravity, m/s²</td>
</tr>
<tr>
<td>h</td>
<td>height of bed, m</td>
</tr>
<tr>
<td>k</td>
<td>diffusion coefficient (eq.4)</td>
</tr>
<tr>
<td>L</td>
<td>bed height, m</td>
</tr>
<tr>
<td>P</td>
<td>Pressure, Pa</td>
</tr>
<tr>
<td>Re</td>
<td>Reynolds number, ρ/νD/μ</td>
</tr>
<tr>
<td>u</td>
<td>superficial velocity, m/s</td>
</tr>
<tr>
<td>y</td>
<td>length along y-direction</td>
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</table>
Greek-letters
\( \varepsilon \) \hspace{1cm} \text{void fraction}
\( \psi \) \hspace{1cm} \text{interstitial angle}
\( \phi \) \hspace{1cm} \text{sphericity}
\( \mu \) \hspace{1cm} \text{dynamic viscosity, Pa s}
\( \gamma \) \hspace{1cm} \text{velocity ratio, } \frac{\nu_{\text{jet}}}{\nu_{\text{inlet}}}
\( \rho \) \hspace{1cm} \text{density, kg/m}^3

Subscripts
\( g \) \hspace{1cm} \text{gas}
\( \text{mf} \) \hspace{1cm} \text{minimum fluidization}
\( p/P \) \hspace{1cm} \text{particle}
\( s \) \hspace{1cm} \text{solid}

Two fluid model
The two-fluid model considers each of the phases to be an interpenetrating continua and the governing equations of mass and momentum conservations are solved in ANSYS FLUENT for each of the phases which are local mean averages of the point fluid and particles variables. Details on the derivation of the continuum equations for fluidized beds is provided by Gidaspow (1994b) and several investigators. Thus, in the present study an isothermal bed is provided by Gidaspow (1994b) and several investigators. Thus, in the present study an isothermal Eulerian-Eulerian approximation is used with the particle phase limited to a constant diameter. In this model the necessary continuum equations for volume fraction and velocities are:

Continuity equations
\[
\frac{\partial (\varepsilon \rho_g)}{\partial t} + \nabla \cdot (\varepsilon \rho_g \mathbf{u}_g) = 0 \hspace{1cm} (1a)
\]

Solid phase
\[
\frac{\partial (\varepsilon \rho_s)}{\partial t} + \nabla \cdot (\varepsilon \rho_s \mathbf{u}_s) = 0 \hspace{1cm} (1b)
\]

Momentum Equations
\[
\frac{\partial (\varepsilon \rho_g \mathbf{u}_g)}{\partial t} + \nabla \cdot (\varepsilon \rho_g \mathbf{u}_g \mathbf{u}_g) = -\varepsilon \mathbf{u}_g \cdot \nabla P_g + \nabla \cdot \mathbf{\tau}_g + \beta_{sg}(\mathbf{u}_g - \mathbf{u}_s) + \varepsilon \rho_g \mathbf{g} \hspace{1cm} (2a)
\]

Solid phase
\[
\frac{\partial (\varepsilon \rho_s \mathbf{u}_s)}{\partial t} + \nabla \cdot (\varepsilon \rho_s \mathbf{u}_s \mathbf{u}_s) = -\varepsilon \mathbf{u}_s \cdot \nabla P_s + \nabla \cdot \mathbf{\tau}_s - \beta_{sg}(\mathbf{u}_g - \mathbf{u}_s) + \varepsilon \rho_s \mathbf{g} \hspace{1cm} (2b)
\]

where
\[
\mathbf{\tau}_g = \mu_g [\nabla \mathbf{u}_g + \nabla \mathbf{u}_g^T] - 2 \frac{\mathbf{g}}{3} (\nabla \cdot \mathbf{u}_g) \mathbf{I} \hspace{1cm} (3a)
\]
\[
\mathbf{\tau}_s = \mu_s [\nabla \mathbf{u}_s + \nabla \mathbf{u}_s^T] + \left( \lambda_s - \frac{2}{3} \mu_s \right) (\nabla \cdot \mathbf{u}_s) \mathbf{I} \hspace{1cm} (3b)
\]

and
\[
\varepsilon_s + \varepsilon_g = 1
\]

For the problem to be completely defined the governing equations require closures for the solid-phase pressure \( P_s \), solid phase shear viscosity \( \mu_s \) and the solid-phase bulk viscosity \( \lambda_s \). These constitutive equations are derived from kinetic theory of granular flow (Kumar et al., 2012).

Apart from these closures, the kinetic theory of granular flow requires the solution to the transport equation for the granular temperature. Granular temperature, \( \Theta \) signifies the random motion of the solid particles and is analogous to temperature definition according to the kinetic theory.

\[
\frac{3}{2} \left[ \frac{\partial (\varepsilon \rho_s \Theta_s)}{\partial t} + \varepsilon \rho_s \mathbf{u}_s \cdot \nabla \Theta_s \right] = (\mathbf{P}_s + \mathbf{P}_\Theta) - \nabla \mathbf{P}_\Theta + \gamma \varepsilon_s - \phi_s \hspace{1cm} (4)
\]

Gidaspow’s drag model
Coupling between the solid and fluid phase in the Two-fluid model is through the interphase momentum exchange coefficient, \( F_s \). In the literature, several semi-empirical closures are presented in order to define \( F_s \). Of these, the Gidaspow model (1994a) \((eq.5)\) is the widely used drag law and is a combination of drag laws by Ergun (1952) and Wen and Yu (1966).

\[
F_s = 150 \left( 1 - \varepsilon_g \right) \frac{\mu_g}{\varepsilon_g d_s^2} + 1.75 \frac{\rho_g (1 - \varepsilon_g) (\mathbf{u}_g - \mathbf{u}_s)}{d_s} \text{ for } \varepsilon_g \leq 0.8 \hspace{1cm} (5a)
\]

\[
F_s = \frac{3}{4} C_D \left( 1 - \varepsilon_g \right) \frac{\rho_g \left( \mathbf{u}_g - \mathbf{u}_s \right)}{d_s} \varepsilon_g^{-2.65} \text{ for } \varepsilon_g > 0.8 \hspace{1cm} (5b)
\]

where \( C_D = -\frac{24}{\varepsilon_{max}^2} \left[ 1 + 0.15 (\varepsilon_g Re_g)^{0.687} \right] \)

Slip flow drag model
Under vacuum conditions, as the pressures are reduced below atmospheric, the mean free path of the fluid particles increases. This can be characterized by the Knudsen number, which is the ratio of the mean free path of the fluid to a length scale, \( d \). With a reduction in pressure the Knudsen number \( (Kn) \) increases which brings changes in the nature of the fluid flow, such as Molecular flow \( (Kn \gg 1) \), Intermediate or slip flow \( (Kn \approx 1) \) and Laminar flow \( (Kn \ll 1) \). Llop et al., (1996) derived pressure drop equations for flow through a fluidized bed and predicted the minimum fluidization velocity \( (u_{mf}) \) by considering the slip flow regime. In the present work, the Gidaspow drag law is modified to include the effect of the slip flow regime by incorporating the effect of Kn on fluid flow.

The pressure drop equation through a fluidized bed which accounts for operating pressures is given by Llop et al., (1996) as:

\[
\frac{dP}{dt} = \frac{16}{45} \cos^2 \psi \left( \frac{\varepsilon_s^2 \phi_s d_s}{(1 - \varepsilon_g)^2} \left[ \frac{2}{\rho_s \nu_s} + \frac{\cos^2 \psi \varepsilon_s^2 (\phi_s d_s)^2}{72} \right] \right)
\]

\[
+ 1.75 \frac{(1 - \varepsilon_g)}{\varepsilon_s^3 \phi_s d_s} \rho u^2 \hspace{1cm} (6)
\]

This can be extended to derive the interphase momentum coefficient, \( F_s \), since the pressure drop in a fluidized bed can also be expressed as:
\[
\frac{dP}{dl} = \frac{F_s}{\varepsilon_g} (\bar{v}_g - \bar{v}_s) \tag{7}
\]

This upon comparison with eq. 6 and substituting the constants given by Llop et al., (1996) yields the following expression for \(F_s\):

\[
F_s = \frac{(1-\varepsilon_g)}{0.1356 \phi_d} \left[ \frac{1}{\sqrt{\varepsilon_g}} \frac{d_g}{\varepsilon_g} \right] + 1.75 \frac{\rho_d (1-\varepsilon_g)(\bar{v}_g - \bar{v}_s)}{\phi_d} \tag{8}
\]

**Problem description**

A two-dimensional computational fluidized bed model (570 x 1000 mm) was considered (see Figure 1) and solved by ANSYS FLUENT. Gas was introduced from the bottom of the bed in addition to using a single nozzle. The simulations were computed for various sub atmospheric pressures in the range 50-101,325 Pa. A parametric study was also carried out at a constant pressure for various \(\gamma\), the ratio of jet velocity to inlet velocity \((u_{jet}/u_{inlet})\). The solution with a centrally located nozzle is used to validate the model by comparing with other published data (Fig.2). Gas inflow was maintained at the minimum fluidization velocity at the bottom as shown (Fig. 1) with operating pressure boundary condition at the top of the bed. No slip wall-boundary condition was imposed for the gas-phase whereas partial slip condition, given by Johnson and Jackson (1987), was applied to solid-phase with the specularity coefficient as 0.5 and the particle–wall coefficient of restitution as 0.9. The material used in the present simulation was Geldart group B Glass (size: 500 \(\mu\)m and density of 2660 kg/m\(^3\)). The density of the fluid (air) was maintained constant in the simulation and was calculated by the ideal gas law \(P_{operating}/RT\).

Mesh size influences both the convergence and the computational time. A larger grid spacing increases the numerical diffusion, while the smaller grid spacing increases the computational time (Zhang et al., 2012). Thus, the strategy adopted here was to use a multi-block, unstructured Cartesian grid (with tetrahedral elements), which allows very fine uniform grids near the nozzle for accurately predicting initial growth of the bubble and at the same time providing an adequate size of elements near the walls and the interface. A grid independency test was also carried out to finalize the total number of grids for the base-case (10 m/s nozzle inlet velocity). All simulation results were checked for mass flowrate balance to confirm the convergence of the results. A maximum error of 0.001% in mass flow balance was seen in all results.

**Results and discussion**

A number of simulations were carried out for various pressure ranges below atmospheric. The inlet velocity at each of the pressures was computed from the minimum fluidization velocity expression given by Llop et al. (1996) as shown in Fig.3 (comparison with Wen & Yu (1966) is also provided). Two sets of simulation were computed, each with a different drag model. Minimum fluidization velocity was used as the inlet velocity with a velocity ratio, \(\gamma = 35\) for each pressures (\(\gamma = 35\), is the initial atmospheric parameter for which validation was carried out for the
Figure 4 shows the comparison of the void fraction contours ($H = 0.85$) for the two drag models considered under vacuum pressures. For pressures down to 1000 Pa from atmospheric, no significant difference in prediction of the bubble shapes are seen. However, as the pressures were reduced below 1000 Pa, the bubble shape prediction for the two drag models started to deviate from each other. It is to be noted that the Kn for pressures 100-1000 Pa is in the range of 0.13-0.013, which denotes the presences of slip and transition flow regime for the particle size considered in present study. It can be further seen from the figures that for the range of inlet velocity and jet velocity considered, a proper bubble did not occur inside the fluidized bed below 1000 Pa. Instead, the entire bed acted as a packed bed and was lifted as a single unit along with a bubble shape indentation due to the jet. Nevertheless, both the drag models predicted this behavior, which can be observed from Fig.4.

It can also be observed from the Fig.4 that both the drag models give similar predictions of bed height at the bottom. Bubble shape however, is predicted to be completely different. The Gidaspow drag model predicts an early shearing of the bubble at the top portion for all the pressures below 500 Pa. The slip flow drag model, on the other hand, predict a full near spherical shape of the bubble with almost no sign of shear. At the pressure of 50 Pa, no bubble was predicted to exist by the Gidaspow drag model, whereas the slip flow model clearly shows the occurrence of a bubble caused by the high velocity jet.

Another interesting observation from Fig.4 is that, although both the drag models predict dissimilar bubble contours in the jet region, the dissimilarity reduces away from the center and towards the bottom of the bed. It can be clearly seen from Fig.4 that the porosity contour prediction ($\varepsilon = 0.85$) near the inlet boundary are similar at least down to a pressure of 200 Pa. Only after a certain height does both the drag model predict differently. As the operating pressures reduced below 200 Pa, the porosity contour predictions are different for regions away from the central jet axis. These observations suggest that the predictions by the drag models are not only sensitive to pressures gradient inside the bed but also to the fluid velocity inside the bed.

**Fig.4: Prediction of porosity contours ($\varepsilon = 0.85$) by Gidaspow and Slip flow drag model for various sub-atmospheric pressures**
In order to study the sensitivity of the porosity prediction by the drag models to the fluid velocity, the velocity ratio $\gamma$ was varied by changing the jet velocity and the inlet velocity, separately. The pressure at 100 Pa was chosen for the current parametric study since the results at 100 Pa at $\gamma = 35$ and $u_{\text{inlet}} = u_{\text{mf}}$ showed a huge variation in porosity contour prediction near the jet. Fig. 5 shows the results of porosity prediction by both the drag models for various $\gamma$ values in range of 5-35, which was varied by changing only the jet velocity and maintaining the $u_{\text{inlet}}$ at $u_{\text{mf}}$. It can be observed very clearly how the prediction by both the drag models begin to coincide as the velocity ratio ($\gamma$) is reduced from 35 to 5. The axial as well as lateral prediction of porosity coincides as the jet velocity is reduced at 100 Pa.

The velocity ratio ($\gamma$) was again varied but this time by changing the $u_{\text{inlet}}$ from 1.21 m/s ($u_{\text{mf}}$ at 100 Pa) to 0.6 m/s thus varying the velocity ratio ($\gamma$) from 35 to 70. Further reduction of inlet velocity did not give any converged results. It can be seen from Fig. 6 that as the inlet velocity is reduced, the prediction by the drag models begin to overlap each other. However, it can be clearly seen that the sensitivity of prediction of porosity contour is not as high as was seen when the jet velocity was reduced. As the velocity ratio ($\gamma$) was increased from 35 to 70 (by reducing inlet velocity), the bubble shape prediction by both the models remain significantly different, although contours away from the axis coincided.

**Conclusion**

CFD simulations using the Eulerian-Eulerian model in ANSYS-FLUENT for a lab-scale bubbling fluidized bed containing Geldart type B particle of 500 $\mu$m were used to compare the predictions of porosity contours by Gidaspow drag model and the new proposed slip flow drag model under different sub-atmospheric pressure conditions. It was seen that the porosity prediction by both the drag models started to deviate only at high vacuum pressures, which
were dominated by slip flow regime. Surprisingly, both the drag models predicted similar porosity contours at lower bed heights, lower jet velocity and/or inlet velocity, suggesting that the predictions are sensitive not only to pressures but also to fluid velocity inside the bed. An experimental validation is highly recommended to validate the bubble shape predictions in vacuum conditions.

References


Chapter 6: section 2  
Validation of the slip flow drag model

1 Introduction

Low-pressure fluidisation is different from beds that usually operate at atmospheric or higher pressure as the slip flow overcomes laminar-hydrodynamic-characteristics. In such situation, the continuum drag models are expected to underperform as discussed in previous section. The present section is a continuation of Chapter 4 which deals with experimental validation of the slip flow drag model as well as studying some of the key bubble characteristics affect the heat transfer under vacuum as suggested in Chapter 3.

2 Pseudo 2D vacuum fluidised bed

In order to validate the bubble size predictions of the slip flow drag model, a pseudo two dimensional (2D) vacuum fluidised-bed was fabricated out of polycarbonate material. Many of the closure models such as frictional model, drag models etc. have been previously verified for its validity using a two dimensional fluidised bed [1, 2]. In the case of single jet/nozzle, fluid with high velocity is injected into the column along with the fluid at lower velocities (order of \( U_{mf} \)) from the distributor plate. From modelling point of view a jet in a fluidised bed is advantageous as the jet establishes the flow pattern and is an easier problem than bubbling fluidised bed. The correct predictions of bubble size, physical phenomena, validates the model. Boulliard et.al[3] studied the motion of the bubble around an obstacle in a fluidised bed and validated their hydrodynamic model in which the pressure drop was considered only in the fluid phase. Gidaspow et.al [4, 5] has conducted many experiments with a jet in fluidised bed to validate his proposed models. In literature, two major results used to validate models are the comparison of bubble size and the time averaged volume fractions at various positions with the
experiments in the fluidised bed. Many other hydrodynamic models such as frictional stresses have also been compared with each other using the case of jet in a fluidised bed [2].

2.1 Experimental setup

A 2D bed of dimension 1 m x 0.25 m x 0.012 m was fabricated from polycarbonate material (Fig.1). Polycarbonate was chosen due to its strength which is an essential requirement to build a vacuum chamber and avoid possible implosion. The walls of the 2D bed were 12 mm in thickness which were chemically glued together with high strength adhesives which can seal vacuum. A windbox fabricated with a flange (30 mm) (not shown) connected with the polycarbonate bed. Two sets of O-rings were used to seal the vacuum in the bed. A sintered stainless steel plate was used as a distributor plate. A jet with a diameter of 6mm was placed in the middle of the distributor plate which was regulated with a solenoid valve, flowmeter and an electronic timer. Due to the flange

Fig.1: Two dimensional polycarbonate vacuum fluidised bed
thickness, the jet was placed 30 mm inside the fluidised bed to enable capture of clear images of injected bubbles.

The top of the bed connects to a particle filter and vacuum pump. A high speed camera (Phantom V711) with Nikkon 24-85 Macro lens was used. The camera was mounted on a levelled tripod to capture the bubble evolution at 700 frames per second (fps) with 1280 x 800 pixels which, according to the calibration used in the present study, yields a resolution of 1.4

Fig.2: Pressure drop vs flowrate curves for 350 μm alumina powder at different pressures
mm per pixel. In order to maximise the image contrast, two light sources are placed at a great distance to achieve a diffused and uniform light. The post-processing of the captured image was carried out in ImageJ software. In our case, we used the default threshold setting for the binary image available in the software.

### 2.2 Experiment method

Alumina powder of size 350 μm was used to validate the bubble size for atmospheric and vacuum pressures. The central injection of bubble was done using a solenoid valve (see Fig.1) and electronic timer. The bubbles were injected for a duration of 0.5 s. at various flowrates and at regular intervals of 1 s. Two different inlet flowmeters were used to regulate the flow through the jet and the distributor plate. The superficial inlet pressure was maintained constant (80kPa) for all the pressures. The fluidised bed was operated at minimum fluidisation conditions obtained from the experiments carried out at atmospheric, 250 and 95 mbar (Fig.2). The bed weight correction had to be included as a fraction of powders were trapped in the particle filter.
3 Numerical simulation

ANSYS FLUENT 15 was used to computationally model and solve the 2d fluidised bed numerically (Fig.3). The experimental rig was modelled with a protruding jet inside the bed. The governing equations and boundary conditions are similar to previous sections. A pulsed velocity inlet is used for the jet as boundary conditions. 27333 number of elements was selected after a grid independence study (Fig.4) based on the atmospheric injection of the bubble (0.22 s) at 20\(U_{mf}\) and the bed fluidisation at \(U_{mf}\) for a duration of 0.5 s. Coarser mesh was opted for freeboard region while finer central mesh covered the bed region of the geometry.

![Fig.3: Physical and computational model of the 2d vacuum fluidised bed](image)
4 Results and discussion

4.1 Atmospheric bubbles

The 2d bed was initially operated at atmospheric pressures and the bubble area from the binary image was used to carry out the grid independence study. The equivalent diameter at $t = 0.22$ s for $V_j = 20U_{mf}$ is used as a reference diameter. This was the detachment time of the bubble. Figs. 5 and 6 reports the experiment images for atmospheric pressures ($20$ and $40$ $U_{mf}$) and the simulation results. Similar to Kuipers results [2] the bubble evolution matches accurately up to the detachment of the bubble. The bubble shape is however more spherical in the simulations. It is also seen that the bubble shapes comparison diverge when the bubble travels upwards and hence only the area of the bubble is compared. This is mostly due to the absence of a third dimension resistance in the simulation. Smaller velocity ($20U_{mf}$) are closer in shape comparison than larger ($40U_{mf}$). Fig. 7 plots the bubble size evolution in the bed during the experiment and simulation. The solid lines are the bubble size recorded from the experiments.
and the dashed line is the simulation prediction. The different colours of the experimental bubble size is to indicate bubbles at different time intervals during the course of the experiment for each flow rate. The bubble size is accurately predicted at the bottom. As the splitting and coalescing of bubbles take place towards the top, the size comparison fluctuates. Nonetheless, the size prediction by the numerical simulation is of the same order of magnitude.

Fig. 5: The high speed image and their corresponding binary images for atmospheric pressure and $U_{jet} = 20 \ U_{mf}$
The bubble shape evolution for smaller and higher velocity is found to be quite different. A general trend for smaller velocity is to grow linearly and then travel with a constant size. This is followed by a decline in size. Leakage of gas from the bubble contributes more to the size reduction than splitting. On the contrary the larger velocity bubbles are seen to grow linearly and split as soon as the injection is switched off. The bubble then travels upwards and grows steadily, attain a constant maximum size (Fig.8). The 30 and 50 U$_{mf}$ jet velocity plots (Fig. 8) shows the shift in the splitting phase. 50 and 40U$_{mf}$ jet were seen to split even while growing and split as soon as the injection is switched off. The bubble then travels upwards and grows steadily, attain a constant maximum size (Fig.8). The 30 and 50 U$_{mf}$ jet velocity plots (Fig. 8) shows the shift in the splitting phase. 50 and 40U$_{mf}$ jet were seen to split even while growing
during injection. It is interesting to note that the rate of increase in size of the bubble after splitting is similar to that during injection phase.

Fig. 7: Comparison of the bubble size evolution in the bed for 20 and 40\(U_{mf}\) at atmospheric pressure (Solid lines: Experiment; Dashed line: simulation)
4.2 Sub-atmospheric bubbles

Figs. 9 and 10 compares the bubbles at 95 mbar for jet velocity of 20 and 40 $U_{\text{inf}}$. For pressures down to 250 mbar, no significant difference in the bubble size by Gidaspow and the new slip flow drag model was observed. However, below 100 mbar, the bubble shapes predicted by both
the models diverged significantly. Due to setup limitation, the operating pressures couldn’t be lowered below 90mbar. The plots (Fig.11) compares the size evolution of the bubbles from experiments and simulation. The bed during the experiment had minor bubbles towards the top due to un-homogenous fluidisation. This, however, did not alter the bubble size significantly. The slip flow model can be seen to predict accurate bubble size as compared to the Gidaspow’s model. There were similarity in their prediction at the bottom of the bed which then diverged

Fig.9: The binary images for 95mbar and $U_{jet} = 40 \, U_{mf}$ and the prediction of the numerical solution by (a) Slip flow drag model and (b) Gidaspow’s drag model
towards the top. The size prediction by the slip flow is also higher by at least 50% at the top of the bed. This is mainly due to the empirical constants in the Gidaspow’s model which has been obtained from experiments at atmospheric pressures.

The vacuum bubbles were seen to expand at the top. The growth rate of the bubbles at the top and bottom differed significantly, especially for higher jet velocity. A comparison of the top

Fig.10: The binary images for 95mbar and $U_{jet} = 20 U_{inf}$ and the prediction of the numerical solution by (a) Slip flow drag model and (b) Gidaspow’s drag model
and bottom bubble size growth rate shows that the growth rate at top is at least 4-6 times the growth rate at the bottom (Fig.12). This demonstrates that the bubble characteristics changes significantly in vacuum conditions. Further, the velocity of bubbles can also be observed to vary significantly towards the top (Fig.13). In comparison, the atmospheric bubbles (Fig.14) attain a maximum velocity, which generally fluctuates (due to splitting). The bubble velocity is also compared to Eq.1, which is an empirical correlation for velocity of single bubbles in 2d bed by Krishna et.al [6]. The bubble velocity compares well with Eq.1 for atmospheric pressure. However, in vacuum conditions, the comparison is highly inaccurate at the top of the bed.

\[
V_b = 0.62 \sqrt{gd_b} x 1.1 \exp \left( - \frac{1.55d_b}{D_T} \right) \quad \text{for} \quad 0.07 < \frac{d_b}{D_T} < 0.4
\]  

(1)

A sharp increase in velocity is observed even at atmospheric pressure but the %increase in velocity jump is 250% at 95mbar. The bubble travels through a high static pressure differential and thus expands at the top. This increases the bubble shape and bubble velocity. A direct consequence of this phenomenon is a reduced renewal frequency of the particle phases which affects the heat transfer. The bubble size and velocity increases steadily from bottom to middle. This is followed by a sharp velocity escalation at the top which contributes towards a reduced heat transfer.

5 Conclusions

A two dimensional polycarbonate bed with central nozzle was used to generate bubbles at regular intervals in order to validate the slip flow drag model and study the bubble characteristics in vacuum conditions. A slip flow model predicted closer values to the experiment data as compared to the Gidaspow’s model. In addition, the bubble size and velocity was observed to sharply increase towards the top of the bed which explained the heat transfer observations on effect of location in Chapter 3.
Fig. 11: Comparison of bubble size evolution in the bed for 20 and 50 $U_{mf}$ at 95mbar (note: the solid lines represent different bubbles during the experiment)
Fig.12: Comparison of growth rate of bubble at top and bottom the bed for different pressures and flowrates
Fig. 13: Comparison of 2d bed bubble velocity (solid lines) with Eq. 1 (dashed line) for 10 and 40 $U_{mf}$ at 95 mbar. (Note: the solid lines represent different bubbles from the experiment)
Fig. 14: Comparison of 2d bed bubble velocity (solid lines) with Eq. 1 (dashed line) at atmospheric pressure. (Note: the solid lines represent different bubbles from the experiment)
References


Chapter 7

General Discussion

An understanding of the hydrodynamics of vacuum fluidised bed is essential for investigations of heat and mass transfer study. The motivation behind the present study was to use the benefits of vacuum fluidisation, namely, low mass consumption (for potential economic benefit) and a safe operating environment, for substances that are prone to thermal degradation and spontaneous combustion. Processing of pharmaceutical materials, coal drying and surface treatment of metals are the main applications of vacuum fluidisation. The present chapter is intended to combine the results of the present work on hydrodynamics and heat transfer in vacuum fluidisation. This is the first study of its kind in vacuum fluidisation and is not exhaustive. Operation of fluidised beds under vacuum conditions and an extensive study into optimisation of heat transfer are the novel aspects which are addressed in the present work. Due consideration is given to effect of pressure on fluidisation quality. In addition, a new modelling approach is adopted to incorporate the slip flow conditions to enable accurate predictions of hydrodynamic aspects under vacuum.

Apart from the commercial benefits, vacuum fluidisation is an interesting problem for classical fluidisation theorists. While hydrodynamics and heat transfer aspects in atmospheric and high pressure fluidisation have been widely investigated, little research exists in vacuum fluidisation [1, 2]. Thus, this presents a great opportunity to investigate the combined unique properties of vacuum and fluidisation physics.

The present work is structured to firstly investigate the general hydrodynamics aspects of vacuum fluidisation such as quality, minimum fluidisation velocity and fluidisation maps. These aspects are important to carry out optimisation of heat transfer. In addition, modelling of hydrodynamics require extensive information of effect of pressure on hydrodynamic
behaviour. The resulting heat transfer study based on hydrodynamic investigations revealed unique phenomenon exclusive to vacuum conditions. The reasons for this is further investigated by using CFD analysis and 2d fluidised bed.

Atmospheric (or high-pressure fluidisation) and vacuum fluidisation can be individually treated as separate branches of fluidisation on the basis of how each phase in the bed is treated for analysis. Solid-gas fluidisation (atmospheric or higher) constitutes agitation of a discrete medium (the particles) using a continuous medium (air, argon, N₂ etc.). The fact that this discrete medium begins to mimic the behaviour of a continuous medium has been the subject of intense research for decades. In fact, most fluidisation physics assumes the discrete media (particles) as “continuous” for simplification of analysis and the accuracy of the predictions justifies these assumptions. In other words, the approximation of the entire fluidisation phenomenon lies well within the continuum approximation of kinematic and mechanical behaviour of materials (particles and the fluid). Conversely, vacuum fluidisation, challenges the continuum approximation for the fluid medium instead. This fact places vacuum fluidisation in a completely new light, which contests the fundamental physical laws of momentum conservation. In other words, vacuum fluidisation is to gas-solid fluidisation what microfluidics is to pipe and channel flow.

The present work, therefore, addresses the nature of hydrodynamics and heat transfer in vacuum conditions where the fluidisation behaviour, at least macroscopically, resembles atmospheric fluidisation when operated in slip/transitional flow regime. The slip/transitional flow regime lies between a pure continuum and a pure discrete (also known as molecular flow) regime and is characterised by Knudsen no. (Kn), which compares the characteristic length of the medium (the particle size) to the mean free path of the fluid. It is known that vacuum fluidisation has a poorer quality of fluidisation than atmospheric, and that the heat transfer is of poor quality. It was previously argued [2] that the degradation of the thermal property of the
air is responsible for poor heat transfer in vacuum (slip flow regime). Literature on vacuum technology [3] disagrees with this theory because the thermal properties of a real gas remain unaltered until molecular flow is encountered in the system. Thus, it is not the degradation of thermal properties that degrades the heat transfer performance in vacuum fluidisation.

The present study observed a new effect of vacuum pressure on heat transfer. The axial location of the heat source in the bed affects the heat transfer and this correlation strengthens as pressures are reduced below the critical “slip flow” regime. A delta (Δ) effect is observed where the heat transfer increases from bottom up to middle and then plummets towards the top of the bed. In addition to the heat transfer and hydrodynamics experiments in vacuum conditions, the numerical predictions of the non-continuum nature of the fluid is also an intriguing problem. Several computational techniques exist that consider the non-continuum nature of the both the particles and fluid (namely, DEM, Monte-Carlo method etc.). However, these demand enormous computational resources and consequently they are not suitable for a pilot/laboratory scale model. An alternative solution for realistic prediction is to account for the non-continuum effect in conservation of momentum during particle-fluid interaction. This was achieved by mutating the existing Gidaspow drag model with non-continuum effect.

An indication, as to what can affect the heat transfer in vacuum conditions and why the location effect is magnified in vacuum, can be obtained from studies at higher pressures (greater than atmospheric), where the bubble dynamics are seen as an underlying cause of the improved heat transfer [4]. The bubble size compresses as pressures are increased and this greatly increases the heat transfer capacity of the fluidised bed. In vacuum conditions, the opposite of this behaviour is observed. The bubbles are seen to increase in size at sub-atmospheric pressure, which is the primary cause of heat transfer degradation in vacuum. However, the bubble dynamics are not similar to those of atmospheric conditions. Unique to vacuum conditions is the fact that the pressure gradient in the bed induces expansion of the fluid, which affects the
growth rate of the bubbles. At atmospheric conditions, the bubble attains a constant size after initial growth, mainly due to coalescence. In addition to coalescence, the bubble growth takes place due to vacuum pressures at the top, which causes the bubble to continue to increase in size. This “delta effect” in heat transfer is due to the unique bubble characteristics of the passage of a bubble in a high pressure-gradient environment.

The discovery of this effect in vacuum translates to a more considered approach towards exploiting the benefits of vacuum fluidisation. With the help of the information on the quality of fluidisation, the fluidisation map (utilising the GR-PG model) and the location effect, it is now possible to optimise the operation parameters to obtain the best possible thermal performance from vacuum conditions. The pressure, flowrate, particle size and size distribution need to be optimised, in addition to the location of the immersed surface inside the bed. The quality of fluidisation is of a similar order of magnitude near the atmospheric pressures, which could be readily utilised without sacrificing the thermal performance. If the slip flow regime is avoided for a given particle size, the vacuum fluidisation can offer a competitive heat transfer environment in comparison with atmospheric pressure.

Since this study was conducted at low temperature (about 200 °C), there is the possibility of the radiation heat transfer being independent of pressure, if operated at high temperatures (around 800°C). In addition, a mass transfer study to investigate if the controlled input of fluidising gas can be utilised as a mass-transfer quality control is desirable, especially for surface treatment of metals where precise depth of coating is needed. Plasma coatings readily utilise the vacuum fluidisation technique to obtain fine coatings on nano-materials. This study will be beneficial to optimise the operating parameters for above-mentioned processes. In addition, investigations can be carried out to study the interaction of bubbles with immersed objects. Validation of existing particle convective models can be readily carried out at very high vacuum pressures, since the gas convection approaches a nil-value at these pressures.
The numerical predictions of the bubble characteristics matched well when the mutated Gidaspow model was used to obtain the drag force. Thus, the non-continuum “technical fix” to the continuum differential equations, to numerically solve the hydrodynamics, functioned accurately in the slip flow regime without the need to alter the basic assumptions of the Eulerian-Eulerian continuum model. This allows us to extend the advantages of the continuum models to carry out more complex simulations such as heat and mass transfer to/from an immersed surface. To efficiently utilise the heat transfer capacity of the bed along with lower mass consumptions, the following sets of condition prove favourable:

- The powder size distribution (for Group B powders) corresponding to intermediate segregation can be safely utilised without additional quality loss due to particle segregation under vacuum pressures.
- Fluidisation maps drawn from experiments or estimated by the GR-PG model will reveal the range of flowrates and location inside the bed that is favourable for highest heat transfer.
- Wherever possible, the immersed object should be kept at location between bottom and middle during the bubbling regime.

1. Conclusions
A parametric study to investigate hydrodynamic and heat transfer behaviour of fluidised bed under vacuum conditions was carried out successfully with emphasis on effect of pressure on quality of fluidisation, bubble characteristics and heat transfer. The key findings from the present work are:

a. The trend of heat transfer coefficient resembled a delta (Δ) for various axial locations in the bed with a minimum heat transfer at the top and maximum in the middle. This was found to be
due to the expansion of the bubbles towards the top of the bed caused by the presence of vacuum pressures. Bubble size and velocity peaked as the bubbles passed the middle of the bed. This phenomenon is unique to vacuum conditions.

b. The slip flow drag model estimated the bubble size accurately than the existing Gidaspow model emphasising that the slip flow conditions can be simulated by the Eulerian-Eulerian models for gas-solid fluidisation.

c. The degradation of quality is caused by the presence of significant pressure gradient in comparison to the operating pressure for a minimally and intermediate segregated powder. For highly segregated powder, segregation plays and additional role in poor quality.

d. Fluidisation maps can be depicted for vacuum pressures using the GR-PG model for a segregated and non-segregated bed. These maps can be used to locate bubbling regimes for known flowrates and pressures.

e. Optical flow technique can be readily utilised to capture the fluidisation behaviour in a transparent fluidised bed. The velocity of particle at the walls can be estimated if proper calibration techniques are available.

2. Recommendations for future work

a. Correlations for bubble characteristics in literature are not valid in vacuum conditions. Future work on developing correlations for growth, velocity of bubbles that includes effect of gas expansion is needed.

b. Study of bubble flow and distribution, effect of wall on bubble trajectory in vacuum conditions (extension of Appendix II), bubble interaction with immersed surface under vacuum, estimation of kinematic viscosity of particles, estimation of particle renewal etc. can be carried out to improve the understanding of hydrodynamics of vacuum fluidisation.
c. Investigations into mass transfer in vacuum fluidised beds.

References


Bubble–wall interaction for asymmetric injection of jets in solid–gas fluidized bed

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HIGHLIGHTS

• Numerical model used to solve 2D gas–solid fluidised bed with jet injected in proximity to the wall.
• Effect of wall analysed on bubble shape, detachment time and bubble trajectory.
• Bubble shape and size significantly affected near the wall.
• Detachment time for bubbles show significant difference as bubbles are injected in proximity to the wall.
• Considerable affect of wall induced forces in bubble trajectory found.

ABSTRACT

A hydrodynamic model describing gas–solid two phase flow has been used to numerically study the effect of wall on the bubble shape, size, detachment time and the bubble trajectory in a two-dimensional fluidized bed. Using the numerical model, the influence of the wall is analysed by moving the jet from the centre of the bed towards the wall. The bubble characteristics are compared for different nozzle locations, thus presenting the effect wall has on asymmetrical injection as compared to symmetrical injection. A 30% increase in the bubble detachment time for an asymmetric injection (jet velocity of 10 m/s) is found when the nozzle is displaced from the centre of the bed towards the wall (80 mm offset from wall). In addition, the bubble evolution reveals an asymmetric wake formation during detachment indicating an early onset of mixing process. The wall forces acts tangentially on the bubble and has a significant impact on the bubble shape, neck formation during detachment, holdup time near the wall and its trajectory through the bed.

1. Introduction

Bubble formation in a gas–solid fluidized bed takes place near the distributor plate when the fluidisation velocity exceeds the requirement for minimum fluidisation. After its inception, the bubble rises and coalesces with other bubbles, increasing in size and finally erupting at the surface. Bubble size and shape are greatly influenced by their close proximity to either side of the bed walls (Glicksman and McAndrews, 1985; Werther, 1974a). The upward motion of the bubbles affects the movement of the emulsion phase and there is considerable mixing through different layers in the bed (Davidson and Harrison, 1971) causing high degree of temperature uniformity in heat and/or mass transfer applications. Many experimental and theoretical cases in literature have been studied that deals with external injection of bubbles in order to induce or enhance turbulence intensity and subsequently aid the heat transfer processes (Christensen et al., 2008; Gilliland and Mason, 1952; Li et al., 2009).

A significant amount of work in the area of bubble characteristics in gas–solid fluidized beds, over a wide range of parameters has been reported in the literature in last few decades. Of various operational parameters, the geometrical parameters such as the bed width/diameter and the gas distributor configuration play an important role in predicting the bubble dynamics, especially in scale up of laboratory bed (Werther, 1974b). It is well established in flow of bubbles in viscous fluid that the containing wall greatly affects the velocity (Collins, 1967), shape elongation and wake formation (Bhaga and Weber, 1981). Wardag and Larachi (2012) have used corrugated wall for the fluidized container to show its impact on bubble size and distribution. The wall has also been shown to affect the minimum fluidisation velocity (Saxena and Vadivel, 1988).
Glicksman and McAndrews (1985) experimentally studied the effect of bed thickness on the hydrodynamics of large particles in fluidized beds and concluded that the two-dimensional beds exhibit larger bubbles, higher bubble voidage, higher bubble flow rate than their three-dimensional counterpart for similar bed height and superficial velocity.

Another aspect of bubble motion namely, rotation and translation motion across the bed when injected in closer proximity to a wall has also been studied (Werther, 1974b). Recently, Das et al. (2011) showed in their experiment that the presence of a vertical wall at closer proximity to one side of the bubble has a strong influence on the bubble shape, orientations and trajectory. Bokkers et al. (2006) reported that bubble which is closer to wall tends to move towards the centre of the fluidized bed as it rises. van Lare et al. (1997) conducted experiments to obtain information on bubble characteristics using an electrical capacitance probe. It was observed that at different heights of the bed the visible bubble flow is almost nil near the wall and increases away from it. Thus, with increasing height of the bed the bubbles tend to move away from the wall. This effect of wall was more predominant at higher superficial velocity in excess of the minimum fluidisation velocity. Spiral flow path of the bubbles near the wall has also been reported in experiments conducted by Miyahara et al. (1988) on solid–liquid fluidized beds.

In recent years, CFD methods have become quite current in solving the classical Navier–Stokes equation for solid–gas fluidisation in order to predict the complex interaction of the bubbles with the emulsion phase. Of the many models which exist, the kinetic theory of dense gas model, known also as the two-fluid model is widely used along with various closures (Gidaspow, 1994a). Over the years, the Eulerian–Eulerian continuum model has been scrutinisingly evaluated and compared with experimental results and it has been seen that the continuum models predict the bubble characteristics like size and shape quite accurately as compared to experimental results (Mougin and Magnaudet, 2002). In addition, numerical models of 2D beds also predict the general behaviour of bubbles similar to the numerical 3D beds (Wu and Gharib, 2002). Lately, Discrete Element Methods (DEM), although limited in their application for predicting fluidisation behaviour due to computational cost, have become the new research tool to validate many of the constitutive closures used in continuum models (Shew and Pinton, 2006). In all the above studies, wall effect has been generally analysed from particle interactions point of view. For instance, the effect of wall is often characterised by coefficient of restitution (e), generally incorporated as a boundary condition in numerical solutions (Mougin and Magnaudet, 2002; Patil et al., 2005). Single bubble injection studies in literature although deal with bubble flow characteristics (Patil et al., 2005) but a wall effect cannot be estimated due to the symmetrical nature of the problem.

Literature, although replete with experimental studies of effect of the wall, lacks any systematic numerical study on the behaviour of bubbles in proximity to the walls despite the advantages offered by the two-fluid model. The numerical models have come a long way in their implementation to predict the qualitative nature of bubbling fluidized beds accurately. Thus, in the present work, an attempt is made to study the effect of the wall on the motion of a bubble in gas–solid fluidized bed with the help of the existing two-fluid model. Such cases also reflect the motion of bubbles near immersed surfaces, generally used in heat and mass transfer applications. The objective of the work is to model and analyse the impact of a vertical wall on the bubble rising characteristics like shape, size and detachment time and to the best of authors’ knowledge, no study is presently available in the literature that has numerically studied the hydrodynamics of bubble injected asymmetrically i.e. in proximity to the wall.

### 2. Two-fluid model and drag laws

Two-fluid model considers each of the phase to be interpretrating continua and the governing equations of mass and momentum conservations are solved in ANSYS-Fluent for each of the phases which are local mean averages of the point fluid and particles variables. Details on the derivation of the continuum equations for fluidized beds are provided by Gidaspow (1994b) and several investigators. Thus, in the present study an isothermal Eulerian–Eulerian approximation is used with the particle phase limited to a constant diameter. In this model the necessary continuum equations for volume fraction and velocities are as follows:

#### 2.1. Continuity equations

- **Gas phase:**
  \[
  \frac{\partial (\rho_g \phi_g)}{\partial t} + \nabla \cdot (\rho_g \phi_g \mathbf{V}_g) = 0
  \] (1a)

- **Solid phase:**
  \[
  \frac{\partial (\rho_s \phi_s)}{\partial t} + \nabla \cdot (\rho_s \phi_s \mathbf{V}_s) = 0
  \] (1b)

#### 2.2. Momentum equations

- **Gas phase:**
  \[
  \frac{\partial (\rho_g \phi_g \mathbf{V}_g)}{\partial t} + \nabla \cdot (\rho_g \phi_g \mathbf{V}_g \mathbf{V}_g) = -\nabla \cdot \mathbf{P}_g + \nabla \cdot \mathbf{f}_g + \rho_g \mathbf{g} \frac{\phi_g}{\phi_s} \mathbf{V}_s
  \] (2a)

- **Solid phase:**
  \[
  \frac{\partial (\rho_s \phi_s \mathbf{V}_s)}{\partial t} + \nabla \cdot (\rho_s \phi_s \mathbf{V}_s \mathbf{V}_s) = -\nabla \cdot \mathbf{P}_s + \nabla \cdot \mathbf{f}_s + \rho_s \frac{\phi_s}{\phi_g} \mathbf{g} \frac{\phi_g}{\phi_s} \mathbf{V}_s
  \] (2b)

where

\[
\tau_g = \mu_g \left( \mathbf{V}_g + \mathbf{V}_g^T \right) - \frac{2}{3} \mu_g \nabla \cdot \mathbf{V}_g \mathbf{I}
\]

\[
\tau_s = \mu_s \left( \mathbf{V}_s + \mathbf{V}_s^T \right) + \left( \lambda_s - \frac{2 \nu_s}{3 \eta_s} \right) (\nabla \cdot \mathbf{V}_s) \mathbf{I}
\]

#### Table 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_n )</td>
<td>( \rho_n \phi_n ), ( 2(1+\phi_n)\rho_g \phi_g ), ( \phi_n )</td>
</tr>
<tr>
<td>( \mu_n )</td>
<td>( \frac{4}{3} \rho_n \phi_n \rho_g \phi_g (1+\phi_n) \left( \frac{\phi_n}{\phi_g} \right) \left( \frac{\phi_g}{\phi_n} \right) )</td>
</tr>
<tr>
<td>( \lambda_s )</td>
<td>( \frac{4}{3} \rho_s \phi_s \rho_g \phi_g \left( \frac{\phi_s}{\phi_g} \right)^{1/2} )</td>
</tr>
<tr>
<td>( g_s )</td>
<td>( \frac{1}{\rho_s} \left( \frac{3 \eta_s}{\phi_n} \right)^{1/2} )</td>
</tr>
</tbody>
</table>

Srivastava and Sundaresan (2003) frictional model:

\[
\beta_s(\epsilon) = \left\{ \begin{array}{ll}
0 & \epsilon < \epsilon_{\text{min}} \\
\frac{\epsilon_{\text{min}} - \epsilon}{\epsilon_{\text{max}} - \epsilon_{\text{min}}} & \epsilon_{\text{min}} < \epsilon < \epsilon_{\text{max}} \\
1 & \epsilon > \epsilon_{\text{max}}
\end{array} \right.
\]

\[
\beta_s(\epsilon) = \sqrt{\frac{\phi_s}{\phi_n}} \sqrt{\frac{\phi_n}{\phi_s}} \frac{3 \eta_s}{\phi_n} \frac{3 \eta_s}{\phi_s}
\]

where \( \epsilon \) is the internal angle of friction, \( \phi_n \) is the volume fraction of the particle phase, and \( \phi_s \) is the volume fraction of the solid phase.
and
\[ \epsilon_s + \epsilon_g = 1 \]

where \( \epsilon \) is the volume fraction.

For the problem to be completely defined the governing equations require closures for the solid-phase pressure \( (P_s) \), solid phase shear viscosity \( (\mu_s) \) and the solid-phase bulk viscosity \( (\lambda_s) \). These constitutive equations are derived from kinetic theory of granular flow (Kumar et al., 2012) and are presented in Table 1. Apart from these closures, kinetic theory of granular flow requires the solution to transport equation for the granular temperature. Granular temperature, \( \Theta \), signifies the random motion of the solid particles and is analogous to temperature definition according to the kinetic theory (Kumar et al., 2012):

\[
\frac{3}{2} \left[ \frac{\partial (\epsilon_s \rho_s \Theta_s)}{\partial t} + \epsilon_s \rho_s \mathbf{V_s} \cdot \nabla \Theta_s \right] = (-P_s I + \epsilon_s) : \nabla \mathbf{V_s} + \nabla \cdot (k_s \nabla \Theta_s) - \gamma + \phi_i \quad (4)
\]

2.3. Gas–solids drag coefficients

Several semi-empirical closures exist in literature to define the gas/solid momentum transfer coefficient. Of these, Gidaspow's model (Gidaspow, 1994a) is the widely used drag law and is a combination of drag law by Ergun (1952) and Wen and Yu (1966).

Gidaspow's model can be summarized as follows:

\[
\beta = \begin{cases} 
150 \frac{(1-\epsilon_g)^2 \rho_g}{\epsilon_g d_s^2} + 1.75 \rho_g (1-\epsilon_g)(\mathbf{V_s} - \mathbf{V_g}) & \text{for } \epsilon_g \leq 0.8 \\
\frac{3}{4} C_D \left( 1-\epsilon_g \right) \rho_g \left( \mathbf{V_s} - \mathbf{V_g} \right) \epsilon_g^{-2.65} & \text{for } \epsilon_g > 0.8 
\end{cases} \quad (5a)
\]

\[
C_D = \frac{24}{\epsilon_g \text{Re}_g} \left[ 1 + 0.15 (\epsilon_g \text{Re}_g)^{0.687} \right] \quad (5b)
\]

where

Fig. 1. Schematic representation and computational domain of the fluidized bed used in the present study.
2.4. Frictional stress model

At high particle volume fractions, individual particles interact with multiple neighbours through sustained contact. Under such conditions, the normal reaction forces and the associated tangential frictional forces at these sliding contacts are dominant. It is assumed that the granular material is non-cohesive and follows a rigid-plastic rheological model of the type proposed by Tardos, 1997. Shuyan et al. (2009) show the influence of the frictional stresses in the gas-solid spouted beds. They have reported that the use of frictional stress in the simulations produce results more physically accurate compared with experimental data. In this model the frictional stress model of Srivastava and Sundaresan (2003) is used, which has been shown in literature to accurately predict the shape and size of a bubble in fluidized bed (Patil et al., 2005).

3. Problem description

A two-dimensional computational fluidized bed model (570 x 1000 mm) is considered (see Fig. 1). A high-speed jet is diffused inside the bed to form definite size bubbles while maintaining the rest of the bed at minimum fluidisation conditions. The solutions are obtained for different positions of nozzle: from symmetrical position (centrally located nozzle) to...
asymmetric positions (closer proximity to the right side of the wall) as shown in Fig. 1. The centrally located jet model is used to validate the results with other published data. Inlet velocity is maintained at the minimum fluidization velocity as shown (Fig. 1) with ambient atmospheric pressure-boundary-condition at the top of the bed. No slip wall-boundary condition is imposed for the gas-phase whereas partial slip condition, given by Johnson and Jackson (1987), is applied to solid-phase with specularity coefficient as 0.5 and particle–wall coefficient of restitution as 0.9.

3.1. Grid independence study

Initially, a uniform grid distribution (mapped mesh) was considered to study the bubble shape during its detachment, where only five elements/cells ($\Delta x = 0.003$ m) were being used to map the jet (0.015 m) at the inlet. The bubble shape ($\varepsilon_g = 0.85$) was predicted to be unphysical pointed-shaped-bubble at detachment, which was attributed to larger grid spacing. The larger grid spacing increases the numerical diffusion, while the smaller grid spacing increases computational time (Zhang et al., 2012). In the present case, increasing the number of elements on the jet (nozzle entry) with a uniform grid throughout the domain is computationally challenging. Thus, the strategy adopted here is to use multi-block, unstructured Cartesian grid (with quadrilateral elements), which allows fine uniform grids near the nozzle for accurately predicting initial growth of the bubble and at the same time providing an adequate size of elements near the walls and the interface. Three specific regions: jet region, freeboard and wall were selected where mesh were generated in doubling the grid spacing, respectively (Fig. 1). Grids with total numbers of elements from 6000 to 30,000 were tested for bubble shape during detachment and compared with equivalent bubble diameter data of Kuiper et al. (Patil et al., 2005). The results of the grid independence test are given in Fig. 2(a) and (b). Results become grid independent when the number of elements increases from 18,000 to 30,000. Computational time for these elements is
4. Results and discussion

4.1. Model validation and wall effect on bubble shape

4.1.1. Central/symmetrical injection

The numerical model is intended to predict the wall effect on the bubble shape and trajectory when the gas is injected asymmetrically. Thus, simulations are carried out for five different nozzle positions (wall offset = 80, 100, 120, 140 and 160 mm) with respect to the side vertical wall as shown in Fig. 1. Gas injection rate, ranging from 2 to 10 m/s is used to form the bubbles in the incipient fluidized bed. A particle diameter of 500 μm with a density of 2660 kg/m$^3$ is used in the present simulation. Geldart type-A materials have small mean size and low particle density, which exhibit a rapid mixing (Geldart, 1973). Bubbles in such bed appear to split and coalesce very frequently. This may cause relatively more hindrance to a shape-study that arises from the wall-effect than Geldart type-B powders (Gidaspow, 1994a). The present numerical model uses experimental results of Kuiper et al. (Patil et al., 2005) for the validation of the predicted data (Fig. 3).
who had used glass particles of size 500 μm. Thus, same material properties are used to study the wall effect for asymmetric injection.

The equivalent diameter of the bubble is assumed as the diameter of a circle with an area equal to that of the bubble, which is given as

\[ D_{eq} = \sqrt{4A/\pi} \]

(6)

For the purpose of analysis of simulation results, a bubble is defined as a void with \( \varepsilon \geq 0.85 \).

It is apparent from Fig. 3 that the prediction agrees well with the experimental data. For a centrally injected jet (offset = 285 mm), it is observed that the evolution of bubble occurs by outward radial displacement of the dense media caused by almost spherical growth up to detachment. A time history of the bubble-evolution up to detachment is given in Fig. 4. A very close spherical shape of the top surface of the bubble is seen, which transforms to an elliptical during detachment. This pattern of bubble evolution has been observed experimentally (Kuipers et al., 1992b). It can be inferred from the symmetrical shape of the bubble that the forces that govern the bubble formation (buoyancy, lift and drag forces) act symmetrically around the bubble. This symmetrical action of governing forces depends significantly upon the position of the jet inside the bed and the fluidisation conditions prevailing during bubble evolution, as will be shown in the present study. For the case of central injection of jet at 10 m/s, the bubble detachment occurred at 0.195 s when the equivalent diameter is 161.2 mm.

The velocity of injection is varied from 2 to 10 m/s to analyse the shape of bubble. Fig. 4 (a) and (b) shows the bubble formation and its detachment at 2 m/s and 10 m/s gas injection rate. The bubble size increases with increase of the jet velocity as shown in Fig. 4. The detachment time also increases with gas injection rates (0.15 s for 2 m/s and 0.195 for 10 m/s). This is because the bubble takes a finite time to break from the orifice. It is also known that the detachment occurs at the instance when the buoyancy force is balanced by the weight of the bubble. Thus, higher the inlet velocity higher the drag forces that keep the bubble attached (Brucker, 1999; Patil et al., 2005). It is also observed that at lower gas injection rate the bubble sizes are flattened considerably to form an elliptic shape during detachment. Fig. 5 shows the velocity vector and the stream-tracers for the dense phase at different time steps during bubble formation. Necking of the bubble begins as the bubble grows and the neck width decreases with the advancement of the time step. During the neck formation, the surrounding fluid (gas-phase) rushes to the detachment point thus creating a strong localised velocity of the dense phase at the trailing edge of the bubble (Das and Das, 2009). It can be seen that the dense phase begins to slide down along the bubble periphery toward the bottom of the bubble. Thus, the onset of wake begins much before the detachment of the bubble. However, it is to be noted that the bubble shape, stream-tracers and velocity field remain symmetrical due to centrally located nozzle (wall-offset = 285 mm).

![Fig. 8. Evolution of bubble width with gas injection rate for centrally injected bubbles.](image)

![Fig. 9. Variation of detachment time with jet injection for symmetric and asymmetric injected bubbles.](image)

![Fig. 10. Variation of bubble equivalent diameter with time and gas injection rate for wall offset of 80, 100 and 285 mm.](image)
4.1.2. Asymmetrical jet injection

Solutions are also obtained for different wall offsets ranging from 80 mm to 160 mm (see Fig. 1). A wall offset of 80 mm refers to the jet’s position at 80 mm from the right side wall. Fig. 6 shows the comparison of the bubble formation for symmetrically (wall offsets of 285 mm) and asymmetrically (wall-offsets: 80 and 140 mm) located jets. Bubble contours ($\varepsilon=0.85$) are shown at different time steps, $\Delta t=0.08$ s and for jet velocity of 10 m/s. It is seen that the bubble drifts away from the vertical jet axis when injected in proximity to the wall. It is also interesting to see how

![Stream-tracers for the dense phase (solid lines) and gas phase (dashed lines) and vector plot (right) for the dense phase at 2 m/s gas injection rate. (a) 0.4 s (b) 0.6 s and (c) 0.8 s.](image-url)
the proximity of the bubble to the wall affects the overall neck formation. The neck elongates in an oblique fashion indicating a tangential force generated due to perturbation caused in the flow patterns by the wall. Fig. 7 shows the dense phase velocity vector for the central and asymmetrical bubble injection indicating the impact of tangential force caused by the wall, which initiates the oblique bubble growth. The asymmetric flow field is generated when the available flow area decreases between the bubble surface and the wall. The dense phase then accelerates resulting in a tangential entry that forces the diagonal growth. The centrally injected bubble shows symmetrical flow pattern of the dense phase thus proving the perturbation effect of wall in case of an asymmetrical injection.

In addition, the shape distortion can be understood from the dense phase velocity vectors. The closer the bubble is injected to the wall, more distorted the bubble shape becomes. Fig. 7 also shows that for wall offset of 140 mm the bubble retains its spherical shape but for the 80 mm wall offset the bubble shape is distorted especially on the wall side. An elongated curvature is predicted because of the increased magnitude of the velocity vector. This causes the shearing of the bubble curvature.

4.2. Effect of wall on detachment time

Detachment of bubble from the nozzle is governed by the interaction of buoyancy and body forces. Drag force on the bubble acts downward and increases with the rising bubble velocity (Brucker, 1999). Thus, it is expected that the detachment time would increase at higher gas injection rate and the predicted data in Fig. 8 confirms higher detachment time at higher gas injection rates. The figure also shows the variation of bubble size ($D_h$) vs. time (up to detachment time) for a centrally located nozzle.

Fig. 9 shows the plot of detachment time vs. gas injection rate for wall offset 80 mm and 285 mm (central injection). It can be seen that the detachment time for bubbles injected near the wall is higher than those injected centrally. Min et al. (2010) also reported that the gas holdup is comparatively higher near the wall in their experiments. However, the possible explanation for this cause may be attributed to the asymmetrical nature of the velocity field around the bubble. There is a shift of centre of gravity, as the bubble deflects away from the wall (Fig. 6), causing a misalignment of the buoyant and body forces. This misalignment may result in both rotational and linear displacements for the rising bubble causing higher holdup time. The delay in detachment

Fig. 12. Stream-tracers for the dense phase (solid lines) and gas phase (dashed lines) and vector plot (right) for the dense phase at 5 m/s gas injection rate. (a) 0.4 s and (b) 0.6 s.
thereby not only increases the gas holdup but also the size in bubble. It can be seen from Fig. 10 that the bubble size is higher for jet located near to the wall for all flowrates.

4.3. Effect of wall on bubble trajectory

4.3.1. Symmetrical injection

Bubble motions, after the detachment, are also predicted for all range of gas injection rates (2–10 m/s). Figs. 11 and 12 show the trajectory of the bubble for two different flowrates (2 and 5 m/s) along with stream-tracers for both the phases. The dense phase begins to move downward relative to the bubble motion. The bubble is predicted to rise with a slight indentation at the base that grows bigger as the bubble rises. The wake zone i.e. the space between the concave edge of the bubble (indented base) and the bubble sphere is occupied by solid particles. These particles are drawn from the upper surface of the bubble into the trailing zone of the bubble, where vortices containing the particles are shed from the wake. This movement of the surrounding fluid around the rising bubble is the main cause of particle mixing (Tsuchiya et al., 1990). At higher gas injection rate (Fig. 11), the velocity field indicates more particle entrainment into the central part of the wake illustrating the increase in mixing process. At such flowrates, smaller bubbles split at the end of the nozzle with irregular shapes, indicating a jetting regime.

Thus, the factor that contributes to the bubble motion is the force induced by the wake behind the bubble plus the inertia caused by injected air velocity. The wake becomes larger as the bubble rises forming two identical vortices at symmetrical locations (Fig. 10). For centrally located nozzle, both the bubble and the wake may have been influenced by the close proximity to either side of the bed walls but their morphology remains nearly symmetrical throughout.

Figs. 13 and 14 show the predicted bubble shapes ($\epsilon=0.85$, $\Delta t=0.1$ s) along its trajectory at lower (2 and 3 m/s) and higher (4, 5 and 6 m/s) velocities, respectively. Shapes are remarkably different when injected at higher gas injection rate. However, for all the cases, the bubble shapes tend to be more flattened at the upper part of the bed due to decrease in jet momentum flux as the distance of bubble from the base increases. Similar observations were also reported by Miyahara et al. (1988). Bubble becomes larger, elongating in the direction perpendicular to the motion, and proceeds by a slight flattening of the bubble initiated near the frontal surface in the vicinity of the stagnation point. More flattened the bubble shape, the more the interfacial force holding the bubble to a single body is weakened (Tsuchiya et al., 1989). Thus, the vertical indentation is predicted to grow from the roof of the bubble, as shown in Fig. 14, and often travels downwards due to higher velocity (of large bubble) followed by necking and consequently splitting into two similar size of bubbles. The figures also predict that the rising path of the bubble remains symmetrical, which may be considered as a linear translation for the bubble along the vertical axis, for all range of gas injection rates.

4.3.2. Asymmetrical injection

Fig. 15 shows the stream-tracers for both phases at wall-offset of 100 mm. When bubble injected nearer to wall, the velocity of the dense phase increases in the gap as discussed earlier. The figure clearly shows a higher velocity of the dense phase at the right side of the bubble. Thus, the net effect of the surrounding field remains tangential below the bubble causing a significant change in its orientation even before its detachment.

Fig. 15 also plots the variation in bubble shapes and vortices at different time steps (0.2–0.8 s). The vortices are expected to experience a significant number of oscillations due to presence of asymmetric wake, which in turn would make the flow-field non-uniform. At higher position from the inlet boundary, when the bubble becomes larger, the oscillation stabilises resulting a trajectory path (deviating from the mid vertical axis of nozzle). Thus, in general, it is predicted that bubble will have a longer path-coverage when injected asymmetrically that would have significant influence on overall heat transfer characteristic of the fluidized bed. Kuipers et al. (1992a) discussed that local heat transfer characteristics are relatively higher in the wake region if gas is injected asymmetrically. The bubble tends to exhibit both linear and rotational displacement due to the presence of asymmetric forces caused by a solid wall in near vicinity. The rotation of the bubble can be seen by the change in the orientation of the bottom surface in Fig. 15(b)–(d).

Fig. 16 compares the trajectory of bubble when injected at different gas injection rates for a wall offset of 80 mm. For a
smaller size bubble i.e. at a low gas injection rate, the wake-flow is predicted to have periodic oscillation, which is intimately related to bubble shape, size and body forces. Fig. 17 shows the predicted vector plots of the dense phase and the oscillatory vortex shedding at wake region. The bubble mass centre oscillates about the vertical axis due to the wall perturbation giving rise to a spiral path of trajectory. For a larger bubble, injected at high jet velocity, such oscillatory trajectory is not observed (Fig. 15) due to higher rise velocity and rate of size increase. It can be seen that the left and right vortices always accompany the bubble all the way to the surface (Fig. 15). In addition, due to significant increase in bubble size during its rise the bubble may not have an oscillatory motion even if the length of the bed is increased.

5. Conclusion

CFD simulations using the Eulerian–Eulerian model in ANSYS-FLUENT for a lab-scale bubbling fluidized bed containing Geldart type B particle of 500 μm are used to study the effect of wall on the shape of bubble, bubble-wake during formation and the bubble-rise for both symmetric (centrally located nozzle) and asymmetric injections. The rising characteristics of bubble for an asymmetric injection are remarkably different in terms of its size, shape, wake and position of vortices. The following conclusions can be made based on the numerical simulations carried out in the present work.

1. The presence of wall in proximity to a rising bubble causes increase in bubble shape in addition to causing the bubble to drift away from the wall. The wall closer to one side of the bubble changes the flow field around the bubble to develop a non-uniform wake behind the bubble.
2. The presence of wall also increases the detachment time of the bubble mainly due to the misalignment of the governing buoyant and body forces. A maximum of 30% increase in detachment size was found (gas injection rate 10 m/s).
3. Bubble of intermediate sizes, generated at lower gas injection rate, are predicted to have a spiral path and the bubble with larger shape exhibits a trajectory increasing total path coverage for the bubble.

More studies need to be carried out in order to analyse the oscillatory motion of the bubble and validate the numerical results.
by experiments. Effect of asymmetric injection on the heat transfer characteristics for immersed objects also needs special attention.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>area, m$^2$</td>
</tr>
<tr>
<td>$d/D$</td>
<td>particle diameter, m</td>
</tr>
<tr>
<td>$e$</td>
<td>restitution coefficient</td>
</tr>
<tr>
<td>$g$</td>
<td>acceleration due to gravity, m/s$^2$</td>
</tr>
<tr>
<td>$h$</td>
<td>height of bed, m</td>
</tr>
<tr>
<td>$k$</td>
<td>diffusion coefficient (Eq. (4))</td>
</tr>
<tr>
<td>$L$</td>
<td>bed height, m</td>
</tr>
</tbody>
</table>

Re | Reynolds number, $\rho_f V_d/\mu$

$u$ | superficial velocity, m/s

$y$ | length along $y$-direction

$\nu$ | velocity vector, m/s

Greek-letters

$\epsilon$ | volume fraction

$\psi$ | interstitial angle

$\phi$ | sphericity

$\mu$ | dynamic viscosity, Pa s

$\rho$ | density, kg/m$^3$

Sub-scripts

$g$ | gas

$\text{mf}$ | minimum fluidisation

$p$ | particle

$s$ | solid

References


