Professional learning in mathematical reasoning: Reflections of a primary teacher

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Abstract

Reasoning is an important aspect in the understanding and learning of mathematics. This paper reports on a case study presenting one Australian primary teacher's reflections regarding the role played by a professional learning program in her developing understanding of mathematical reasoning. Examination of the transcripts of two interviews identified changes in her perceptions of mathematical reasoning by mapping interview responses against the Mathematical Reasoning Framework (Herbert et al., 2015). This change indicates that a well planned program of professional learning based on a demsonstration is efficacious in developing teachers' understanding of mathematical reasoning.

Introduction

Currently, in the Australian Curriculum (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2015) reasoning is one of the four key proficiencies where it is described as "logical thought and actions, such as analysing, proving, evaluating, explaining, inferring, justifying and generalising" (p. 2). Clarke, Clarke and Sullivan (2012) claimed there is wide variation in the degree of teachers' understanding of this statement. This indicates that professional learning in this important aspect of mathematics requires careful consideration. ACARA (2015) further explained that:

> [s]tudents are reasoning mathematically when they explain their thinking, when they deduce and justify strategies used and conclusions reached, when they adapt the known to the unknown, when they transfer learning from one context to another, when they prove that something is true or false, and when they compare and contrast related ideas and explain their choices.

This paper reports on one teacher's growth in understanding of reasoning as a result of engagement in the Mathematical Reasoning Professional Learning Research Program (MRRLRP), a professional learning program designed for this purpose. It included several activities based around a demonstration lesson (Clarke et al., 2013) focused on reasoning. The teacher was interviewed before and after the program, observed the demonstration lesson, engaged in discussions before and after the lesson and finally trialled the lesson with her own class.

This paper begins with a review of the literature relating to mathematical reasoning, pedagogical content knowledge and teacher professional learning in mathematics. This is followed by a discussion of variation theory as the theoretical framework for this study, the presentation and discussion of the results and then the conclusion draws attention to insights gained by considering this teacher's reflections on factors influencing her developing understanding of mathematical reasoning.

Background

"Mathematical reasoning is a key element of mathematics and thus is central to learning mathematics in school" (Brodie, 2009, p. 11). Teaching to foster the development of mathematical reasoning especially in primary schools presents many challenges. Long, De Temple and Millman (2012) asserted:

> Mathematical reasoning develops in classrooms where students are encouraged to put forth their own ideas for examination. Teachers and students should be open to questions, reactions and elaborations from others in the classroom. Students need to explain and justify their thinking and learn how to detect fallacies and critique others' thinking. They need to have ample opportunity to apply their reasoning skills and justify their thinking in mathematical discussions. They will need time, many varied and rich experiences, and guidance to develop the ability to construct valid arguments and to evaluate the arguments of others (p. 49).

Stein, Engle, Smith and Hughes (2008) stressed the importance of orchestrating mathematical discussions "that promote conceptual understanding and the development of thinking, reasoning, and problem-solving skills" (p. 314). They proposed five practices for making classroom discussions more effective:

> (1) anticipating likely student responses to cognitively demanding mathematical tasks, (2) monitoring students' responses to the tasks during the explore phase, (3) selecting particular students to present their mathematical responses during the discuss-and summarize phase, (4) purposefully sequencing the student responses that will be displayed, and (5) helping the class make mathematical connections between different students' responses and between students' responses and the key ideas (p. 321).

In addition to providing opportunities for students' reasoning, teachers' own mathematical content knowledge (MCK) underpins their teaching of mathematical reasoning as a sound MCK is required to make choices relating to appropriate teaching strategies which address the content and sequence of presentation of concepts. It has long been acknowledged (Shulman, 1987; Sowder, 2007) that there is an important difference between the knowledge of the content of instruction (CK) and pedagogical content knowledge (PCK), that is, knowledge on how the content may be taught. Sowder (2007) emphasised the importance of PCK in the manner in which experienced teachers are able to plan to scaffold knowledge, so that later stages of development of a complex concept are supported by a foundation to the concept. Teachers' competency with content knowledge directly impacts on their confidence to teach it (Darling-Hammond, 2000). Deepened PCK and CK has potential for quality improvement in teachers' practices, so it is important that teachers' MCK is correct and complete.

One avenue of strengthening teachers' MCK is through professional learning. However, teachers will only change their practice if they are compelled internally or externally to make that change (Chapman, 1996). The explicit inclusion of reasoning as one of the proficiencies of the Australian Curriculum (ACARA, 2015) is an external incentive for teachers to attend to the meaning of mathematical reasoning and to develop strategies for assessing and reporting on the development of reasoning of their students. Sowder (2007) asserted that a successful professional learning program includes modelling of different strategies of instruction; the provision of support for implementation of changes in strategies of instruction; collaborative problem solving; and continuity over time.

Guidance in the preparation of professional learning programs is provided by Clarke's (1994) ten key principles: address issues of concern and interest; involve groups of teachers; recognise and address impediments to teachers' growth; model desired classroom approaches; teachers' commitment to active participation; provide opportunity to validate new ideas by observing positive student learning; allow time for planning reflection and feedback; enable teachers' ownership as partners in change; provide ongoing support; and encourage goal setting. Many aspects of these key principles are embedded in the MRRLRP undertaken by the teacher in this case study. The aim of this program was to enhance teachers' reasoning capacity; their knowledge of reasoning and its relationship to learning mathematics and other mathematics proficiencies; their approach to teaching reasoning. This paper investigates how this teachers' knowledge developed through observation and trialling of a demonstration lesson.

Theoretical Framework - Variation Theory

In *variation theory* learning is viewed as a change in an individual's conceptions of the phenomenon being explored (Cope, 2000). Learning is characterised by an expansion of awareness in that learners – in this case a primary teacher – become aware of additional features or aspects of a concept or skill that they had not previously discerned (Bowden & Marton, 2004). From this perspective, learning can be said to have taken place if the learner comes to "see the phenomenon differently" (Bowden & Green, 2005, p. 30). Therefore, the providers of professional learning can assist learners by focusing their attention on particular aspects of the phenomenon. For instance, when teachers are observing a demonstration lesson alerting them to focus and report on students' reasoning they might observe.

Variation theory emphasises the importance of a person's awareness of critical aspects of a concept (Cope, 2000) and proposes that full understanding can only take place if these critical aspects are discerned, requiring explicit exposure to the variation in the critical aspect, for example, different situations where reasoning may be detected in a student's thinking. However, Kaput (1992) warns that it cannot be guaranteed that learners will attend to the variation simply because it exists.

In this way, variation theory can be seen as a guide for designing activities which expose learners to the greatest possible variation in the widest range of critical aspects of the *object of learning* (Cope, 2000, p. 41). The learning environment can then be arranged so that learners come "to experience something in a qualitatively new and more powerful way, so that it can be accomplished in different circumstances, in different ways, and facilitate doing altogether new things" (Booth, 2004, p. 10). The teacher in this study was provided opportunities to become aware of the critical aspects of mathematical reasoning through the professional learning program (MRRLRP) focusing on mathematical reasoning.

Method

A case study approach is appropriate because this paper presents an examination of single case. It is "an intensive, holistic description and analysis of a single instance, phenomenon or unit" (Merriam, 1988, p. 21), that is, one teacher's engagement in MRRLRP. It is intended that by utilising this approach a deeper understanding may be gained of specific issues associated with the implementation of the program. Changes in the language used by the teacher to describe their understanding of mathematical reasoning are documented to determine the impact of observing and trialling a mathematical reasoning demonstration lesson on their awareness of the critical aspects of mathematical reasoning.

The primary teacher in this case study was a vice principal at a primary school in a country town in Victoria. Olive[1] had been employed at this school for seven years and was team teaching across Grade 3-4 classes, but had experience teaching at all levels of the school. She had attended external professional learning in mathematics, even travelling considerable distance to do so, and claimed that she "always had a passion for maths"; had undertaken her own reading in terms of mathematics; and was keen to participate in the

[1] Pseudonym

MRRLRP. The prinicipal is proactive in encouraging professional discussions amongst the teachers and attendance at external professional learning opportunites. These factors influence teachers' willingness to learn more about teaching mathematics (Beswick & Jones, 2011).

The MRRLRP was intended to expose teachers to variation in aspects of mathematical reasoning (Booth, 2004). The program was built around a demonstration lesson, conducted in Olive's Grade 3-4 classroom utilising Small's (2011) "What else belongs?". The launching task challenged students to find other numbers which belonged with {30, 12, 18}. When they were brought together to discuss their results, selected pairs were asked to share their numbers and convince the class that their reasoning justified their choice of numbers. Then the pairs created their own group of 3 numbers justifying why these belonged together. The lesson closed with a final class discussion and individual written reflections on their learning from the lesson (see Bragg et al., 2013, for details of this lesson).

Olive was interviewed a couple of weeks prior to the observation lesson and before receiving the lesson plan. On the day of the demonstration lesson she participated in a pre-lesson briefing about the lesson to be taught, observed the lesson taking notes about students' thinking and work and teacher's actions, and finally, participated in a post-lesson discussion conducted by one of the researchers. This discussion focused on teachers' observations of students' reasoning and the way the lesson structure and the demonstrating teacher fostered the students' reasoning. Both the pre-lesson briefing and post-lesson discussion were audio-recorded and the demonstration lesson was video-recorded. The post-lesson discussion and audio-recorded interviews provided Olive the opportunity to discuss and reflect on the practice of others as well as her own practice.

Two weeks later she trialled the lesson in the other Grade 3-4 classroom at her school. After this experience she was interviewed again with questions about her students' learning and reasoning and her experiences of teaching the lesson. Olive's trialling of the demonstration lesson provided her with practical experience of teaching a lesson where mathematical reasoning had been specifically embedded. It was anticipated that Olive would benefit from the opportunity to implement tasks designed to illicit student's reasoning and hence gain insight into students' responses and perceptions of reasoning and experience teacher actions relevant to the teaching and learning of reasoning.

Olive's responses at the two interviews were mapped against the Mathematical Reasoning Framework (Herbert, Vale, Bragg, Loong, Widjaja, 2015) (see Table 1) to determine if there had been a change in her perception of reasoning.

Table 1
Mathematical Reasoning Framework: Primary Teachers' Perceptions of Mathematical Reasoning (Herbert et al., 2015).

Category	Perception of mathematical reasoning
Category A	Reasoning is perceived to be thinking.
Category B	Reasoning is perceived to be communicating thinking
Category C	Reasoning is perceived to be problem solving
Category D	Reasoning is perceived to be validating thinking
Category E	Reasoning is perceived to be forming conjectures
Category F	Reasoning is perceived to be using logical arguments for validating conjectures

Category G	Reasoning is perceived to be connecting aspects of mathematics

This paper reports on data from the second interview which provides evidence of the aspects of the program Olive felt were influential in her growing awareness of mathematical reasoning, including trialling of the demonstration lesson with her own class.

Results and Discussion

Olive was chosen for this case study because she demonstrated substantial progress in the development of her understanding of mathematical reasoning. When Olive was interviewed before the commencement of MRRLRP her understanding was judged to be consistent with Category A (see Table 1) where reasoning is seen to be an action which is conducted only in an individual's head, involving choices and personal reflection on those choices because when asked "what do you see reasoning to be?" Olive replied:

> The thinking [pause] – the thinking behind why things work.

Olive reported that reasoning was not something they would usually think about when planning lessons, perhaps because of this limited view of reasoning.

> I don't think when we're planning we purposely think about reasoning. We do a lot of work in number and if reasoning happens to happen within that.

The second interview revealed a change in her understanding, then consistent with Category G as her awareness of many aspects of reasoning (see Herbert et al., 2015) were evident when asked again "what do you see reasoning to be?" Olive replied:

> The thinking about the thinking … analysing what they're actually thinking about. … [students] talk about thinking and infer with evidence. … So within reasoning it's getting them to think about their own thinking and why does this work. … It's [reasoning] how they think and how well they think and what they know [and bring to their thinking], if they know more they can think more … if you can give your reasons you seem to have a higher level of [reasoning].

This quote indicates that Olive considered mathematical reasoning occured when students brought together and discussed different aspects of prior mathematical knowledge to solve problems and make sense of mathematics. Key features of Olive's inderstanding are seen in her reference to inferring from evidence and justifying solutions, bringing to the discussion with others their prior knowledge, selecting from that knowledge and applying it to a problem.

Olive's reflections noted in the transcript of her second interview provide evidence of her perceptions of MRRLRP. Illustrative quotes were chosen to demonstrate influences on Olive's growing awareness and knowledge about mathematical reasoning.

Demonstration Lesson

Olive was adamant that the demonstration lesson was a valuable experience because:

> just reading someone's notes [lesson plan] you can interpret it differently, but watching someone do it, you've seen the conversations, you've seen how they've done it so you get so much more out of it … when I read it [lesson plan] maybe I didn't understand it very well but … we had sat down with you before the lesson and you'd asked us what we thought some of the answers might be and we said hopefully they'll say 2 digit numbers and hopefully they'll say even numbers and hopefully they'll see this. But even though we were giving these answers, I was thinking, some of the kids in this room might not actually be able to see anything. So I was pleasantly surprised that they were all able to have a level of success within that lesson.

This suggests that she felt she needed to see this lesson taught to be able to understand how to follow the lesson plan. She was interested in the way the whole group discussion was orchestrated:

> when [demonstration presenter] put it on the board you could see it set out within the levels. So this was you know 2 digit numbers was probably the simplest part of it or even numbers and then, yeah working your way through to that higher level thinking.

This quote illustrates Olive's learning about one strategy which has the potential to foster students' ability to reason mathematically, that is, orchestrating discussions. The discussion she described is consistent with Stein et al.'s (2008) five practices of orchestrating discussions i.e. "anticipating, monitoring, selecting, sequencing and making connections between student responses" (p. 314). It demonstrates her growing awareness of teacher actions which facilitate the development of students' mathematical reasoning particularly her awareness of the importance of purposeful sequencing of her student responses and helping her students to make mathematical connections between different ideas to advance their mathematical thinking. These teacher actions she had seen modelled in the demonstration lesson.

When asked about the post-lesson discussion, Olive commented:

> Yes … the discussion on reasoning and the lesson itself, you were hearing ideas from someone else that you didn't necessarily have, but it also confirmed what we were thinking. … we got a lot out of the discussion, all the questions were answered and everyone had a chance to speak and everyone did speak.

Olive valued the post-lesson discussion in facilitating a broader range of views and experiences to be shared and allowing collegial collaboration that support her growing understanding of reasoning. She reported that these discussions continued with the other teacher at the school who observed the lesson.

Description of the trial

Olive trialled the demonstration lesson with students in Grade 3-4 who had not been involved in the demonstration lesson mostly because they had not returned permission forms, so the profile of this group was differernt from the demonstration lesson. Olive commented that:

> Some of the children found the activity really really difficult. … [we] were left with a lower group - a different thinking group. So it was really interesting.

In describing her conduct of the trial Olive reported that she had followed the lesson plan closely; orchestrated the discussion in a similar fashion she had seen in the demonstration lesson; and Olive used questioning techniques she had seen modelled in the demonstration lesson.

> We followed the script [lesson plan] … We orchestrated discussions in the same way. … We had walked around the tables taking copious notes and picked a few of their reasons, put them in order according to easiest reason, hardest or more developed reasoning … I asked a mix of children to get a variety of answers. … We had the table already drawn on the board, same as what XXXXX did… while they were speaking we filled it in.

> We had even numbers … one of the children said it's [even] … if you had lollies and 2 people, you could give them the same amount each, I said, so I've got 5 lollies they can have 2½ each, they said no, I said what do you mean and … they said they meant without any fractions. Someone said, counting by 2's and I said, oh great 1, 3, 5, 7, and then they had to re-establish their definition say starting from zero. Then the next pair said counting by 5's and I said 1, 6, 11, 16 and they said no, I said what do you mean when you say counting by 5's, so then they could explain that a bit better.

Olive's most powerful strategy to support and encourage reasoning was the use of counter examples in her probing questions. This explicitly highlighted for students critical aspects of the concept of even numbers because students then had other examples of when a number is not even thereby deepening their understanding of this concept (Cope, 2000). It is also illustrates Olive's enhanced PCK as a aresult of the demonstration lesson where she saw this modelled by the demonstration teacher who used student answers to elicit reasoning.

When reflecting on the trial of the demonstration lessons, Olive commented:

> We weren't necessarily teaching them something, so from a teacher's point of view I found that hard, I wasn't teaching them anything. I was trying to get information from them, so to understand their reason. Well yeah we were [teaching] but the learning was, the teaching was coming from each other not me as a facilitator, not me as the person standing at the front. I didn't necessarily feel that we were teaching the children something, it was more collecting data and seeing how they think which was useful for us, but it wouldn't be how we teach all the time. I think I'll read Ausvels and Vels documents more closely and actually have a better understanding and talk to our team about teachers having a better understanding of that part of maths [reasoning], it's not just about place value, it's not just about number, this [reasoning] is another important part of it.

This quote indicates a paradigm shift since she has realised that it is not about her teaching the students per se but that the learning occurs as a result of her facilitation of the elicitation of ideas and reasoning that makes the learning happen. It is an 'aha' moment for her about what she should now do to develop the reasoning proficiency as called for in the Australian Curriculum. She now intends to be an ambassador of that idea to other teachers.

When asked to comment about the main ideas or strategies that she had learnt about teaching reasoning, Olive replied:

> Giving the kids more chance to talk to as partner…. I would set more activities that were problem solving type activities, where you are working with a partner and discussing how and why you're going to do it the way you are, and working together … I actually liked the journal entry at the end and I think that would be useful because we don't always have time for that share time. And then it becomes a record of their learning as well. So I think it would be a powerful thing, and they would get better at that the more times they saw that.

It is clear from this quote that Olive had learnt new teaching strategies which facilitate the students' development of mathematical reasoning. She saw regular interactive engaging group discussions coupled with written reflections in journal entries as influential steps to improving reasoning in her students. Trialing the same lesson as the demonstration lesson was instrumental in developing Olive's understanding of reasoning and how to embed reasoning in her lesson by using probing questions to challenge and extend students' thinking. Olive's reflection of the trial of the demonstration lesson suggested to some extent she believed that teaching involved telling as indicated in her comment "I wasn't teaching them anything". There has been a shift in the thinking about teaching from telling to facilitating thinking and reasoning. The variation in the critical aspects of mathematical reasoning she experienced in MRRLRP were influential in her growth in CK and PCK of mathematical reasoning and perhaps may extend to teaching mathematics more generally.

Conclusion

The key factors which appear to have been influential in Olive's improvement in her understanding of the nature of mathematical reasoning are: observing the demonstration lesson; discussions with peers; her growing awareness of mathematical reasoning; her interest and intent to learn more; trialling of the lesson and her reflection on the trial. This case study of Olive suggests that investing in quality planning through a demonstration lesson with a focus on advancing students' reasoning leads to improvements in teachers' CK and PCK about mathematical reasoning.

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